A low-cost Michelson wavemeter with picometer-level accuracy

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An inexpensive Michelson interferometer-based wavemeter was developed for use in atomic spectroscopy experiments, with the goal of attaining wavelength measurements with picometer accuracy. A commercial frequency stabilized helium-neon laser was used as a reference laser. The wavemeter was tested using an external cavity diode laser referenced to resonances in iodine at 636 nm. Measurements were within one picometer of the known vacuum wavelengths.

I. Introduction

Wavemeters are useful tools for precision spectroscopy, where a tunable laser is used to excite a particular transition of interest in an atom. In order to use the laser to excite this transition, however, its wavelength should be known with an accuracy that can resolve an atomic transition, a few parts in 10^6 [3]. The wavemeter provides an accurate and convenient method for determining the wavelength of the laser.

There are various designs for a wavemeter setup, including designs based on a Fizeau (Fabry-Perot) interferometer and a Michelson interferometer. The Fizeau interferometer-based wavemeter design utilizes a Fizeau wedge, where two optical flats are inclined with respect to one another such that their inner faces form a small angle. Monochromatic light passing into the Fizeau wedge will form interference fringes with a constant spacing and with a certain phase. If the fringe period and phase are analyzed, the wavelength of the light can be accurately determined [5, 8].

Another design operates on the principle of a Michelson interferometer that is simultaneously detecting interference fringes for two copropagating laser beams as the length of one interferometer arm is varied. One of the copropagating lasers is a reference laser with a well-defined wavelength, while the other is a laser of unknown wavelength. The unknown wavelength can be determined through the ratio of the number of fringes and the wavelength of the reference laser [3].

A commercial wavemeter can be an expensive piece of equipment for an undergraduate laboratory. The purpose of this project was to develop a wavemeter that is affordable for use in experiments in our lab, while still attaining an accuracy that enables us to resolve atomic transitions. Our goal was to achieve a wavelength accuracy of one picometer. A picometer resolution would allow us to distinguish between nearby atomic transitions. For example, the two required laser frequencies for cooling ^{7}Li are separated by ~800 MHz (1.2 pm) at ~671 nm.

II. Theoretical Principles

The wavemeter described below is a Michelson interferometer. A Michelson interferometer, Fig. 1, is composed of a beamsplitter and two mirrors. The beamsplitter divides the light equally into two parts between the two arms of the interferometer and then recombines them to form interference patterns at the detector. Without the introduction of other polarizing optics, it is necessary that the beamsplitter be

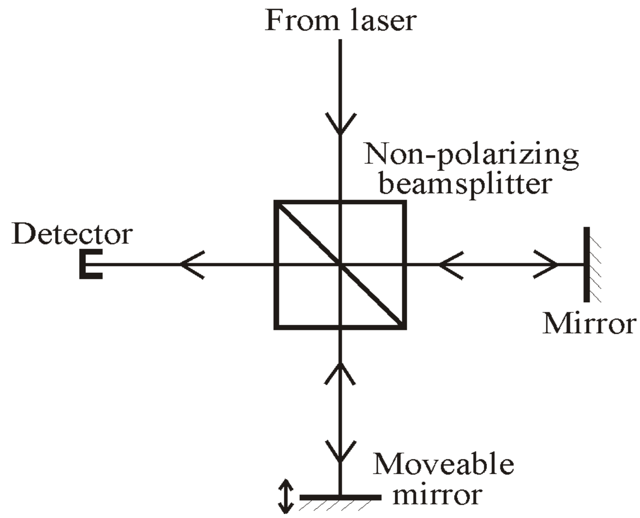

Figure 1: Michelson interferometer. A laser beam is split between two arms. After recombination the irradiance corresponds to the difference in path lengths of the two arms.

non-polarizing to get the interference effects when the partial beams are recombined. One of the mirrors in the interferometer is movable, and if the mirror is moved through a distance d, interference fringes will pass at the detector as there is constructive and destructive interference, depending on the relative path difference between the two arms. The number of fringes that pass at the detector as the mirror is moved a distance d can be expressed as $\mathrm{N} = \frac{2\mathrm{nd}}{\lambda_0}$, where λ_0 is the vacuum wavelength of the light and n is the index of refraction.

The conditions for interference at the detector based on the relative path difference can be seen by analyzing the electric fields for the partial beams from the two arms. The two fields can be expressed $\mathbf{E}_1 e^{i\phi_1(x,t)}$ and $\mathbf{E}_2 e^{i\phi_2(x,t)}$, where $\phi_1(x,t)$ and $\phi_2(x,t)$ are the phases of the two waves, and the total field incident on the detector is the sum of the two individual fields, $\mathbf{E}_T = \mathbf{E}_1 e^{i\phi_1(x,t)} + \mathbf{E}_2 e^{i\phi_2(x,t)}$. The intensity incident on the detector is proportional to the square of the total field, $I \propto |\mathbf{E}_T|^2 = E_1^2 + E_2^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \cos\theta$, where $\theta = k\Delta x$, k is the wave number $\frac{2\pi}{\lambda}$, and Δx is the relative path difference between the two arms. If $|\mathbf{E}_1| = |\mathbf{E}_2|$ and the field polarizations are identical, the terms in the intensity add to produce constructive interference when θ is a multiple of 2π and Δx is correspondingly a multiple of half the wavelength of the light. The terms cancel producing destructive interference when θ is an odd multiple of π and Δx is then a multiple of a fourth of the wavelength. Therefore, the condition for constructive interference in the interferometer is that the path difference between the two arms is a multiple of half of the air wavelength of the light, and the condition for destructive interference is that the path difference is an odd multiple of a fourth of the air wavelength [10].

A wavemeter is a Michelson interferometer that is simultaneously counting fringes for two counter-propagating beams that travel identical paths. One laser is of unknown wavelength, while the other is a reference laser with a well-defined wavelength. As the mirror is moved a distance d, a number of fringes will pass at the detector for the laser of unknown wavelength, $\mathrm{N_U} = \frac{2\mathrm{d}}{\lambda_\mathrm{U}}$, and a number of fringes will pass at the detector for the reference laser, $\mathrm{N_R} = \frac{2\mathrm{d}}{\lambda_\mathrm{R}}$, where the wavelengths of the two lasers are the wavelengths in air. Since the beams are propagating through the same interferometer and have the same relative path difference d, the unknown wavelength can be determined from the ratio of the number of fringes and the wavelength of the reference laser,

$$\lambda_\mathrm{U} = \frac{N_\mathrm{R}}{N_\mathrm{U}} \lambda_\mathrm{R}. \tag{1}$$

The calculation for the unknown wavelength will be more precise as more fringes are counted. It is beneficial to the overall uncertainty, therefore, to have a longer movable arm which allows for the counting of more fringes. We expected that our movable arm of the interferometer, being 1.4 m in length, would allow for many fringe counts ($\sim 10^6$ for $\lambda = 632.8\,\mathrm{nm}$) and contribute to the accuracy of the our wavemeter measurements.

III. Setup

Our wavemeter layout can be seen in Fig. 2 and was based on a design by Fox and coworkers [3]. In the setup, a reference laser and a laser of unknown wavelength are collimated before entering the interferometer. The non-polarizing beamsplitter divides the reference and the unknown laser beams into two beams that enter the separate arms of the interferometer. One set of partial beams travel the constant path from the beamsplitter to mirrors M3 and M4, then back to the beamsplitter. The other set of partial beams travel to mirror M5 and are reflected to the corner cube, which retro-reflects the beams back to M5 and the beamsplitter. The corner cube ensures that each beam is reflected parallel to the incoming beam, but with a transverse displacement [2]. The corner cube is mounted on a cart that is situated on a standard introductory-phyiscs laboratory air track of length 1.4 m. After returning to the beamsplitter, the partial beams are recombined to

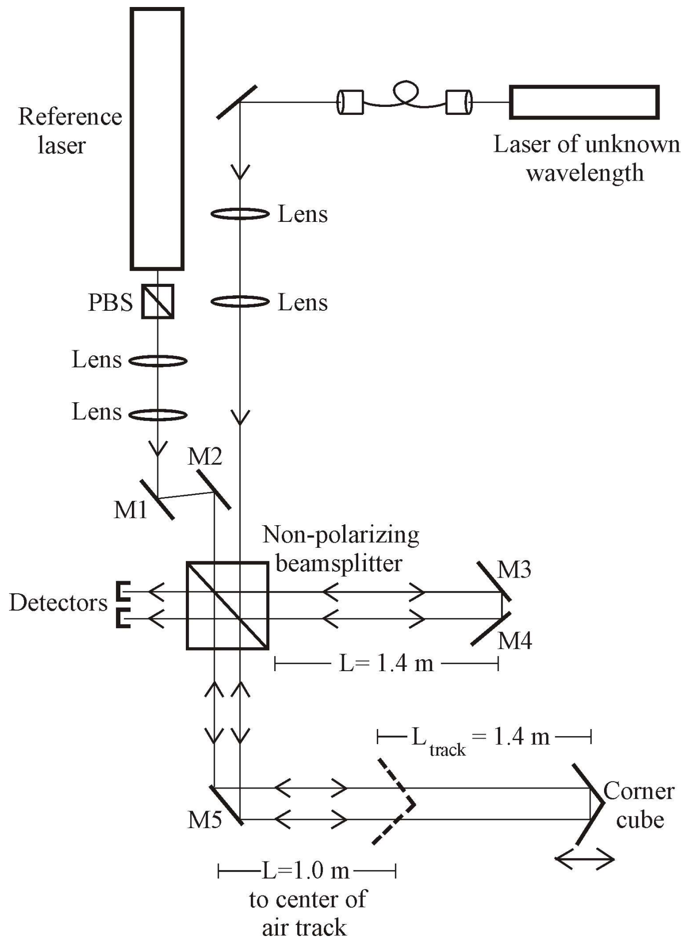

Figure 2: Wavemeter layout. The reference laser was a commercial HeNe laser. The laser of unknown wavelength was an external cavity diode laser that was coupled to the wavemeter via fiber optic coupling. The corner cube was mounted on a cart straddling an air track. The lenses were used to collimate and enlarge both beams. PBS: Polarizing beamsplitter.

produce two collinear output beams. The output reference and unknown beams are incident on detectors connected to an oscilloscope where the interference fringes are observed. The detectors are also connected to a high resolution programmable counter (Fluke PM6681) that counts the number of fringes for both the reference laser and the laser of unknown wavelength.

The reference laser used for our wavemeter was a frequency stabilized helium-neon (HeNe) laser (Spectra-Physics Model 117A), the specifications of which include a beam diameter of 0.5 mm and a divergence of 1.8 mrad [11]. The laser of unknown wavelength was a tunable diode laser (New Focus Velocity Model 6300) that had a wavelength of approximately

636 nm. The diode laser light was brought into the wavemeter by an optical fiber. This coupling had the benefit of producing a spatially clean beam.

IV. Alignment

Collinear output beams are essential to the accuracy of the wavemeter. Our procedure began with aligning the wavemeter layout for the HeNe, then using the HeNe beam as a reference for aligning the laser diode.

Alignment for the HeNe began with the air track arm. It was necessary to adjust mirrors M2 and M5 until the retro-reflected beam did not wander as the cart glided back and forth on the air track. This was accomplished through the use of a diaphragm as a reference at different locations on the air track as the mirrors were adjusted. Once the retro-reflected beam from the air track arm was stable, mirrors M3 and M4 of the stationary arm were adjusted until the beam from that arm was collinear with the beam from the air track arm.

To verify that the beams were in fact collinear and not simply overlapping at one point in space, the beams were analyzed at a close location, just on the other side of the beamsplitter, and at a location several meters downstream. The detector was removed from its location in the setup and the beams were reflected off one mirror across the room to another and ultimately were observed on a card close to the beamsplitter. By observing the HeNe beams at two locations, it was possible to ensure that they were collinear.

In order to align the diode laser, the aligned HeNe beam was used as a reference. With the diode beam blocked from entering the optical fiber, part of the HeNe beam from the beamsplitter was mode-matched and aligned into the fiber, and the precision mount holding the fiber was adjusted until the HeNe coupling through the fiber was optimized. The diode beam was then coupled through the fiber in the other direction. This process produced a pre-aligned port for the light of unknown wavelength. In the future, light need only be coupled into the fiber with slight collimation adjustments.

V. Operation of the wavemeter

In order to take a measurement of the unknown wavelength once the wavemeter was aligned, the air pump was turned on, and the cart was given a push to travel the length of the air track. The oscilloscope displayed the interference fringes as the cart traveled back and forth. The counter had a pre-programmed math function that computes the ratio of the number of laser diode and HeNe fringes and multiplies it by the wavelength of the HeNe, a number input by the user. The counter then displayed the wavelength of the diode laser. For our measurements, we used a HeNe wavelength of 632.991 nm, which is the vacuum wavelength of an I_2-stabilized HeNe laser [6, 9]. The displayed measurement therefore corresponds to the vacuum value of the unknown wavelength. The indices of refraction of air for the laser diode and the HeNe were not important, since the wavelengths of the laser diode and the HeNe laser are so close that the change in the index of refraction of air between them is negligible. However, in other cases when the unknown wavelength is much different from the reference wavelength, the indices of refraction of air must be taken into consideration, as discussed below and is shown in Eqn. 4.

The wavelength measurement displayed on the counter was not always stable, and depended greatly on the quality of the signal from the detectors. If the signal was noisy or too small, the counter struggled to accurately count fringes. For example, the reading for the wavelength was unstable when the speed of the cart was too fast, or when the cart bounced off the ends of the air track. An approximate speed of the cart needed to obtain stable readings was around 20-40 cm/s. On the counter, the gate time could be adjusted, changing the time interval over which the device counted fringes. We found that a gate time producing stable readings was 1 second. As discussed below, improvements to the air track to obtain more stable measurements are envisioned and include a new custom-made air track and cart.

VI. Results

We compared a wavemeter measurement to a known wavelength. Our absolute frequency reference came through the use of iodine spectroscopy. Previous work done in a lasers course (PHY 430) lab project involved first using the diode laser to develop a map of resonances in iodine, then labeling the peaks with the known wavelengths for those resonances as published in an atlas by S. Gerstenkorn [4]. The resulting map is shown in Fig. 3. We used the map of the iodine resonances as a guide to tune the diode laser to a particular peak.

The wavemeter was tested against three known resonances. The first was for the peak labeled number 1 in Fig. 3, which has a known wavelength of 636.817400 nm. For this peak, the wavemeter measurement read 636.816 ± 0.001 nm. The second was for

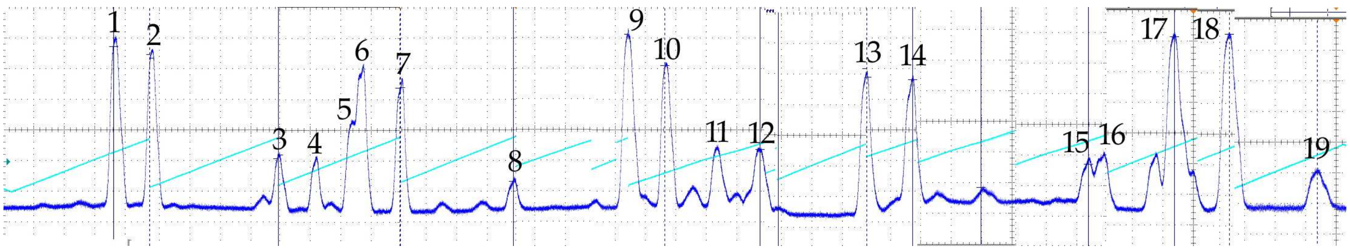

Figure 3: Map of the iodine absorption resonances made by the lasers course lab group. Peaks one and 19 are 636.817400 nm and 636.940963 nm, respectively [4].

the peak labeled number 9, which has a known wavelength of 636.872266 nm. The wavemeter measurement read 636.871 ± 0.001 nm. The third resonance was labeled number 18 in the figure, with a known wavelength of 636.934455 nm. For this resonance, the wavemeter measurement read 636.933 ± 0.001 nm.

VII. Error

The accuracy of the wavemeter has several limitations with contributions from wavefront curvature, misalignment of the beam, uncertainty in the frequency of the reference HeNe, uncertainty in the index of refraction of air, and fringe counting error. The total relative uncertainty is a quadrature sum of these individual relative systematics [3]:

$$\left(\frac{\Delta\lambda}{\lambda}\right)^2 = \left(\frac{\Delta\lambda_{\mathrm{WC}}}{\lambda}\right)^2 + \left(\frac{\Delta\lambda_{\mathrm{align}}}{\lambda}\right)^2 + \left(\frac{\Delta\lambda_{\mathrm{HeNe}}}{\lambda}\right)^2 + \left(\frac{\Delta\lambda_{\mathrm{air}}}{\lambda}\right)^2 + \left(\frac{\Delta\lambda_{\mathrm{count}}}{\lambda}\right)^2.$$

Wavefront curvature adds a contribution to the error because of diffraction effects, and the contribution is discussed at length by Monchalin et al. [7]. The wavefront curvature error can be estimated by diffraction theory to be

$$\frac{\Delta\lambda_{\mathrm{WC}}}{\lambda} = \frac{\lambda^2}{4\pi^2\omega_0^2} = \frac{\Delta\theta^2}{4} \tag{2}$$

where ω_0 is the beam waist and $\Delta\theta$ is the divergence half-angle of the beam [3]. The measured divergences of our HeNe and the laser diode beams were so small that this contribution to the error is negligible.

Misalignment of the system will contribute to error in the measurement of the wavelength since the reference and unknown lasers will travel different path lengths. The error can be expressed as

$$\frac{\Delta\lambda_{\mathrm{align}}}{\lambda} = \frac{\Delta L}{L} = \frac{1}{2}\left(\frac{\Delta x}{L}\right)^2, \tag{3}$$

where L is the optical path length and Δx is a transverse displacement [3]. The optical path length for our wavemeter layout is 6.64 m, and a conservative estimation for Δx is 1 mm. Therefore, the contribution to the error for our wavemeter from misalignment of the system is

$$\frac{\Delta\lambda_{\mathrm{align}}}{\lambda} = 3 \times 10^{-9}.$$

The uncertainty in the HeNe wavelength has two factors: the Doppler-broadened emission linewidth and the unknown mixture of isotopes in the gain medium.

First, there is an uncertainty in the HeNe frequency due to the Doppler broadened linewidth of the transition. Direct frequency measurements performed on HeNe lasers report a frequency of 473.612 THz, and we used the corresponding vacuum wavelength for our measurements as discussed above [6, 9]. However, these frequency measurements were made using I_2-stabilized HeNe lasers, and it is unclear in the reporting papers where this iodine stabilized line falls in comparison to the emitted lines of our reference laser. An estimate of the uncertainty in the HeNe frequency due to the unknown location of the I_2-stabilized frequency measurement is then the full width at half maximum of the gain curve, which is calculated to be approximately 1.2 GHz (1.6 pm) [10].

Second, there is an uncertainty in the frequency due to the unknown mixture of neon isotopes in the gain medium of the HeNe laser. Slight differences in the number of neutrons in the nucleus cause a difference in the energy levels of the atom, which in turn shifts the center line frequency of a transition. The estimated uncertainty due to the unknown mixture of Neon isotopes in the gain medium is therefore the isotope shift between Ne^{20} and Ne^{22}, which is 1 GHz [10].

Therefore, the total uncertainty in the HeNe frequency can be conservatively estimated by the addition of the uncertainty in the isotope mixture and the uncertainty in the location of the iodine stabilized laser line.

$$\Delta\nu_{\mathrm{HeNe}} = \Delta\nu_{\mathrm{isotope}} + \Delta\nu_{\mathrm{FWHM}}$$

$$\Delta\nu_{\mathrm{HeNe}} = 1\mathrm{GHz} + 1.2\mathrm{GHz} = 2.2\mathrm{GHz}$$

The contribution to the error due to uncertainty in the frequency of the HeNe is therefore

$$\frac{\Delta\lambda_{\mathrm{HeNe}}}{\lambda} = 5\times10^{-6}.$$

It is important to note, however, that if it were possible to eliminate the uncertainty due to the unknown mixture of isotopes in the laser, the error would be greatly reduced. The isotope of neon used in the direct frequency measurements of the I_2-stabilized HeNe laser was Ne^{20} [6]. This fact, coupled with the observation that our test results of the wavemeter were within 1 pm of the known values, indicates that it is probably valid to eliminate the Ne^{22} isotope from consideration, thus decreasing the HeNe uncertainty.

Our frequency stabilized HeNe laser cavity produces two orthogonally polarized modes that are symmetric about the gain curve due to the particular method of stabilization. In our layout, a polarizer situated at the output of the HeNe allows us to select between the two oppositely polarized modes. By rotating the polarizer while observing a beat signal from a high-speed (2 GHz) photodetector, we were able to measure the free spectral range of the laser cavity to be 640 MHz.

Although we can distinguish the polarizations of the two possible modes from the reference laser as well as their frequency separation (640 MHz), we do not yet know their relative absolute frequencies. A measurement of these frequencies is envisioned for the wavemeter. In this test, a wavemeter measurement is taken using one mode from the reference laser, then the polarizer is rotated and another wavemeter measurement taken using the other mode. A highlight of this test is that if the wavemeter is able to resolve a difference in the wavelength between the two measurements, then we have demonstrated that our wavemeter is capable of obtaining high precision (subpicometer) values.

The fourth contribution to error in the unknown wavelength is uncertainty in the index of refraction of air. Eqn. 1 yields the unknown wavelength in air; however, in order to find the unknown wavelength in a vacuum, the indices of refraction of air must be taken into account.

$$\lambda_{\mathrm{U}_0} = \frac{N_{\mathrm{R}}}{N_{\mathrm{U}}}\lambda_{\mathrm{R}_0}\left(\frac{n_{\mathrm{U}}}{n_{\mathrm{R}}}\right) \tag{4}$$

The equation modified to find the unknown wavelength in a vacuum is shown in Eqn. 4, where n_{U} and n_{R} are the indices of refraction and λ_{U_0} and λ_{R_0} are the vacuum wavelengths of the unknown and reference lasers [3].

The index of refraction of air depends on the pressure, the temperature, and the partial pressures of H_2O and CO_2 [2, 1]. The error due to uncertainty in the indices of refraction of air is [1]

$$\frac{\Delta\lambda_{\mathrm{air}}}{\lambda} = \frac{\Delta r}{r} = (4\times10^{-9}+2\times10^{-9})\left|\frac{1}{\lambda_{\mathrm{U}}^2} - \frac{1}{\lambda_{\mathrm{R}}^2}\right|, \tag{5}$$

where r is the ratio of the indices of refraction of air, $\mathrm{r} = \frac{\mathrm{n}(\lambda_{\mathrm{U}})}{\mathrm{n}(\lambda_{\mathrm{R}})}$, and where λ_{U} and λ_{R} are in μm. The two terms in Eqn. 5 are due to uncertainty in the air density and uncertainty in the water vapor density respectively [2, 1]. For our wavemeter, λ_{U} is 0.6368 μm and λ_{R} is 0.6328 μm, and therefore the uncertainty due to the unknown index of refraction of air is

$$\frac{\Delta\lambda_{\mathrm{air}}}{\lambda} = 2\times10^{-10}.$$

The final contribution to the overall error is the fringe counting error. For a typical run of the wavemeter, there are $700,000\pm1$ fringes counted per second for the HeNe laser, and $704,000\pm1$ fringes counted per second for the laser diode. This corresponds to an uncertainty of $\frac{1}{700,000} = 1.43\times10^{-6}$ for the HeNe and $\frac{1}{704,000} = 1.42\times10^{-6}$ for the laser diode. The total contribution due to error in the number of fringe counts is then

$$\frac{\Delta\lambda_{\mathrm{count}}}{\lambda} = 3\times10^{-6}.$$

The accuracy of the fringe counting is limited in our current wavemeter setup since in order to get a stable measurement from the wavemeter, the cart must be gliding relatively slowly. Whenever the cart bounces off the ends of the air track, the counter struggles to count fringes due to the extra noise on the signal, and it takes some time for the counter to recover its stable reading after bouncing off the ends. When the cart is moving at a fast speed, the reading does not have time to recover its stability the reading before the cart hits the other end of the air track.

Summing all the error terms, the total relative uncertainty of the wavemeter is 5×10^{-6}, which corresponds to an uncertainty of about 3.5 pm in our measurement of the laser diode wavelength. The uncertainty in the frequency of the HeNe and the error in fringe counting have the largest contributions to the overall error. The fact that our results are better than our estimated error may have origins in the isotope contribution to the error, as discussed previously. If the isotope contribution to the error is eliminated, we obtain an uncertainty in our measurement of about 2.5 pm.

VIII. Further Work

Improvements envisioned for the wavemeter include adding a second corner cube to the cart and a new, custom-made air track and cart [3]. The addition of another corner cube would enable the currently stationary arm of the interferometer to change in path length. In this setup, the second corner cube would be mounted on the cart to face in the opposite direction of the current corner cube. The beam from the now stationary arm of the interferometer would be brought around to the second corner cube, where it would be retroreflected back to the beamsplitter. This change from one stationary arm to two variable arms would allow for more fringe counts and thus improve the precision of the wavemeter measurements.

A new design for the air track and cart could improve the accuracy in two ways. First, the current track is bowed slightly in the middle which interferes with the alignment of that arm. Also, the current air track has holes across its surface through which the air blows and floats the cart. Turbulence from the blowing air along the axis of the track could alter the index of refraction of air, causing a change in optical path length of the beam and producing an inaccurate calculation of the wavelength. In the new design for the air track and cart, the air would blow through holes in the interior of the cart, instead of through the track itself. This method would reduce the turbulence along the length of the track.

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