Phys 420

Spatial Light Modulator Activities

Introduction

Spatial Light Modulators (SLMs) are liquid crystal (LC) devices that are used to encode patterns into the wavefront of a beam of light. In the series of activities in this document, we will work with a Jasper Display SLM (officially their Educational Development Kit) to explore the behavior of the device, use it to gain a better understanding of the phase of light, and see some applications of SLMs.

Background

Liquid Crystals are molecules with elongated shapes that are often represented as short lines. Light incident on one of these molecules will interact strongly (causing the electrons in the molecule to move more) if the polarization of the light is parallel to the long axis of the molecule. If the polarization is perpendicular to the long axis, then the light will not interact with the molecule as strongly.

LC molecules get their name because they take on characteristics of both liquids (molecules able to move or rotate) and crystalline solids (molecules are oriented in regular, ordered patterns). While there are many difference phases that LC can be in, the *nematic* phase is the one used most often in SLMs and Liquid Crystal Displays (LCDs – used in must current computer and phone displays). In the nematic phase, the LC molecules all try to line up in the same direction as their neighbors, but their locations are random (see Fig. 1).

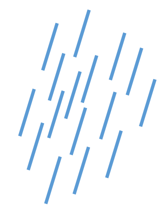


Figure 1. The nematic phase of LCs. All the molecules line up along a single direction (the director), but each molecule is located randomly.

Because of the polarization-dependent interaction between light and the molecules, light that is polarized parallel to the director (the direction the LCs are pointing) travels slower than light polarized perpendicular to the director. This makes LCs *birefringent* – with a refractive index that depends on the polarization direction of the light.

The whole point of an SLM is to be able to use the LCs to control the how much the light slows down while traveling through the SLM, thereby controlling the *optical path length* that the light experiences. This is accomplished by applying an electric field to the LCs in a direction parallel to the direction the light is traveling (and perpendicular to the director of the LC).

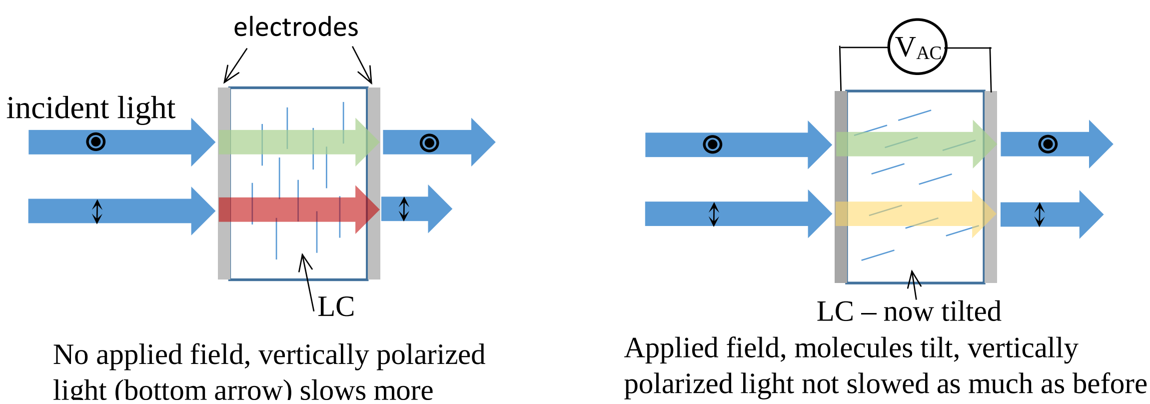


Figure 2. Side view of a LC cell in a transmissive SLM. When an electric field is created between the two electrodes, the LC molecules tilt in the direction of the field. This decreases the refractive index for light polarized parallel to the director.

The magnitude of the electric field determines how far the molecules tilt. The more tilted they are, the less the light will interact with them, causing the light to not be slowed as much as they pass thought the LC. See Figure 2.

In an SLM, there are thousands of separate pairs of electrodes (one pair for each pixel), and each one can have a different amplitude electric field applied to it, giving us the ability to slow down different parts of the beam more than others. Figure 3 shows a front view of the SLM.

In the Jasper Display SLM, the light does not pass through the SLM, but is reflected off the back surface. This LCOS (Liquid Crystal on Silicon) geometry has two advantages: first, the light passes through the LC twice, effectively doubling the width of the LC layer and doubling the change in the phase of the light; second, the front electrode can be a single grounded transparent electrode, and the pixel structure can all be defined on the reflective Silicon surface at the back, which improves the light throughput. The disadvantage is that the light needs to be reflected off the SLM, complicating some of the optical setups.

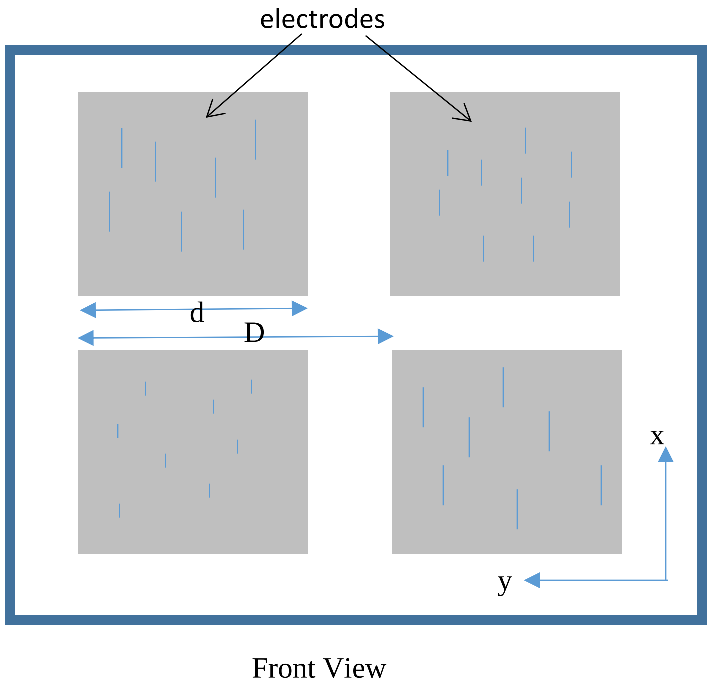


Figure 3. Front view of an SLM showing four pixels (defined by the location of the electrodes (gray squares). The electrode in the lower left has the largest amplitude applied voltage, causing the molecules to tilt into the page the most. Light passing through that pixel will travel faster than through the other pixels (assuming that the incident light is all polarized vertically).

Some nomenclature: the axis parallel to the original director direction is called the extraordinary axis, and the orthogonal axis is the ordinary axis. The refractive indices for light polarized along those two directions are n_e and n_o, respectively. So, as the applied voltage is increased, n_e will decrease, but n_o will not change.

Transmission of light through the SLM

In order to make the best use of an SLM, the device is paired with two linear polarizers, one in the beam of incident light, and the other in the beam coming out of the SLM (see Figure 4). Somewhat confusingly, they are traditionally given the respective names: 'polarizer' and 'analyzer.' The first polarizer determines the polarization of the light incident on the SLM (relative to the axes determined by the LC director direction).

The normalized Jones vector for the light incident on the SLM is: $\begin{pmatrix}\cos(\beta)\\ \sin(\beta)\end{pmatrix}$

The matrix that describes the phase shift of the SLM is: $\begin{pmatrix}e^{-ik_e l} & 0\\ 0 & e^{-ik_o l}\end{pmatrix}$ where l is the effective thickness of the SLM (twice the actual thickness in the case of a reflective SLM) and k_e and k_o are the wavenumbers of light aligned with the extraordinary and ordinary axes, respectively.

Therefore, the Jones vector of the light exiting the second polarizer is:

$$1)\qquad \begin{pmatrix}E_{xf}\\ E_{yf}\end{pmatrix} = \begin{pmatrix}\cos^2(\theta) & \sin(\theta)\cos(\theta)\\ \sin(\theta)\cos(\theta) & \sin^2(\theta)\end{pmatrix}\begin{pmatrix}e^{-ik_e l} & 0\\ 0 & e^{-ik_o l}\end{pmatrix}\begin{pmatrix}\cos(\beta)\\ \sin(\beta)\end{pmatrix}$$

where the first matrix describes the second polarizer.

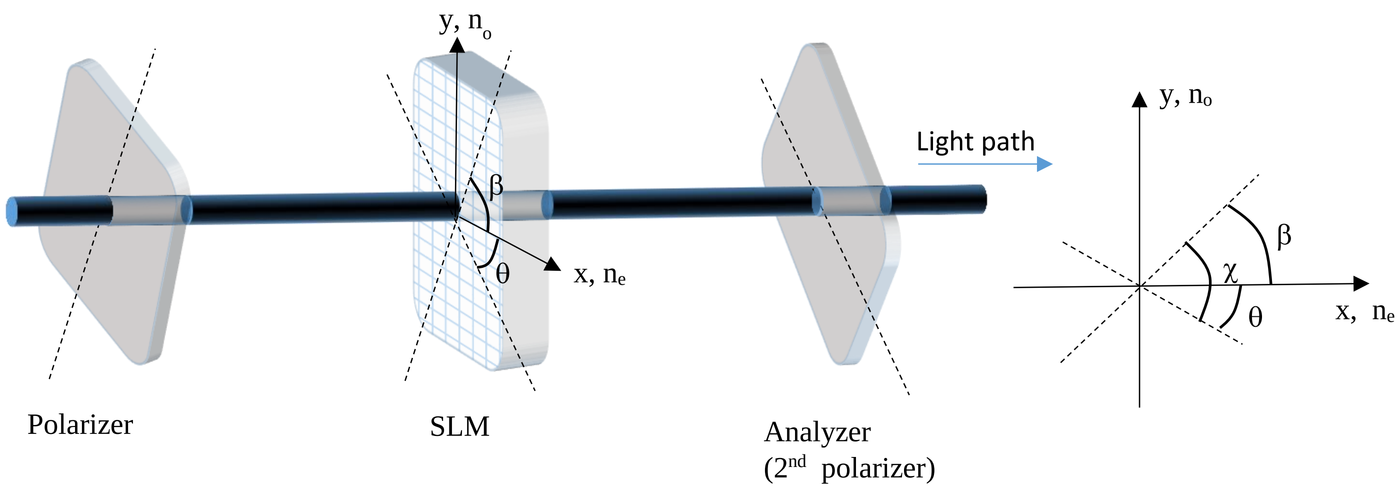


Figure 4. Schematic of light passing through a polarizer, SLM, and a 2nd polarizer. The transmission axes of the polarizers are shown as dashed lines. The angles shown are measured from the x axis and are positive toward the y axis.

The Irradiance of the exiting light is found by:

2) $$I = \begin{pmatrix} E_{xf}^* & E_{yf}^* \end{pmatrix} \begin{pmatrix} E_{xf} \\ E_{yf} \end{pmatrix} = \cos^2(\theta-\beta) - \sin(2\theta)\sin(2\beta)\sin^2\left(\frac{k_e-k_o}{2} l\right)$$

or

3) $$I = \cos^2(\theta-\beta) - \sin(2\theta)\sin(2\beta)\sin^2\left(\frac{n_e-n_o}{\lambda}\pi l\right)$$

Note, since the initial light was normalized, this expression for the exiting intensity is equivalent to the *Transmission* of the polarizer/SLM system. Also, the length l of the SLM is really twice the SLM thickness due to the reflection.

There are two special cases that are of great importance when using the SLM:

Case A) $\beta = 45^o$ and $\theta = -45^o$

Case B) $\beta = \theta = 0^o$

For Case A), the amplitude of the exiting light is determined by the varying value of n_e in the SLM. This is the Amplitude Modulation condition.

For Case B), the amplitude of the exiting light is unchanged, but the phase of the light is varied by the value of n_e. This is the Phase Modulation condition.

Exercise 1: Starting from equation 1), show that the Irradiance is given by equation 3). Dust off your list of trig identities!

Exercise 2: Find the expression for the Irradiance for Case A).

Exercise 3: Find the expression for the Irradiance and the expression for the phase shift for Case B).

Reference:

A nice online discussion of LCs related to SLMs can be found at (http://laser.physics.sunysb.edu/~melia/SLM_intro.html)

Activity #1: Diffraction from the structure of the SLM pixels

As shown in Figure 3, the SLM electrodes form a 2-dimensional periodic structure with a period D. This means the light reflecting off the SLM will form a diffraction pattern. Looking at a 1-dimensional version of this structure, it looks very much like an N-slit aperture. The Irradiance of the diffracted light from an N-slit aperture is:

4) $$I = I_0 \left(\frac{\sin(\beta)}{\beta}\right)^2 \left(\frac{\sin(N\alpha)}{\sin(\alpha)}\right)^2$$

where $\alpha = (1/2)k\,D\sin(\theta)$, and $\beta = (1/2)k\,d\sin(\theta)$, θ is the angle of the diffracted light measured from the normal to the SLM surface, and k is the wavenumber of the light. [See Pedrotti & Pedrotti for the derivation of equation 4).]

The first term in parentheses in equation 4) is the intensity profile due to the diffraction from a single slit. The last term describes the interference pattern produced by the N slits. If N is sufficiently large (as in a diffraction grating or the SLM), then the only locations with significant intensity occur when the final term = 1, which corresponds to the condition:

5) $D\sin(\theta_m) = m\lambda$, where m is an integer

The *relative intensities* of these bright diffraction spots is determined by the $\left(\frac{\sin(\beta)}{\beta}\right)^2$ term. Combining this with equation 5) we find:

6) $$\left(\frac{\sin(\beta)}{\beta}\right)^2 = \left(\frac{\sin\left(\pi m\frac{d}{D}\right)}{\pi m\frac{d}{D}}\right)^2$$

So, by measuring the locations of the diffraction spots, we can determine D, and by measuring the ratio of the intensities of the diffraction spots, we can determine the ratio d/D, which tells us what fraction of the area of the SLM is actively involved in changing the phase of the light.

The experiment:

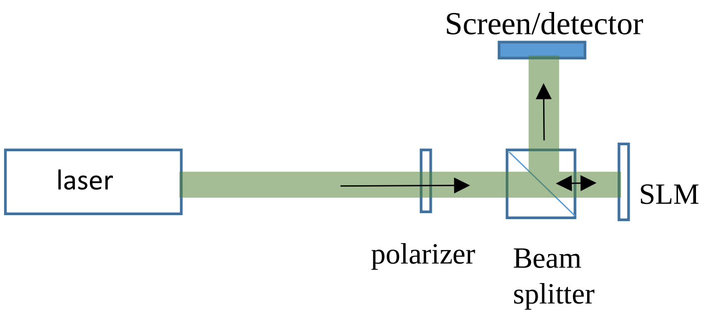


Figure 5. Experimental setup for measurement of diffraction from the pixel structure of the SLM.

Set up the components as shown above. Make sure that the beam maintains a constant height above the optics table, and travels parallel to the holes in the table. When you add the SLM, be sure that the zeroth order diffraction spot travels back along the incident beam. The polarizer should be set at 45^o.

Measure the angular spread of the diffraction spots and the intensity of the diffraction spots in order to determine D and d/D. Note: the manufacturer's specifications are: D = 6.5µm, d/D = 0.97.

Reference:

F.L. Pedrotti and L.S. Pedrotti, Introduction to Optics, 2nd Ed. (New Jersey, Prentice-Hall, 1993) pp341-342.

Activity #2: Using the SLM in Amplitude Modulation Mode and Calibrating the Gray Level to the Phase Shift

As far as the computer is concerned, the SLM is nothing more than another screen that isn't very high resolution and most importantly, only can display a grayscale image. Each pixel in the SLM can be given a gray value between 0 (black) and 255 (white) – it is an 8 bit display (2^8 = 256 possible values).   Those values are proportional to the applied voltage across the LC molecules at each pixel. Calibrating the SLM system involves determining the relationship between the grayscale value and the corresponding phase shift given to the light. That is the end goal of this activity.

Step 1:

First, we need to determine the 'alignment axis' of the SLM – the direction of the extraordinary axis. This can be done fairly simply using the polarizer and analyzer. If in equation 3) $\theta = \beta$ (both polarizers aligned so that their transmission axes are in the same direction), then the Transmission (equation 3) becomes:

7) $$T = 1 - \sin^2(2\beta)\sin^2\left(\frac{n_e - n_o}{\lambda}\pi l\right)$$

When $\beta = 0^o$ or 90^o, T = 1, no matter how n_e changes with the voltage applied to the electrodes. So if a pattern is displayed on the SLM by the computer, it should not be visible in the reflected beam if the two polarizers are *both* aligned either parallel or perpendicular to the extraordinary ('alignment') axis of the SLM.

Step 2:

Once the SLM axes are determined as above, the two polarizers can be rotated as described in Case A) on page 3, where $\beta = 45^o$ and $\theta = -45^o$, in order to have the system work in the Amplitude Modulation mode. Make sure you come up with the equation for T for this situation.

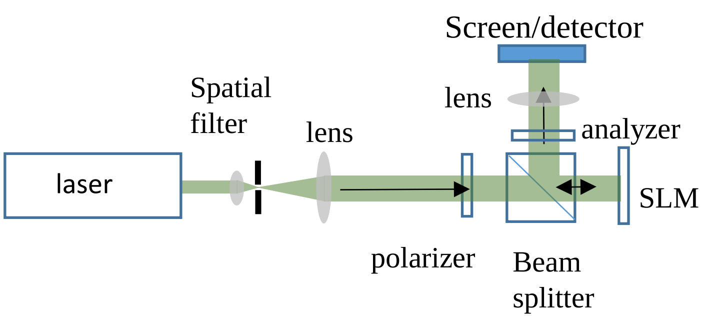


Figure 6. Experimental setup to use the SLM in amplitude modulation mode.

The experiment:

Set up the optical system as shown above, using the spatial filter and the next lens to produce a 'clean' collimated beam of light. The polarizer and analyzer should be positioned as needed (Case A)) to produce amplitude modulation with the SLM. [This will require you to first determine the alignment axis of the SLM as described above.] *The final lens needs to be positioned so that the pattern produced on the SLM will be imaged onto the screen. [So the SLM is in the object plane of the lens and the screen is in the image plane.]*

Using a uniform image (all pixels set to the same grayscale value) and a power meter in place of the screen, measure the power for a wide variety of grayscale values of the image starting at 0 and ending at 255.

Plot your data, fit it to a function of the form expected for Case A), and find the parameter from your fit that describes the relationship between the grayscale value and the apparent phase shift produced in the laser beam by the SLM.

Things to report:

Report the angle of the polarizers you determined in Step 1 (these tell us the angles of the n_e and n_o axes).
Report your data (the graph), the fit equation, and the parameter that describes the phase shift.

Activity #3: Using the SLM in Phase Modulation Mode and Making a Second Calibration

As we saw in the two beam interference lab previously, when two coherent beams of light cross at an angle θ, we get an interference pattern whose intensity varies as:

8) $$\mathrm{I} = 2\mathrm{I}_0\left(1 + \cos(2\mathrm{k}_y\mathrm{y})\right) = 2\mathrm{I}_0\left(1 + \cos\left(2\mathrm{k}\sin\left(\frac{\theta}{2}\right)\mathrm{y}\right)\right)$$

where y is the distance across the interference pattern. We will set up a similar experiment (see below) in the form of a Michelson interferometer, where one of the 'mirrors' is the SLM. This will allow us to add a controllable phase shift (ϕ) to one of the two split beams. This means we need to allow for a phase shift term in the intensity function:

9) $$\mathrm{I} = 2\mathrm{I}_0\left(1 + \cos\left(2\mathrm{k}\sin\left(\frac{\theta}{2}\right)\mathrm{y} + \phi\right)\right)$$

By determining ϕ over a wide range of grayscale values applied to the SLM, we can come up with a separate relationship between the two quantities and compare that to what we determined in the previous activity.

The experiment:

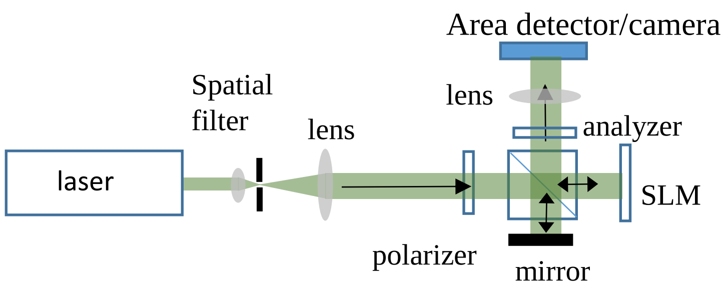


Figure 7. Experimental setup to use the SLM in phase modulation mode.

Set up the optical system as shown above, using the spatial filter and the next lens to produce a 'clean' collimated beam of light. The polarizer and analyzer should be positioned as needed (Case B)) to produce phase only modulation with the SLM. Use your determination of the alignment angle from Activity 2 to properly orient the polarizers.

Adjust the orientation of the mirror in order to produce a set of horizontal interference fringes on the camera. There should be at least 5 sets of fringes visible, but not more than about 30.

Display a pattern on the SLM where one side is completely black (grayscale =0) and the other side will be changed from black to white (from 0 to 255). This will cause the interference pattern to be split – one side will remain fixed, and the other will shift as the grayscale value changes.

First, measure the 'distance' from one bright fringe to the next in units of pixels in the camera image (measure over multiple periods to get a more accurate value). This corresponds to a 2π phase shift.

At each grayscale setting of the second half of the SLM, measure the distance the fringes shift from their original position. Using the period value measured above, convert this to the equivalent phase shift.

Make a plot of phase shift vs. grayscale value and compare that relationship to what you determined in Activity #2. Are the two relationships in agreement with each other?

Activity #4: Producing Plane, Spherical, and Cylindrical Waves by Modifying the Wavefront with the SLM

Finally, we can now start using the SLM to make more interesting modifications to the wavefronts of a laser beam. The collimated beam that is incident on the SLM has wavefronts that are very nearly planar. We want to modify the shape of the wavefronts in order to produce plane waves that travel in different directions, and spherical waves that 'focus' down to a point.

The equation describing the electric field for a plane wave is:

10) $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = \vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}$

If we assume that the wave is travelling in the x-z plane (where the z axis is in the direction of the optical axis of the system – the path followed by a beam sent straight through the middle of all the elements and ending up in the middle of the screen – so the coordinate system orientation changes at each reflection). The incident beam on the SLM will be directed parallel to the z-axis.

We will assume that the SLM is located at $z = z_0$. If we want the light leaving the SLM to travel at an angle θ from the z axis (measured toward the x axis), then we need the wavevector to equal:

$$\vec{k} = k\sin(\theta)\hat{x} + k\cos(\theta)\hat{z}$$

and equation 10 becomes (at $z = z_0$):

11) $\vec{E} = \vec{E}_0 e^{i(k\sin(\theta)x + k\cos(\theta)z_0 - \omega t)}$

So, if we want the beam to be traveling in this direction after it leaves the SLM, we need the SLM to tilt the wavefront toward this new direction. Equation 11) shows us that to accomplish this we need the phase shift produced by the SLM ($\delta(x)$) to vary linearly across the SLM as:

12) $\delta(x) = k\sin(\theta)x = \frac{2\pi}{\lambda}\sin(\theta)x$

where the slope depends on the wavelength of the light and the angle with which we want the beam to travel.

In the previous two activities, we obtained calibrations that allow us to convert the desired phase shift to grayscale values, so we can use those with equation 12). However, we also saw that the largest phase shift we can obtain is about 2p. Across the whole width of the SLM, equation 12) is likely to require phase shifts much larger than that! There is an easy solution to this – we can 'wrap' the phase back into the $0 - 2\pi$ domain because a 3π phase shift is the same as π phase shift.

There is a limitation for this, however – if the slope becomes too great, the distance in the x direction over which the phase changes by 2π will become very small. Eventually, we'd reach the limit of the resolution of the SLM – where the sawtooth pattern would be reduced to just a square wave with one pixel at 0 and the next at π. This will occur when $\delta = 2\pi$ for x = 2D. Of course, well before that, the efficiency of the SLM will be reduced as too few pixels are used in one period of the pattern.

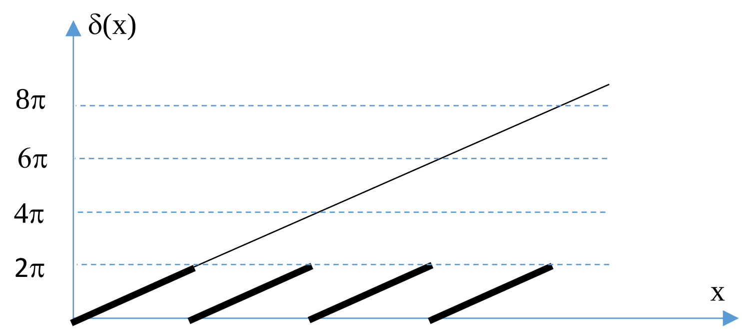
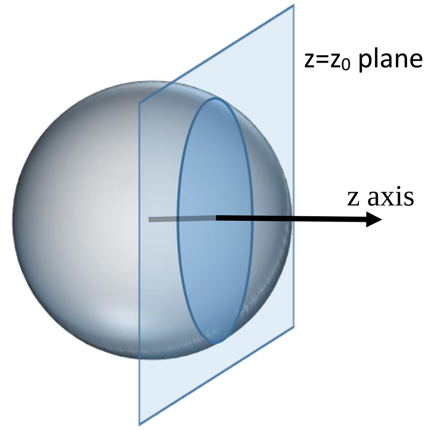


Figure 8. Phase wrapping. As the phase shift δ increases above 2π, a multiple of 2π is subtracted (or added) to bring δ back to with $0 - 2\pi$.

Producing a Spherical wavefront:

The equation of a spherical wave at a distance r from the origin (where the wave was produced) is given by:

13) $\vec{E} = \frac{\vec{E}_0}{r} e^{i(kr-\omega t)}$



In this case, we want to produce the appropriate phase pattern on the SLM to create a wave of this form. The factor of r in the amplitude poses some problem, as the amplitude of the wave would change across the face of the SLM. Or, if r is large enough, and the SLM's area is small enough, then the amplitude will not vary much across the surface, and we can treat the 1/r factor as a constant.

The factor of r in the exponent cannot be treated as a constant, however, as it is within the phase factor of the wave and determines how the wave propagates. Converting r to Cartesian coordinates,

14) $\vec{E} = \frac{\vec{E}_0}{r_0} e^{i\left(k\sqrt{x^2+y^2+z_0^2}-\omega t\right)} = \frac{\vec{E}_0}{r_0} e^{-i\omega t} e^{i\left(kz_0\sqrt{1+\left(\frac{x}{z_0}\right)^2+\left(\frac{y}{z_0}\right)^2}\right)}$

When the SLM is far from the center of the sphere, $z_0 >> x,y$, so we can expand the exponent:

15) $\vec{E} = \frac{\vec{E}_0}{r_0} e^{i\left(k\sqrt{x^2+y^2+z_0^2}-\omega t\right)} = \frac{\vec{E}_0}{r_0} e^{-i\omega t} e^{i\left(kz_0\left(1+\frac{1}{2}\left(\left(\frac{x}{z_0}\right)^2+\left(\frac{y}{z_0}\right)^2\right)\right)\right)} = \frac{\vec{E}_0}{r_0} e^{i(kz_0-\omega t)} e^{i\left(\frac{k}{2z_0}(x^2+y^2)\right)}$

The phase factor that we are concerned with is the part depending upon x and y:

16) $\delta(x, y) = \frac{k}{2z_0}(x^2 + y^2)$

Unlike equation 12), this describes a circular pattern on the SLM

Happily, a cylindrical wave will have a phase relationship just like equation 16), with dependence only on x or y, not both.

The experiment:

Set up the optical system as shown to the right, using the spatial filter and the next lens to produce a 'clean' collimated beam of light. The polarizer and analyzer should be positioned as needed (Case B)) to produce phase only modulation with the SLM. Use your determination of the alignment angle from Activity 2 to properly orient the polarizers.

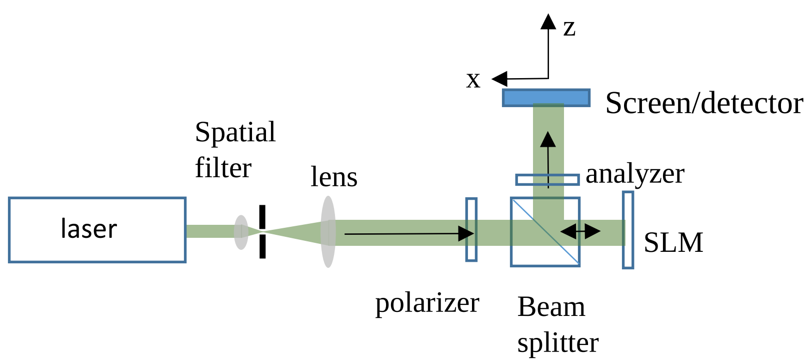


Figure 9. Experimental setup to produce deflected, spherical, or cylindrical waves by modifying the phase of the wavefront with the SLM.

Create phase patterns on the SLM to:

a) Deflect a plane wave to the left and to the right. Explore the upper limit on the angle of deflection.
b) Create converging and diverging spherical waves.
c) Combine the phase patterns to create a converging spherical wave that is also deflected to the left or right.
d) Create a converging cylindrical wave.

Activity #5: Fourier Optics with the SLM

In this activity, we will use the SLM in amplitude modulation mode in order to use it as an easily configurable spatial filter element.

As discussed in class, and as you setup in lab, we can make optical Fourier transforms, and inverse transforms of 2-d apertures with a two lens system in the "4f" configuration. The transform of the aperture appears at the overlapping focal points between the two lenses. We will place the SLM at this location and use it to selectively block spatial frequencies in the transform to filter the image. This is just what we did in lab, but now we will have the freedom of being able to have nearly complete control over the filter, based on whatever image we display on the SLM.

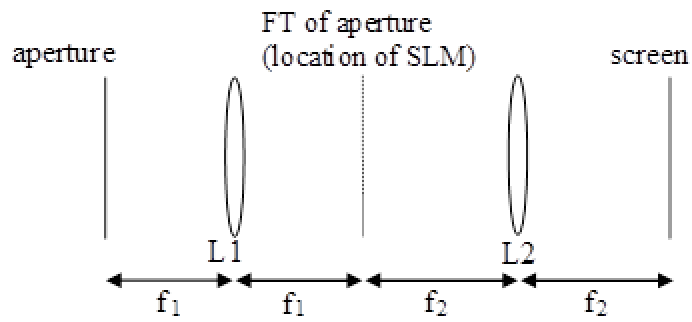


Figure 10. Schematic setup for a 4f spatial filtering experiment. In this case, the SLM will be placed at the plane of the Fourier transform.

The critical part of this experiment is determining the proper size of the 'filter' pattern on the SLM.

If the aperture has a pattern with a periodic spacing of size A, the light diffracted into the first order will head off at an angle θ determined by the diffraction grating relationship (Equation 5, with D = A). When that light is focused by lens L1, it will end up a distance h from the center of the Fourier transform pattern on the SLM. Figure 11 shows the relationship between h, f, and θ. Using simple trigonometry, Equation 5, and the small angle approximation, we can show that:

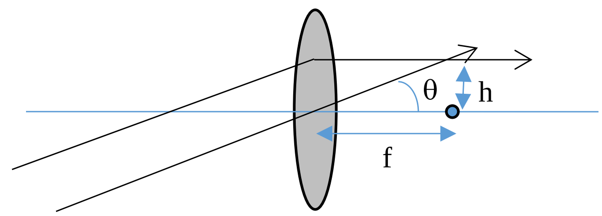


Figure 11. Diagram relating where the image will form in the focal plane for parallel rays entering the lens at an angle θ.

17) $$h = \frac{f\lambda}{A}$$

Knowing h and the size of the pixel spacing on the SLM (D), we can determine the size and placement of the pattern we need on the SLM to spatially filter the transform.

The experiment:

Set up the optical system as shown in Figure 12 to the right. Lenses L1 and L2 need to have focal lengths that are long enough to place them one focal length from the SLM as shown. If f2> f1, then the filtered image on the screen will be magnified.

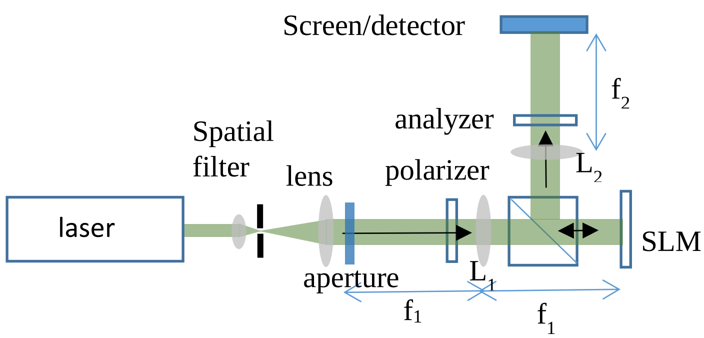


Figure 12. Experimental setup to create the 4f spatial filtering experiment with the SLM. The polarizer and analyzer need to be placed in position so the SLM will act as an amplitude modulator.

Use an adjustable slit as the aperture. With a uniform gray-scale image displayed on the SLM you should see the image of the 2-d slit in focus on the screen.

Display a line on the SLM in order to remove one direction of the diffraction pattern (the image of the slit should have defined edges only for either the vertical sides or horizontal sides).

Bonus Activity #6: Creating beams with orbital angular momentum

In this activity, we will use the SLM in phase modulation mode to create waves with helical wavefronts. To do this, create a pattern based on the following electric filed description of the wave traveling in the z direction:

18) $\vec{E} = \vec{E}_0(\rho)e^{im\phi}e^{i(kz-\omega t)}$

where m is an integer, ρ is the radial distance from the origin in the x-y plane, and ϕ is the regular azimuthal angle in the x-y plane. If m=1, the phase of the wave varies from 0 to 2π as we rotate from ϕ = 0 to ϕ = 2π.

The phase function we want to record on the beam with the SLM is then just:

19) $\delta(\rho, \phi) = m\phi$

Make this pattern for (m = 1, 2, and some other integers), and see what happens to the laser beam after you encode this phase pattern onto it. Note, you need to make sure that the laser beam is centered at the origin of the pattern on the SLM. This is easiest to do by moving the pattern around on the SLM, probably.

Being able to do this to a beam may become technologically very important, as photons with entangled spin (left and right circular polarization) and orbital angular momentum (helical wavefronts) are possible qubits for use in quantum computing.