Are flows electromagnetically forced in thin stratified layers two dimensional?

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The relaxation of three-dimensional perturbations in flows generated in thin density stratified layers is discussed. It is found that the relaxation time for such perturbations is small compared to the other characteristic times of the system. These results offer a basis to assess the two dimensionality of freely decaying turbulence prepared in such configurations. © *1997 American Institute of Physics.* [S1070-6631(97)02510-5]

It is known that density stratification and rotation favor two dimensionality. These effects have been used by many experimentalists since Taylor[1] to investigate two-dimensional flows in the laboratory. There exists other methods for constraining flows to two dimensionality: for example the use of a geometrical confinement[2] or of a magnetic field for liquid metals.[3] Recently, a new configuration has been proposed for investigating two-dimensional flows: it consists of a system of thin fluid layers, confined in a container and driven by electromagnetic forces.[4] This configuration has been used to study the equilibrium states of two-dimensional turbulence,[5] the phenomenon of vortex stripping,[6] and the decay of two-dimensional turbulence.[4] Dispersion phenomena have also been investigated in a similar configuration.[7]

There are several good reasons to consider that in such a configuration, the flow ''lives'' in a two-dimensional world: elementary experiments (such as dipole propagation, dipole collisions) show that we do recover, at a qualitative level, the known phenomenology of two-dimensional vortex dynamics.[8] Moreover, in the few cases where a comparison with two-dimensional direct numerical simulations has been done, good quantitative agreement has been obtained.[9] However, the issue is still unsettled, because, for this particular system, the physical process leading to the suppression of three-dimensional effects is rather unclear. Moreover, the temporal and spatial range of scales within which the system can be treated as two dimensional is not determined. The purpose of this brief communication is to unravel the physical origin of this mechanism and to estimate the boundaries of the time domain within which the system can be considered as two dimensional.

In the experiments we report, the flow is generated in a PVC cell, 20 cm$\times$27 cm. As in previous studies,[5] the cell is filled by two 3-mm-thick layers of NaCl solution, in a stable configuration, i.e., the heavier underlying the lighter. Permanent magnets are located just below the bottom of the cell. Their magnetization axis is vertical and they produce a magnetic field, 0.3 T, of maximum amplitude, which decays over a typical length of 3 mm. An electric current is driven through the cell from one side to the other. Here we study the relaxation regimes following the application of a short current impulse (typically between 1 and 2 s in our experiments). Since the magnetic field decays over a length which is within the 6 mm fluid depth, only the lower layer is stirred during the impulse. One can then, by studying the relaxation regime, get information on the momentum diffusion process across the two fluid layers. This is the principle of the experiments we report here.

As in previous studies, the system is analyzed by particle image velocimetry techniques.[4,10] However, in the present case, we perform the measurements both on the free surface, and on the interface separating the two fluid layers. We achieve that by using two sets of particles, chosen so that the first set of particles stays on the free surface, whereas the other set stays at the interface separating the two fluids. In practice, for each flow configuration, we perform two series of experiments, each one using a particular set of particles. We measure the velocity fields at the two levels separately, for each series.

In a first series of experiments, the magnets are arranged so as to get a single vortex. We measure the kinetic energy E defined by

$$E=\frac{1}{2}\int\int_S \underline{u}^2 ds$$

at the free surface (E_f) and at the inner interface (E_i). The time evolution of these two quantities is shown in Fig. 1. It is convenient to introduce (somewhat arbitrarily) two domains; for $0<t<4$ s, E_f increases, whereas E_i decreases up to a point where the two energies become equal. Then, for 4 s $<t<20$ s, the two quantities decrease exponentially in time, according to the laws

$$E_i=E_i^0 \exp(-t/\tau), \quad E_f=E_f^0 \exp(-t/\tau).$$

This asymptotic regime is thus characterized by a single time constant $\tau=14.3$ s. The ratio $r=E_i/E_f$ is constant in the major part of this time domain, and is approximately equal to 0.52. Thus, in the asymptotic regime, the velocity profile is nonuniform across the layer. There are velocity gradients, the amplitudes of which are consistent with a Poiseuille profile (assuming such a profile would lead to $r=0.56$). Moreover, the estimate of the characteristic time associated to the free decay of a Poiseuille flow, i.e., $2b^2/\pi^2\nu$ (b is the total depth of the fluid layer, ν is the kinematic viscosity), gives 14.6 s, which also agrees with our measurement of τ. All this indicates that in the asymptotic regime, i.e., for $t/\tau>0.3$, the flow structure within the layer is close to a Poiseuille flow.

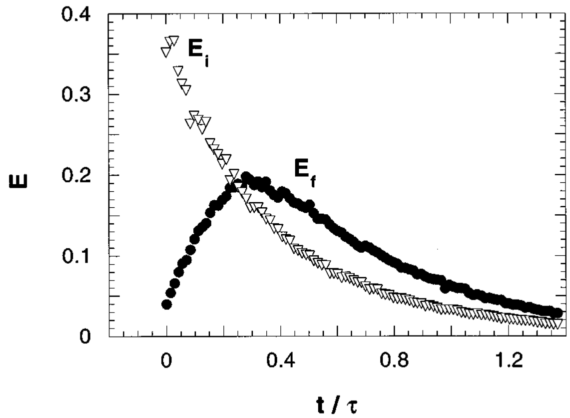

FIG. 1. Compared evolutions of kinetic energies measured at the free surface (black dots) and at the interface (triangles) for the single vortex flow.

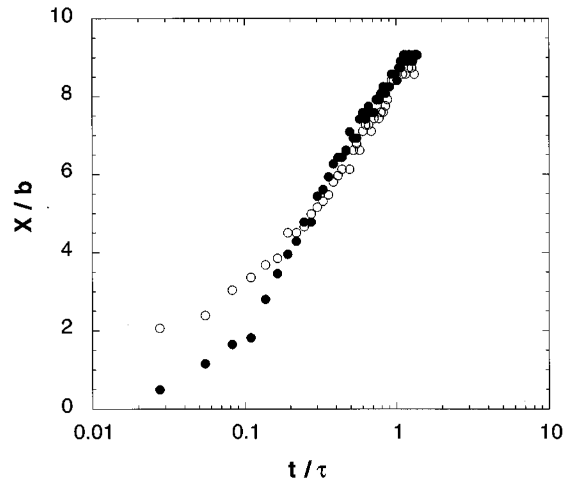

FIG. 3. Trajectories of the dipole centers at the free surface (black dots) and at the interface (circles). Positions X are rescaled by the total depth b.

The first period of time displayed in Fig. 2—i.e., for $0 < t/\tau < 0.03$—is worth investigating more precisely. We consider here the difference of energy between the lower and the upper layer, i.e., the quantity $\Delta E = E_i - E_f$.
Our measurements are well approximated by the law

$$\Delta E = \Delta E^0 \exp(-t/\tau_1) + C, \quad \text{with} \quad \tau_1 = 1.9 \text{ s}.$$

This defines another characteristic time, τ_1, eight times smaller than the viscous time τ. τ_1 characterizes the transfer of momentum across the fluid layers. During the period $0 < t/\tau < 0.3$, the velocity profile relaxes from a strongly inhomogeneous profile, towards a Poiseuille-like profile, which defines the asymptotic state.

We now consider the propagation of a pair of equal strength counter-rotating vortices. Here, we use a single magnet to produce the dipole. As we do for the isolated vortex, we apply an impulsional electrical current and investigate the decaying regime. For this series of experiments, we measure four quantities: the total kinetic energies at the inner interface, E_i, and at the free surface, E_f, and the instantaneous position $X(t)$ of the dipole center along the propagation axis, at the interface $[X_i(t)]$ and at the free surface $[X_f(t)]$. Here again, we find two time domains: one for t between 0 and 4 s and another one for t between 4 and 20 s. As for the monopole, there is an asymptotic regime, located at the same place in the time domain, for which the energies E_f and E_i decay exponentially, with the same characteristic time $\tau = 14.3$ s; the limiting value of the ratio $r = E_i/E_f$ is 0.46, which is close to the value obtained for the monopole. In this asymptotic regime the dipole trajectories merge (see Fig. 3): we have thus reached a columnar flow, for which both fluid layers propagate at the same speed; these two characteristics (same propagation velocity and Poiseuille-like profile across the layer)—which are compatible if the interface between the two layers is not flat—characterize the asymptotic regime for the dipole.

To go further into the analysis, we have followed the evolution of the separation $\Delta X = X_i(t) - X_f(t)$ between the dipole centers. This is shown in Fig. 4, on which we have plotted the quantity ΔX.

One sees that the relaxation of the separation fits an exponential function reasonably well. The corresponding characteristic time is 1.6 s, which is close to τ_1. It is remarkable (although not so surprising) that we find the same characteristic times for the dipole and the monopole; this indicates that a single relaxation mechanism is involved in both cases.

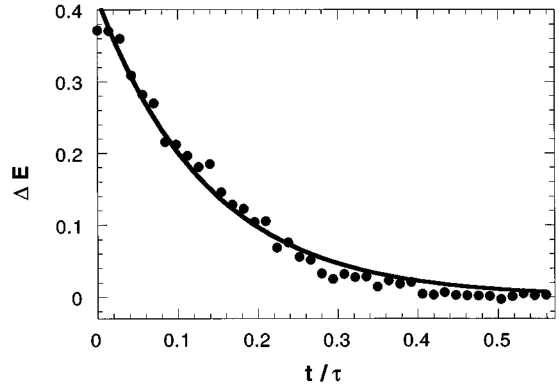

FIG. 2. Temporal evolution of $\Delta E = E_i - E_f$ during the initial period (dots) compared to an exponential fit (line).

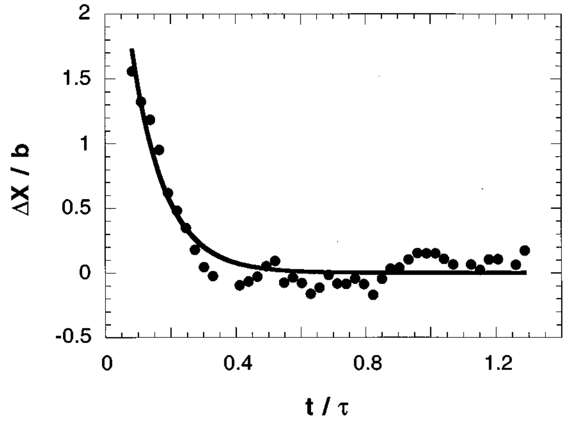

FIG. 4. Evolution of $\Delta X = X_i - X_f$ (dots) compared to an exponential fit (line). ΔX is rescaled by the total depth b.

The physical mechanism leading to the rapid transfer of momentum from the lower to the upper layer is certainly the displacement of the inner interface under the action of gravitational forces. Visual observations show that the interface between the two layers moves rapidly during the transient period and further becomes stationary as we reach the asymptotic regime. This suggests that, as in ordinary stratified flows, the underlying physical mechanism at work is the relaxation of isodensity surfaces (see Ref. 11, chapter 6). Expressed differently, the interface generates waves which radiate away the energy of three-dimensional perturbations. A characteristic time associated with this process is given by

$$\tau_2 = 2a/\sqrt{gb(d\rho/\rho)},$$

where a is the vortex size, ρ is the fluid density, $d\rho$ is the density difference between the two layers, g is the gravity, and b is the layer thickness. τ_2 is the time necessary for a wave to propagate across a distance equal to the vortex radius. One gets $\tau_2 \approx 0.4$ s, which is somewhat lower, albeit consistent, with the measurement. The above expression for τ_2 indeed provides a crude estimate based on a long-wave approximation, which probably fails for the three-dimensional perturbations we consider. Moreover, the expression for τ_2 neither takes into account the presence of vorticity nor the bottom wall friction.

More specifically, one may propose the following mechanism for the rapid transfer of momentum across the layers. During the application of the current impulse, momentum is mainly injected at the bottom of the fluid layer so that the velocities are higher in the lower layer than in the upper one. Low pressures appear in the lower layer, at the vortex centers and this in turn generates a strong deformation of the inner interface. When the current is switched off, the interface relaxes upward, thus transferring momentum from the bottom layer to the upper one. The deformation rapidly decreases and an asymptotic state takes place.

To conclude, we have determined the time constant associated with the transfer of momentum across the layer, for two configurations—the monopole and the dipole; it is found smaller than the characteristic time for the decay of energy and also smaller than the turn-over time of the vortices we consider (the latter is typically 7 s in our experiments). One can thus infer that in these two cases, after a short transient state, the flow can be treated as two dimensional. We finally suggest that this conclusion can be generalized to previous turbulence decay experiments[5] carried out in this configuration.

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