Light scattering in daily life

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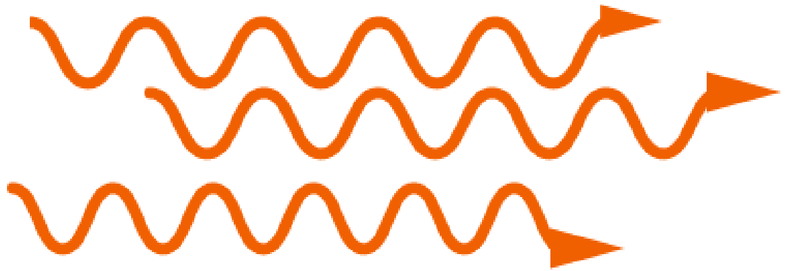
Outline

- What is the light itself?
- Blackbody radiation
- Why the sky has different colors?
- Mie scattering
- Quantum dots

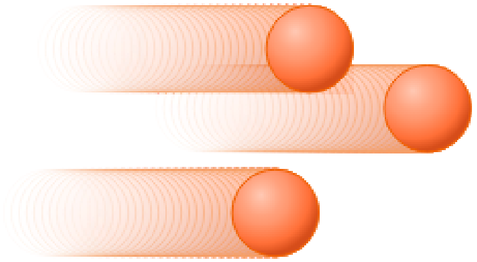
WHAT DO WE KNOW ABOUT PHOTONS AND LIGHT?

(the prerequisites to the emergence of quantum mechanics)

WHAT IS THE LIGHT ITSELF?

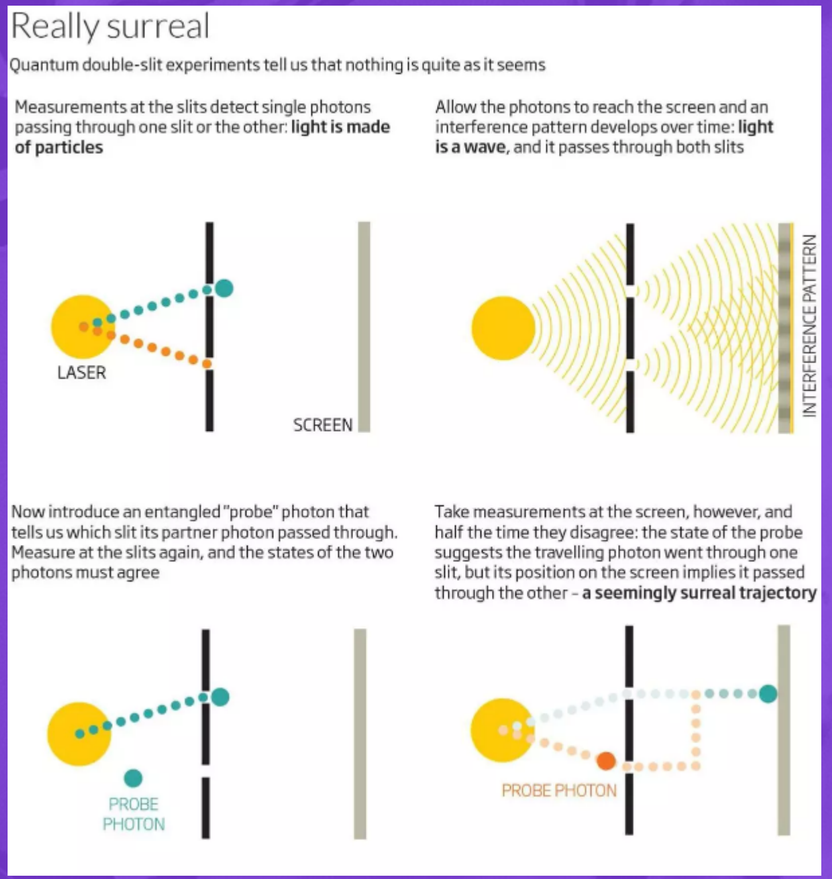


Waves

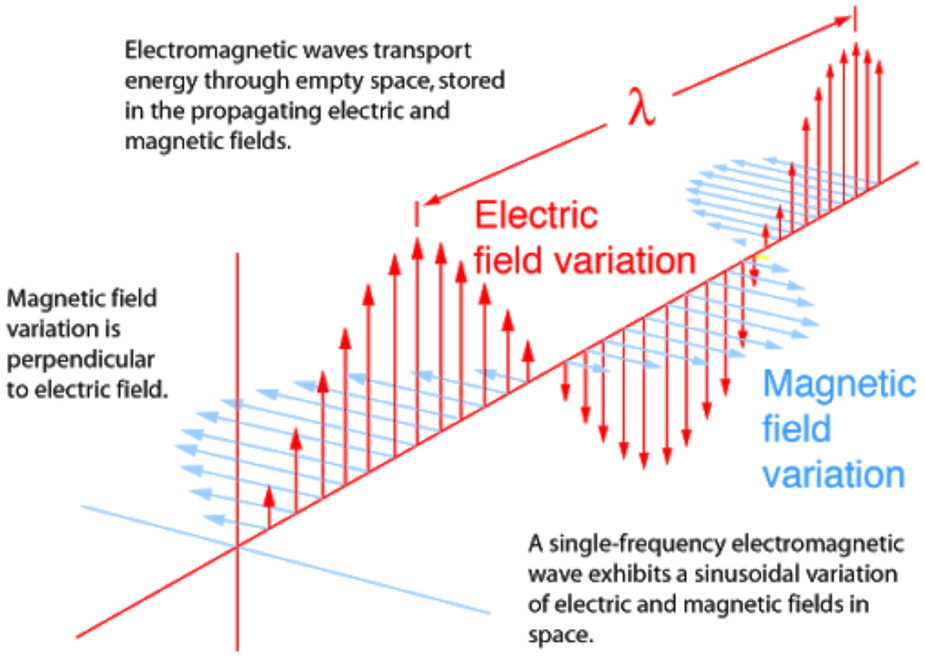


Particles

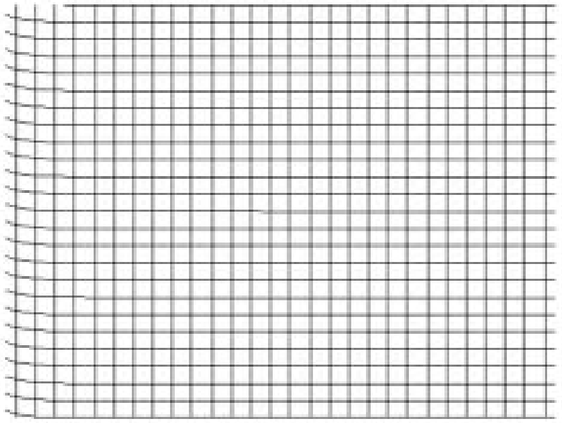
WAVE-PARTICLE DUALITY



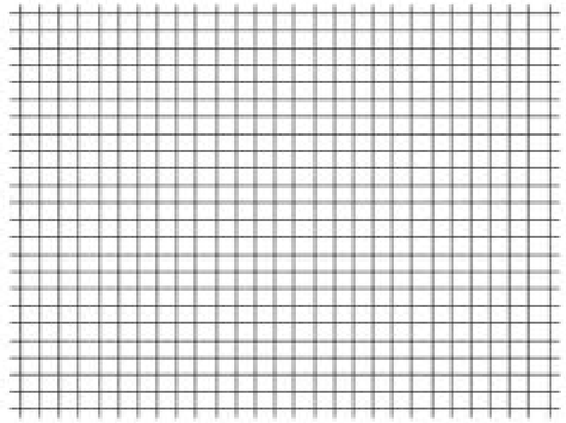
PHOTONS (TRANSVERSE ELECTROMAGNETIC WAVES)



- k (wave vector) - is the direction of wave propagation. It is connected to the wavelength $k=2\pi/\lambda$.
- Frequency $\omega=ck$ (energy $E = \hbar\omega$).
- Energy and wavelength are inversely proportional $E = \hbar ck = 2\pi\hbar c/\lambda$.
- Polarization of light (linear, circular, elliptical)

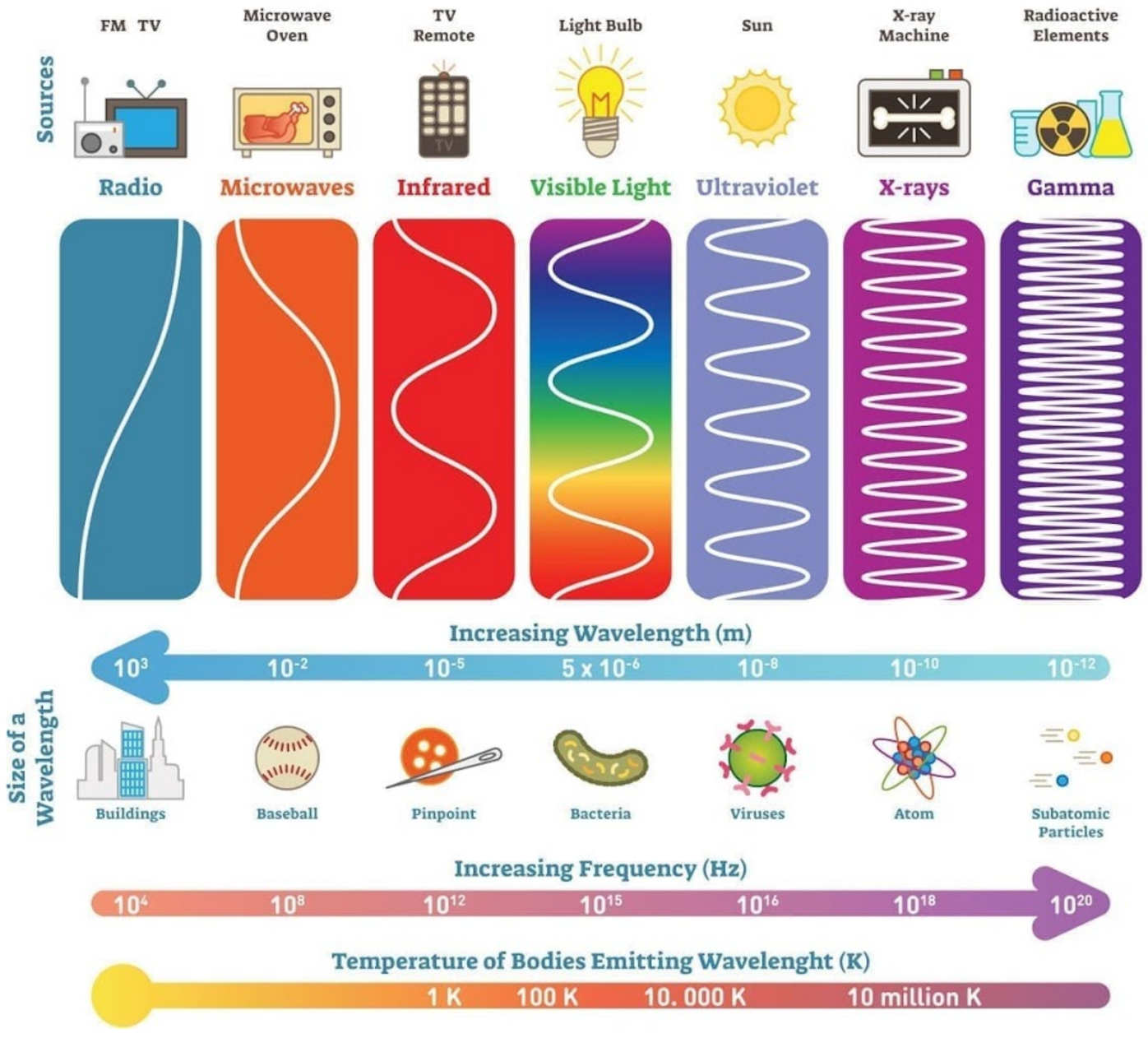


Transverse waves (oscillation is perpendicular to the direction of propagation)

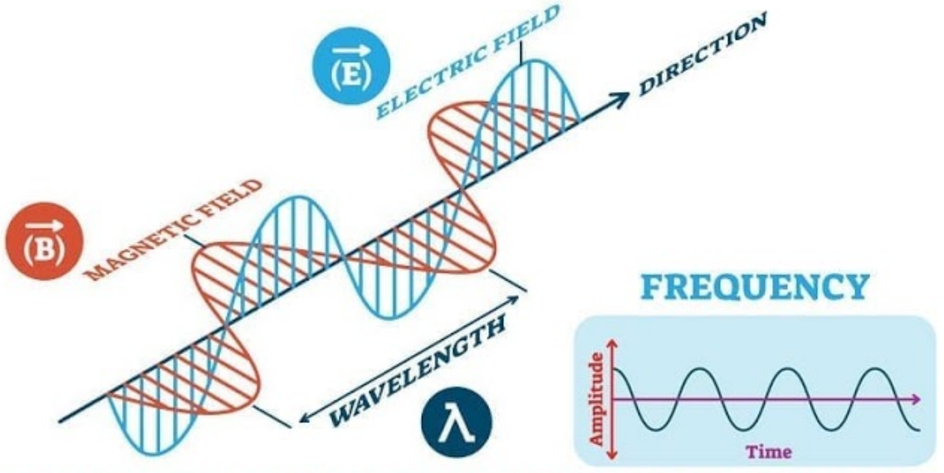


Longitudinal waves (oscillation is parallel to the direction of propagation, e.g. acoustic wave)

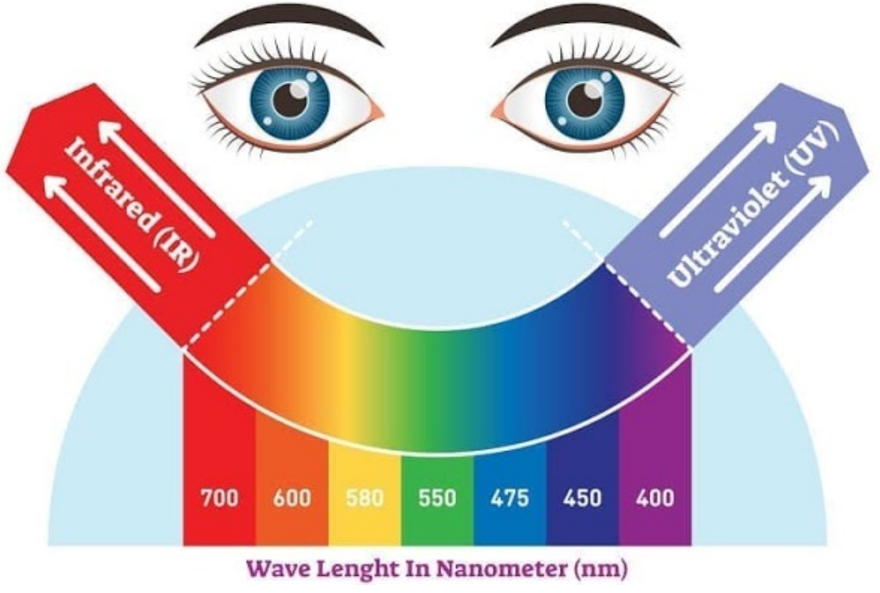
ELECTROMAGNETIC SPECTRUM



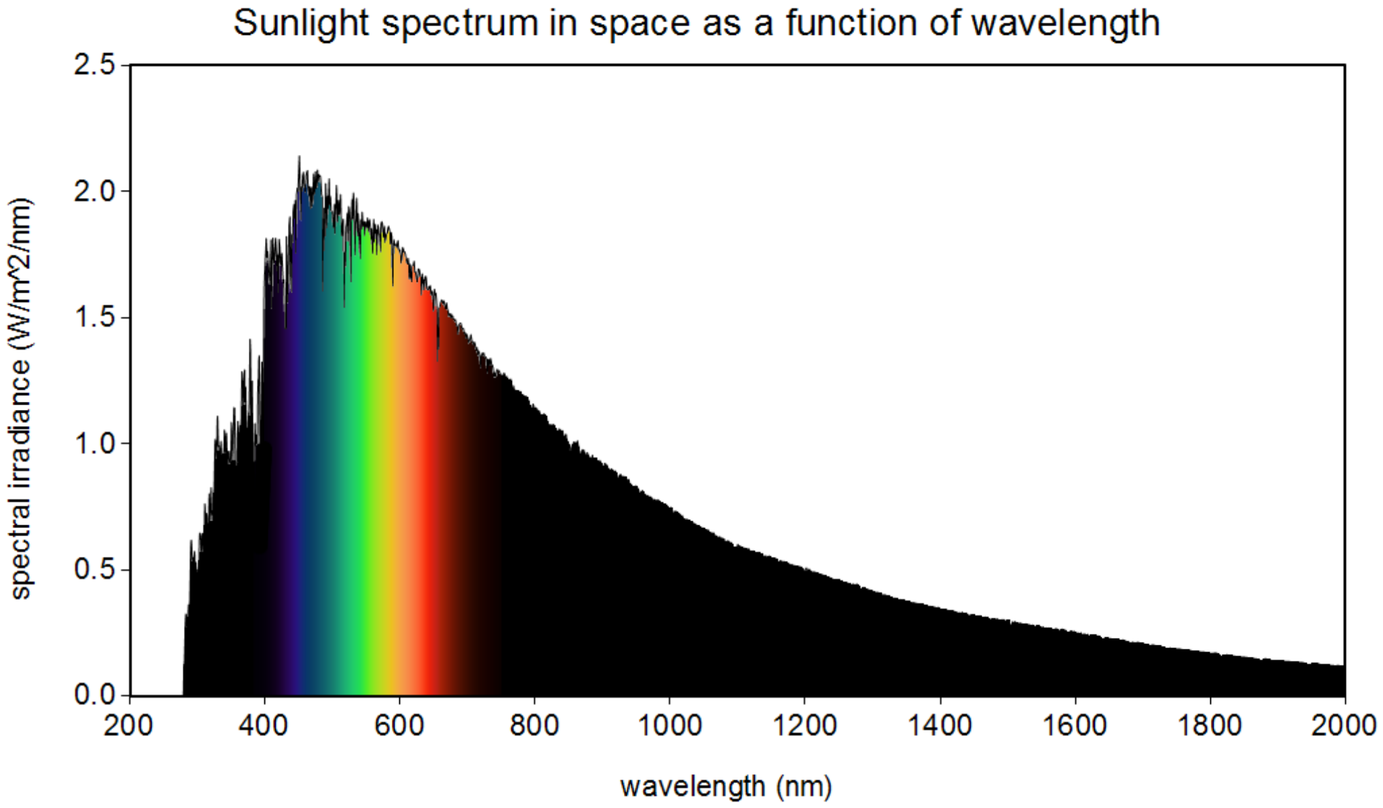
ELECTROMAGNETIC WAVES



VISIBLE SPECTRUM

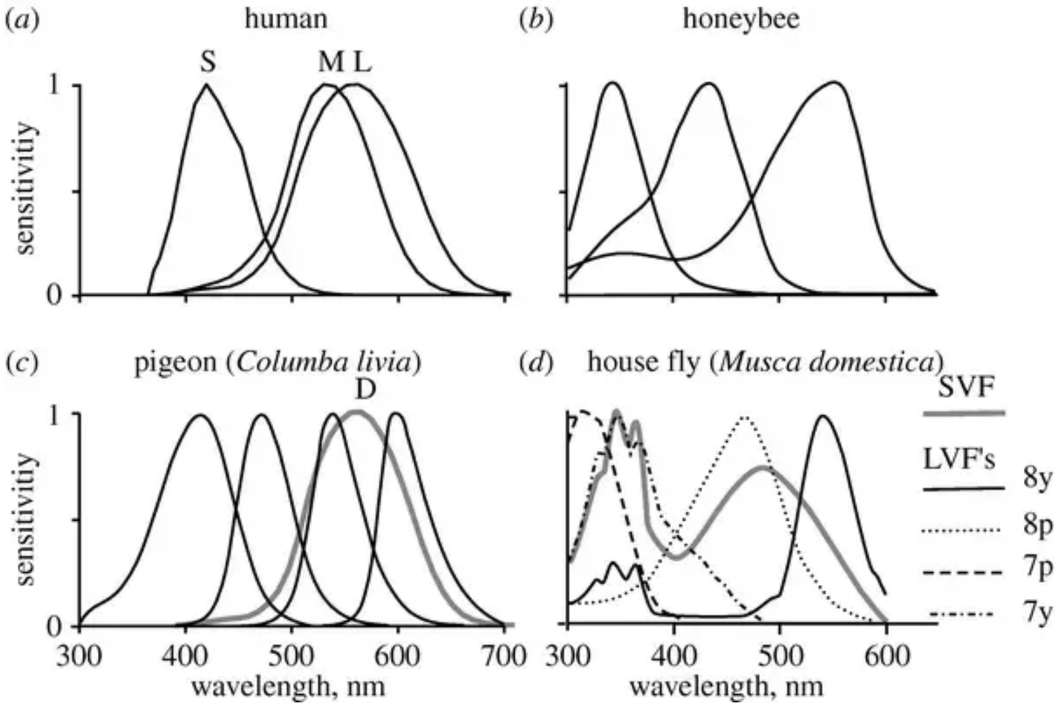


EXAMPLES

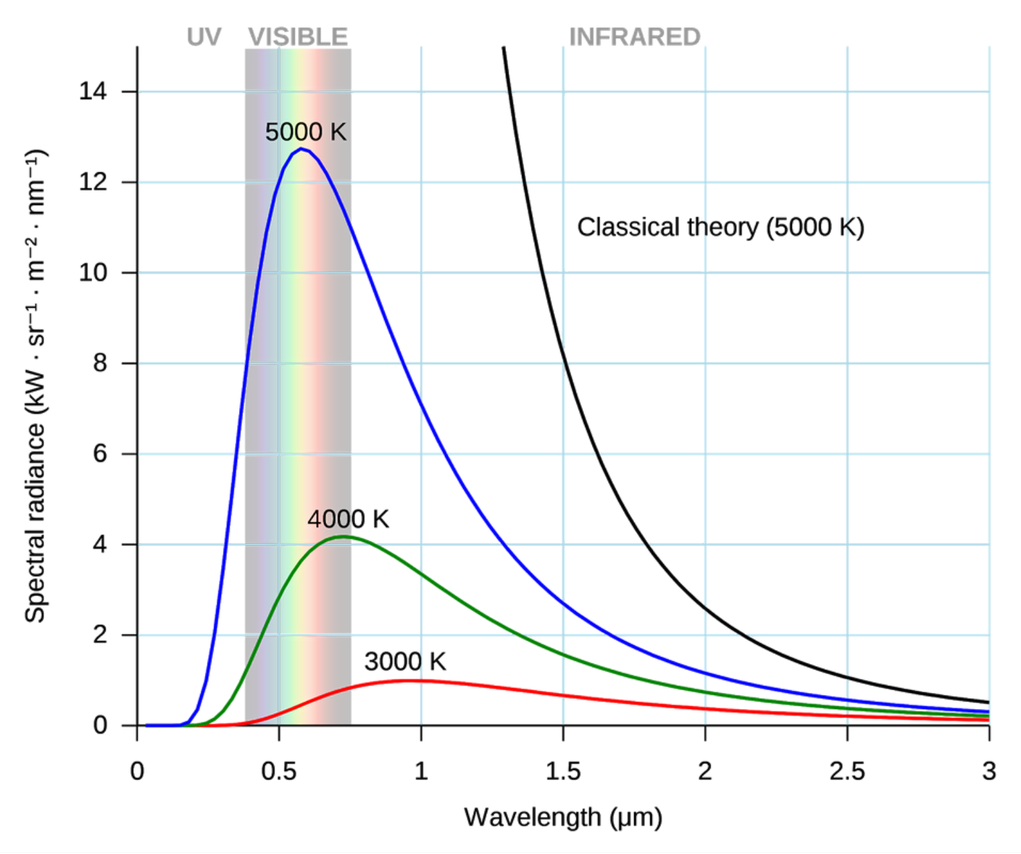


- Experimental spectrum of the sun

- Experimental spectra of the eyes



BLACKBODY RADIATION

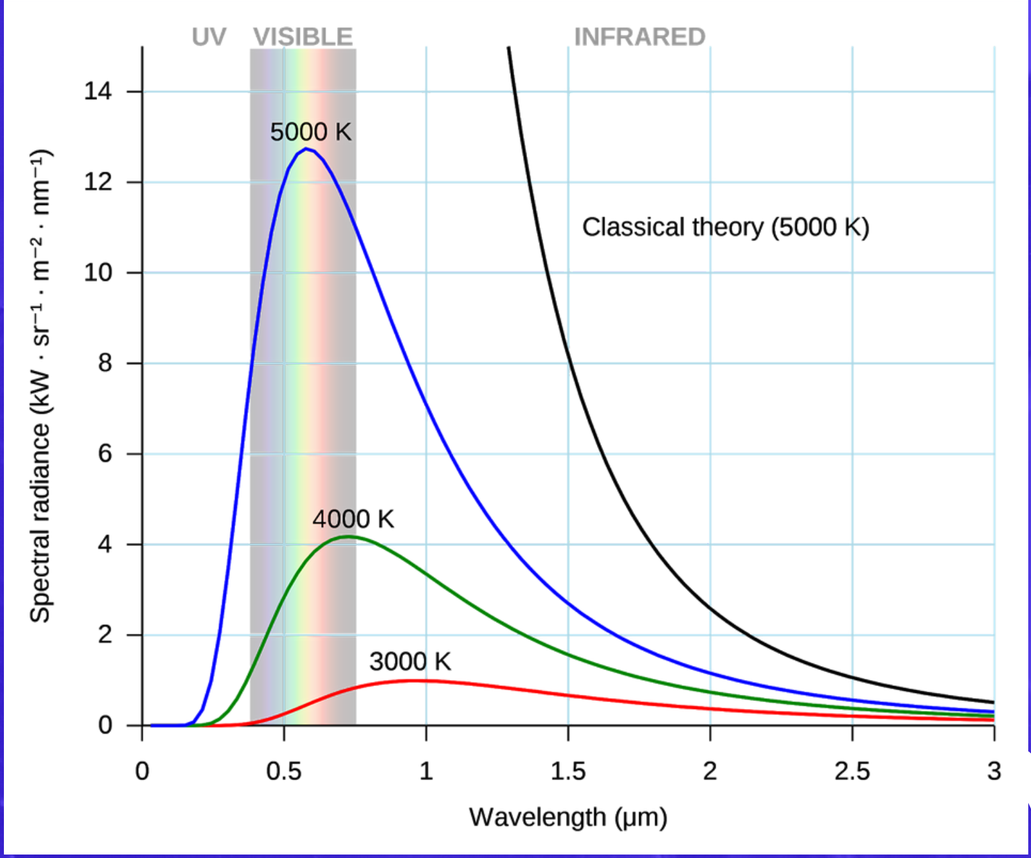


- Black body is a model of a body that can absorb/emit continuous set of wavelengths
- Classical theory - Rayleigh-Jeans law describe a power emitted from the body at a constant wavelength with a constant temperature

$$B_\lambda(T) = \frac{2ck_\mathrm{B}T}{\lambda^4},$$

- “**Ultraviolet catastrophe**” - infinite energy at a small wavelengths? NOT POSSIBLE!

BLACKBODY RADIATION



Here quantum mechanics comes up! Already!
Max Planck in 1900 suggested that maybe energy of particles are quantized and not continuous $E=\hbar\omega$?

Empirical result: $$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\,\frac{1}{e^{\frac{hc}{\lambda k_\mathrm{B} T}} - 1},$$

At the long wavelength limit (use Taylor series):

$$e^{\frac{hc}{\lambda k_\mathrm{B} T}} \approx 1 + \frac{hc}{\lambda k_\mathrm{B} T}.$$

$$\frac{1}{e^{\frac{hc}{\lambda k_\mathrm{B} T}} - 1} \approx \frac{1}{\frac{hc}{\lambda k_\mathrm{B} T}} = \frac{\lambda k_\mathrm{B} T}{hc}.$$

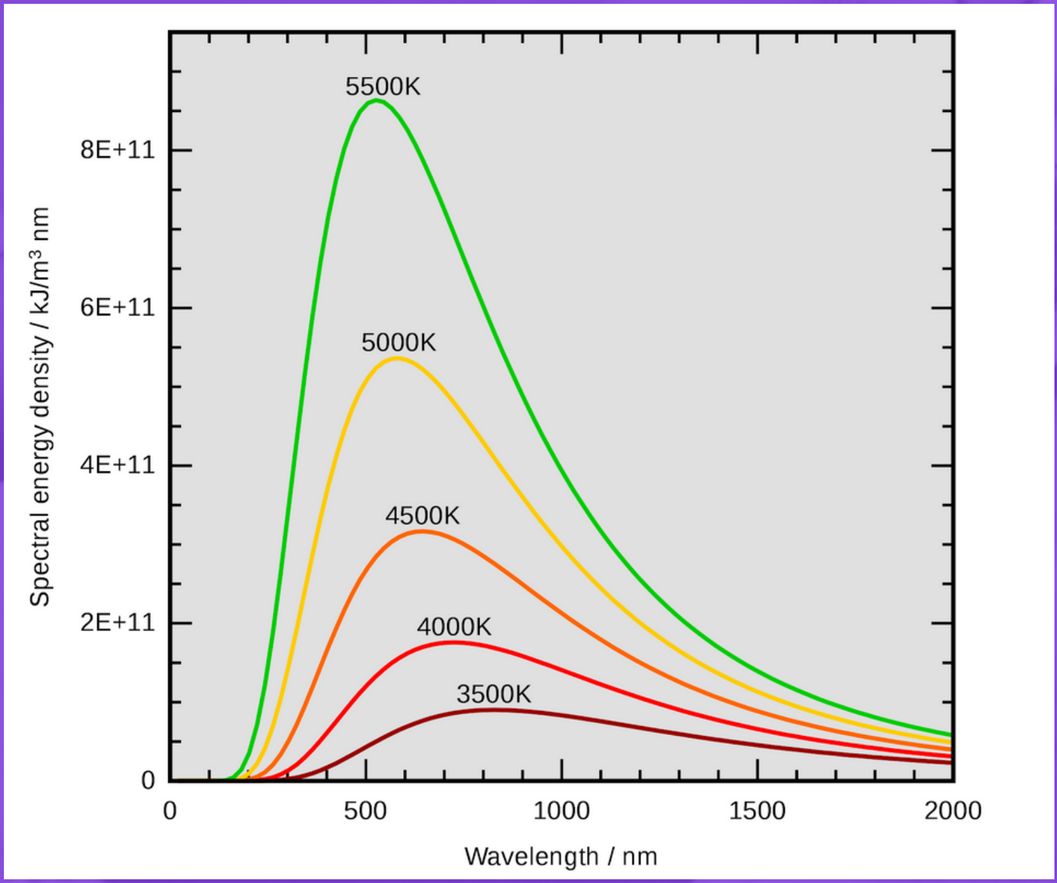
$$B_\lambda(T) = \frac{2ck_\mathrm{B} T}{\lambda^4},$$

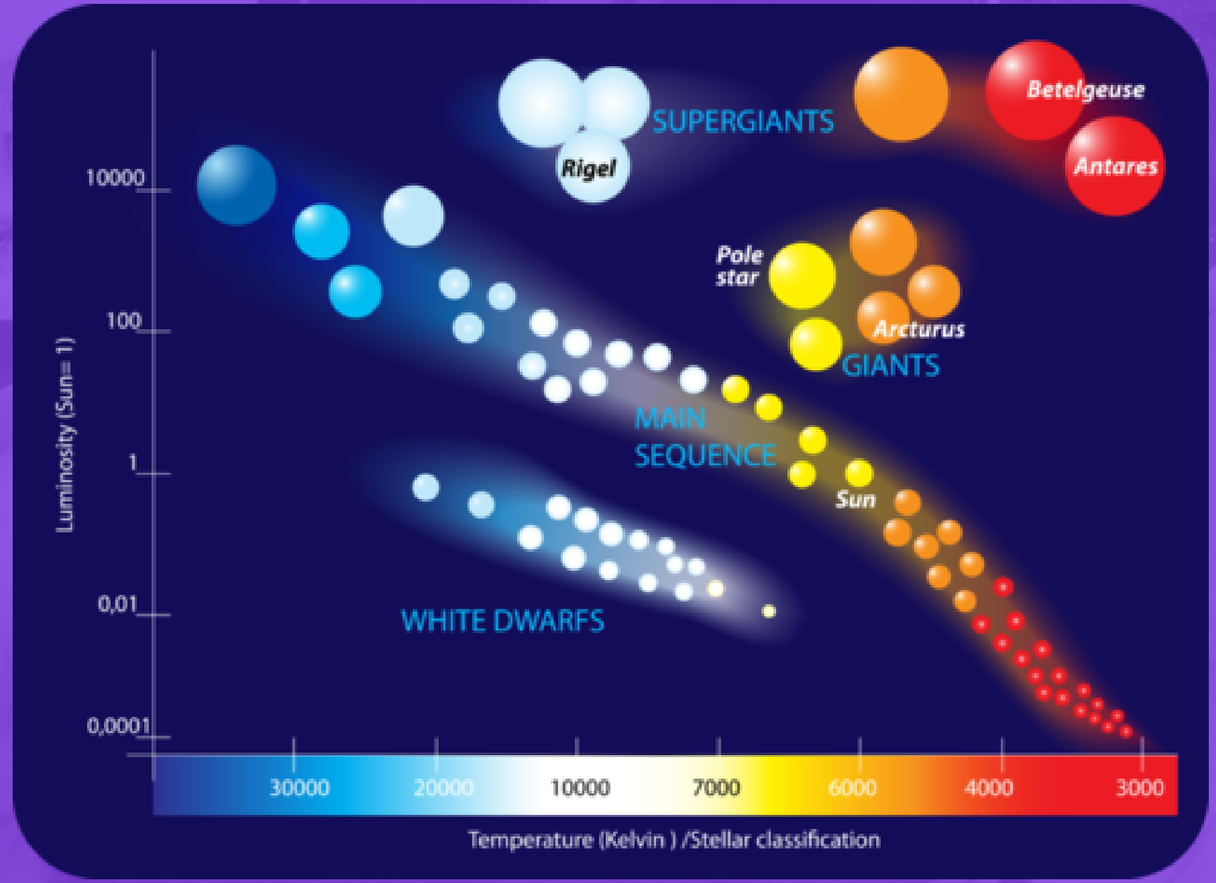
UNEXPECTED SLIDE!

Unexpected slide!

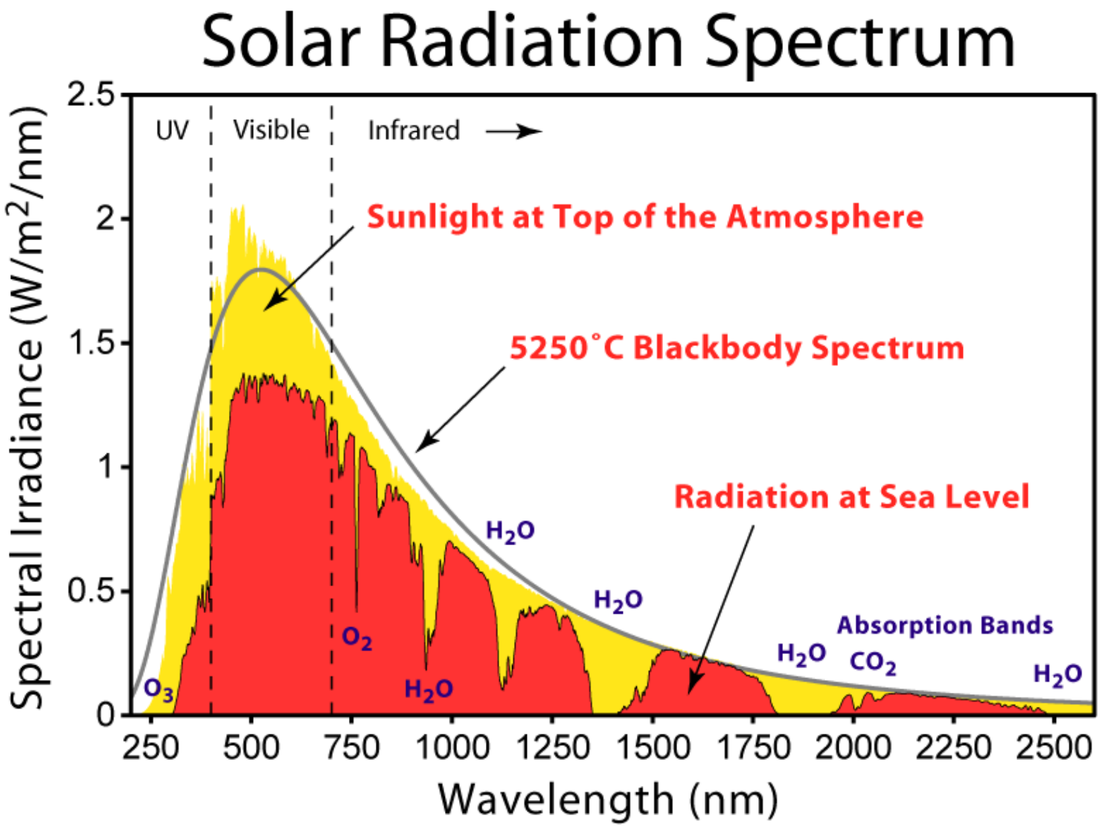
Colors of stars

The higher is the temperature,
the more bluish is the item.





SPECTRUM OF THE SUN



- Yellow color fill shows the spectrum of the sunlight without any atmospheric absorption
- Spectrum (red color fill) takes into account absorption bands in the atmosphere

That is why sunlight in the Earth and sunlight in the Universe differs (and solar panels as well)

ELASTIC SCATTERING OF LIGHT

Elastic scattering - means that the wavelength does not change after the scattering process (the energy does not change)

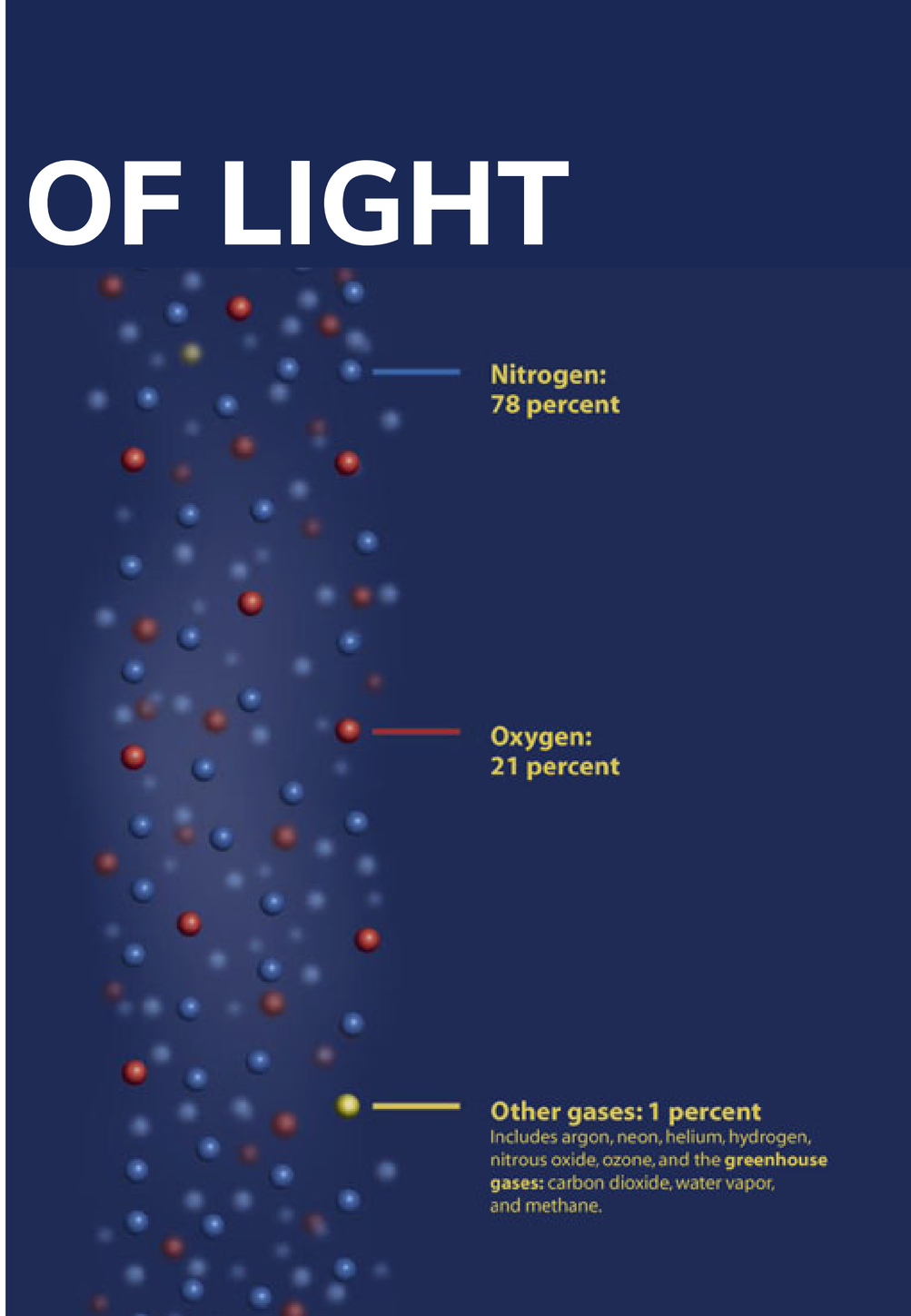
Three types of elastic scattering:

no.1 When the size of the particle is much smaller than the wavelength (**Rayleigh scattering**)

no.2 Almost the same size (**Mie scattering**)

no.3 Much larger than the wavelength (**geometrical optics**)

Atmosphere - Rayleigh scattering! Nitrogen size (0.3 nm) is much smaller than $\lambda \sim 450$ nm. Actually too small

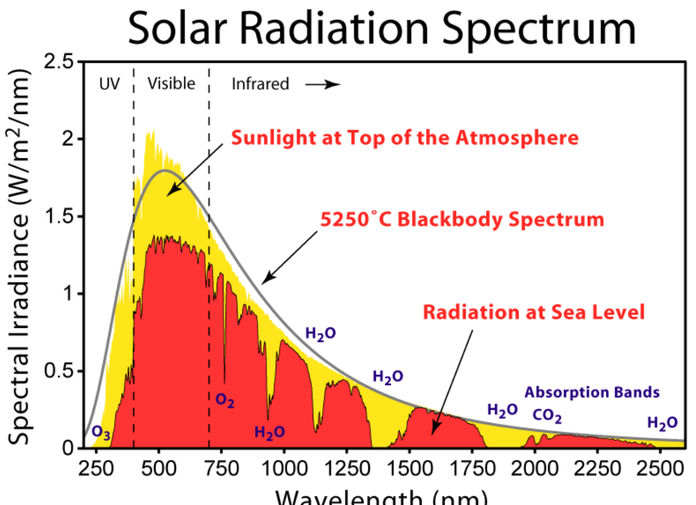


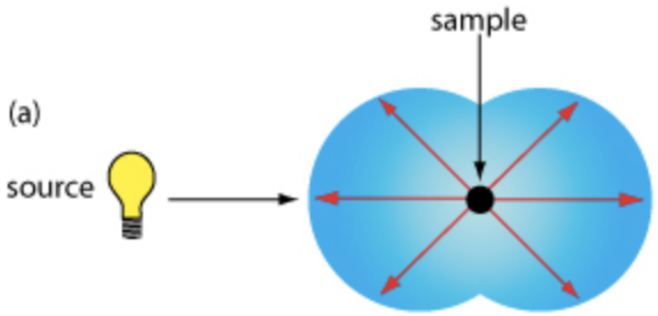
WHY THE SKY IS BLUE?

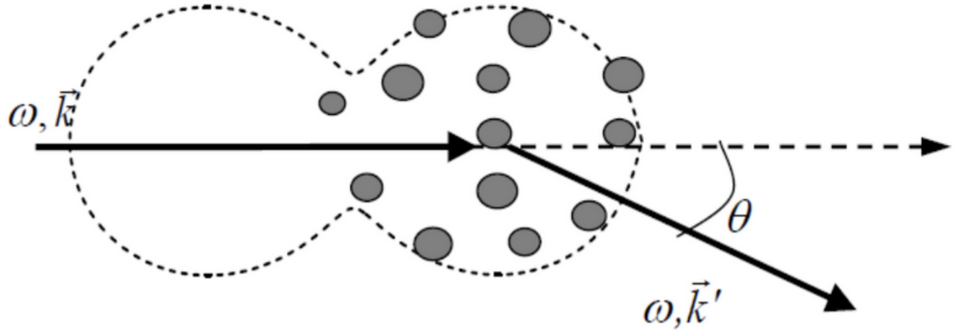
There are a lot of particles and the Rayleigh scattering occurs due to the scattering on the density fluctuations of the atmosphere (it is also less the 0.1λ)

Intensity is $I(\theta) \sim \lambda^{-4}$ Blue light scatters stronger

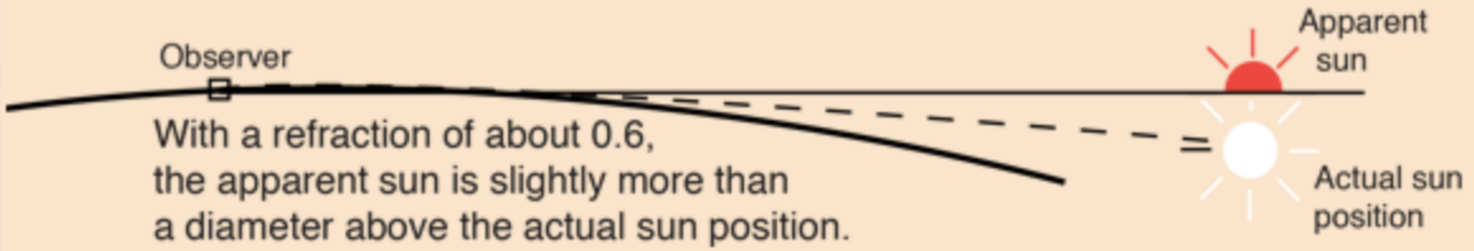
But why is it blue and not magenta?
Lower intensity from the sun, lower sensitivity in the eye

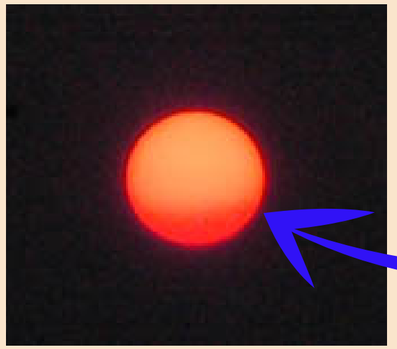




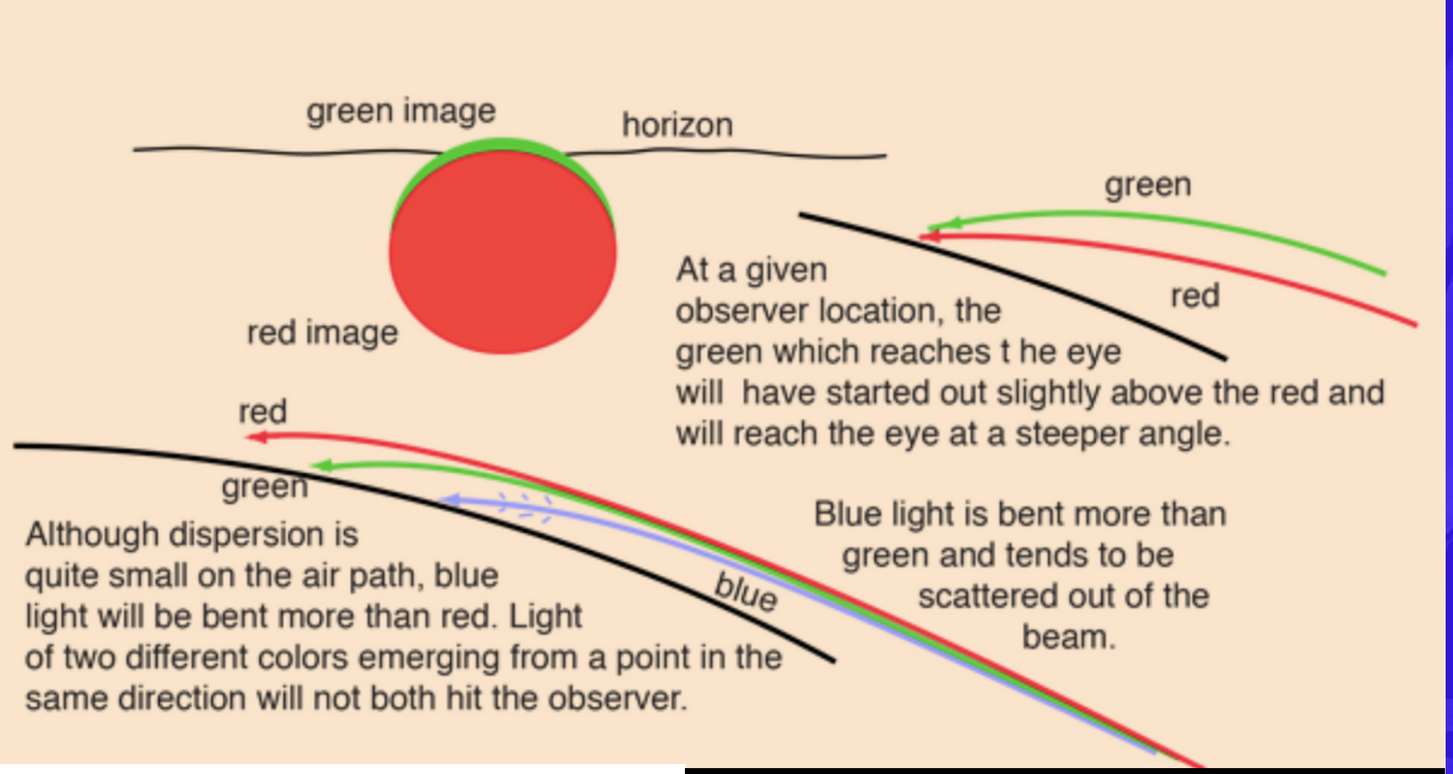


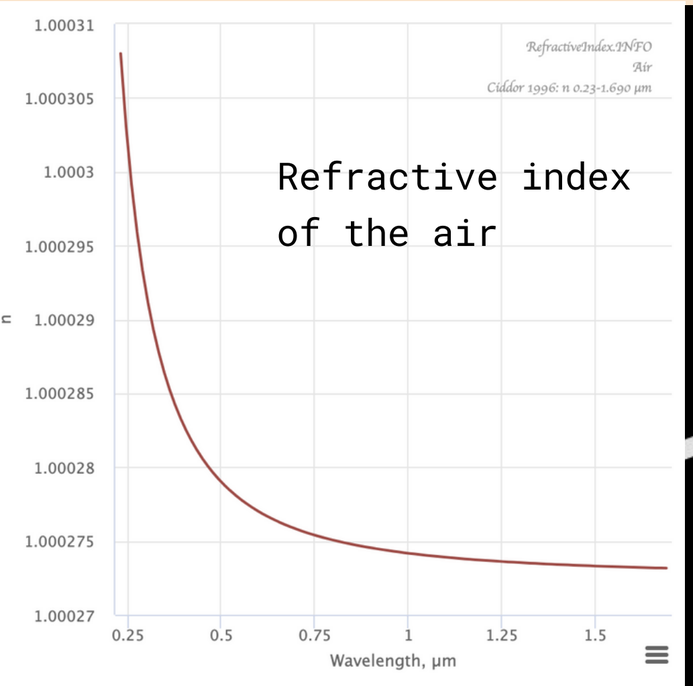
Why is the sky red at the sunset?

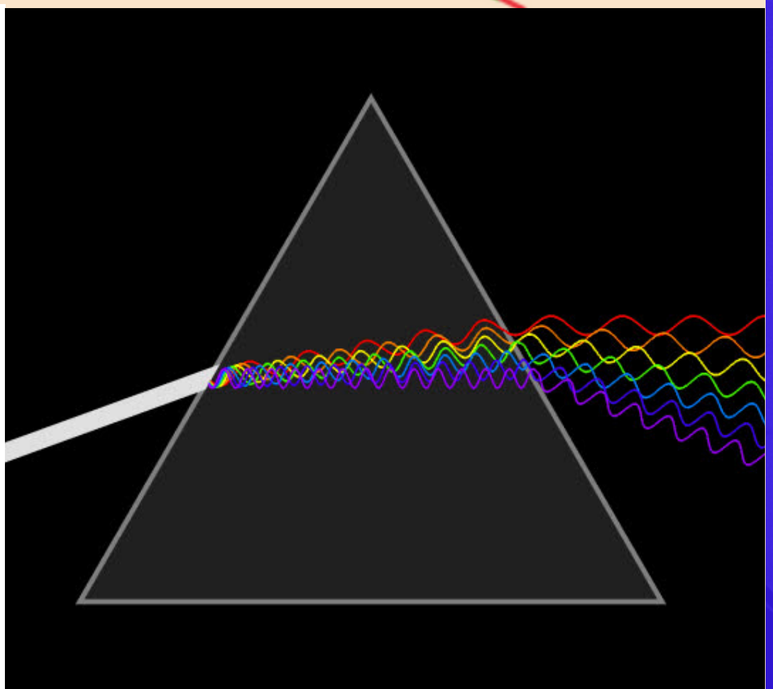




Even stronger effect

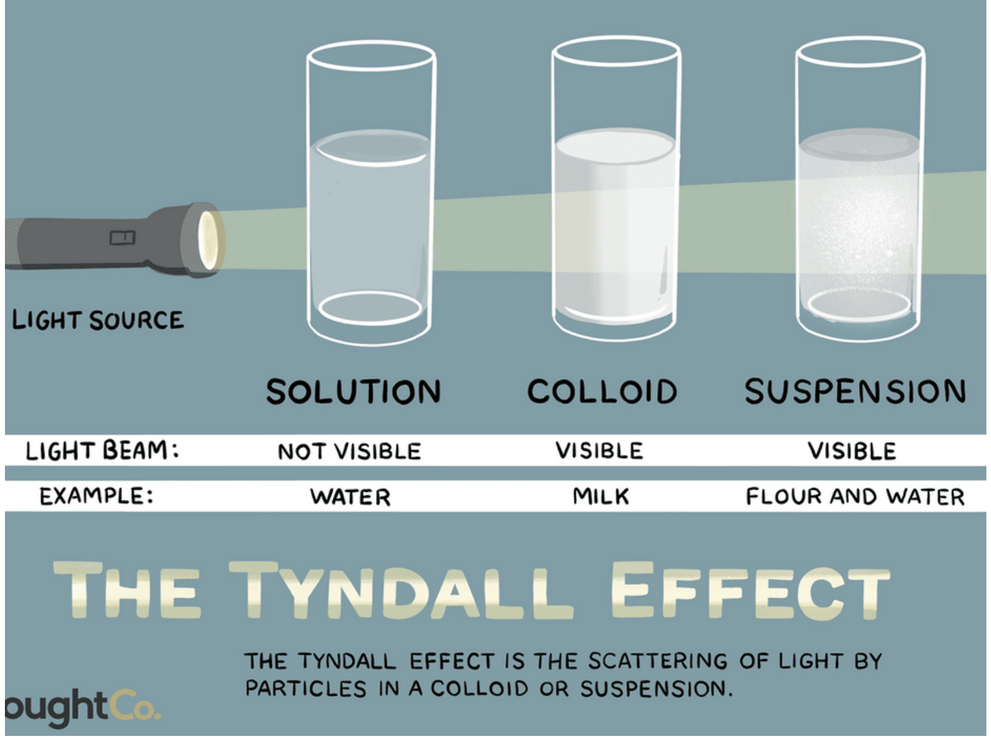






Why is the sun green?! Green Flash

TYNDALL EFFECT (LARGER PARTICLES)



It depends on frequency of light and density of suspension

- Longer wavelength light is transmitted
- Shorter wavelength light is scattered

MIE SCATTERING

HOW TO DESCRIBE AND FIND ELECTROMAGNETIC FIELDS (COMMON APPROACH)

Maxwell's equations

$$\nabla \cdot \mathbf{D} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

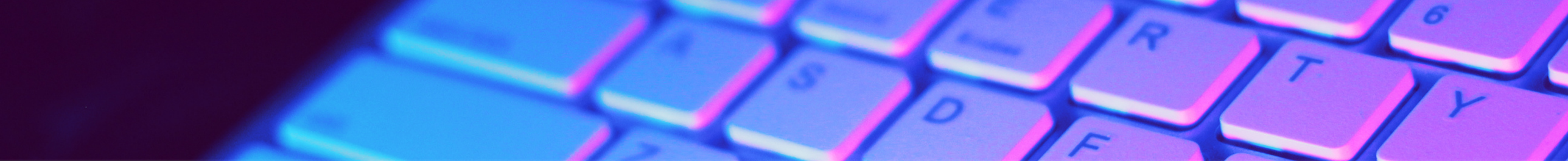
Helmholtz equations

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0, \quad \nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

You can solve them for any case,
do not forget to use physical boundary conditions

$\Delta\psi + k^2\psi = 0$ as follows

$$\mathbf{L} = \nabla\psi; \qquad \mathbf{M} = \nabla \times (\mathbf{a}\psi); \qquad \mathbf{N} = \frac{1}{k}\nabla \times \mathbf{M}$$



MIE SOLUTION (FOR SPHERICAL PARTICLES)

Vector spherical harmonics

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}+k^2\psi=0$$

$$\psi_{e\ell m}=P_\ell^m(\cos\theta)z_\ell(kr)\cos m\varphi$$
$$\psi_{o\ell m}=P_\ell^m(\cos\theta)z_\ell(kr)\sin m\varphi;\quad \text{Here } m \text{ is an integer and } m\geq 0$$

Here $z_\ell(kr)$ is the spherical Bessel function of 1st, 2nd, or 3rd kind.

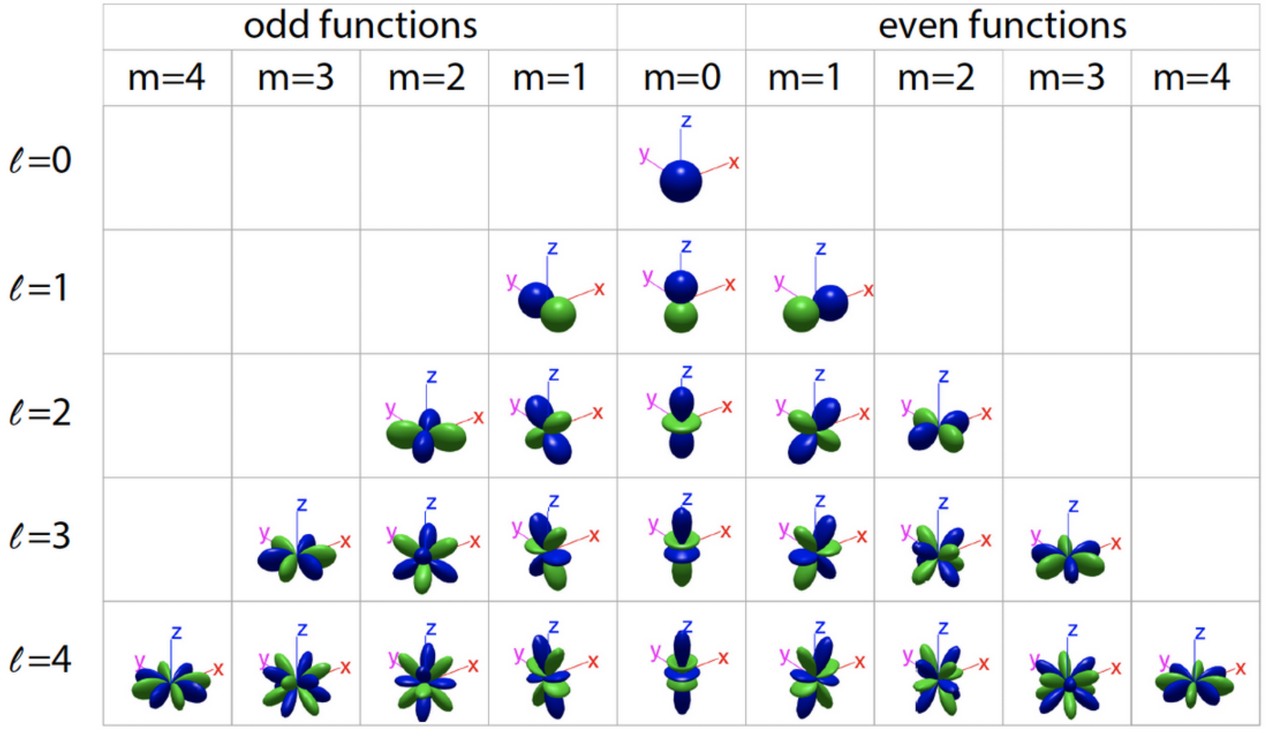
From scalar harmonics to vector harmonics

$$\mathbf{M}_{e\ell m}=\nabla\times(\mathbf{r}\psi_{e\ell m}) \qquad \mathbf{M}_{o\ell m}=\nabla\times(\mathbf{r}\psi_{o\ell m})$$

$$\mathbf{N}_{e\ell m}=\frac{\nabla\times\mathbf{M}_{e\ell m}}{k} \qquad \mathbf{N}_{o\ell m}=\frac{\nabla\times\mathbf{M}_{o\ell m}}{k}$$

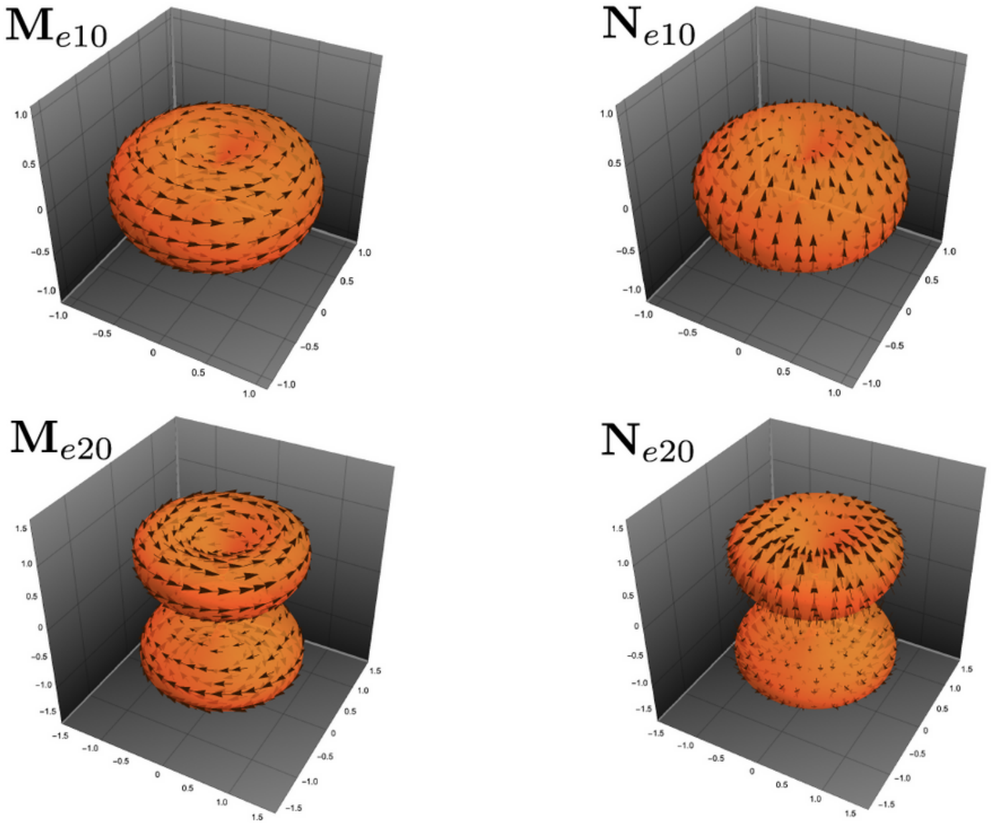
Mie solution (for spherical particles)

$P_\ell^m(\cos\theta)\sin m\varphi$ $P_\ell^m(\cos\theta)\cos m\varphi$



Blue colour corresponds to positive values, Green colour corresponds to negative values.

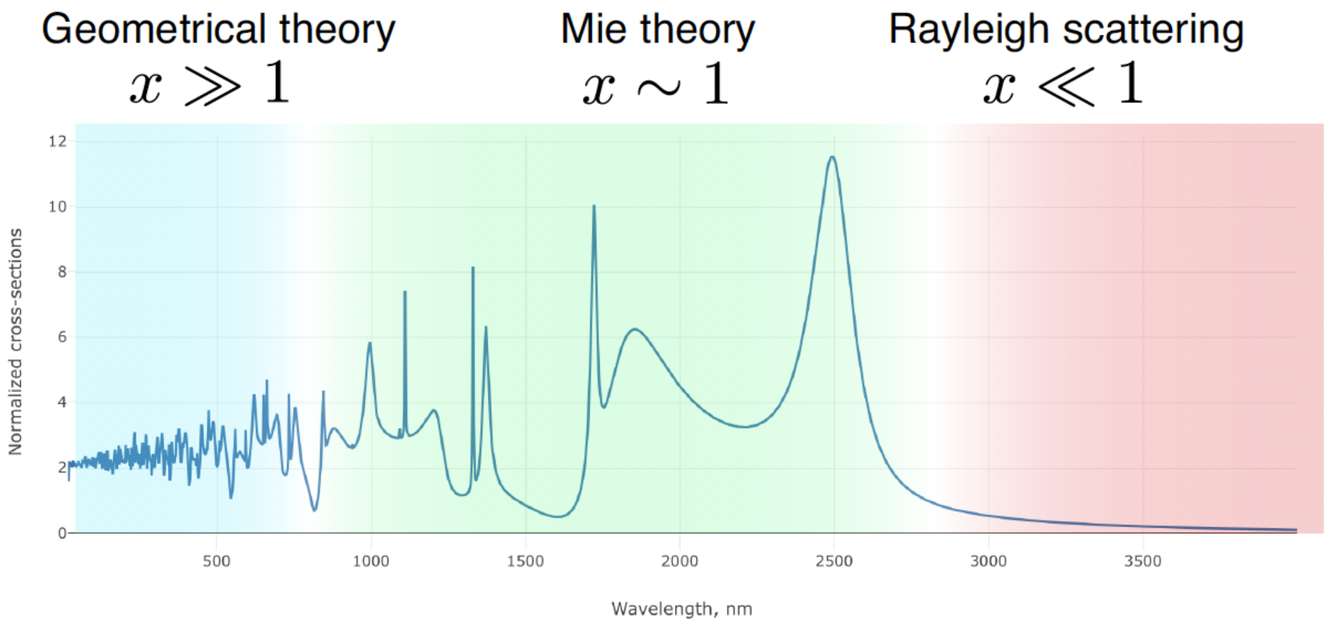
Mie solution (for spherical particles)



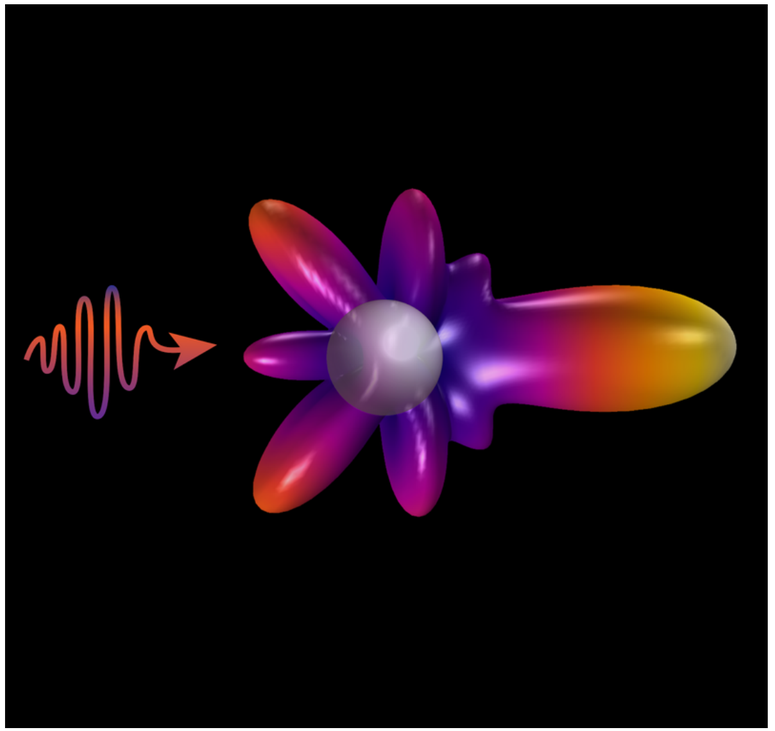
LIGHT SCATTERING

Mie theory - size of a particles is of an order of wavelength of light.

Interference between light inside and outside of the particle

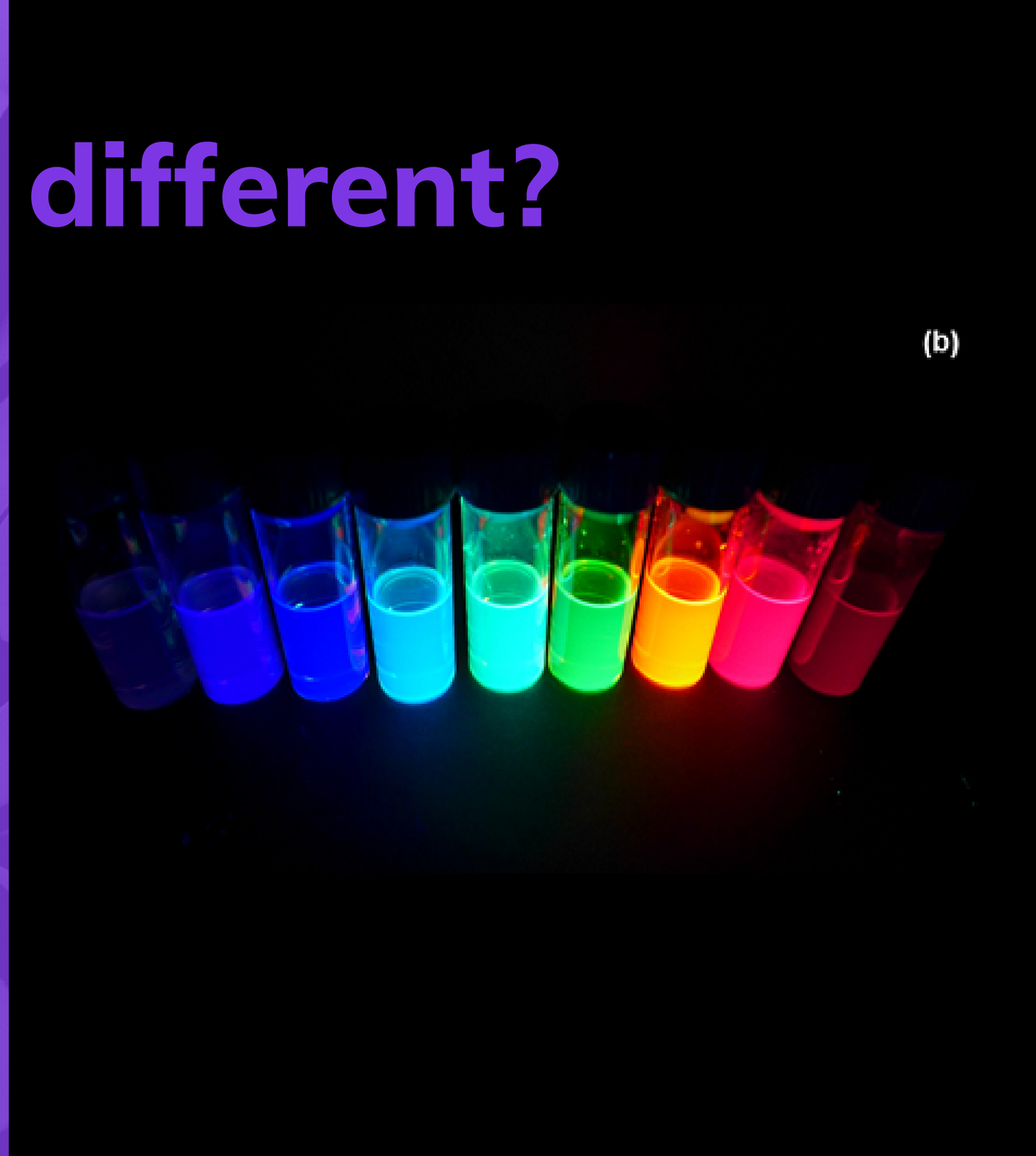


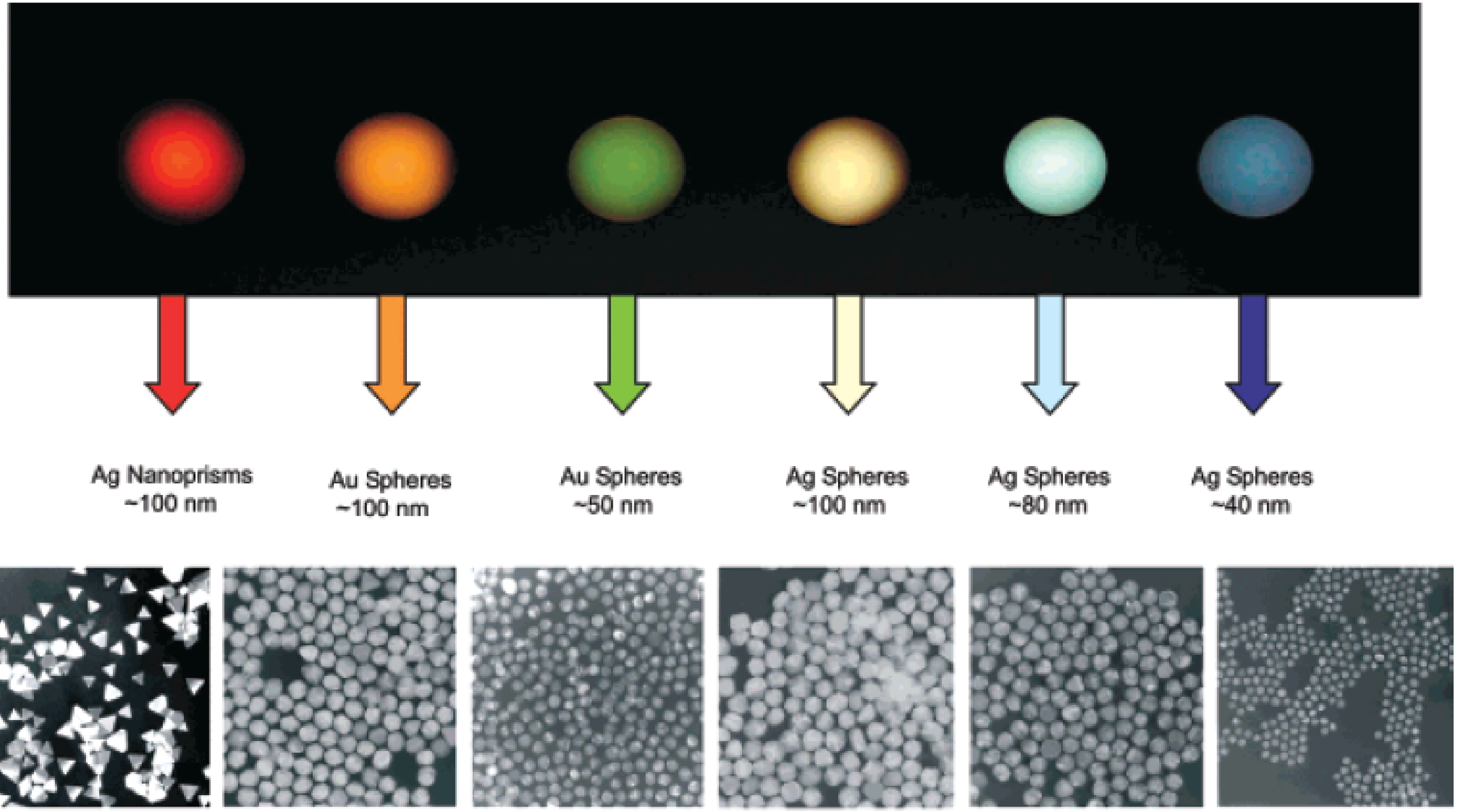
Radius of the sphere $a = 300$ nm; permittivity $\varepsilon = 16$



Why the color is different?

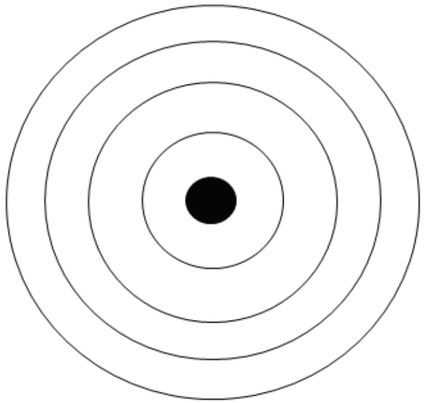
Why the color is different?

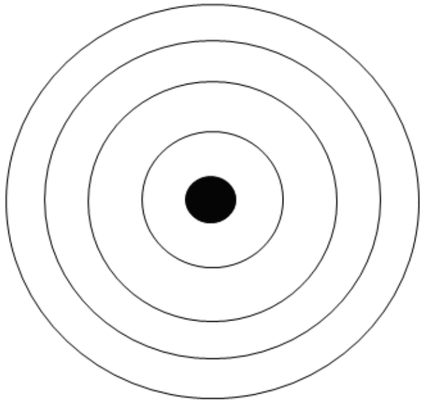


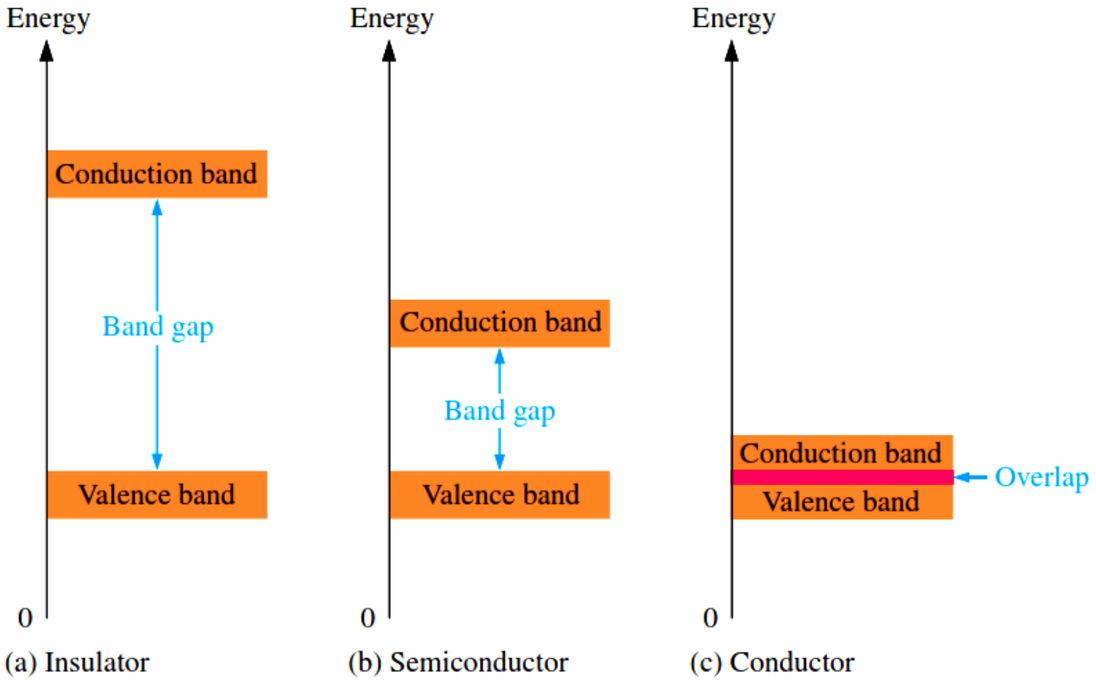

Ag Nanoprisms
~100 nm
Au Spheres
~100 nm
Au Spheres
~50 nm
Ag Spheres
~100 nm
Ag Spheres
~80 nm
Ag Spheres
~40 nm
200nm (same for all the images)

SEMICONDUCTORS

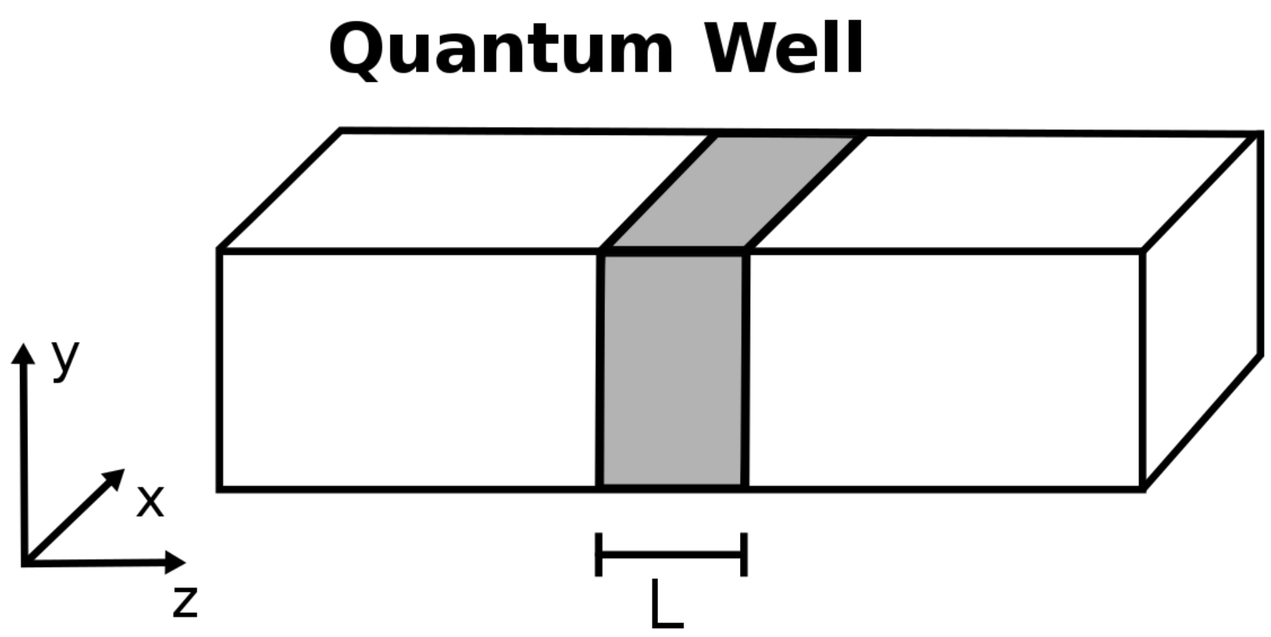
Atom have electrons on the orbit.Between orbits there are no electrons.Interacting atoms create a zone of electrons and zone of empty spaceLet’s look on the last level with electrons and first level where are no electrons.We should call this empty space a band gap

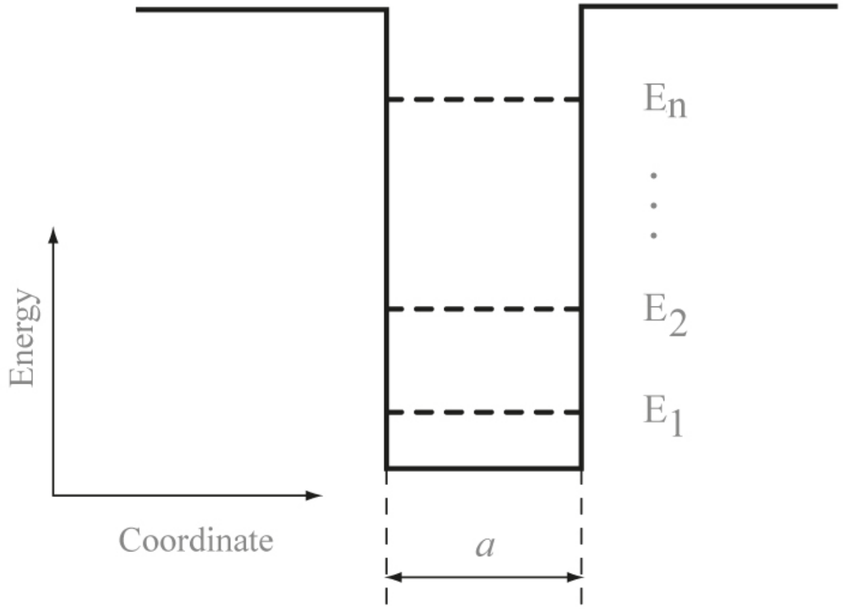


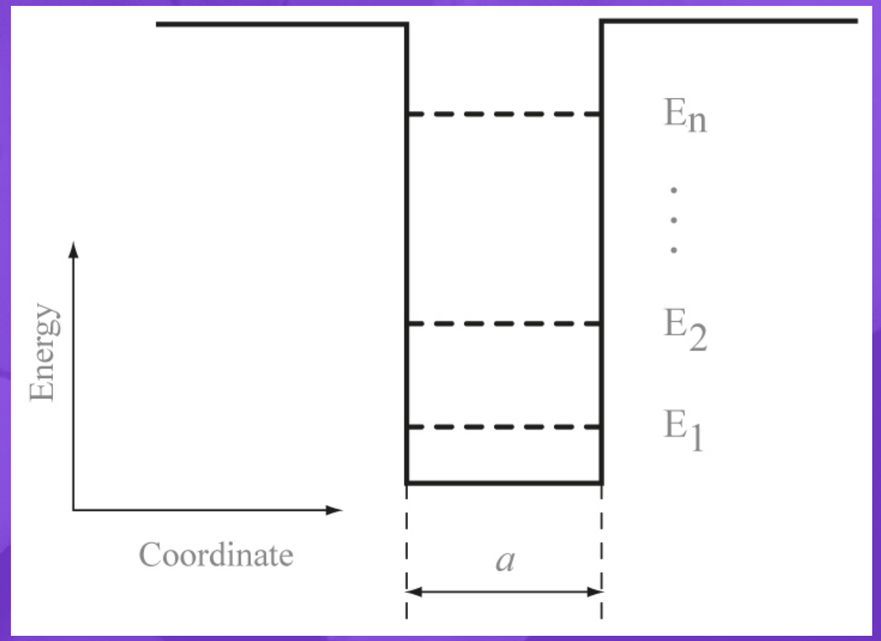




QUANTUM WELL OR QUANTUM DOTS







Schrodinger equation

$$\hat{H}\Psi = E\Psi$$

Time-independent Schrödinger equation (*single nonrelativistic particle*)

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

Looks similar to Helmholtz equation

With Schrodinger equation we can find energy level of electron. It can feel the “borders”.

$$E_n = \frac{\hbar^2 k_n^2}{2m_w^*} = \frac{\hbar^2}{2m_w^*}\left(\frac{n\pi}{d}\right)^2.$$

If we think of it as a wave - it should be clamped inside of that potential, thus, should have smaller wavelength (higher energy)

CONCLUSIONS

no.1 Quantum mechanics says that electromagnetic wave is a particle that can have discrete energies. Problem with ultraviolet catastrophe of blackbody radiation solved

no.2 Elastic scattering plays an important role in everyday life - from blue sky and smoke to different colors in colloidal solution

no.3 Ratio between wavelength of light and the size of a particle is crucial for understanding the interaction

no.4 Quantum mechanics talks on the basis of particles, electromagnetism on the basis of waves.