Sheaves in Geometry and Logic — Solutions

zin3724

Chapter 1

1.1

1.2

1.3

1.4

1.5

1.6

1.7 zin3724

Let G be the Lie group $S^1 := \{\mathbb{R} \mod 2\pi, +\}$. Then each θ induces a map $S^1 \xrightarrow{+\theta} S^1$ as G-spaces, given by

$$+\theta(\omega) = \omega + \theta \mod 2\pi$$

for all $\omega \in S^1$. Here, G acts by left multiplication in both cases. The equalizer of $\{+\theta \mid \theta \in [0, 2\pi)\}$ in the category of G-spaces is $S^1 \xrightarrow{\mathrm{id}_{S^1}}$, but each nonzero $+\theta$ has no fixed points, so in **Sets**, the equalizer is $\emptyset$, which isn't the underlying set of the G-set S^1, so we have a counterexample to the claim that the forgetful functor $U\colon \mathbf{B}G \to \mathbf{Sets}$ preserves limits.

1.8

1.9

1.10

1.11

Chapter 2

2.1

2.2

2.3

2.4

2.5

2.6

2.7

2.8

2.9

2.10