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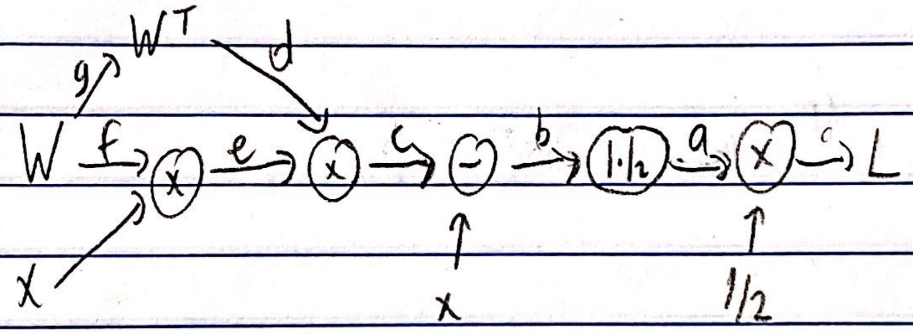
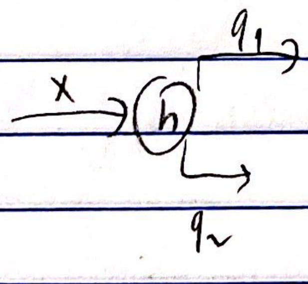
ECE C247 HW3

1) a) We can think of the Wx operation as projecting the x vector into another dimension, as in PCA. Then, W^TWx is simply the reconstruction. To minimize the error, the reconstruction must be as close as possible to the original x. In order to have this, the projection Wx must retain as much information about x as possible. Therefore, minimizing the loss should find a W that ought to preserve information about x.

b)



c) From the law of total derivatives, we know that when the variable we are taking the derivative for is affected by other variables, we should take the derivative with respect to these other variables and sum the results. In other words:



$$\frac{dL}{dx} = \sum_{i=1}^{2} \frac{dL}{dq_i} \cdot \frac{dq_i}{dx}$$

$$= \frac{dL}{dq_1} \cdot \frac{dq_1}{dx} + \frac{dL}{dq_2} \cdot \frac{dq_2}{dx}$$

So, we need to take the derivative with respect to the two paths and sum them up.

d) By looking at the graph:

$$\frac{dL}{da} = 1/2 \qquad \frac{db}{dc} = \frac{d(c-x)}{dc} = 1$$

$$\frac{da}{db} = 2b \qquad \frac{dc}{dd} = \frac{d(W^TWx)}{dW^T} = x^TW^T$$

$$\frac{dc}{de} = \frac{d(W^TWx)}{d(Wx)} = W \qquad \frac{de}{dW} = \frac{d(Wx)}{dW} = x^T$$

$$\frac{dL}{dd} = \frac{dL}{da} \cdot \frac{da}{db} \cdot \frac{db}{dc} \cdot \frac{dc}{dd}$$

$$= 1/2 \cdot 2b \cdot 1 \cdot x^TW = bx^TW^T$$

$$\frac{dL}{de} = \frac{dL}{da} \cdot \frac{da}{db} \cdot \frac{db}{dc} \cdot \frac{dc}{de}$$

$$= 1/2 \cdot 2b \cdot 1 \cdot W$$

$$\frac{dL}{dW} = \frac{dL}{dd} \cdot \frac{dd}{dW} + \frac{dL}{de} \cdot \frac{de}{dW}$$

$$= (bx^TW^T)^T + Wbx^T$$

$$= Wxb^T + Wbx^T$$

$$\Downarrow \; b = (W^TWx - x)$$

$$\frac{dL}{dW} = Wx(W^TW - x)^T + W(W^TWx - x)x^T$$

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2)

a)

$$\begin{tikzcd}
 & & & \alpha \arrow[d] & & & & & \\
X \arrow[rr, "d"] \arrow[d] & & (\times) \arrow[r, "c"] & (\times) \arrow[r, "b"] & (+) \arrow[r, "K"] & (\log|\cdot|) \arrow[r, "a"] & (\times) \arrow[r] & L_1 \\
X^T \arrow[urr, "e"] & & & & \beta^{-1}I \arrow[u] & & -\frac{D}{2} \arrow[u] &
\end{tikzcd}$$

b) $\frac{dL_1}{da} = \frac{-D}{2}$ $\quad \frac{da}{dK} = \frac{\log|K|}{dK} = K^{-T}$ $\quad \frac{dK}{db} = 1$

$\frac{db}{dc} = \alpha$ $\quad \frac{dc}{dd} = X$ $\quad \frac{dc}{de} = X^T$

$$\frac{dL_1}{dc} = \frac{dL_1}{da} \cdot \frac{da}{dK} \cdot \frac{dK}{db} \cdot \frac{db}{dc}$$

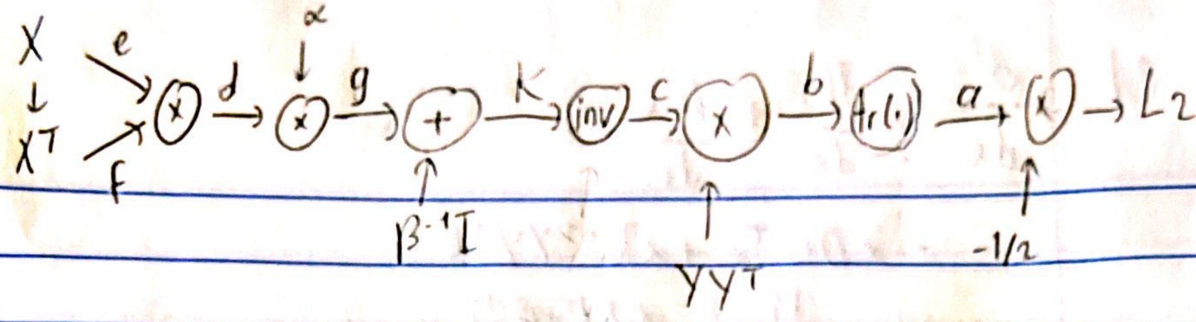
$$= \frac{-D}{2} \cdot K^{-T} \cdot 1 \cdot \alpha$$

$$\frac{dL_1}{dx} = \frac{dL_1}{dc} \cdot \frac{dc}{dx} + \frac{dL_1}{dc} \cdot \frac{dc}{de} \cdot \frac{de}{dx}$$

$$= \frac{-D}{2} K^{-T} 1 \cdot \alpha \, (X + X)$$

$$= -\alpha D K^{-T} X$$

c)



d) $\dfrac{dL_2}{dK} = -K^{-T} \dfrac{dL_2}{dK^{-1}} K^{-T}$

$\dfrac{dL_2}{dK^{-1}} = \dfrac{dL_2}{da} \cdot \dfrac{da}{dK^{-1}}$

$\dfrac{dL_2}{da} = -1/2 \qquad \dfrac{da}{dK^{-1}} = \dfrac{d(tr(K^{-1}YY^T))}{dK^{-1}} = YY^T$

$\dfrac{dL_2}{dK^{-1}} = \dfrac{-1}{2} YY^T$

So, $\dfrac{dL_2}{dK} = -K^{-T} \dfrac{-1}{2} YY^T K^{-T}$

$\dfrac{dK}{dg} = 1, \ \dfrac{dg}{dd} = \alpha, \ \dfrac{dd}{de} = X \quad \dfrac{dd}{df} = X^T$

$\dfrac{dL_2}{dX} = \dfrac{dL_2}{dK} \cdot \dfrac{dK}{dg} \cdot \dfrac{dg}{dd} \left(\dfrac{dd}{de} \cdot \dfrac{de}{dX} + \dfrac{dd}{df} \cdot \dfrac{df}{dX} \right)$

$= -K^{-T} \dfrac{-1}{2} YY^T K^{-T} \cdot \alpha \cdot 2X$

$= \alpha K^{-T} YY^T K^{-T} X$

e) $\frac{dL}{dx} = \frac{dL_1}{dx} + \frac{dL_2}{dx}$

$= -\alpha D K^{-T} x + \alpha K^{-T} Y Y^T K^{-T} x$

$\Downarrow \quad K = \alpha X X^T + \beta^{-1} I$

$= -\alpha D (\alpha X X^T + \beta^{-1} I)^{-T} x + \alpha (\alpha X X^T + \beta^{-1} I)^{-T} Y Y^T (\alpha X X^T + \beta^{-1} I)^{-T} x$