Nuclear Reactions and Energy Generation

Energy Generation

- We discussed energy sources to a shell of stellar material:

$$\frac{\partial L}{\partial M} = \epsilon_{\rm nuc} + \epsilon_\nu + \epsilon_{\rm grav}$$

  - The gravitational term represents local expansion or contraction
  - For the nuclear term, we focused (so far) on non-resonant reactions
  - Neutrinos are generally an energy sink (except for neutron stars)

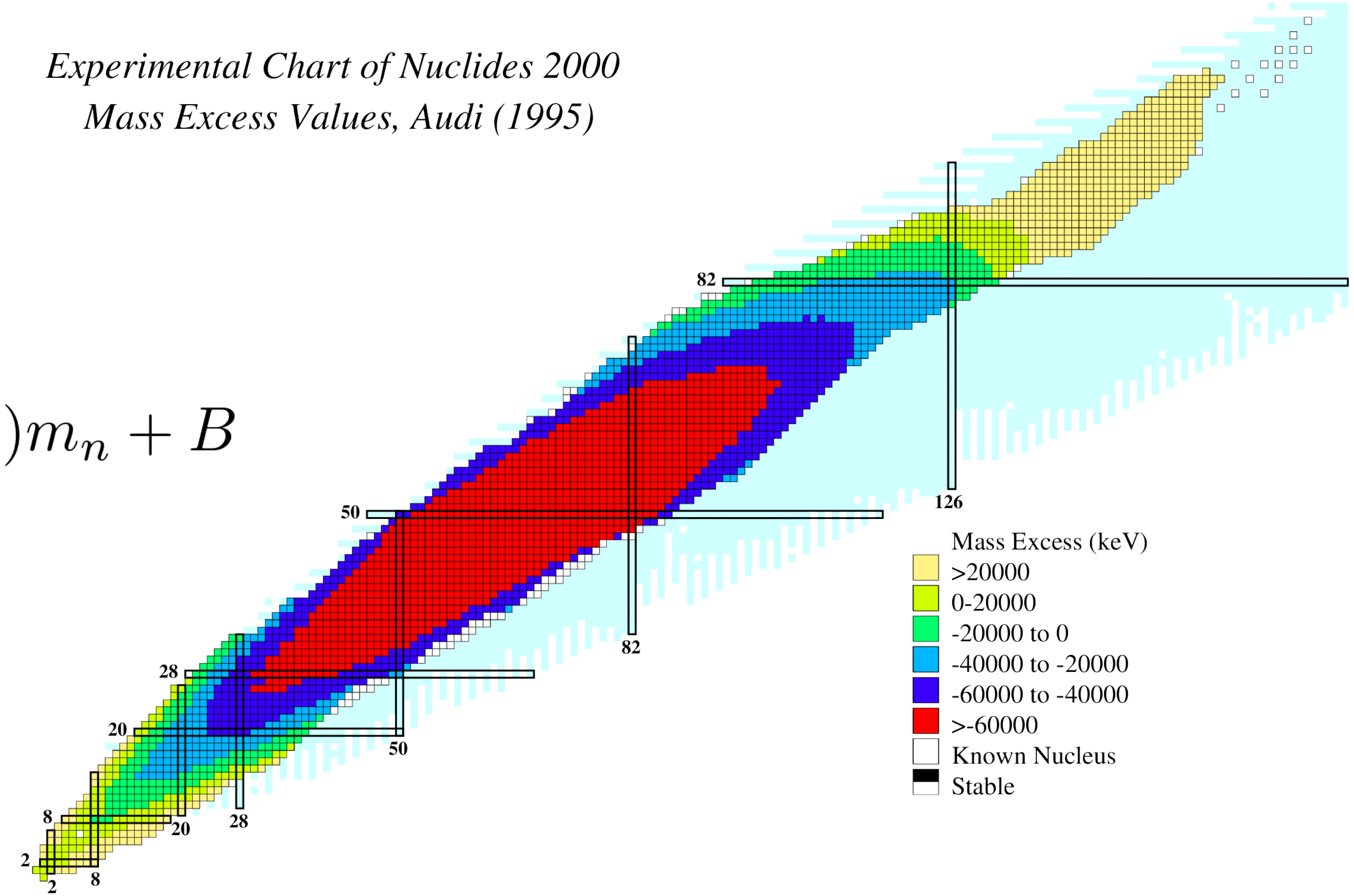
(Hansen et al.)

Mass Excesses

$$\Delta m_{\mathrm{AZ}} = \left(\frac{m_{\mathrm{AZ}}}{m_u} - A \right) m_u c^2$$

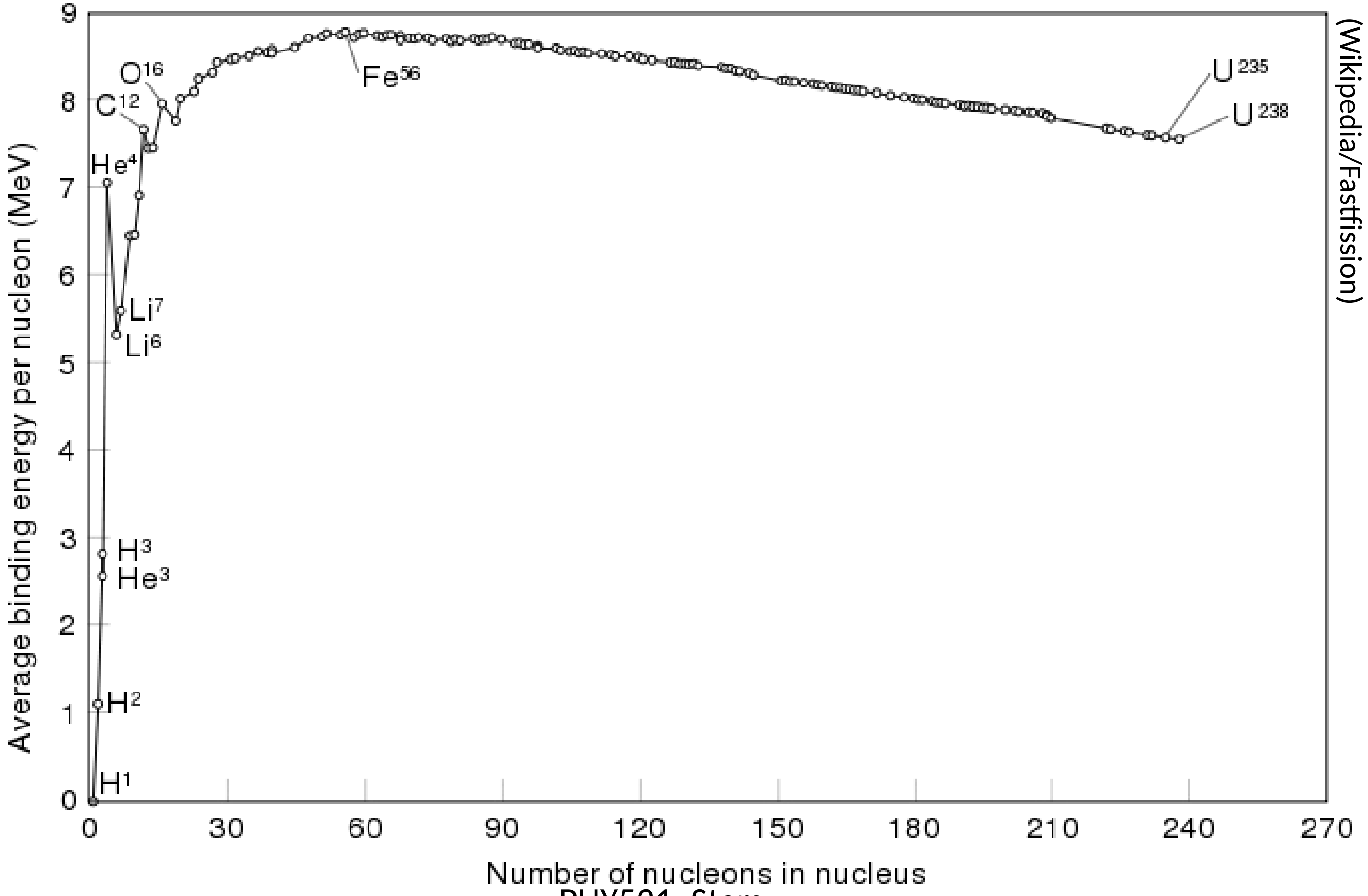
$$M = Am_u + \Delta M = Zm_p + (A - Z)m_n + B$$
$$B = \Delta M - Z\Delta M_p - (A - Z)\Delta M_n$$

Experimental Chart of Nuclides 2000
Mass Excess Values, Audi (1995)



Binding Energies

- The binding energies are related to the mass excesses
- Models for the binding energy exist: e.g. liquid drop model



(Wikipedia/Fastfission)

Non-Resonant Rates

- We argued that the rate (# of reactions/time/volume) is

$$r = n_X n_a \sigma v$$

- In the CM frame, the relative velocity obeys a Maxwell-Boltzmann distribution:

$$r = \frac{n_X n_a}{1+\delta_{Xa}} \int_0^\infty v\sigma(v)\phi(v)dv$$
$$\equiv \frac{n_X n_a}{1+\delta_{Xa}} \langle \sigma v \rangle$$

- The cross section will have strong dependence due to tunneling and the de Broglie cross section, which we factored out, giving:

$$\sigma(E) = \frac{S(E)}{E} e^{-bE^{-1/2}}$$

- This gives:

$$\langle \sigma v \rangle = \left(\frac{8}{\mu\pi}\right)^{1/2} \int_0^\infty S(E) e^{-E/kT - bE^{-1/2}} dE$$

Cross Section

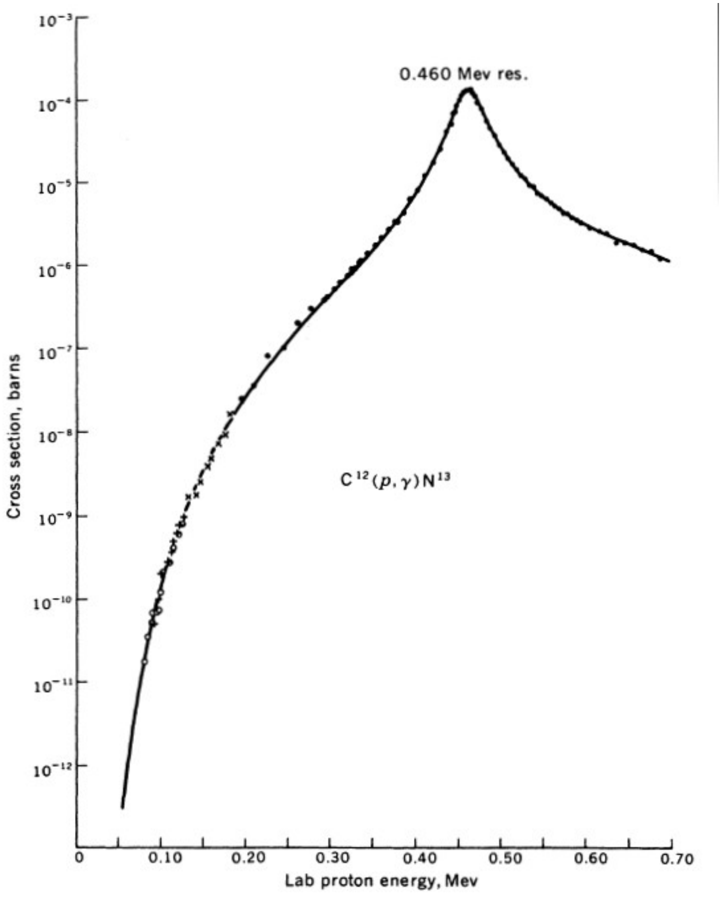


Fig. 4-4 The measured cross section for the reaction $C^{12}(p,\gamma)N^{13}$ as a function of laboratory proton energy. A four-parameter theoretical curve has been fitted to the experimental points. An extrapolation to $E_p = 0.025$ Mev, which is an interesting energy for this reaction in astrophysics, appears treacherous. (*Courtesy of W. A. Fowler and J. L. Vogl.*)

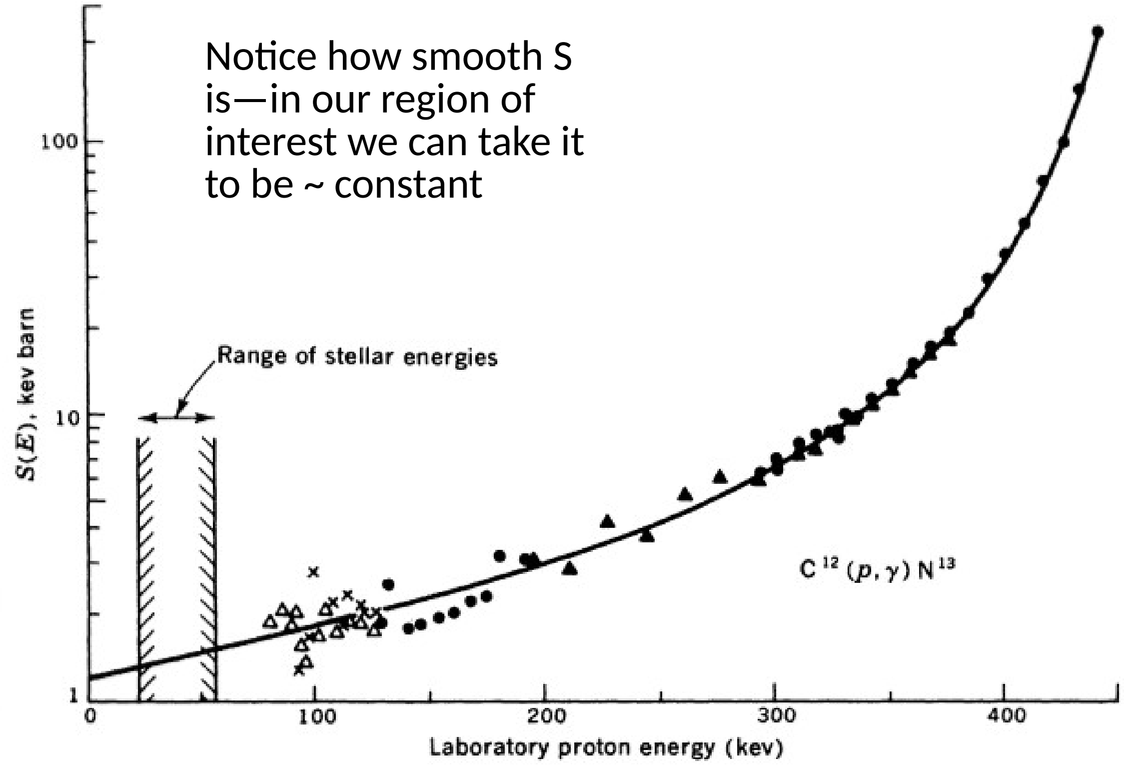


Fig. 4-5 The cross-section factor $S(E)$ for the radiative capture of protons by C^{12}. The differing types of data points represent five different experiments performed at different times and laboratories by the workers indicated. Detailed references and discussion may be found in D. F. Hebbard and J. L. Vogl, *Nucl. Phys.*, **21**:652 (1960). This curve is more readily extrapolated than the one in Fig. 4-4.

(Clayton)

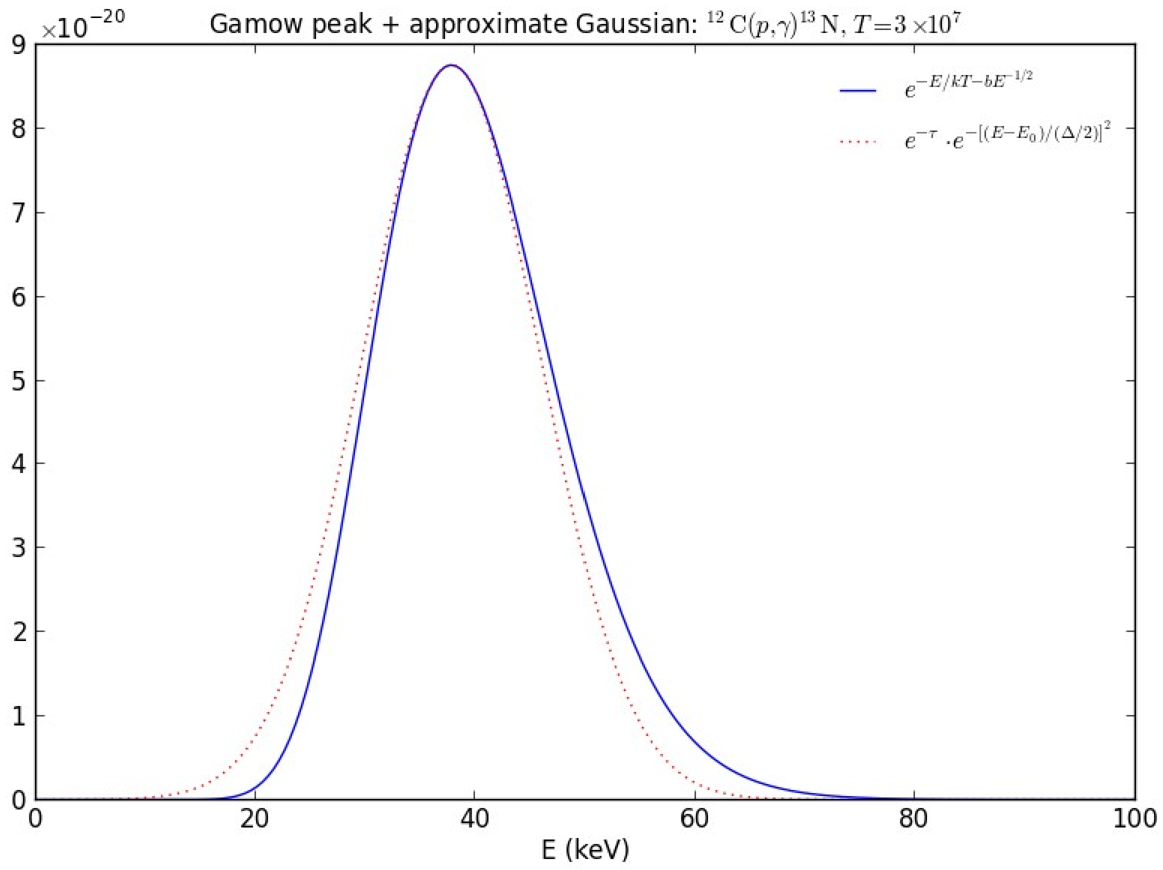
Gamov Peak

- Note how well the Gaussian approximation fits the Gamow peak

$$E_0 = \left(\frac{bkT}{2}\right)^{2/3}$$

$$\Delta = \frac{4}{\sqrt{3}}(E_0 kT)^{1/2}$$

$$\tau = \frac{3E_0}{kT}$$



Non-Resonant Rates

- We can assume that S is constant
- Integrate the Gaussian (extending lower limit to -∞)

$$r \sim \frac{n_a n_X}{1+\delta_{aX}} S_0 \tau^2 e^{-\tau}$$
$$\sim \frac{n_a n_X}{1+\delta_{aX}} \frac{S_0 e^{-aT^{-1/3}}}{T^{2/3}}$$

- Energy generation rate:

$$\epsilon = \frac{Qr}{\rho}$$

- Finally, we estimated the temperature sensitivity in the form:

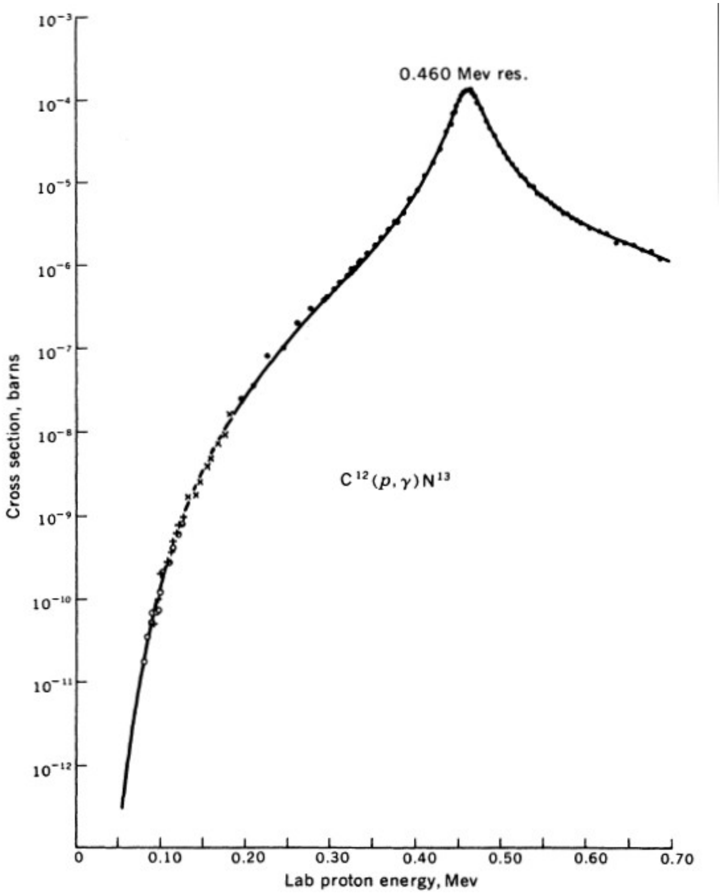
$$r = r_0 \rho T^{\nu} \; ; \quad \nu = \frac{\partial \ln r}{\partial \ln T}$$

Resonant Rates

- At a resonance, the cross section is strongly peaked
  - Energy corresponds to an energy level in the compound nucleus resulting from X + a
  - Dominates the cross section

$$\sigma(E) \sim \frac{\Gamma^2}{(E - E_r)^2 + (\Gamma/2)^2}$$

  - Integrate assuming the M-B term doesn't change much in this range
    - Different T dependence results in the rate



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(Clayton)

$^{12}C(p,\gamma)^{13}N$

- Q = 1.944 MeV (just difference the mass excesses)
- Two parts to the rate
  - Low temperature (note the dominant terms):

$$\langle\sigma v\rangle_{p,\gamma} = 3.39\times 10^{-17}(1+0.0304T_9^{1/3}+1.19T_9^{2/3}+0.254T_9 + 2.06T_9^{4/3}+1.12T_9^{5/3})\times T_9^{-2/3} \times \exp\left[-13.69/T_9^{1/3}-(T_9/1.5)^2\right]$$

  - Resonance: if $E_0 + \Delta/2 \sim E_r$ ($T_9 \sim 0.6$ for this reaction)

$$\langle\sigma v\rangle_{p,\gamma} = \frac{1.8\times 10^{-19}}{T_9^{3/2}} e^{-4.925/T_9}$$

  - Total rate is the sum for $0.25 < T_9 < 7$

Weak Rates

- That reaction leaves us with ^{13}N
- We could do $^{13}N(p,\gamma)^{14}O$ (Q = 4.628 MeV) next
  - But ^{13}N is unstable and will beta decay
  - We need to consider timescales
- Consumption via $^{13}N(p,\gamma)^{14}O$ changes ^{13}N number density as:

$$\frac{dn(^{13}\mathrm{N})}{dt} = -r = -n_p n(^{13}\mathrm{N})\langle\sigma v\rangle_{p,\gamma}$$

  - Destruction timescale is

$$\tau \sim \frac{n(^{13}\mathrm{N})}{r} = \frac{1}{n_p\langle\sigma v\rangle_{p,\gamma}}$$

  - From your text, for $T_6 = 20$, $X \sim 1$, and $\rho \sim 10$, we have $\tau \sim 10^7$ yr.

Weak Rates

- But what about beta decay?
  - Consider:

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^{+} + \nu_e$$

    - half-life of 10 min
    - We can ignore the ^{13}N(p,γ)^{14}O reaction in this case
- Evolution of ^{13}N consists of creation and destruction (via beta decay) as:

$$\frac{dn(^{13}\mathrm{N})}{dt} = + n_p n(^{12}\mathrm{C}) \langle \sigma v \rangle_{^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}} - \lambda_{^{13}\mathrm{N}} n(^{13}\mathrm{N})$$

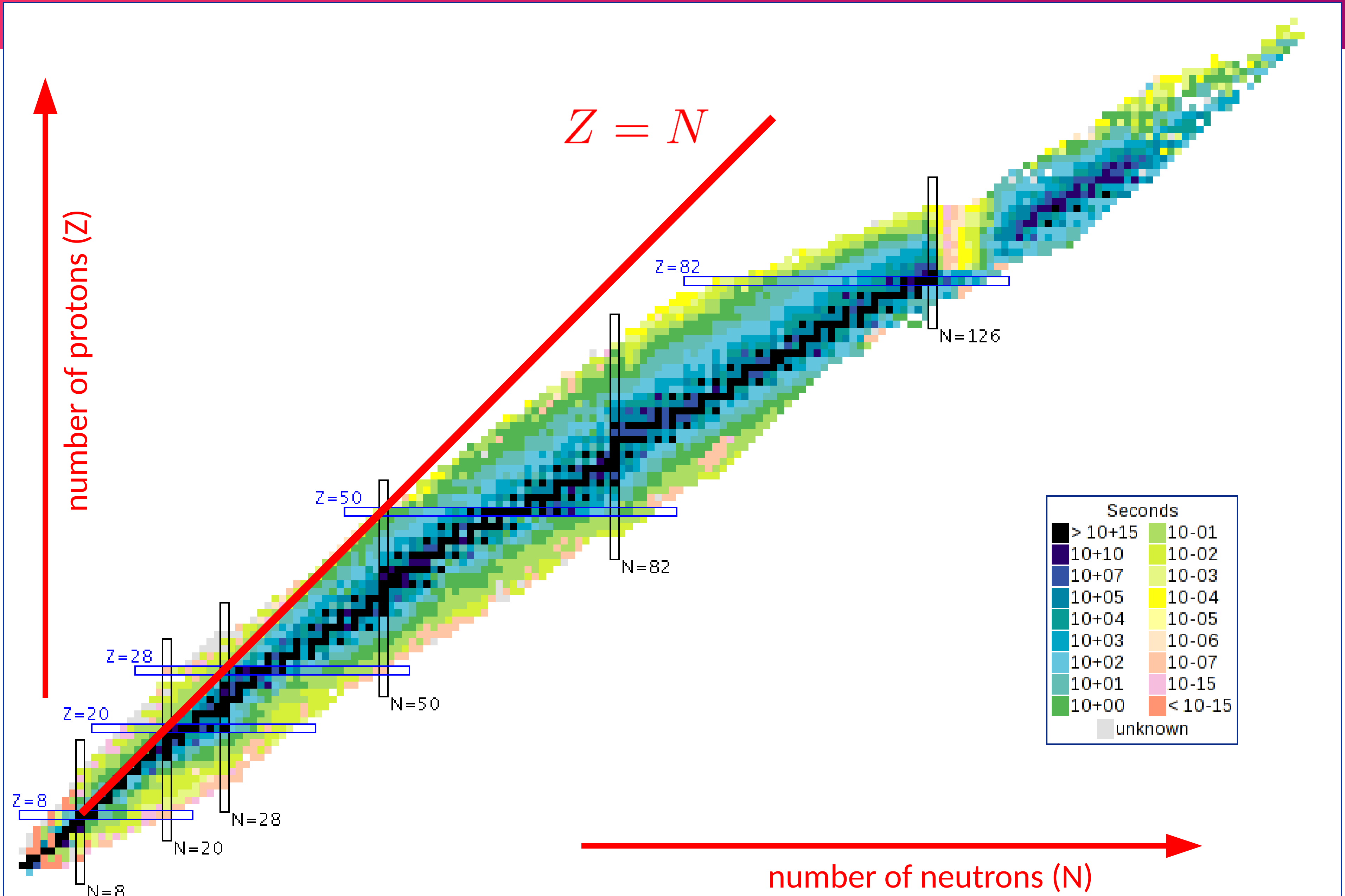
  - Here,

$$\lambda_{^{13}\mathrm{N}} = \ln(2)/\tau_{1/2}$$

  - Note: there will be corresponding evolution equations for p, ^{12}C and ^{13}C.
  - Even through ^{13}N is unstable, an equilibrium amount will be present, given by:

$$dn(^{13}\mathrm{N})/dt = 0$$

Chart of Nuclides



(Taken from National Nuclear Data Center/BNL)

Weak Rates

- There are also electron captures
- Consider pp chain:

$$e^- + {}^7\mathrm{Be} \rightarrow {}^7\mathrm{Li} + \nu_e$$

  - Rate requires detailed modeling and understanding of the weak interaction
  - Your text lists the following effective rate:

$$\langle \sigma v \rangle_{e^-,\nu_e} = \frac{2.23 \times 10^{-34}}{T_9^{1/2}}$$

  - Q value would be 0.862 MeV but neutrinos carried almost all of this out of the star
- Electron captures also important in core-collapse supernovae

Integrating Reaction Networks

- See the Jupyter notes here: https://zingale.github.io/comp_astro_tutorial/reaction_networks/reactions.html

Reaction Rate Libraries

- JINA ReacLib

  (Cyburt et al. 2010)

- >4500 nuclei w/ > 75000

2. THE JINA REACLIB DATABASE

The JINA REACLIB database is completely public and accessible to the community via the World Wide Web. The interface[11] is PHP-driven[13] and connected to a MySQL database[14]. The current version of the database stores reaction rates as a function of temperature in the seven-parameter rate parameterization of Thielemann et al. (1987) and F.-K. Thielemann (1995, private communication):

$$\lambda = \exp\left[a_0 + \sum_{i=1}^{5} a_i T_9^{\frac{2i-5}{3}} + a_6 \ln T_9\right]. \tag{1}$$

These rates go into a set of stiff coupled differential equations, and are then evolved to solve the abundance changes of the nuclides in the network. For a single reaction channel ($A + B \to C + D$), the equations take the form

$$\begin{aligned} -\frac{1}{\mathcal{N}_A}\partial_t Y_A &= -\frac{1}{\mathcal{N}_B}\partial_t Y_B = \frac{1}{\mathcal{N}_C}\partial_t Y_C \\ &= \frac{1}{\mathcal{N}_D}\partial_t Y_D = \frac{Y_A^{\mathcal{N}_A}}{\mathcal{N}_A!}\frac{Y_B^{\mathcal{N}_B}}{\mathcal{N}_B!}\rho_{\text{baryon}}^{\nu}\lambda, \end{aligned} \tag{2}$$

where Y_i are the molar abundances per gram and $\mathcal{N}_i$ is the number of nuclides of type i produced or destroyed in the reaction and $\nu = \mathcal{N}_A + \mathcal{N}_B - 1$. By definition, the reaction rate or "rate of reaction" is given by the entire right-hand side of Equation (2), but the term reaction rate is used synonymously for the "reduced" reaction rate or reactivity, λ, throughout this paper and in the REACLIB database. For a network of reactions, each $\partial_t Y_i|_{A+B\to C+D}$ is summed over all participating reactions, including their reverse rates. For unary rates, $\lambda = 1/\tau$ has units of s^{-1}, inversely proportional to the mean lifetime. For binary rates, $\lambda = N_A\langle\sigma v\rangle$ has units of $cm^3\ s^{-1}\ mol^{-1}$, while for trinary rates, λ has units of $cm^6\ s^{-1}\ mol^{-2}$. Multiple sets of parameters can be added to fit more complex temperature dependencies.

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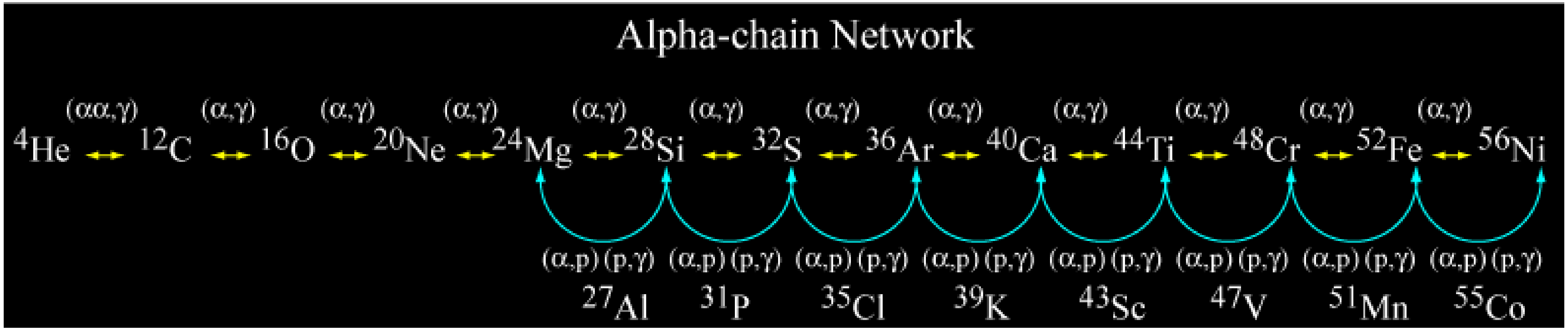
13 http://www.php.net

14 http://www.mysql.com

Approximate Networks

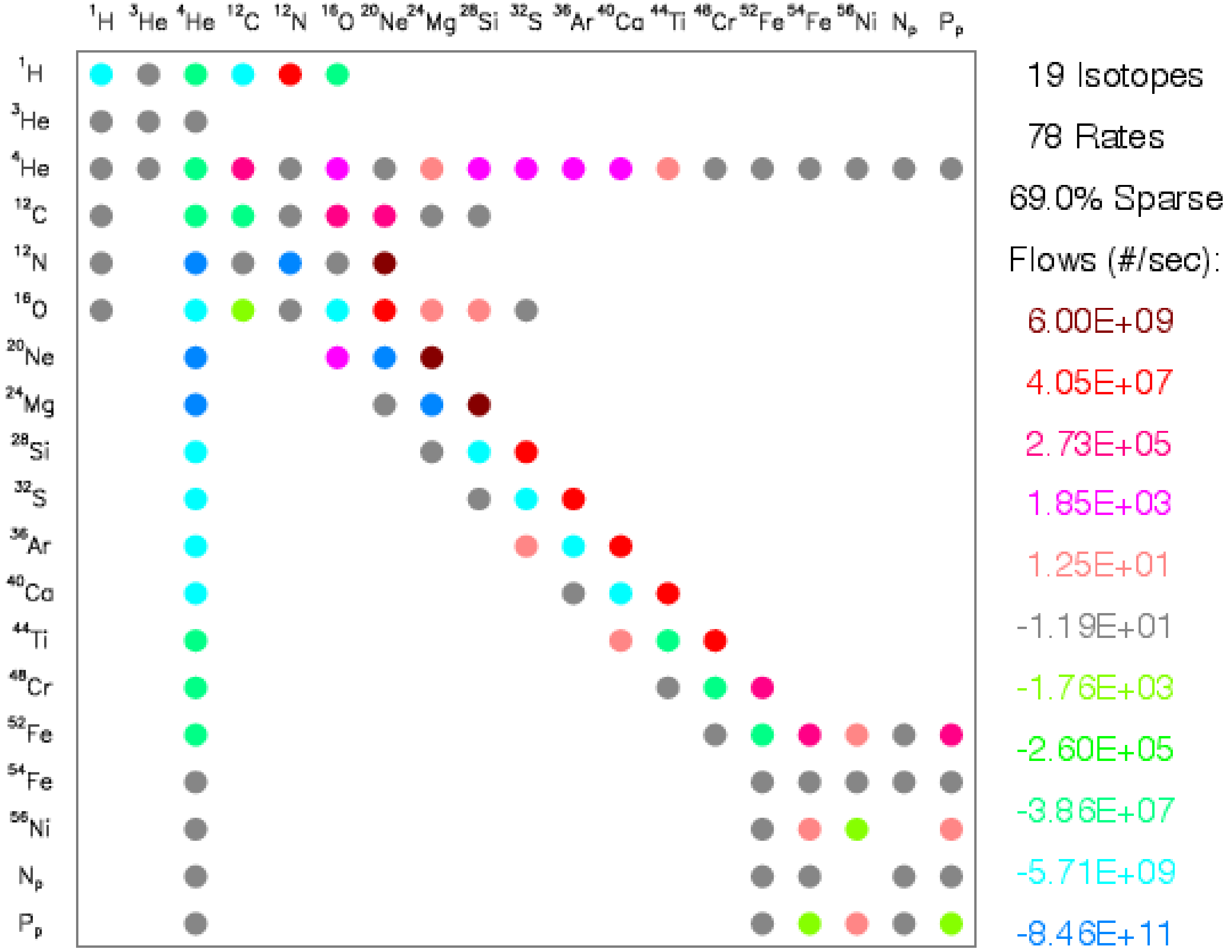
- Carrying lots of nuclei is computationally expensive
- For helium-rich environments, alpha-chains provide the bulk of the energetics, but…
  - (α,p)(p,γ) may be faster than (α,γ)
  - Intermediate nuclei doesn't last long, so we can just carry the end result using the rates for the sequence (α,p)(p,γ)



(Frank Timmes)

Approximate Networks

- Jacobian for a 19-isotope network (handles pp, CNO, and alpha)



(Frank Timmes)

Screening

- Electron screening: cloud of electrons shield the nuclear charges—easier to overcome the Coulomb potential
- Two important scales:
  - Wigner-Seitz radius (inter-ion spacing):

$$\frac{4\pi}{3}a^3 = \frac{1}{n_I}$$

  - Debye radius (measure of the distance over which electrostatic effects act):

$$\lambda_D^{-1} = \kappa_D = \left[\frac{4\pi e^2}{kT}(Z^2 n_I + n_e)\right]^{1/2}$$

- Weak screening: $\lambda_D \gg a$, or equivalently:

$$\frac{Z^2 e^2}{a} \ll kT$$

Screening

- Strong screening is harder to deal with
- Ex: carbon burning in pre-SNe Ia WDs (see Woosley et al. 2004)

2.1. *Nuclear Energy Generation Rate and Timescale*

Nuclear energy generation during carbon ignition is given entirely by the (highly screened) fusion of two ^{12}C nuclei to form, chiefly, ^{20}Ne and ^{24}Mg. The approximate energy generation rate (assuming carbon burns to a mixture of 3 parts ^{20}Ne and 1 part ^{24}Mg; Woosley 1986) is

$$\dot{S}_{\rm nuc} \approx 6.7 \times 10^{25} X^2(^{12}{\rm C})\rho_9 F_{\rm sc}\lambda_{12,12} \text{ ergs g}^{-1}\text{ s}^{-1}, \quad (1)$$

where $\lambda_{12,12}$ is the carbon fusion reaction rate (Caughlan & Fowler 1988), $F_{\rm sc}$ is the electron screening function, and $X(^{12}{\rm C})$ is the mass fraction of carbon. For a range of temperatures $T_8 = 6-8$,

$$\lambda_{12,12} \approx 7.6 \times 10^{-16}\left(\frac{T_8}{7}\right)^{30}. \quad (2)$$

The electron screening function (Alastuey & Jancovici 1978) is given by ($\rho_9 = 1-3$, $T_8 = 6-8$)

$$F_{\rm sc} \approx 1100\left(\frac{\rho_9}{2}\right)^{2.3}\left(\frac{T_8}{7}\right)^{-7}, \quad (3)$$

so the energy generation rate for a composition of 50% carbon, 50% oxygen is

$$\dot{S}_{\rm nuc} \approx 2.8 \times 10^{13}\left(\frac{T_8}{7}\right)^{23}\left(\frac{\rho_9}{2}\right)^{3.3} \text{ ergs g}^{-1}\text{ s}^{-1}. \quad (4)$$

The specific energy available is

$$q_{\rm nuc} = 4.0 \times 10^{17} X(^{12}{\rm C}) \text{ ergs g}^{-1}. \quad (5)$$

Neutrino Emission

- Neutrinos can be created by means other than weak reactions
  - Except for NS, these are an energy loss
- Pair Annihilation: $e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e$
  - High temperatures ($kT \sim m_e c^2$) generate $\gamma + \gamma \Longleftrightarrow e^- + e^+$
- Photoneutrinos and Bremsstrahlung neutrinos
  - Electron-photon scattering that gives neutrino/antineutrino pair instead of photon
- Plasma neutrinos
  - Electromagnetic waves become “plasmons” that can decay into neutrino/antineutrino pairs
- Many references exist which give rates for these different mechanisms