# 50.051 Programming Language Concepts

## W10-S2 Context Free Grammars (CFG)

Matthieu De Mari

# Let us start with a problem

Consider the mathematical expressions below. Which ones are **valid**?

A. 2+7+9
B. (3+4)+7
C. ((6+9)+8)*2
D. 6+2++7
E. 8+3+
F. ((4+3)+8
G. (7+2))(+6

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# Let us start with a problem

**Question:** Let us consider mathematical expressions, consisting of

- single digit numbers,
- + operations,
- along with opening and closing parentheses.

**Can you write a RegEx to check whether such a given mathematical expression is valid or not?**

Some examples of **valid** and **invalid** expressions (same expressions as before):

- **2+7+9**
- **(3+4)+7**
- **((6+9)+8)+2**
- **6+2++7**
- **8+3+**
- **((4+3)+8**
- **(7+2))(+6**

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**Can you write a RegEx to check whether such a given mathematical expression is valid or not?**

No, unfortunately, RegEx is not powerful enough to do so.

Checking that requires to keep track of:

- how many parentheses have been opened after having read n characters of the given input string,
- how many have been closed after having read n characters of the given input string,
- and in which order opened parentheses have been closed.

# Let us start with a problem

**Can you write a RegEx to check whether such a given mathematical expression is valid or not?**

No, unfortunately, RegEx is not powerful enough to do so.

It is not possible to do so with a **finite state machine**.

After all, we could technically have **any number of parentheses in the expression, not just any finite number of them**. So how could we keep track of this with a RegEx that runs on **finite** state machines?

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**→Important lesson: Syntax analysis tasks, e.g. checking parentheses, will typically require something more powerful than RegEx!**

**(But what then?)**

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**Let us do some chemistry!**

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**Let us do some chemistry!**

(Nani the hell is going on here?!)

# A chemical problem

Consider the chemistry expressions below (no ions).
Which ones are **valid**?

A. $\mathrm{CO}$

*B.* $\mathrm{H_2O}$

*C.* $\mathrm{H^2O}$

*D.* $\mathrm{C_{12}H_{22}O_{11}}$

*E.* $\mathrm{C_2\ J_8}$

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A. **$CO$ (carbon monoxyde)**

*B.* **$H_2O$ (water)**

*C.* **$H^2O$ (exponents are not allowed, except for ions)**

*D.* **$C_{12}H_{22}O_{11}$ (sugar)**

*E.* **$C_2\ J_8$ (no J element in periodic table)**

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**Question:** Is there a RegEx for checking if these formulas are valid?

No, because parentheses can be used in chemistry formulas for compounds, e.g. $Ca_3(PO_4)_2 = 3\text{Ca} + 4\text{P} + 8\text{O}$

# Introducing Context Free Grammars

**Definition (Context-Free Grammars):**

A **Context-Free Grammar (CFG)** is **a formal system used to generate and describe sets of strings based on a specific set of syntax rules**.

It is particularly useful for defining the syntax of programming languages and the structure of natural languages.

The term "context-free" means that the **production rules** of the CFG are applied independently of the surrounding context.

A context free grammar is defined by **four elements**: a set of **terminals** and **non-terminals**, a **start symbol** and a set of **production rules**.

# Elements of Context Free Grammars

**Definition (Terminals and Non-Terminals):**

**Terminals** are the **basic symbols in a language that cannot be further divided**.

In the case of our chemistry formulas, these would be elements symbols such as C, O, H, He, etc.

**Non-terminals**, on the other hand, **represent syntactic patterns or intermediate structures in the language.**

These can be **further decomposed into sequences of terminals and non-terminals**.

# Elements of Context Free Grammars

**Definition (Start Symbol):**

The **Start Symbol** is a special **non-terminal** symbol **from which the derivation of strings begins**. It represents the main structure of the language, and all other rules ultimately derive from it.

**Definition (Production Rules):**

**Production Rules** define how **non-terminal** symbols can be replaced by sequences of **terminals** and **non-terminals**.

Written in the form $\boldsymbol{A} \rightarrow \boldsymbol{B}$, where $\boldsymbol{A}$ is a **non-terminal**, and $\boldsymbol{B}$ is a sequence of **terminals** and/or **non-terminals** that can replace $\boldsymbol{A}$.

# Back to our Chemistry

Consider the CFG below.

$$Formula \rightarrow Molecule$$

$$Molecule \rightarrow Element$$
$$Molecule \rightarrow Element_{Count}$$
$$Molecule \rightarrow MoleculeElement$$
$$Molecule \rightarrow MoleculeElement_{Count}$$

$$Element \rightarrow C\ or\ O\ or\ H\ or\ He\ or\ \ldots$$
$$Count\ \rightarrow 1\ or\ 2\ or\ 3\ or\ \ldots$$

# Back to our Chemistry

The symbol *Formula* is a **Non-Terminal**, which serves as a **Start Symbol**.

Consider the CFG below.

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Any symbol written as "*Word*" is a **non-terminal**.

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Consider the CFG below.

All the **elements of the periodic table** and the **non-zero integer numbers** can be used as **terminals**.

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$$Molecule \rightarrow Element$$
$$Molecule \rightarrow Element_{Count}$$
$$Molecule \rightarrow MoleculeElement$$
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$$Element \rightarrow C \ or \ O \ or \ H \ or \ He \ or \ \ldots$$
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# Back to our Chemistry

Consider the CFG below.

We have defined 7 **production rules**.
Several production rules may appear and start with the same non-terminal.

$$Formula \rightarrow Molecule$$

$$Molecule \rightarrow Element$$
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$$Molecule \rightarrow MoleculeElement$$
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$$Element \rightarrow C \text{ or } O \text{ or } H \text{ or } He \text{ or } \ldots$$
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# Back to our Chemistry

We may also use the keyword “or” for convenience

Consider the CFG below.

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# Back to our Chemistry

Consider the CFG from earlier

$$Form \rightarrow Mol$$
$$Mol \rightarrow Elem$$
$$Mol \rightarrow Elem_{Count}$$
$$Mol \rightarrow MolElem$$
$$Mol \rightarrow MolElem_{Count}$$
$$Elem \rightarrow C\ or\ O\ or\ H\ or\ He\ or\ \ldots$$
$$Count\ \rightarrow 1\ or\ 2\ or\ 3\ or\ \ldots$$

Is the formula CO valid? Yes.

Because, it can be **derived** from the CFG production rules, starting from *Formula*.

$Form \rightarrow Mol$ (rule 1)

$Mol \rightarrow MolElem$ (rule 4)

$MolEle \rightarrow ElemElem$ (using rule 2 on *Mol* symbol)

$ElemElem \rightarrow CElem$ (using rule 6 on first *Elem* symbol)

$CElem \rightarrow CO$ (using rule 6 on second *Elem* symbol)

# Practice 1 and 2

Consider the CFG from earlier

$$Form \rightarrow Mol$$
$$Mol \rightarrow Elem$$
$$Mol \rightarrow Elem_{Count}$$
$$Mol \rightarrow MolElem$$
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$$Elem \rightarrow C\ or\ O\ or\ H\ or\ He\ or\ \ldots$$
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**Practice 1:** Using the same logic, can you prove that

- $H_2O$ is a valid expression,
- $C_{12}H_{22}O_{11}$ is a valid expression,
- $H^2O$ is not a valid expression,
- And $C_2\ J_8$ is not a valid expression?

**Practice 2:** Which additional production rule(s) would you add to cover for $Ca_3(PO_4)_2$?

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**Answer 1: To be shown on board.**

# Practice 1 and 2

Consider the CFG from earlier

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**Practice 2:** Which additional production rule(s) would you add to cover for $Ca_3(PO_4)_2$?

**Answer 2:** Probably something like

$$Mol \rightarrow (Mol)_{Count}$$

**Observation:** It seems that CFGs are capable of checking parentheses!

CFG would reject $Ca_3PO_4)_2$.

# Derivation and Syntax Validity

**Definition (Derivation):**

In CFG, a **derivation** is a **sequence of production rules** that

- starts from the start symbol,
- and rewrites non-terminal symbols using production rules,
- until only terminal symbols remain.

The resulting sequence of terminal symbols forms a string, called the **result of the derivation**.

# Derivation and Syntax Validity

**Theorem (Syntax Validity):**

**A given string *x* of terminal symbols (e.g. $x = H_2O$) has a valid syntax, according to a given CFG,**

if and only if,

**There exists a derivation for the given CFG, which produces the given string *x* as the result of the derivation.**

# How is that useful for compilers?

Let us assume we have used our tokenizer on a given source code and we have obtained a tokens stream of some sort.

**Which of the two tokens streams below shows that the code has a syntax problem of some sort?**

A. Token(KEYWORD_INT, “int”), Token(IDENTIFIER, “x”), Token(EQSIGN, “=”), Token(INT_LITERAL, “1023”), Token(SEMICOL, “;”).

B. Token(KEYWORD_INT, “int”), Token(INT_LITERAL, “1023”), Token(EQSIGN, “=”), Token(IDENTIFIER, “x”), Token(SEMICOL, “;”).

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**Equivalent question:** Can you write "int 1023 = x;" in C?

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**Equivalent question:** Can you write "int 1023 = x;" in C?
No, this statement is incorrect because it does not follow the proper syntax for declaring and initializing a variable in C, which is "int x = 1023;" .

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A. **Token(KEYWORD_INT, “int”), Token(IDENTIFIER, “x”), Token(EQSIGN, “=”), Token(INT_LITERAL, “1023”), Token(SEMICOL, “;”).**

**This is fine.**

B. **Token(KEYWORD_INT, “int”), Token(INT_LITERAL, “1023”), Token(EQSIGN, “=”), Token(IDENTIFIER, “x”), Token(SEMICOL, “;”).**

**This one has a syntax problem (identifier appears on right hand side of the equal sign and literal value on the left hand side).**

# How is that useful for compilers?

**Property: Programming languages are ruled by syntax rules, which can be described as CFGs.**

For instance, when declaring a variable of type integer (using no arithmetic operations on the right hand side of the equal sign, only literals) the stream of tokens should follow a specific syntax described by the CFG on the right.

Declar as start symbol

$$Declar \rightarrow Type\ TOKENID\ TOKENEQ\ Literal\ TOKENSEMICOL$$

Several possible keywords for integer variables, to decide on number of bits

$$Type \rightarrow TOKENINT\ or\ TOKENSHORT\ or\ TOKENLONG$$

Could technically have decimal and exponential notations for int literals

$$Literal \rightarrow TOKENINTLITERALDEC\ or\ TOKENINTLITERALEXP$$

# Practice 3

**Question:** Let us consider mathematical expressions, consisting of

- single digit numbers,
- + operations,
- along with opening and closing parentheses.

**Can you write a CFG to check whether such a given mathematical expression has a valid syntax or not?**

Some examples of **valid** and **invalid** expressions (same expressions as before):

- **2+7+9**
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- single digit numbers,
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**Can you write a CFG to check whether such a given mathematical expression has a valid syntax or not?**

**Answer:** Probably something along the lines of

$$Expr \rightarrow Term$$
$$Expr \rightarrow Expr + Term$$
$$Term \rightarrow Num$$
$$Term \rightarrow (Expr)$$
$$Num \rightarrow 0 \; or \; 1 \; or \; 2 \; or \; \ldots or \; 9$$

This is also something we could use to check if an arithmetic expression in our source code has a valid syntax!

# Parse tree of a derivation

**Derivation (parse tree of a CFG derivation):**

For a given CFG derivation, we can build a parse tree,

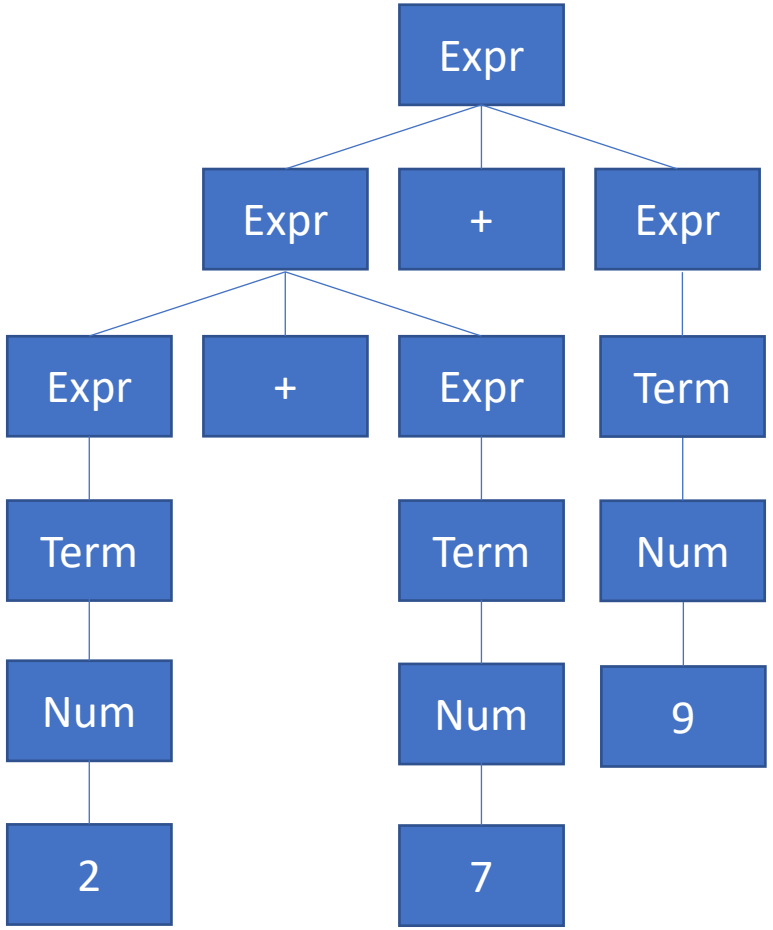
- Whose root is the start symbol,
- Where every production rule, $X \rightarrow Y_1 \ldots Y_N$ in the derivation sequence, adds children nodes $Y_1, \ldots, Y_N$ to the node $X$.

# Parse tree of a derivation

**Reusing the production rules below**

$$Expr \rightarrow Term$$
$$Expr \rightarrow Expr + Expr$$
$$Term \rightarrow Num$$
$$Term \rightarrow (Expr)$$
$$Num \rightarrow 0\ or\ 1\ or\ 2\ or\ \dots or\ 9$$

**We can define the parse tree for 2+7+9, as shown on the right.**



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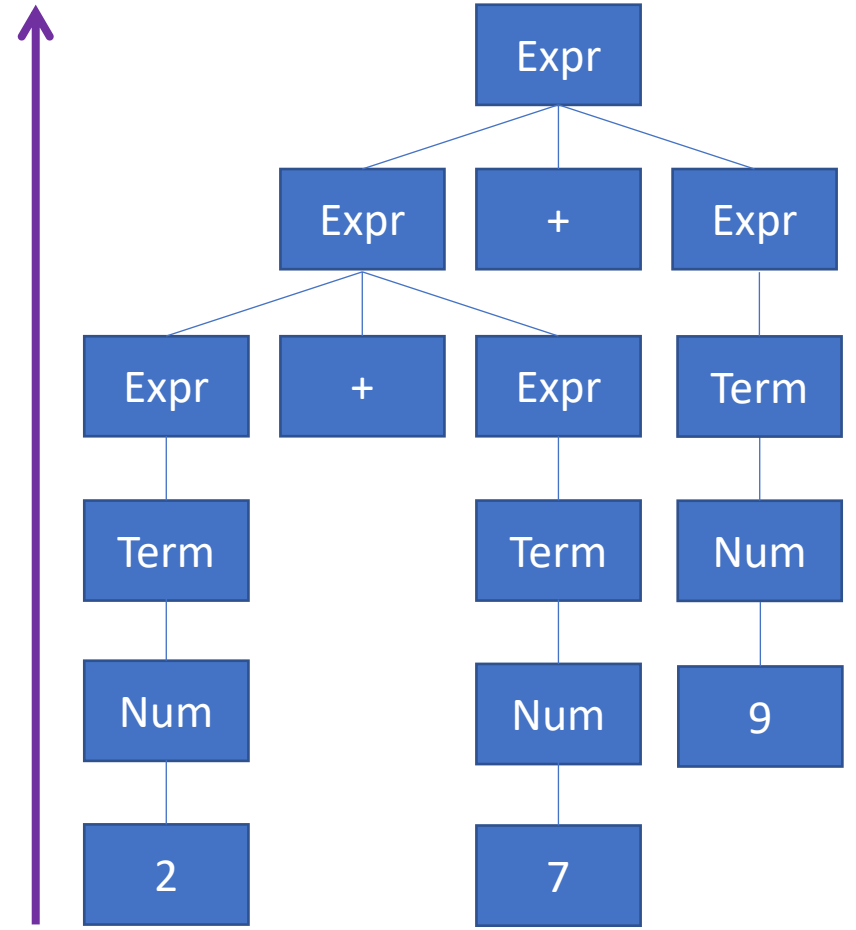


This parse tree is interesting because it shows the **order** in which we should compute the different operations, starting with 2 and 7, then 2+7, and finally (2+7)+9.

# Quick question

Assuming that a given string *x* has a valid syntax for a given CFG and admits a valid derivation…

→ **Is the valid derivation unique?**

→ **Is there only one parse tree that could have been defined?**

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Assuming that a given string *x* has a valid syntax for a given CFG and admits a valid derivation…

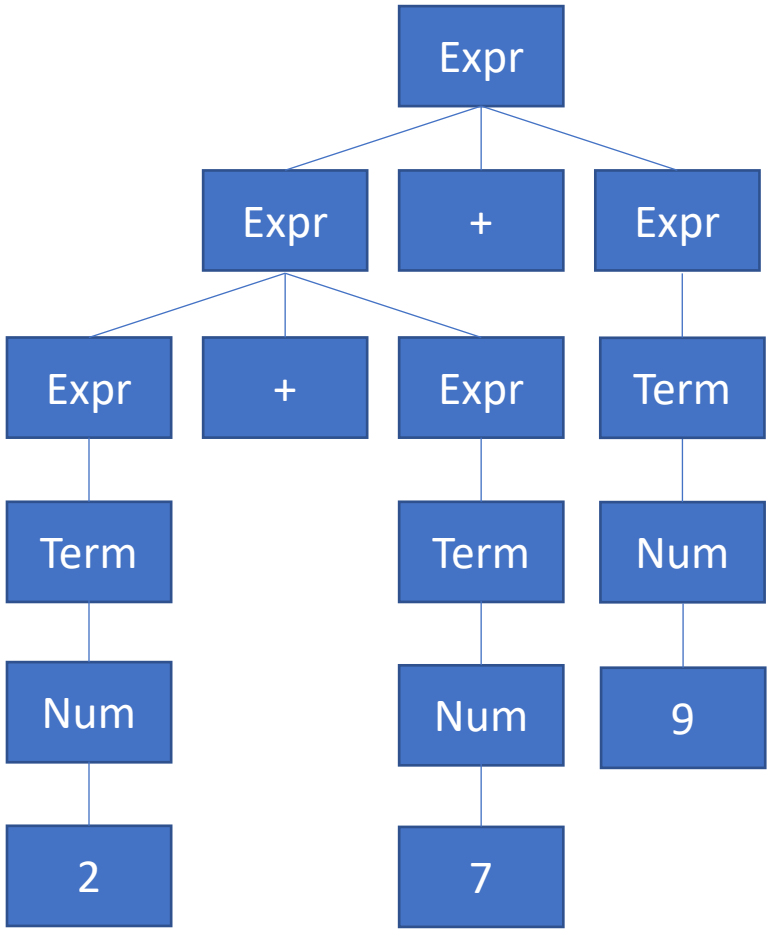
→ **Is the valid derivation unique?**

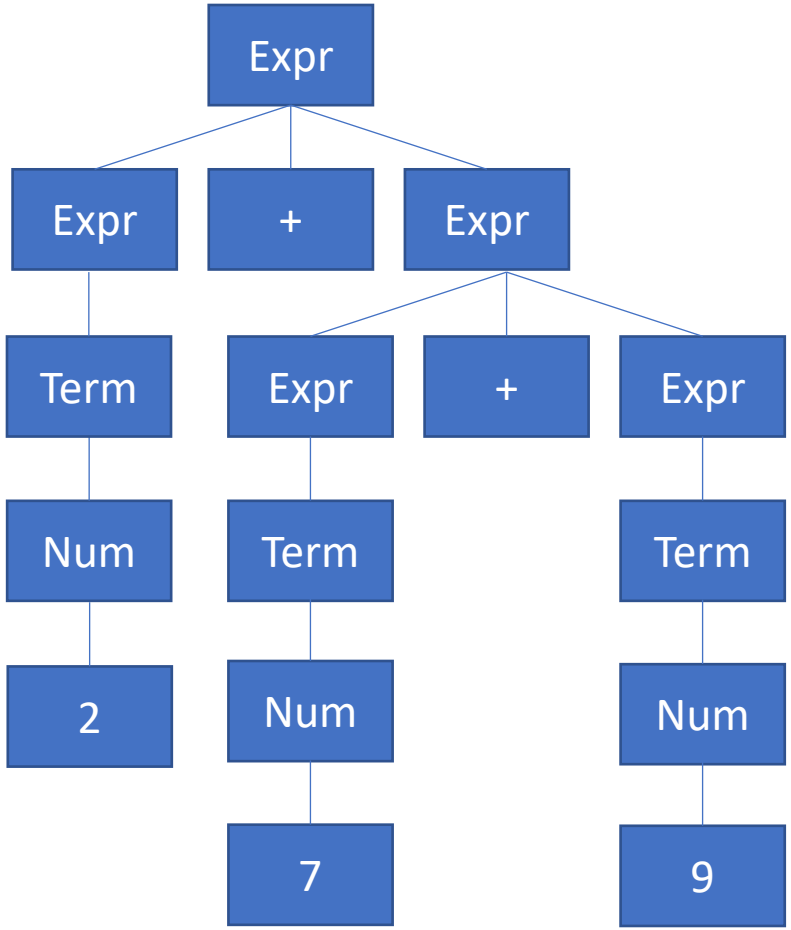
→ **Is there only one parse tree that could have been defined?**

In general, no, multiple valid derivations might do the trick…

And that ambiguity might even be a problem in certain scenarios…

# Two parse trees for 2+7+9



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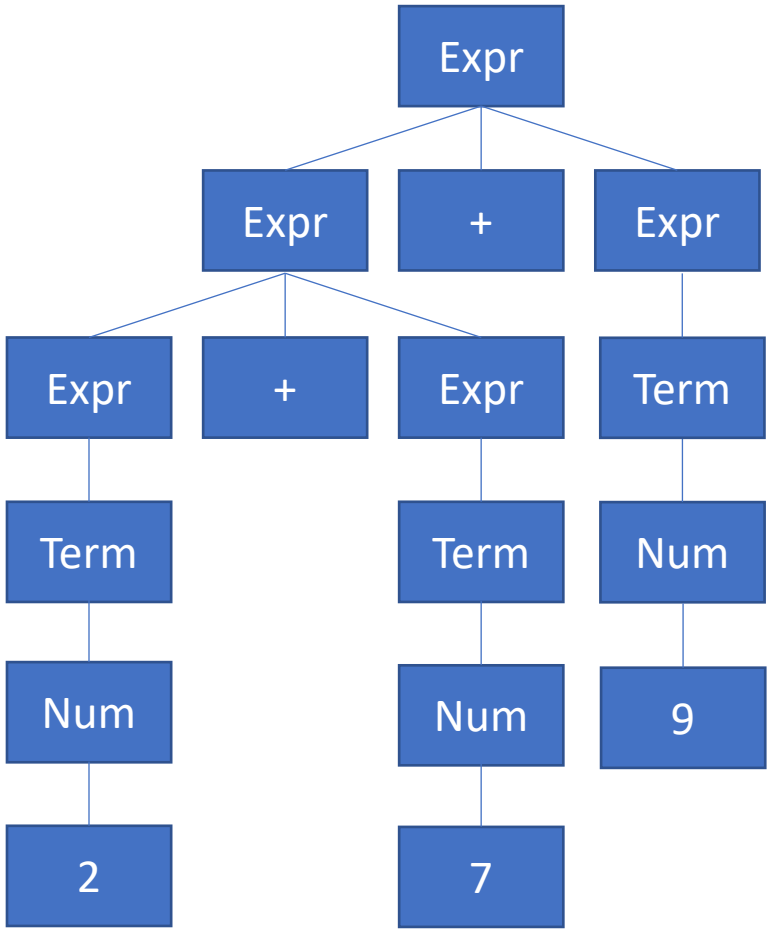
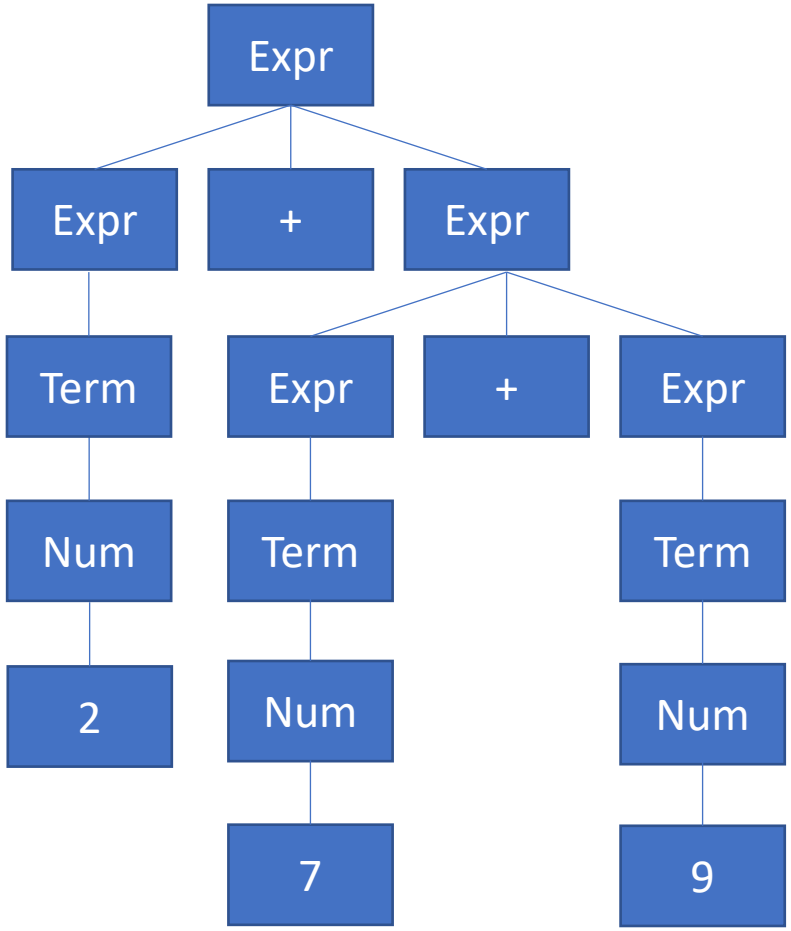



**Question: Does that make a difference?**



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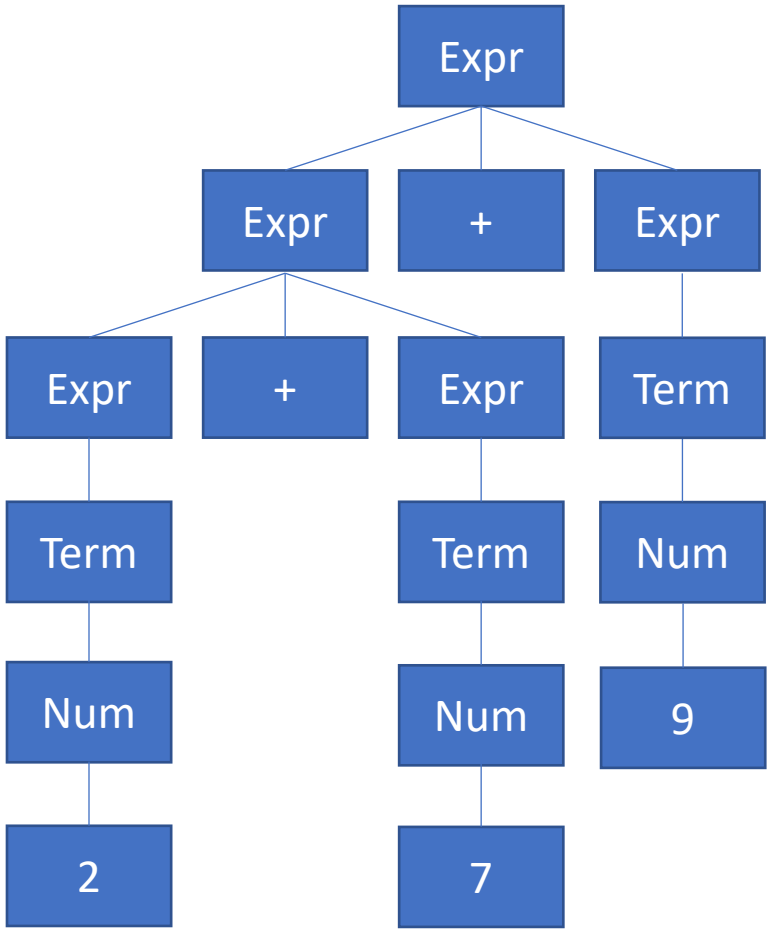
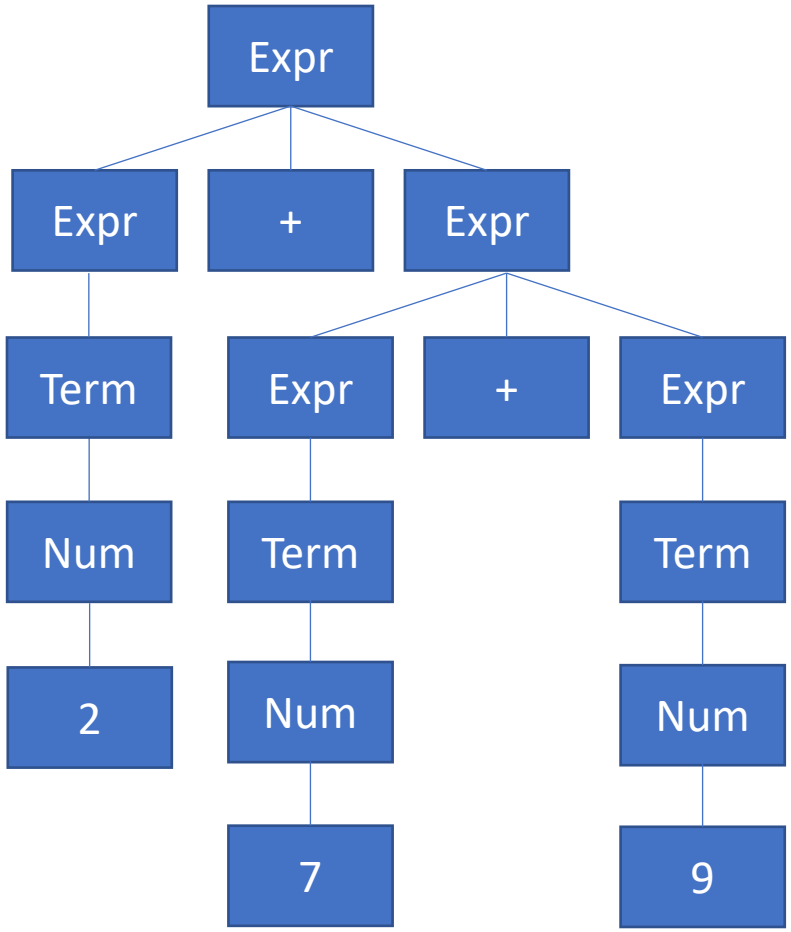
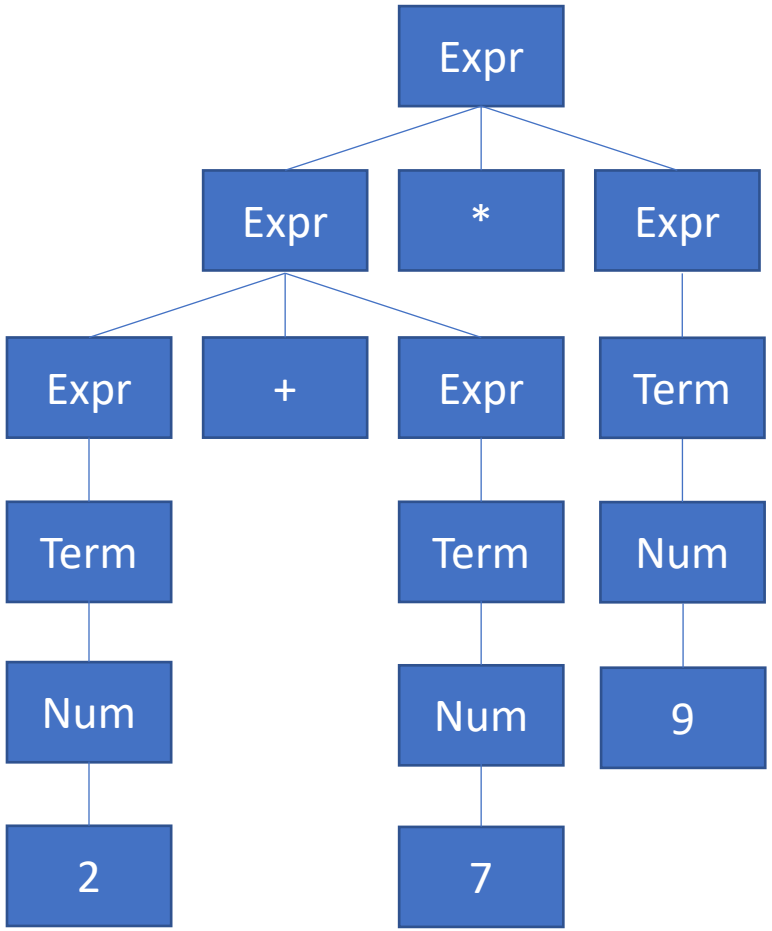


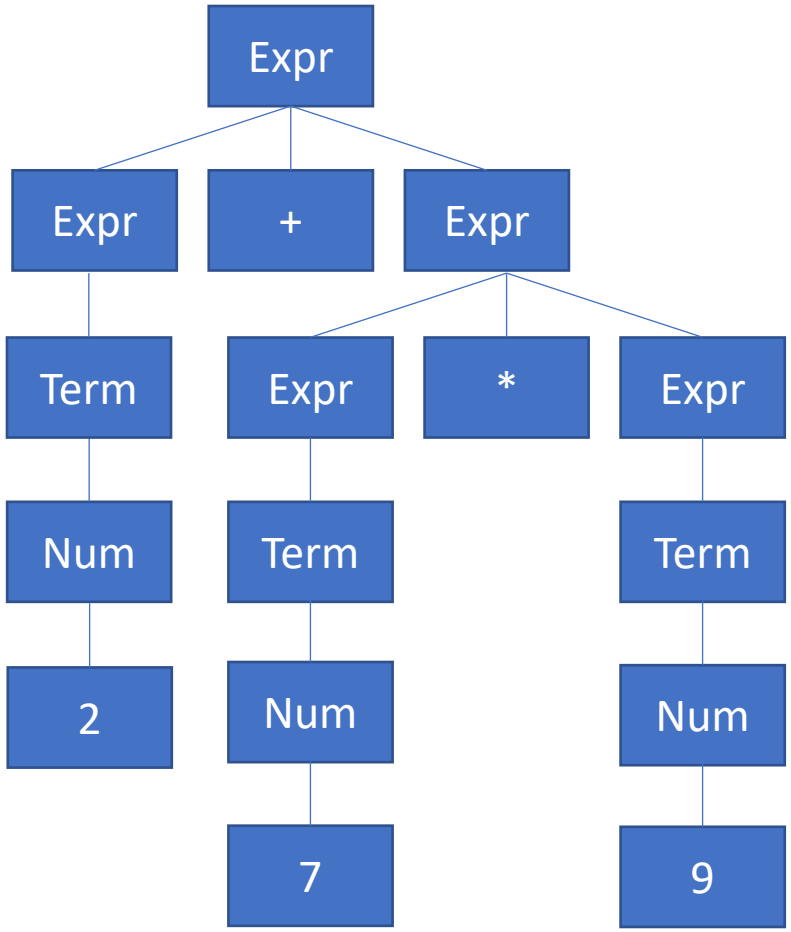

**Question: Does that make a difference?**
At the moment, no, because the order does not matter.



# Two parse trees for 2+7+9




**But what if we were building parse trees for 2+7*9 instead?**



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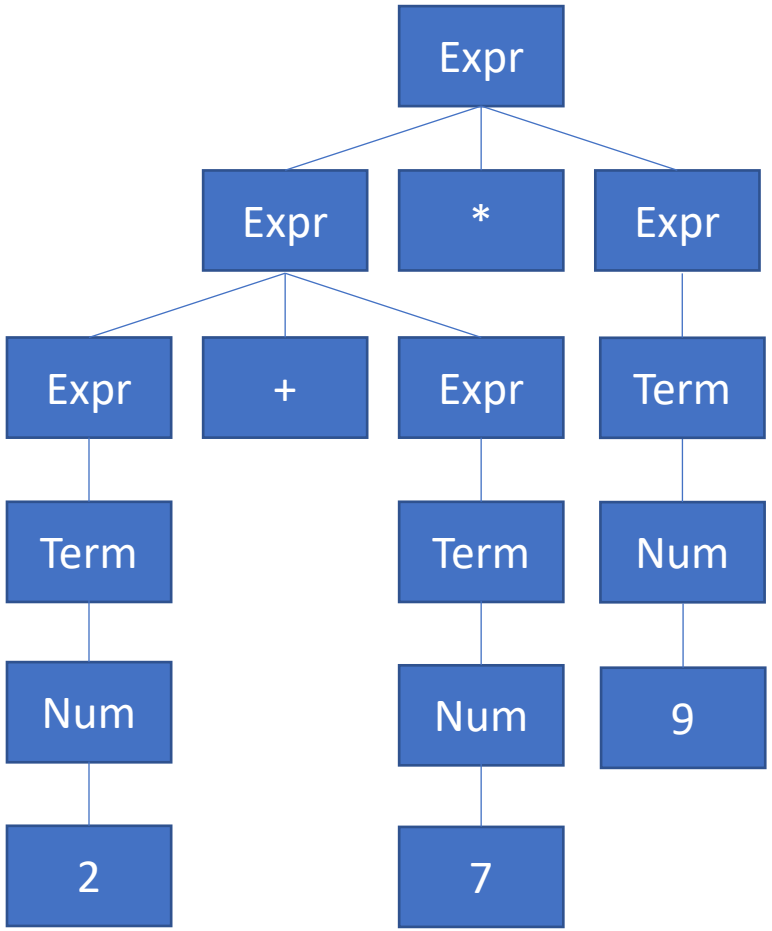
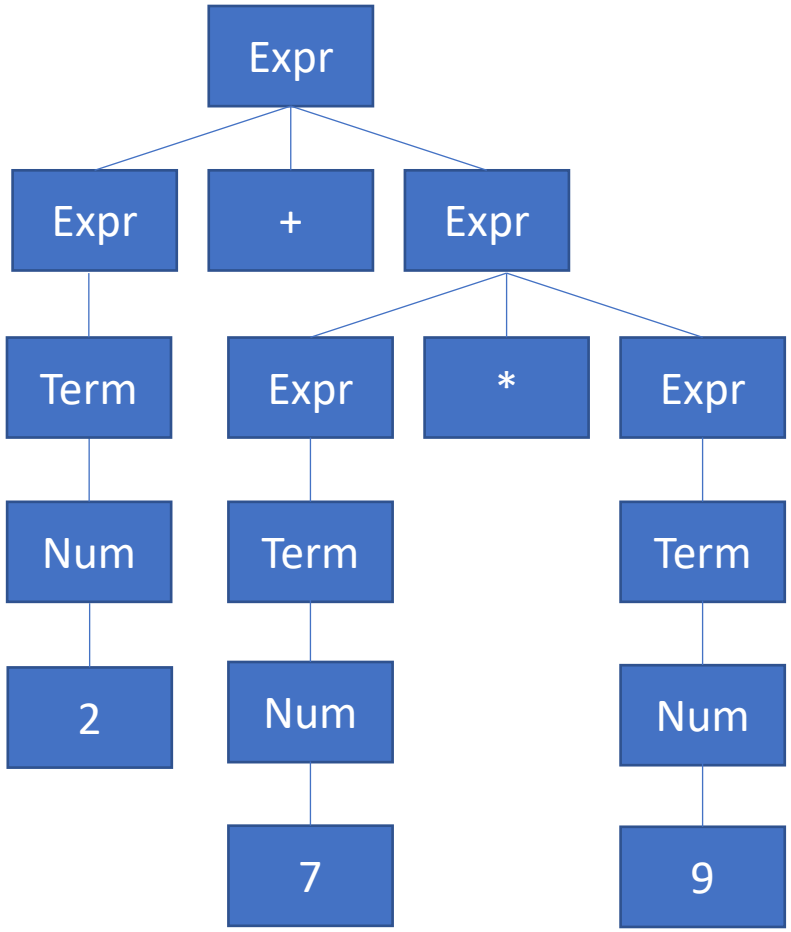



**But what if we were building parse trees for 2+7*9 instead?**

The right tree represents 2+(7*9). The left one represents (2+7)*9.

The order matters in that case!



# Ambiguity

**Definition (Ambiguity in a CFG derivation):**

When using a CFG to check the syntax validity of an expression and building a parse tree, we say that a **CFG is ambiguous** if it can lead to two different parse trees with different results.

In the case of arithmetic expressions and programming languages, this means that

- Two different derivations might exist,
- Producing two different parse trees,
- And the result of both operations following the two parse trees might differ and lead to different outcomes for a given program (not good!).

# Checking ambiguity algorithmically

**Theorem (On checking the ambiguity of a CFG algorithmically):**

Let us consider a given CFG.

**There is no algorithm to check if a given CFG is ambiguous or not.**

This is known as the ambiguity problem for context-free grammars, and it is proven to be an undecidable problem.

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*(**Note:** Similarly, there is no general algorithm that can determine whether a given program contains an infinite loop. This is known as the Halting problem, and means that you cannot define a compiler program that can check for the presence of infinite loops in the compiled source code.)*

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There are however **manual methods** for designing CFGs that will be non-ambiguous.

We will investigate them on the next lecture.

# Quiz time!

**What is a Context-Free Grammar (CFG)?**

A. A set of rules that can generate all strings in a language
B. A method for tokenizing source code
C. A formal system for describing the structure of a language
D. A technique for optimizing compiler performance

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B. A rule for optimizing the generated code
C. A rule for reducing the number of steps in a computation
D. A rule for replacing a non-terminal symbol with a sequence of terminal and non-terminal symbols

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**What is a derivation in the context of CFGs?**

A. The process of breaking down a string into its constituent tokens
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C. The process of applying production rules to generate a string in the language
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**What is a parse tree?**

A. A data structure for representing the structure of a language
B. A tree used for optimizing compiler performance
C. A tree representing the derivation of a string using a CFG
D. A tree used for breaking down a string into tokens

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**What is an ambiguous context-free grammar?**

A. A grammar that can generate two different strings for the same derivation

B. A grammar whose production rules cannot produce a result string consisting of terminal symbols only

C. A grammar that generates only one parse tree for each string

D. A grammar that can generate more than one parse tree for the same string

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