# CS/ECE/ME532 Activity 3

*Estimated Time: 15 min for P1, 15 min for P2, 15 min for P3, 15 min for P4*

1) You collect ratings (out of 100) of three classes from four of your friends.

Bao: (CS760, 36), (ECE533, 40), (MATH521, 20)
Julia: (CS760, 72), (ECE533, 80), (MATH521, 40)
Vivek: (CS760, 90), (ECE533, 100), (MATH521, 50)
Jamal: (CS760, 54), (ECE533, 60), (MATH521, 30)

a) $X = \begin{bmatrix} 36 & 72 & 90 & 54 \\ 40 & 80 & 100 & 60 \\ 20 & 40 & 50 & 80 \end{bmatrix}$

   a) Express these ratings in a matrix $\boldsymbol{X}$ where the column indicates the friend (order the columns as Bao, Julia, Vivek, Jamal), the row indicates the class (ordered as CS760, ECE533, MATH521), and the value is the corresponding rating.

   b) Suppose $\boldsymbol{X}$ is expressed as the outer product of a taste vector $\boldsymbol{t}$ and an affinity weight vector $\boldsymbol{w}$, that is, $\boldsymbol{X} = \boldsymbol{t}\boldsymbol{w}^T$ and assume $\boldsymbol{t} = \begin{bmatrix} 9 \\ 10 \\ 5 \end{bmatrix}$. Find $\boldsymbol{w}$.

   $w = \begin{bmatrix} 4 \\ 8 \\ 10 \\ 6 \end{bmatrix}$

   c) Your friend Brianna wasn't able to complete your survey, but did rate ECE533 as 30. Assuming Brianna has the same taste profile as the rest of your friends, what would her ratings for CS760 and Math521 be?

   $\text{Brianna} = 3 \begin{bmatrix} 9 \\ 10 \\ 3 \end{bmatrix} = \begin{bmatrix} 27 \\ 30 \\ 9 \end{bmatrix}$

2) Suppose a $4 \times 5$ rating matrix $\boldsymbol{X}$ reflecting ratings of four movies by five people is decomposed as the product of a rank-2 taste matrix $\boldsymbol{T} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and rank-2 affinity weight matrix $\boldsymbol{W} = \begin{bmatrix} 8 & 6 & 4 & 5 & 4 \\ 2 & 3 & 3 & -2 & -1 \end{bmatrix}$, that is, $\boldsymbol{X} = \boldsymbol{T}\boldsymbol{W}$.

   a) Find $\boldsymbol{X}$.

   b) Write $\boldsymbol{X} = \boldsymbol{t}_1\boldsymbol{w}_1^T + \boldsymbol{t}_2\boldsymbol{w}_2^T$ where $\boldsymbol{t}_1, \boldsymbol{t}_2, \boldsymbol{w}_1, \boldsymbol{w}_2$ are column vectors and the first elements of $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ are given by $[\boldsymbol{t}_1]_1 = [\boldsymbol{t}_2]_1 = 1$. Find one choice for $\boldsymbol{t}_1, \boldsymbol{t}_2, \boldsymbol{w}_1$ and $\boldsymbol{w}_2$.

a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 8 & 6 & 4 & 5 & 4 \\ 2 & 3 & 3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 10 & 9 & 7 & 3 & 3 \\ 6 & 3 & 1 & 7 & 5 \\ 6 & 3 & 1 & 7 & 5 \\ 10 & 9 & 7 & 3 & 3 \end{bmatrix}$

b) $X = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [8\ 6\ 4\ 5\ 4] + \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} [2\ 3\ 3\ -2\ -1]$

3

5

3) Let $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{c}^T & \boldsymbol{d}^T \end{bmatrix}$ where $\boldsymbol{A}$ is $2 \times 3$, $\boldsymbol{B}$ is $2 \times 2$, $\boldsymbol{c}^T$ is $1 \times 3$, and $\boldsymbol{d}^T$ is $1 \times 2$.

a) Express the product $\boldsymbol{R} = \boldsymbol{X}\boldsymbol{X}^T$ in terms of $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{c}, \boldsymbol{d}$.

$R = \begin{bmatrix} A & B \\ c^T & d^T \end{bmatrix} \begin{bmatrix} A^T & c \\ B^T & d \end{bmatrix} =$

$\begin{bmatrix} AA^T + BB^T & Ac + Bd \\ c^TA^T + d^TB^T & c^Tc + d^Td \end{bmatrix}$

b) What are the dimensions of $\boldsymbol{R}$? $3 \times 3$

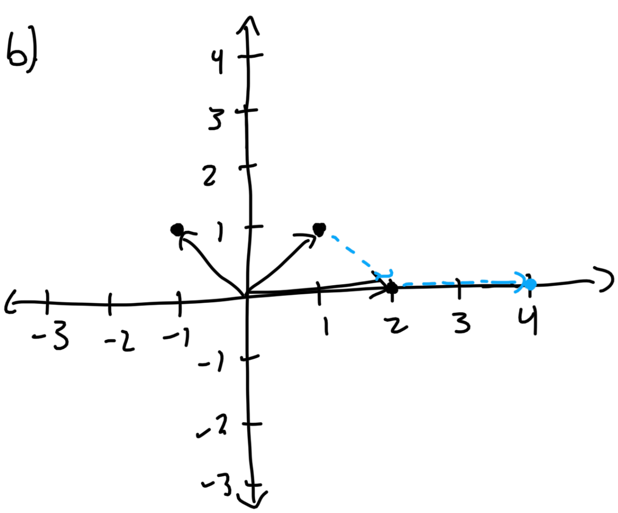
4) **a)** Let $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$. Denote the columns of $\boldsymbol{A}$ as $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n$. Express $\boldsymbol{y}$ as a weighted sum of the columns of $\boldsymbol{A}$.

$y = x_1 \underline{a}_1 + x_2 \underline{a}_2 + \ldots + x_n \underline{a}_n$

**b)** Let $\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$. Consider the columns of $\boldsymbol{A}$ as vectors in $\mathbb{R}^2$, and plot them with the first element on the horizontal axis, and the second element on the vertical axis.

**c)** Let $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$ with $\boldsymbol{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\boldsymbol{A}$ as in part (b). Draw a picture to find $\boldsymbol{y}$ by expressing it as a weighted sum of vectors you ploted in (b).

b)




c) $y = 1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$y = \begin{bmatrix} 1 - (-1) + 2 \\ 1 - 1 + 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$