CSC 255

Unit 6 Programming Assignment 1 [30 points]

Karatsuba Algorithm

We can improve the runtime performance of integer multiplication by using a divide and conquer approach. The generic approach to multiplying two integers has a runtime complexity of $O(n^2)$. Using the Karatsuba algorithm for integer multiplication gives a runtime complexity of $O(n^{1.59})$. The Karatsuba algorithms breaks each of the two large integers (e.g., X and Y) into halves (e.g., $X_{left\text{-}half}$, $X_{right\text{-}half}$, and $Y_{left\text{-}half}$, $Y_{right\text{-}half}$), and performs the following calculation:

$$X*Y = (X_{left\text{-}half}*2^{(n/2)} + X_{right\text{-}half})(Y_{left\text{-}half}*2^{(n/2)} + Y_{right\text{-}half})$$

$$= 2^{n}*X_{left\text{-}half}*Y_{left\text{-}half} + 2^{(n/2)}*(X_{left\text{-}half}*Y_{right\text{-}half} + X_{right\text{-}half}*Y_{left\text{-}half}) + X_{right\text{-}half}*Y_{right\text{-}half}$$

Now, we can manipulate the following sum of the product above:

$$X_{left\text{-}half}*Y_{right\text{-}half} + X_{right\text{-}half}*Y_{left\text{-}half}$$

into the following equivalent expression:

$$[(X_{left\text{-}half} + X_{right\text{-}half})*(Y_{left\text{-}half} + Y_{right\text{-}half}) - X_{left\text{-}half}*Y_{left\text{-}half} - X_{right\text{-}half}*Y_{right\text{-}half}] + X_{right\text{-}half}*Y_{right\text{-}half}$$

so that the Kuratsuba algorithm gains its advantage in runtime. See an example of using the generic approach of integer multiplication and the Karatsuba algorithm; and, examine the steps used to solve the recurrence relation which gives the more efficient runtime complexity.

Implement the Karatsuba algorithm to multiply two integers no larger than 123,456,789.