assignment04

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Mathematical Foundations for Computer Vision and Machine Learning
*** Assignment04 - k-means algorithm (MNIST) ***
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Link to Github

1 Setting up

By changing variable numOfClusters, you can see that "k" in **k-means** changes. But in this assignment, **k is 10**.

Variable `label` is used to store the label of nth image.
Variable `im_average` stores the vector of centroids of each clusters.
Variable `im_count` stores the number of images of each cluster.
Variable `E` stores the energy of each loop.
Variable `A` stores the accuracy of each loop.
Variable `threshold` means the termination condition of this algorithm, which can be written as :

$$\Delta E = E_{n-1} - E_n$$

$$E_{n-1} - E_n < threshold$$

The **maximum k** is **6** since there's not enough color.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

file_data = "mnist_test.csv"
handle_file = open(file_data, "r")
data = handle_file.readlines()
handle_file.close()

# Conditions
numOfClusters = 10
size_row = 28   # height of the image
size_col = 28   # width of the image
dim = size_col * size_row
num_image = 0
```

```
    im_average = np.zeros((dim, numOfClusters), dtype=float)           # centroid of each cl
    im_count = np.zeros(10, dtype=int)
    E = [] # energy of each iteration
    A = [] # accuracy of each iteration
    iter_num = 0
    threshold = 0.005
    a = np.zeros(numOfClusters)
```

2 Normalize

We normalize images since there can be some differences in lightness.

```
In [2]: # Functions

        def normalize(data):
            data_normalized = (data - min(data)) / (max(data) - min(data))

            return (data_normalized)
```

3 Loading Images

Variable `list_image` stores the images as vector.
Variable `list_label` stores the label of each image.
Variable `list_assign` stores the cluster number that image is assigned.
Variable `listOfDist` stores the distance (second-order) from one point to all centroids.

```
In [3]: def load():
            num_image = len(data)
            count = 0  # count for the number of images
            list_image = np.zeros((size_row * size_col, num_image), dtype=float)  # image data
            list_label = np.zeros(num_image, dtype=int)  # given label of image
            list_assign = np.zeros(num_image, dtype=int)
            listOfDist = np.zeros((numOfClusters, num_image))

            for line in data:
                line_data = line.split(',')
                label = line_data[0]
                im_vector = np.asfarray(line_data[1:])
                im_vector = normalize(im_vector)

                list_label[count] = label
                list_image[:, count] = im_vector

                count += 1

            return list_image, list_label, list_assign, listOfDist, num_image
```

4 Define Essential Functions

I define some funtions that is essential to implement k-means algorithm.
`initializeLabel` initializes the whole images' label.
`computeDistance` computes the matrix `listOfDist`
`assignLabel` assigns the whole images to the most close centroid.
`computeCentroid` computes the vector of each centroids using `list_assign` and `list_image`.
`computeEnergy` computes energy of each step and record it to list `E`
`computeAccuracy` computes accuracy of each step and record it to list `A`
`record` calls `computeEnergy` and `computeAccuracy`.

In [4]:
```
def initializeLabel(numOfClusters):
    return np.random.randint(numOfClusters, size = num_image)

def computeDistance():
    # L2 Norm
    for c in range(numOfClusters):
        for i in range(num_image):
            listOfDist[c][i] = sum((im_average[:,c] - list_image[:,i])**2)

def assignLabel():
    for i in range(num_image):
        list_assign[i] = np.argmin(listOfDist[:,i])

def computeCentroid():
    num = np.zeros((numOfClusters))
    c = np.zeros((dim, numOfClusters))

    for i in range(num_image):
        num[list_assign[i]] += 1
        c[:,list_assign[i]] += list_image[:,i]

    for i in range(numOfClusters):
        c[:,i] /= num[i]
    return c

def computeEnergy(): #must decrease
    energy = 0
    for i in range(num_image):
        energy += sum((im_average[:,list_assign[i]] - list_image[:,i])**2)
    energy /= num_image
    return energy

def computeAccuracy():
    corrects = 0
    for k in range(numOfClusters):
        im_count = np.zeros(numOfClusters)
        for i in range(num_image):
```

```
            if list_assign[i] == k:
                im_count[list_label[i]] += 1

        a[k] = np.argmax(im_count)

    for i in range(num_image):
        if list_label[i] == a[list_assign[i]]:
            corrects += 1
    return corrects/num_image

def record():
    E.append(computeEnergy())
    A.append(computeAccuracy())
```

5 Initial Centroid Images

By using `list_assign = initializeLabel(numOfClusters=numOfClusters)` the initial labels are set.

$$c_i = \frac{1}{N}\Sigma x_k$$

where

$$l(x_k) == i$$

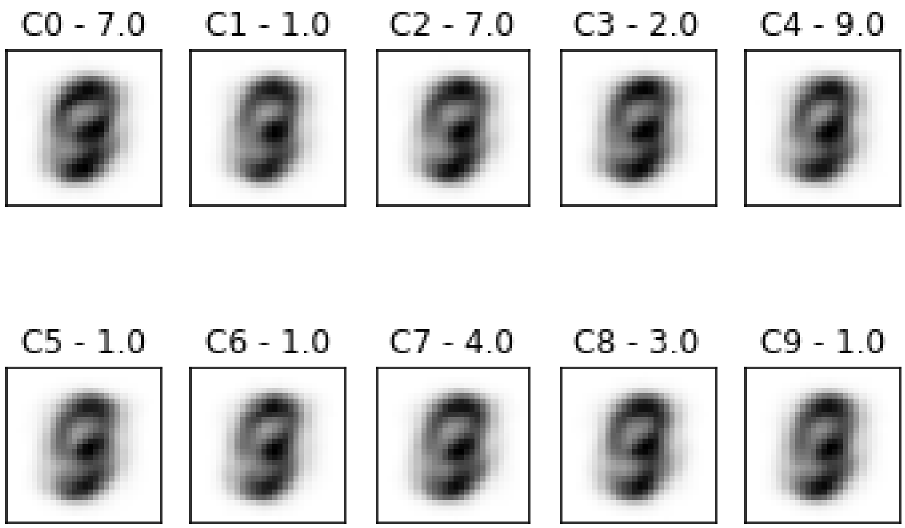
```
In [5]: # Test Code
        list_image, list_label, list_assign, listOfDist, num_image = load()
        list_assign = initializeLabel(numOfClusters=numOfClusters)
        im_average = computeCentroid()
        record()
        iter_num += 1

        plt.figure(1)
        for i in range(numOfClusters):

            plt.subplot(2, 5, i + 1)
            plt.title("C"+str(i)+" - "+str(a[i]))
            plt.imshow(im_average[:, i].reshape((size_row, size_col)), cmap='Greys', interpolati

            frame = plt.gca()
            frame.axes.get_xaxis().set_visible(False)
            frame.axes.get_yaxis().set_visible(False)

        plt.show()
```



6 Final label

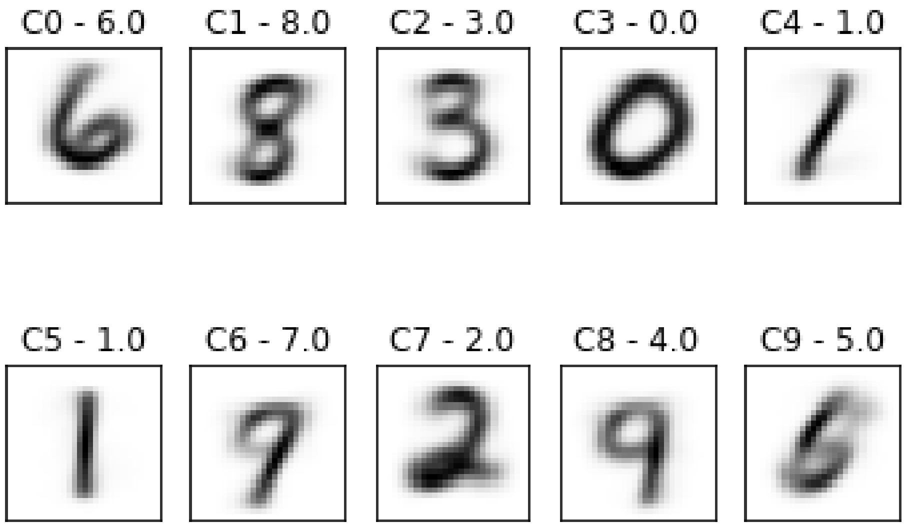
After k-means algorithm, the data are clustered into 'k' clusters. Data are labeled in `list_assign` and answers are stored in `list_label`.

```
In [6]: while True:
            computeDistance()
            assignLabel()
            im_average = computeCentroid()
            record()
            #print(E[iter_num])
            #print(A[iter_num])
            if E[-2] - E[-1] < threshold:
                break
            iter_num += 1

        plt.figure(2)
        for i in range(numOfClusters):
            plt.subplot(2, 5, i + 1)
            plt.title("C"+str(i)+" - "+str(a[i]))
            plt.imshow(im_average[:, i].reshape((size_row, size_col)), cmap='Greys', interpolati

            frame = plt.gca()
            frame.axes.get_xaxis().set_visible(False)
            frame.axes.get_yaxis().set_visible(False)

        plt.show()
```

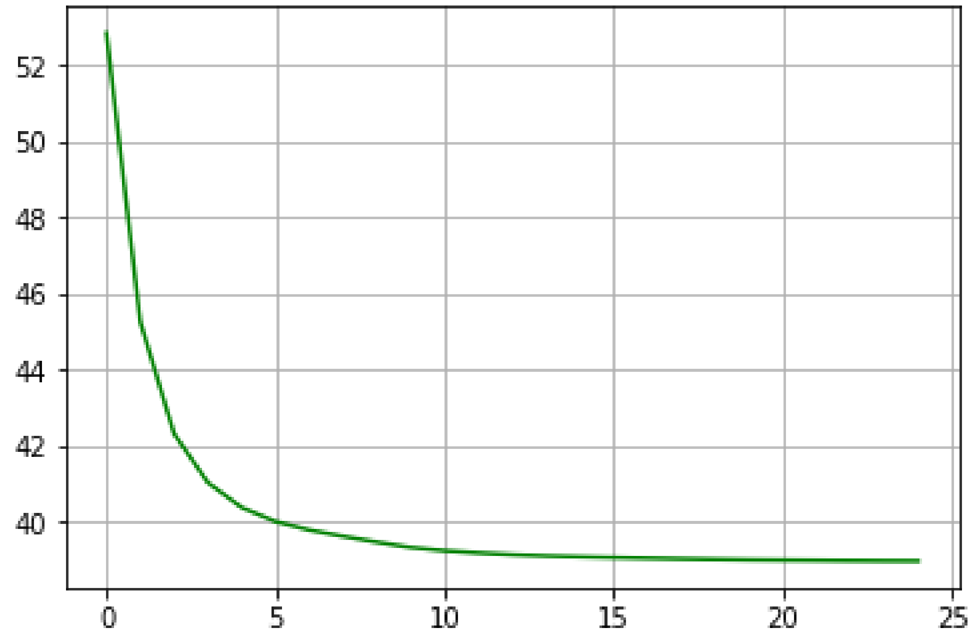


7 Energy per each iteration

$$E = \frac{1}{N}\Sigma\|x_i - c_k\|^2$$

Let's see if energy decreased monotonically. It seems my program works fine.

In [7]:
```
plt.figure(3)
x_range = np.arange(iter_num+1)
plt.plot(x_range,E,"g")
plt.grid(True)
plt.show()
```

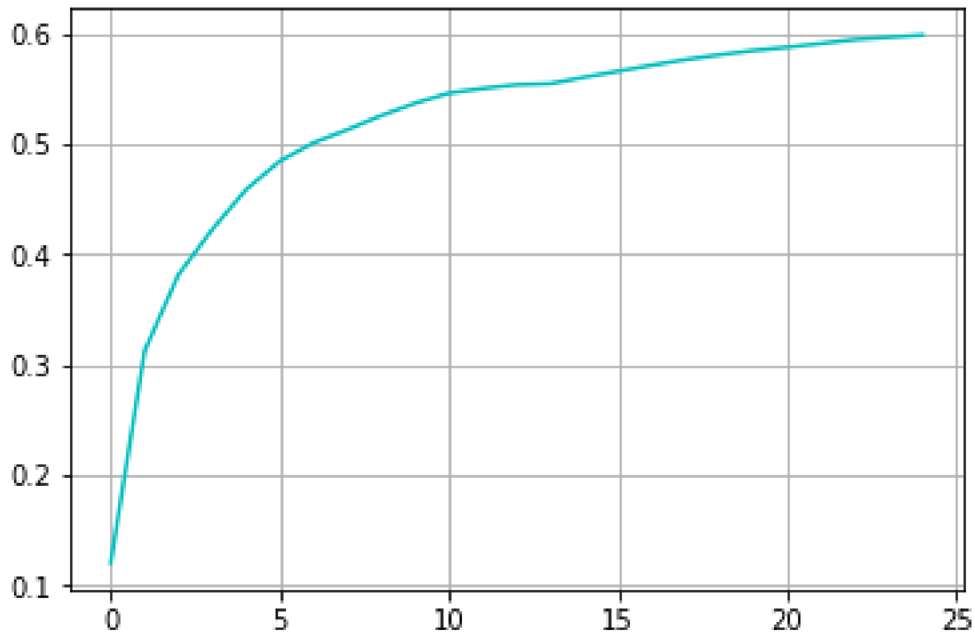


8 Accuracy per each iteration

$$A = \frac{1}{N} \Sigma count(x_i, c_k)$$

We first find what each cluster means. This is defined by argmax function.
Next, we count correct images and divide it into number of images.

```
In [8]: plt.figure(4)
        x_range = np.arange(iter_num+1)
        plt.plot(x_range,A,"c")
        plt.grid(True)
        plt.show()
```



In [ ]: