# assignment07

November 14, 2018

**Mathematical Foundations for Computer Vision and Machine Learning**
*Assignment07 - Polynomial Fit(Least Square Approximate Solution)*
Name: Jinwoo Jeon
Student ID: 20143954
Link to Github

## 1 Setting Up

Now we got 1001 sample data. We define a function that is our answer(clean data).
This function is a function that can not be expressed as a polynomial.

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt

        num = 1001
        std = 5


        # x  : x-coordinate data
        # y1 : (noisy) y-coordinate data
        # y2 : (clean) y-coordinate data

        def fun(x):
            # f = np.sin(x) * (1 / (1 + np.exp(-x)))
            f = np.abs(x) * np.sin(x)

            return f
```

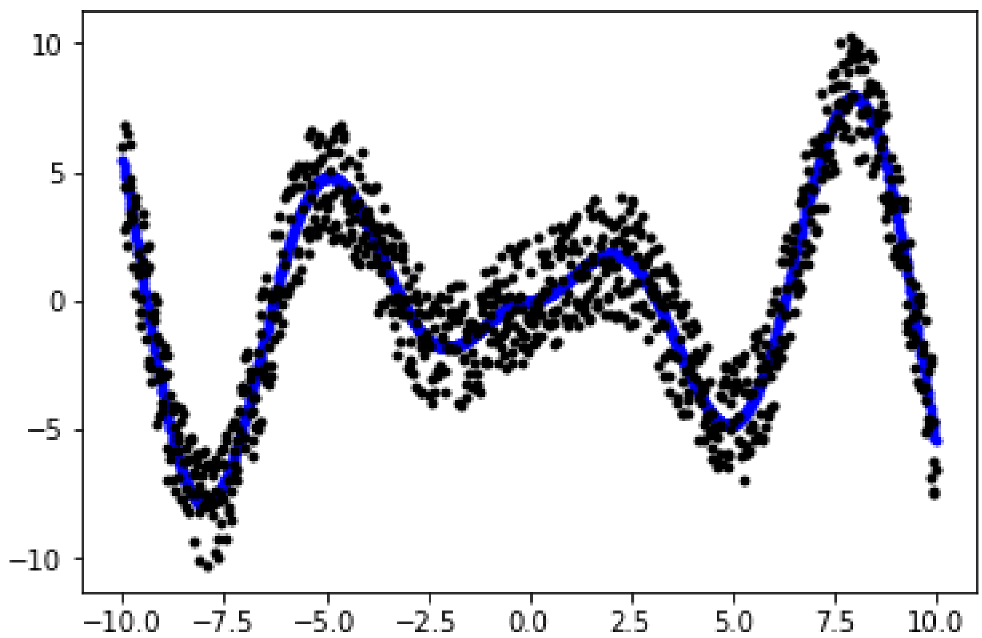
## 2 Clean and Noisy Data

y1 is clean data with x is -10 to 10 divided into 1001 sections.
y2 is noisy data with standard deviation = 5.
Clean data are plotted as **blue dots** and noisy data are plotted as **black dots**.

```
In [2]: n = np.random.rand(num)
        nn = n - np.mean(n)
```

```
x = np.linspace(-10, 10, num)
y1 = fun(x)
y2 = y1 + nn * std

plt.plot(x, y1, 'b.', x, y2, 'k.')
```

Out[2]:
```
[<matplotlib.lines.Line2D at 0x8836e80>,
 <matplotlib.lines.Line2D at 0x88420f0>]
```




## 3 Define Essential Functions

I define some funtions that is essential to implement polynomial fitting.
`defA(p,x)` is a function that makes A matrix in varying p=0,1,2,3,...9.
`getResidual` automatically calculates the sum of residual with varying p=0,1,2,3,...9.
`resi_x` and `resi_y` is to plot the error of model.

In [3]:
```
def defA(p,x):
    res = np.zeros((p+1,num))
    for i in range(p):
        res[i] = x**(p-i)
    res[p] = 1
    return np.matrix(np.transpose(res))

def getResidual(f_hat,y):
    return sum((f_hat - y)**2)
```

```
resi_x = range(0,10)
resi_y = np.zeros((10))
```

## 4 Approximation

Since p varies from 0 to 9, variable `i` indicates the varying p.

Matrix A and B are defined in every loop and theta is obtained by **pseudo inverse**.

We use matrix multiplication to calculate least square.

$$\theta_p x_1^p + \cdots + \theta_1 x_1^1 + \theta_0 x_1^0 = y_1$$
$$\theta_p x_2^p + \cdots + \theta_1 x_2^1 + \theta_0 x_2^0 = y_2$$
$$\theta_p x_3^p + \cdots + \theta_1 x_3^1 + \theta_0 x_3^0 = y_3$$
$$\vdots$$
$$\theta_p x_n^p + \cdots + \theta_1 x_n^1 + \theta_0 x_n^0 = y_n$$

We can write this as matmul.

$$\begin{pmatrix} x_1^p & \cdots & x_1^0 \\ \vdots & \ddots & \vdots \\ x_n^p & \cdots & x_n^0 \end{pmatrix} \begin{pmatrix} \theta_p \\ \vdots \\ \theta_0 \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

We can rewrite this as

$$A\theta = B$$

We can derive theta from equation above by pseudo inverse.

$$\theta = \left(A^T A\right)^{-1} A^T B$$

`approx` indicates the model function which can be written as

$$approx = \hat{f}(x) = \theta_0 x^0 + \theta_1 x^1 + \cdots + \theta_p x^p$$

And the error is defined as

$$resi_y[i] = \sum_{j=1}^{n} r_j^2$$

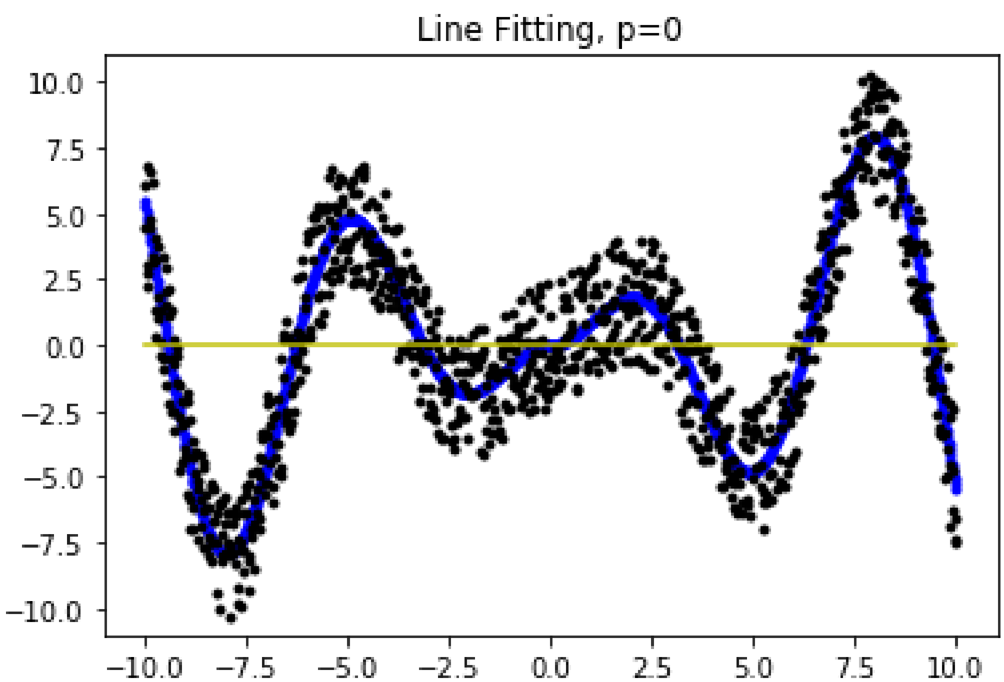
where

$$r_j = y_j - \hat{f}(x_j)$$

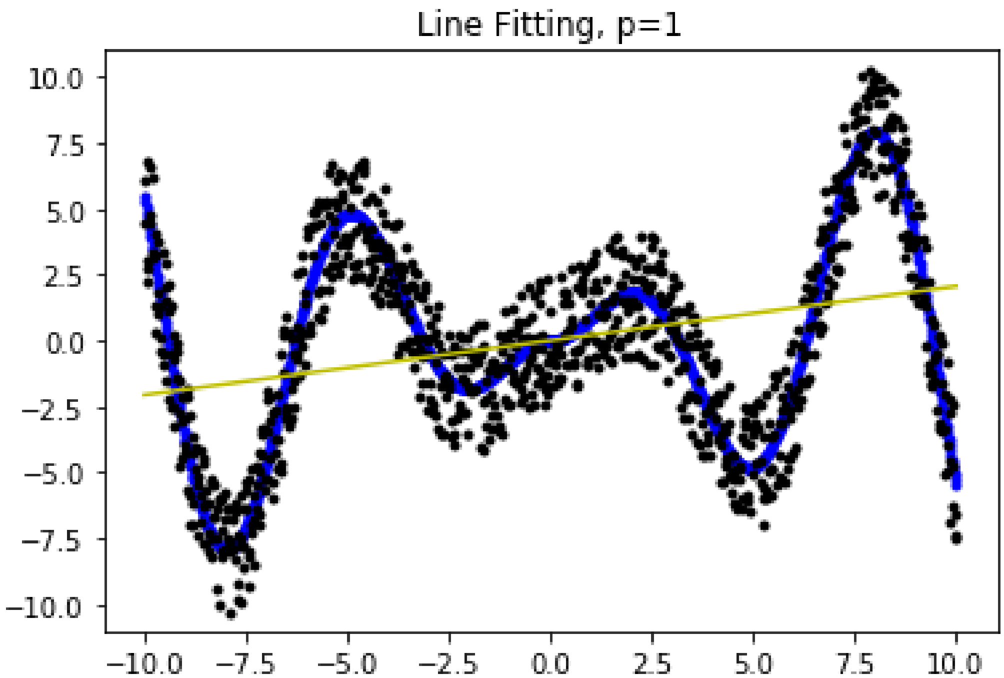
```
In [4]: for i in range(10):
            A = defA(i,x)
            B = np.matrix(y1)

            theta = (A.T * A).I*A.T*B.T
            theta = np.asarray(theta)
```

```python
        print(theta)
        print(theta.shape)

        approx = 0
        for j in range(len(theta)):
            approx += theta[j][0] * x**(i-j)

        resi_y[i] = getResidual(approx,y1)
        plt.plot(x, y1, 'b.', x, y2, 'k.', x, approx, 'y')
        plt.axis([-11,11,-11,11])
        plt.title("Line Fitting, p="+str(i))
        plt.show()
```

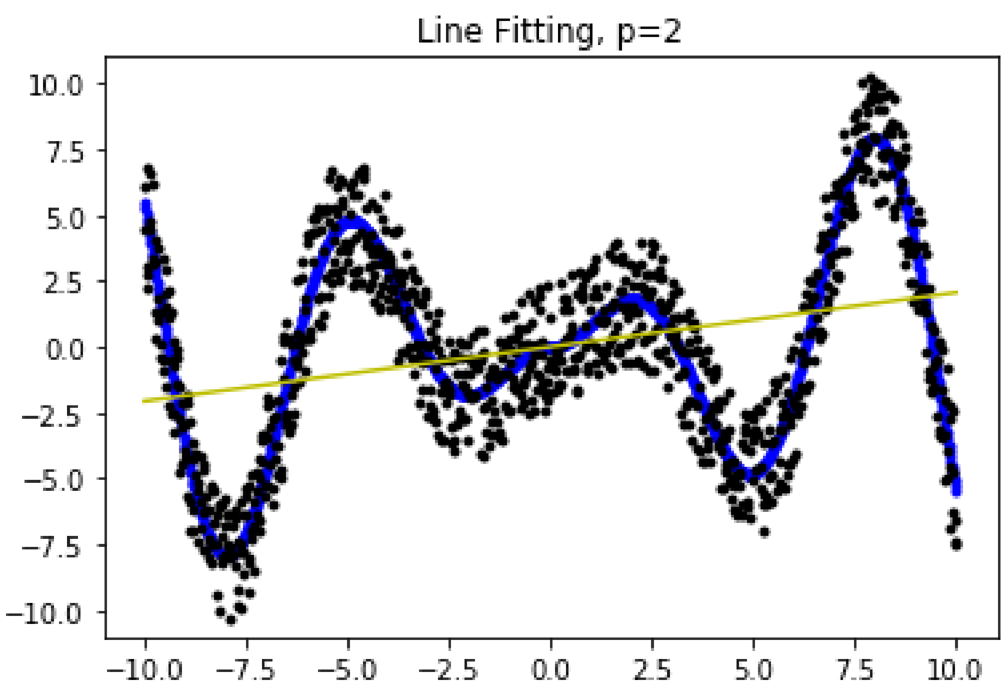
```
[[-1.67400815e-16]]
(1, 1)
```



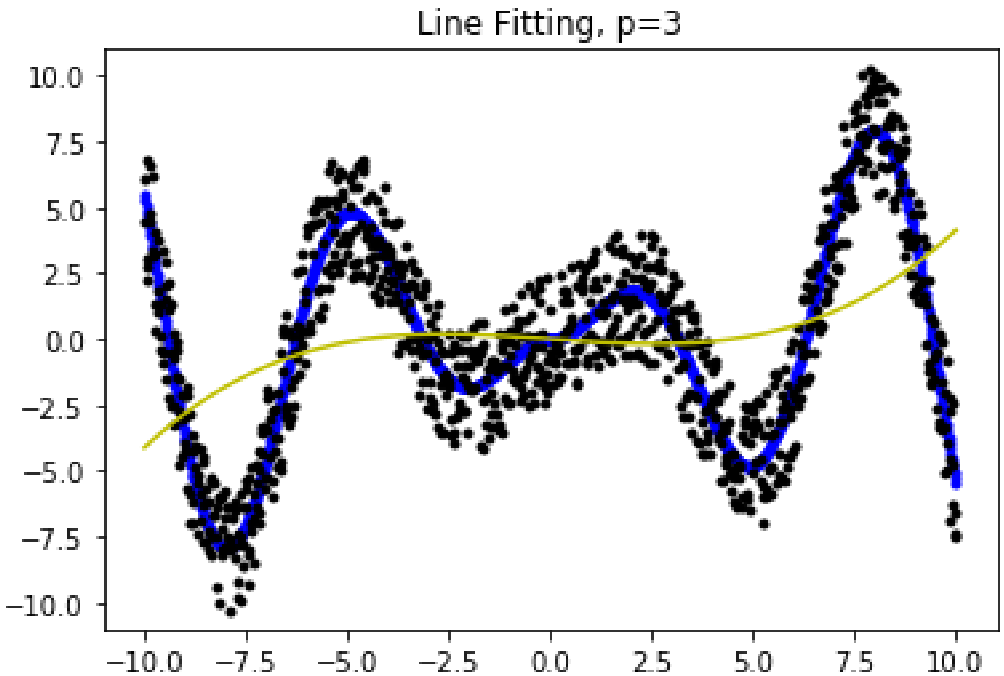

```
[[ 2.05786450e-01]
 [-1.52655666e-16]]
(2, 1)
```



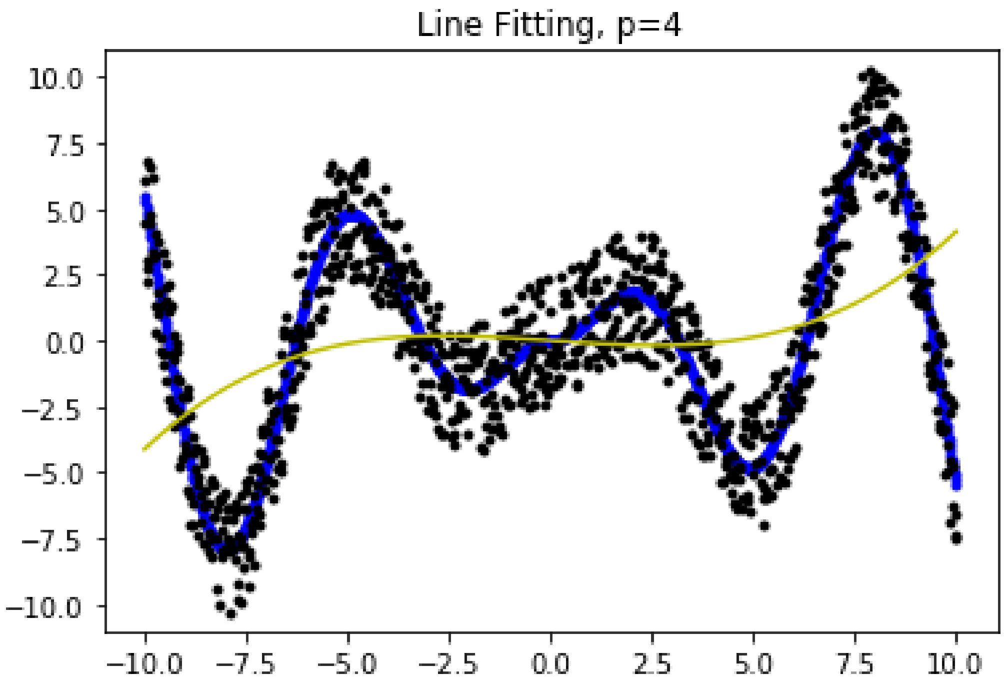

```
[[ 4.06575815e-19]
 [ 2.05786450e-01]
 [-4.70977424e-16]]
(3, 1)
```



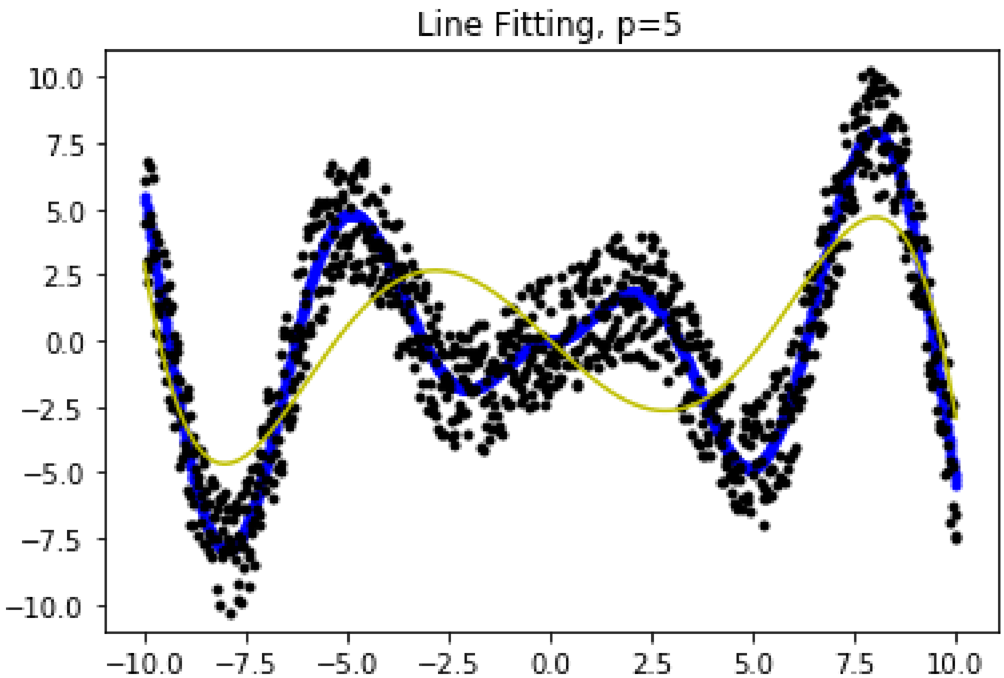
```
[[ 5.17592537e-03]
 [-1.33898968e-17]
 [-1.05389769e-01]
 [ 4.91794105e-16]]
(4, 1)
```



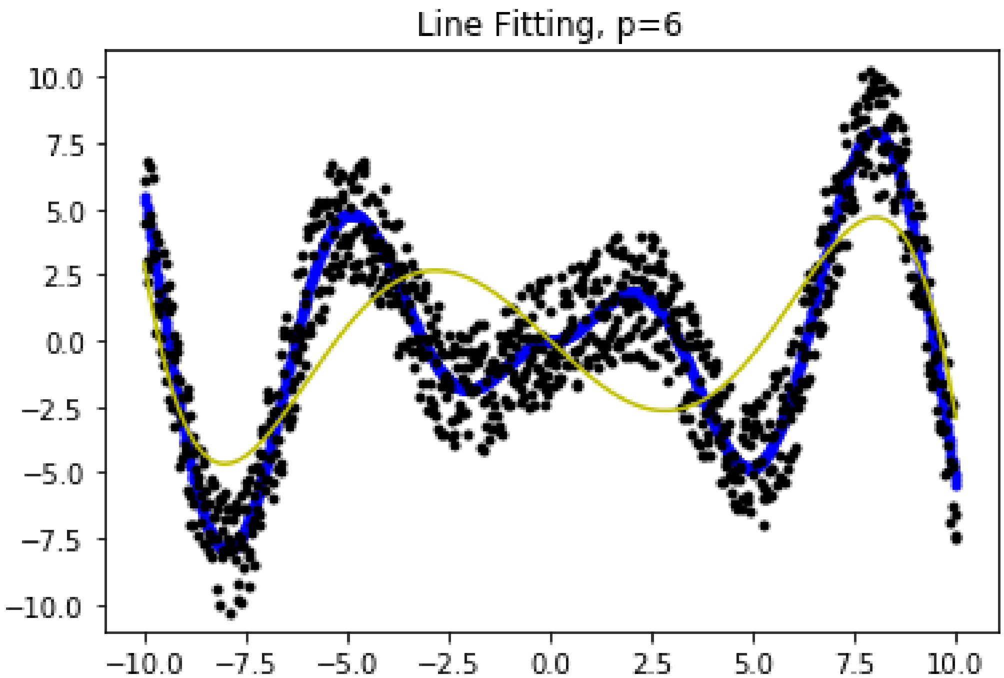

```
[[-1.80756831e-18]
 [ 5.17592537e-03]
 [ 1.73580768e-16]
 [-1.05389769e-01]
 [-2.30544750e-15]]
(5, 1)
```



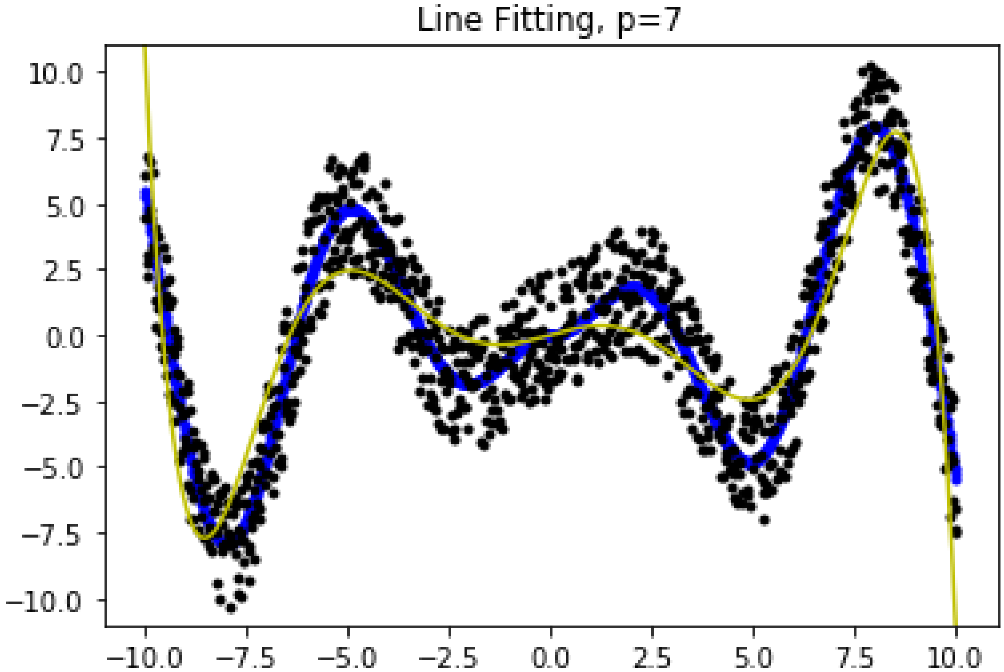

```
[[-5.59862621e-04]
 [-6.44829242e-17]
 [ 6.75069239e-02]
 [ 5.60272315e-15]
 [-1.44371556e+00]
 [-5.92512150e-14]]
(6, 1)
```



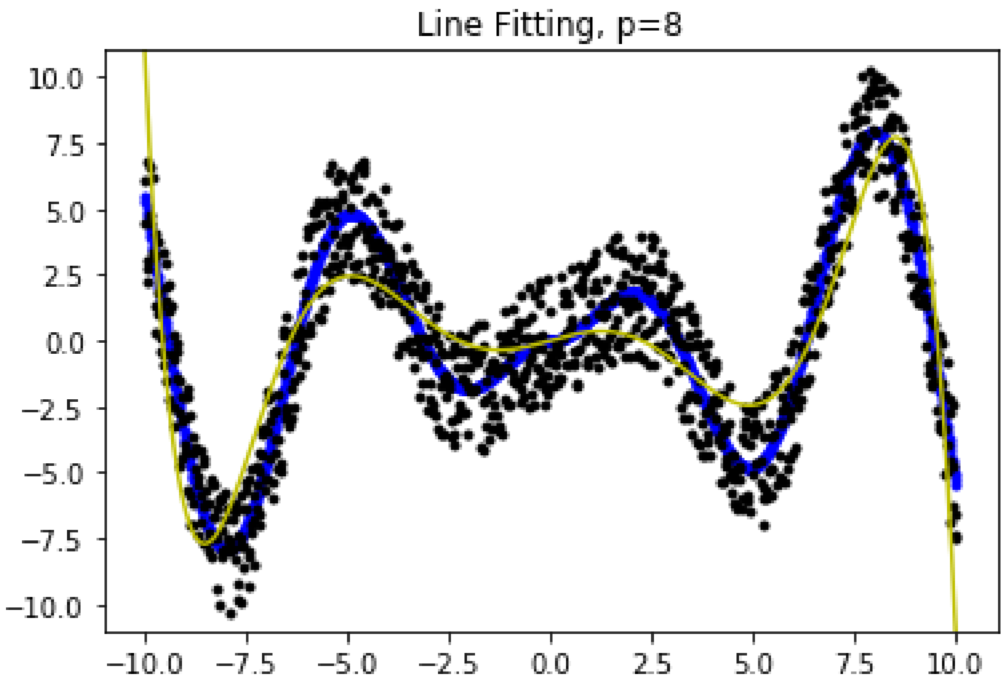

```
[[ 2.15133663e-17]
 [-5.59862621e-04]
 [-3.01294363e-15]
 [ 6.75069239e-02]
 [ 1.04602091e-13]
 [-1.44371556e+00]
 [-5.36573390e-13]]
(7, 1)
```



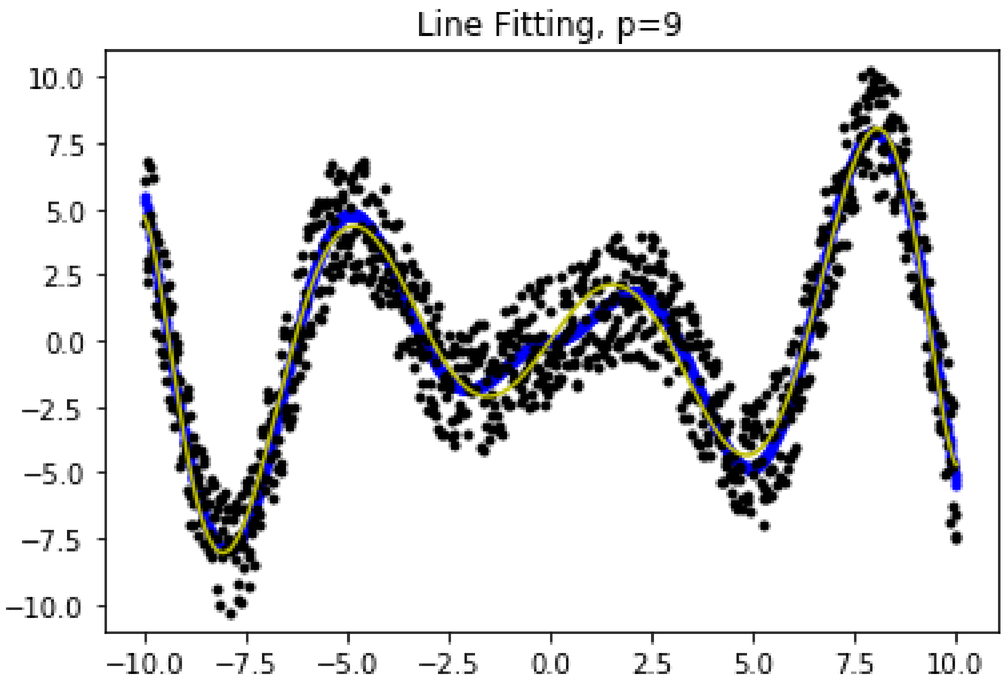

```
[[-2.29143441e-05]
 [-1.39157984e-17]
 [ 3.14903901e-03]
 [ 2.12746216e-15]
 [-1.01414214e-01]
 [-8.46228469e-14]
 [ 4.36909750e-01]
 [ 5.62313217e-13]]
(8, 1)
```




```
[[ 9.93301310e-18]
 [-2.29143441e-05]
 [-1.93508531e-15]
 [ 3.14903901e-03]
 [ 1.18403659e-13]
 [-1.01414214e-01]
 [-2.35029140e-12]
 [ 4.36909750e-01]
 [ 7.59585710e-12]]
(9, 1)
```



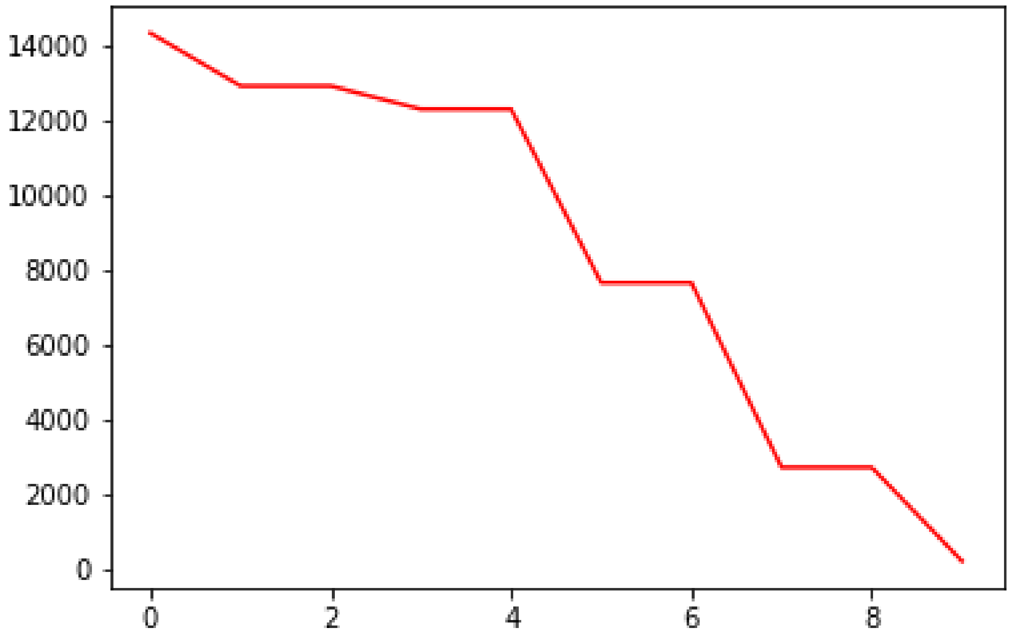

```
[[ 6.48406059e-07]
 [ 4.30506315e-17]
 [-1.60495278e-04]
 [-8.37175644e-15]
 [ 1.27987281e-02]
 [ 5.11339479e-13]
 [-3.49330563e-01]
 [-1.01421193e-11]
 [ 2.13058297e+00]
 [ 3.30059279e-11]]
(10, 1)
```




## 5 Plot Error with Varying p

As we stored sum of residuals in `resi_y`, we can check that error decreases as p goes higher.

```
In [5]: plt.plot(resi_x,resi_y,'r')
        plt.show()
```


14000
12000
10000
8000
6000
4000
2000
0
0
2
4
6
8