assignment09

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Mathematical Foundations for Computer Vision and Machine Learning
Assignment09 - Binary Classifier(Zero Classifier)
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Link to Github

1 Setting Up

We should get ready to read mnist files.

Careful! It's really huge! You should have enough free memory.

```
In [2]: import matplotlib.pyplot as plt
        import numpy as np
        import pandas as pd
        from pandas import Series, DataFrame

        file_data_train = "mnist_train.csv"
        file_data_test  = "mnist_test.csv"

        h_data_train    = open(file_data_train, "r")
        h_data_test     = open(file_data_test, "r")

        data_train      = h_data_train.readlines()
        data_test       = h_data_test.readlines()

        h_data_train.close()
        h_data_test.close()

        size_row    = 28    # height of the image
        size_col    = 28    # width of the image

        num_train   = len(data_train)   # number of training images
        num_test    = len(data_test)    # number of testing images
```

2 Functions

Theres some funtions implemented below.

We should normalize the input data to get rid of bias.
We are implementing binary classifier, so if label is 0, it is the answer. Otherwise, it isn't.

In [3]:
```
#
# normalize the values of the input data to be [0, 1]
#
def normalize(data):

    data_normalized = (data - min(data)) / (max(data) - min(data))

    return(data_normalized)

#
# return 1 only if the value is what we are looking for. Otherwise -1.
#
def bi_partitioning(M,val):
    length = len(M)
    res = np.zeros((length))
    for i in range(length):
        if(M[i] == val):
            res[i] = 1
        else:
            res[i] = -1
    return res

#
# sign funtion
#
def sign(x):
    if(x>=0):
        return 1
    else:
        return -1
```

3 Vectorize the Input Data

We should get first element of MNIST data to get label.
The real image data starts from second element.

In [4]:
```
#
# make a matrix each column of which represents an images in a vector form
#
list_image_train    = np.empty((size_row * size_col, num_train), dtype=float)
list_label_train    = np.empty(num_train, dtype=int)
list_image_test     = np.empty((size_row * size_col, num_test), dtype=float)
list_label_test     = np.empty(num_test, dtype=int)
```

```
count = 0

for line in data_train:

    line_data   = line.split(',')
    label       = line_data[0]
    im_vector   = np.asfarray(line_data[1:])
    im_vector   = normalize(im_vector)

    list_label_train[count]     = label
    list_image_train[:, count]  = im_vector

    count += 1

count = 0

for line in data_test:

    line_data   = line.split(',')
    label       = line_data[0]
    im_vector   = np.asfarray(line_data[1:])
    im_vector   = normalize(im_vector)

    list_label_test[count]      = label
    list_image_test[:, count]   = im_vector

    count += 1
```

4 Feature Function

We define a feature function `featureOf` with varying `p`.

$$\begin{aligned}
\theta_1 f_1(x_1) + \theta_2 f_2(x_1) + \cdots + \theta_p f_p(x_1) &= y_1 \\
\theta_1 f_1(x_2) + \theta_2 f_2(x_2) + \cdots + \theta_p f_p(x_2) &= y_2 \\
\theta_1 f_1(x_3) + \theta_2 f_2(x_3) + \cdots + \theta_p f_p(x_3) &= y_3 \\
&\vdots \\
\theta_1 f_1(x_n) + \theta_2 f_2(x_n) + \cdots + \theta_p f_p(x_1) &= y_n
\end{aligned}$$

We can write this as matmul.

$$\begin{pmatrix} f_1(x_1) & \cdots & f_p(x_1) \\ \vdots & \ddots & \vdots \\ f_1(x_n) & \cdots & f_p(x_n) \end{pmatrix} \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_p \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

Let's say that left tall matrix as `A`. Then,

$$A = \begin{pmatrix} f_1(x_1) & \cdots & f_p(x_1) \\ \vdots & \ddots & \vdots \\ f_1(x_n) & \cdots & f_p(x_n) \end{pmatrix} = \begin{pmatrix} r_1^T x_1 & \cdots & r_p^T x_1 \\ \vdots & \ddots & \vdots \\ r_1^T x_n & \cdots & r_p^T x_n \end{pmatrix} = \left(\begin{pmatrix} r_1^T \\ \vdots \\ r_p^T \end{pmatrix} (x_1 \cdots x_n) \right)^T$$

We can easily implement this feature funtion by using `R` where,

$$R = \begin{pmatrix} r_1^T \\ \vdots \\ r_p^T \end{pmatrix}$$

We can derive theta from equation above by pseudo inverse.

$$\theta = \left(A^T A\right)^{-1} A^T B$$

```
In [30]: R = np.zeros((size_row*size_col,size_row*size_col))
         for i in range(size_row*size_col):
             R[i] = np.random.normal(0,1,size=size_row*size_col)
         print(R)


         def featureOf(x, p):
             r=np.zeros((size_row*size_col,size_row*size_col))
             for i in range(p):
                 r[i] = R[i]
             return np.dot(r,x)
```

```
[[ 0.06209322  1.69742221  0.93905718 ...  0.85790117 -1.1091334
   1.88575297]
 [-0.01382504 -0.87095544  0.39818214 ...  0.76803312 -0.54626269
  -0.00873051]
 [ 1.48064805 -0.19944027 -0.30861911 ... -1.08976679 -0.43483775
   1.06988384]
 ...
 [ 1.22328131 -1.53053215  0.61355726 ...  1.63483972 -1.21843046
   0.60292559]
 [-2.2363228  -0.59812129 -0.44962861 ... -0.35739552  1.42024426
   0.48127517]
 [ 0.18784073 -0.32321428 -0.18496259 ... -0.21928588  1.10832017
   0.48382031]]
```

5 To make it Linearly Indepent

We should erase **zero column** from the input data. This makes Matrix A as **dependent**, which makes impossible to get **pseudo inverse**.

This means that in theta, the **ith value** is **0**.

$$\theta = [\theta_1 \theta_2 \cdots \theta_{784}]$$

$$\theta_i = 0$$

We get the **F1 score** of every p between 100~784.
And set index of maximum F1 score as variable m.

```
In [47]: scores = np.zeros(10)
         B = np.matrix(np.transpose(bi_partitioning(list_label_train,0)))   # (60000, 1)
         B_hat = bi_partitioning(list_label_test,0)

         for j in range(10):
             p = 2**j
             feature = featureOf(list_image_train, p)

             index = np.where(~feature.any(axis=1))[0]
             A = feature[~np.all(feature == 0, axis=1)]
             A = np.matrix(np.transpose(A))

             temp_theta = (A.T * A).I*A.T*B.T

             theta = np.zeros((size_col*size_row))
             count = 0
             for i in range(size_col*size_row):
                 if i not in index:
                     theta[i]=temp_theta[count]
                     count+=1

             table = np.zeros((2,2))

             #im_avg = np.zeros((size_row*size_col,4))

             feature = featureOf(list_image_test, p)
             for i in range(num_test):
                 if sign(theta.dot(feature[:,i])) == 1:
                     if(B_hat[i] == 1):
                         # True Positive
                         table[0][0] += 1
                         #im_avg[:,0] += list_image_test[:,i]
                     else:
                         # False Positive
                         table[1][0] += 1
                         #im_avg[:,1] += list_image_test[:,i]
                 else:
                     if(B_hat[i] == 1):
                         # False Negative
```

```
                        table[0][1] += 1
                        #im_avg[:,3] += list_image_test[:,i]
                    else:
                        # True Negative
                        table[1][1] += 1
                        #im_avg[:,2] += list_image_test[:,i]
            #     data = {
            #         'y_hat = +1' : [table[0][0],table[1][0],sum(table[:,0])],
            #         'y_hat = -1' : [table[0][1],table[1][1],sum(table[:,1])],
            #         'Total' : [sum(table[0]),sum(table[1]), sum(sum(table))]
            #     }
            #     frame = DataFrame(data,columns = ['y_hat = +1','y_hat = -1','Total'], index = ['y
            #     display(frame)
                prec = table[0][0]/(table[0][0]+table[1][0])
                rec = table[0][0]/(table[0][0] + table[0][1])
                F1 = 2*(prec*rec/(prec+rec))



                scores[j] = F1
                print(str(j) + " " + str(prec) + " " + str(rec) + " " + str(F1))
```

```
0 0.20144144144144144 0.5704081632653061 0.2977363515312916
1 0.3291139240506329 0.8755102040816326 0.4783942012824086
2 0.5494117647058824 0.47653061224489796 0.5103825136612022
3 0.6935866983372921 0.5959183673469388 0.6410537870472008
4 0.7749419953596288 0.6816326530612244 0.7252985884907709
5 0.8195232690124858 0.736734693877551 0.7759269210102095
6 0.895855472901169 0.860204081632653 0.8776678813118167
7 0.9331919406150583 0.8979591836734694 0.9152366094643786
8 0.9393939393939394 0.9173469387755102 0.9282395456892102
9 0.9422680412371134 0.9326530612244898 0.9374358974358974
```

```
In [51]: print("Best p = " + str(2**np.argmax(scores)))
         print(max(scores))

         m = 2**np.argmax(scores)
```

```
Best p = 512
0.9374358974358974
```

6 Average Image of Each Table Value

While classifing data, I also calculated average image of each TP,FP, TN, and FN.
Average image of each one is plotted.

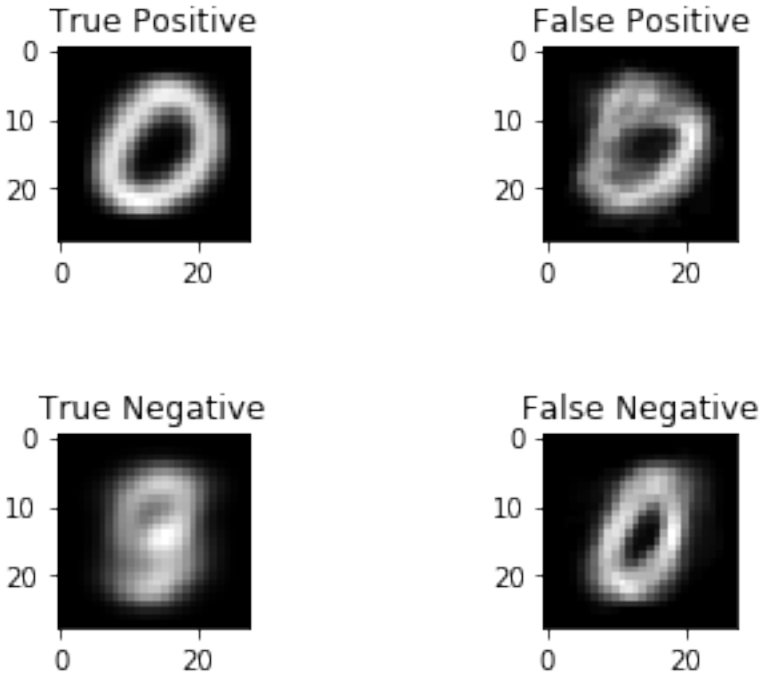
```
In [54]: feature = featureOf(list_image_train, m)
         index = np.where(~feature.any(axis=1))[0]
         A = feature[~np.all(feature == 0, axis=1)]
         A = np.matrix(np.transpose(A))
         B = np.matrix(np.transpose(bi_partitioning(list_label_train,0)))   # (60000, 1)
         # print(A)
         temp_theta = (A.T * A).I*A.T*B.T
         theta = np.zeros((size_col*size_row))
         count = 0
         for i in range(size_col*size_row):
             if i not in index:
                 theta[i]=temp_theta[count]
                 count+=1
         table = np.zeros((2,2))

         im_avg = np.zeros((size_row*size_col,4))
         feature = featureOf(list_image_test, m)
         for i in range(num_test):
             if sign(theta.dot(feature[:,i])) == 1:
                 if(B_hat[i] == 1):
                     # True Positive
                     table[0][0] += 1
                     im_avg[:,0] += list_image_test[:,i]
                 else:
                     # False Positive
                     table[1][0] += 1
                     im_avg[:,1] += list_image_test[:,i]
             else:
                 if(B_hat[i] == 1):
                     # False Negative
                     table[0][1] += 1
                     im_avg[:,3] += list_image_test[:,i]
                 else:
                     # True Negative
                     table[1][1] += 1
                     im_avg[:,2] += list_image_test[:,i]


         im_avg[:,0] /= table[0][0]
         im_avg[:,1] /= table[1][0]
         im_avg[:,2] /= table[1][1]
         im_avg[:,3] /= table[0][1]

         p1 = plt.subplot(2,2,1)
         p1.imshow(im_avg[:,0].reshape((size_row, size_col)),cmap='gray')
         p1.set_title("True Positive")
         p2 = plt.subplot(2,2,2)
         p2.imshow(im_avg[:,1].reshape((size_row, size_col)),cmap='gray')
```

```
p2.set_title("False Positive")
p3 = plt.subplot(2,2,3)
p3.imshow(im_avg[:,2].reshape((size_row, size_col)),cmap='gray')
p3.set_title("True Negative")
p4 = plt.subplot(2,2,4)
p4.imshow(im_avg[:,3].reshape((size_row, size_col)),cmap='gray')
p4.set_title("False Negative")
plt.subplots_adjust(hspace=1)
```



In [ ]: