In [36]:

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Question 1

In [17]:

```
K = 15 # capacity of a shuttle if it is dispatched
env = [1,2,3,4,5] # A random variable giving the number of customers arriving du
ring time interval t
Cf = 100 #The cost of dispatching a shuttle
Ch = 2 #The cost per customer left waiting per time period.
S = list(range(201))
gamma = 0.95 #discount rate
stateMax = 200
unif_dist = 1/len(env)
```

In [111]:

```
def getReward(s, a, Cf, Ch):
    #a -> dispatch
    c = (a*Cf) + Ch*max(s-a*K, 0)
    return -c

def getNextState(s,a,K,Arr,stateMax):
    #new arrivals
    s_prime = min(s + Arr, stateMax)
    #action
    s_prime = s_prime - a * min(K, s_prime)
    return s_prime

def getEV(s, a, K, stateMax, env, unif_dist, V_t1):
    states = [getNextState(s, a, K, Arr, stateMax) for Arr in env]
    values = [unif_dist* V_t1[s] for s in states]
    ev = sum(values)
    return ev
```

A) Enumeration

In [112]:

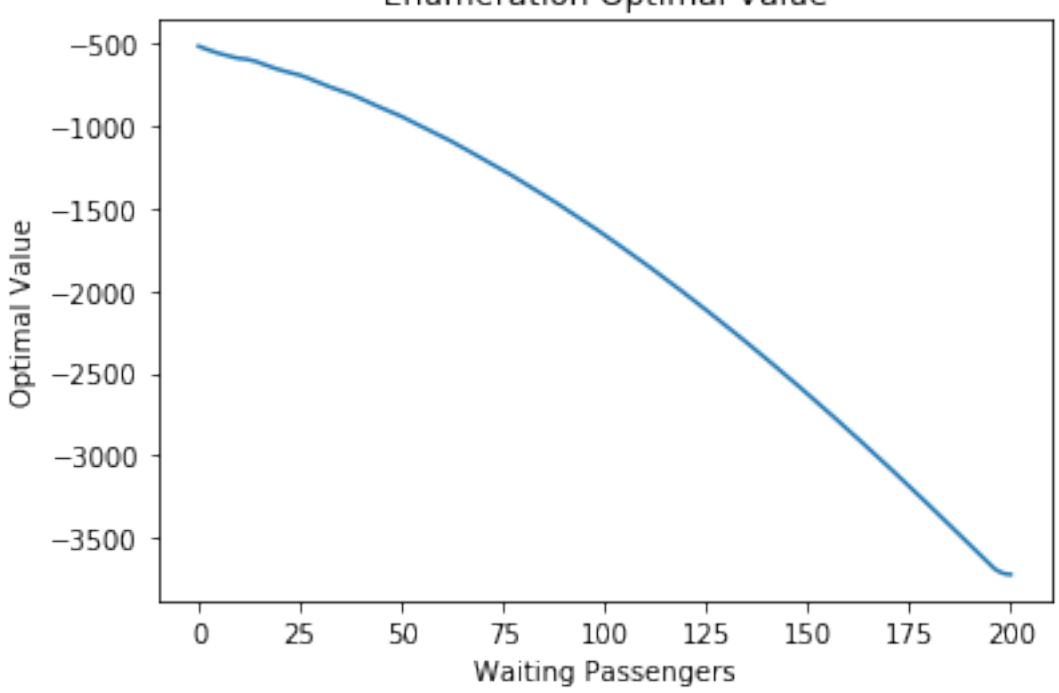
```
def enumeration(T):
    V_t1 = np.zeros(201)
    V_t = np.zeros(201)
    for t in range(T, -1, -1):
        for s in S:
            V_t[s] = max([getReward(s, a, Cf, Ch)
                          + gamma * getEV(s, a, K, stateMax, env, unif_dist, V_t
1) for a in [0,1]])
        V_t1 = V_t
    return V_t
```

In [113]:

```
Vt = enumeration(500)
plt.plot(Vt)
plt.xlabel("Waiting Passengers")
plt.ylabel("Optimal Value")
plt.title("Enumeration Optimal Value")
```

Out[113]:

```
Text(0.5,1,'Enumeration Optimal Value')
```



B) Value iteration

In [200]:

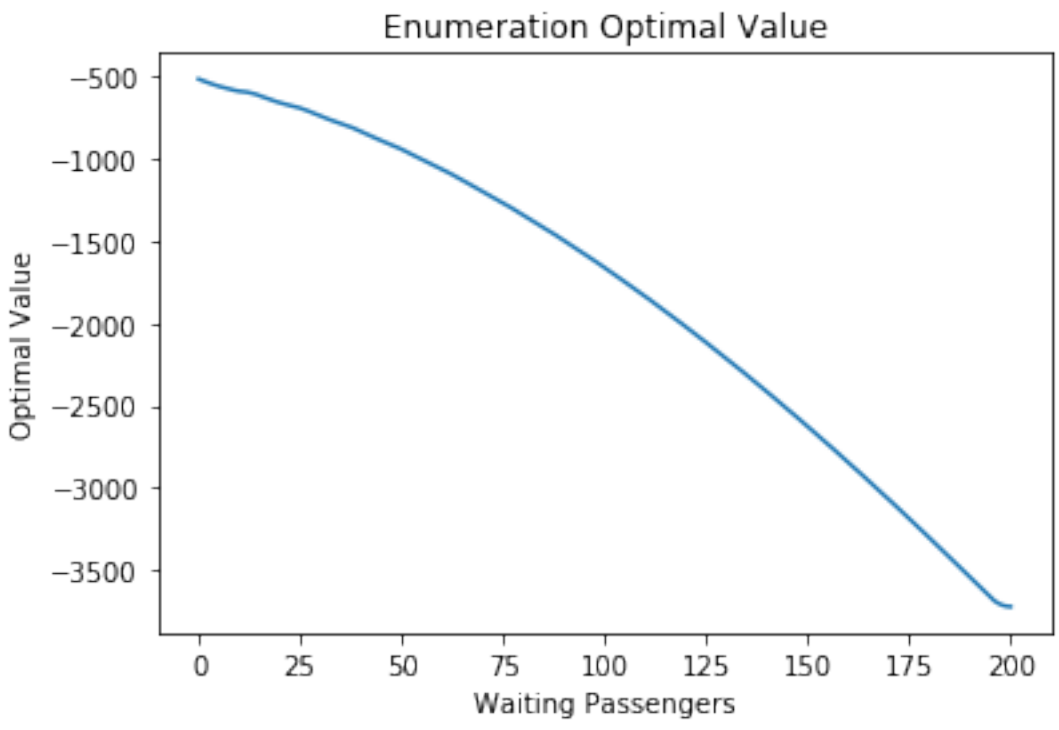
```
V_t1 = np.zeros(201)
V_t = np.zeros(201)
def valueIter(delta, theta):
    while delta > theta:
        V_t1 = V_t.copy()
        for s in S:
            V_t[s] = max([getReward(s, a, Cf, Ch)
                          + gamma * getEV(s, a, K, stateMax, env, unif_dist,
V_t1) for a in [0,1]])
        delta = min(delta,max([V_t1[s]-V_t[s] for s in S]))
    return V_t
```

In [201]:

```
theta = 1e-5
delta = 1
Vt = valueIter(delta,theta)
plt.plot(Vt)
plt.xlabel("Waiting Passengers")
plt.ylabel("Optimal Value")
plt.title("Enumeration Optimal Value")
```

Out[201]:

```
Text(0.5,1,'Enumeration Optimal Value')
```



C) Policy iteration

In [217]:

```
def policyIter(delta, theta):
    V_t = np.zeros(201)
    policy = [np.random.randint(2) for s in S]
    policy_new = [np.random.randint(2) for s in S]
    bestAction = [None for s in S]
    c = 0
    while True:
        c = c+1
        print(c,"th loop")
        #policy evaluation
        while delta > theta:
            V_t1 = V_t.copy()
            for s in S:
                EVs = [getReward(s, a, Cf, Ch)
                                        + gamma * getEV(s, a, K, stateMax, env, un
if_dist, V_t1) for a in [0,1]]
                V_t[s] = EVs[policy[s]]
#                 print("rewards: ", EVs )
#                 print("argMax: ",np.argmax(EVs) )
                bestAction[s] = np.argmax(EVs)
            delta = min(delta,max([V_t1[s]-V_t[s] for s in S]))

        policy_new = bestAction.copy()
#         print("new policy is: ", policy_new)
        #improve policy
        if all([policy_new[s] == policy[s] for s in S]):
            return policy
        else:
            policy = policy_new.copy()
```

In [218]:

```
delta = 1
theta = 1e-5
policy = policyIter(delta,theta)
```

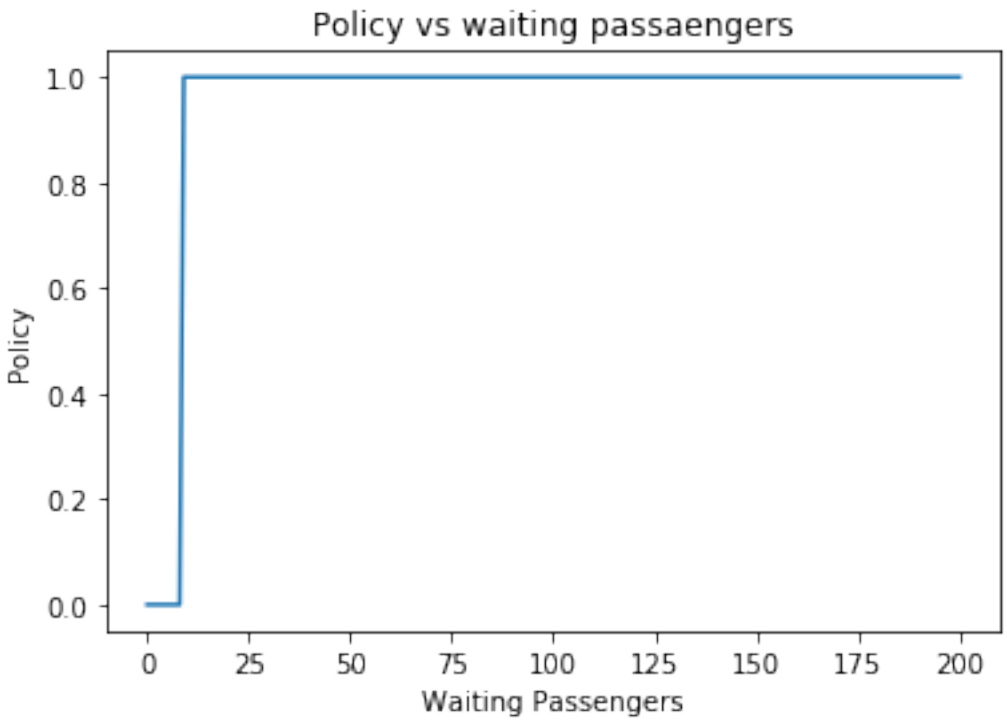
```
1 th loop
2 th loop
```

In [221]:

```
plt.plot(policy)
plt.xlabel("Waiting Passengers")
plt.ylabel("Policy")
plt.title("Policy vs waiting passaengers")
```

Out[221]:

```
Text(0.5,1,'Policy vs waiting passaengers')
```



Question 2

Here are code for illustration. Due to running time and memory issue, do not run the following script.

In [247]:

```
#All possible arrivals in term of type:number
arrivals = {x: env for x in Ch}
keys, values = zip(*arrivals.items())
all_arrs = [dict(zip(key, v)) for v in itertools.product(*value)]
```

In [238]:

```
def createState(cap, arr_cap):
    S = []
    V_t1 = {}
    V_t = {}
    curPolicy = {}
    newPolicy = {}
    for state1 in range(0, cap + 1):
        for state2 in range(0, cap + 1):
            for state3 in range(0, cap + 1):
                for state4 in range(0, cap+1):
                    for state5 in range(0, cap+1):
                        S.append([state1, state2, state3, state4, state5])
                        V_t1[(state1, state2, state3, state4,state5)] = 0
                        V_t[(state1, state2, state3, state4,state5)] = 0
                        curPolicy[(state1, state2, state3, state4,state5)] = np.
random.randint(2)
                        curPolicy[(state1, state2, state3, state4,state5)] = np.
random.randint(2)
    env = []
    for env1 in range(1, maxArrivals + 1):
        for env2 in range(1, maxArrivals + 1):
            for env3 in range(1, maxArrivals + 1):
                for env4 in range(1, maxArrivals + 1):
                    for env5 in range(1, maxArrivals + 1):
                        E.append([env1, env2, env3,env4, env5])

    return S, V_t1, V_t, curPolicy, newPolicy, env
```

In [ ]:

```
def getReward(s, a, Cf, Ch, cap):
    #a -> dispatch
    c = (a*Cf) + sum(Ch*getNextState(s, a, K, cap, env))
    return -c

def getNextState(s,a,K,cap,env):
    #new arrivals
    s_prime = s.copy()
    s_prime = np.clip(np.add(s_prime,env),0,cap)
    #action
    if a == 1:
        for i in range(len(s_prime), -1, -1):
            s_prime[i] =  max(s_prime[i] - K, 0)
            K = K - (s[i] - s_prime[i])
    return s_prime

def getEV(s, a, K, cap, env, unif_dist, V_t1):
    states = [getNextState(s,a,K,cap,env) for e in env]
    values = [(1/len(env))* V_t1[s] for s in states]
    ev = sum(values)
    return ev
```

a) Enumeration

In [ ]:

```
def enumeration(T):
    for t in range(T, -1, -1):
        for s in S:
            V_t[s] = max([getReward(s, a, Cf, Ch)
                          + gamma * getEV(s, a, K, stateMax, env, unif_dist, V_t
1) for a in [0,1]])
        V_t1 = V_t.copy()
```

In [ ]:

```
K = 30
Cf = 100
Ch = [1,1.5,2,2.5,3]
gamma = 0.95
cap = 100
arr_cap = 5

S, V_t1, V_t, curPolicy, newPolicy, env = createState(cap, arr_cap)
Vt = enumeration(T)
```

b) Value Iteration

In [ ]:

```
def valueIter(delta, theta):
    while delta > theta:
        V_t1 = V_t.copy()
        for s in S:
            V_t[s] = max([getReward(s, a, Cf, Ch)
                          + gamma * getEV(s, a, K, stateMax, env, unif_dist,
V_t1)
        delta = min(np.abs(V_k1[s] - V_k[s]), delta)
    return V_t
```

In [ ]:

```
delta = 1
theta = 1e-5

V_t = valueIter(delta, theta)
```

c) Policy Iteration

In [ ]:

```
def policyIter(delta, theta):
    V_t = np.zeros(201)
    policy = [np.random.randint(2) for s in S]
    policy_new = [np.random.randint(2) for s in S]
    bestAction = [None for s in S]
    c = 0
    while True:
        c = c+1
        print(c,"th loop")
        #policy evaluation
        while delta > theta:
            V_t1 = V_t.copy()
            for s in S:
                EVs = [getReward(s, a, Cf, Ch)
                                        + gamma * getEV(s, a, K, stateMax, env, un
if_dist, V_t1) for a in [0,1]]
                V_t[s] = EVs[policy[s]]
                bestAction[s] = np.argmax(EVs)
            delta = min(delta,max([V_t1[s]-V_t[s] for s in S]))

        policy_new = bestAction.copy()
#         print("new policy is: ", policy_new)
        #improve policy
        if all([policy_new[s] == policy[s] for s in S]):
            return policy
        else:
            policy = policy_new.copy()
```

In [ ]:

```
delta = 1
theta = 1e-5
policy = policyIter(delta,theta)
```