

#### Astrofisica Generale II — 5

Maurizio Tomasi (maurizio.tomasi@unimi.it)

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# Interstellar Gas (ISG)



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In the 1930s, astronomers observed stellar spectra with weird optical absorption lines:

- 1. In binary systems, they do not show the Doppler effect;
- 2. They are more pronounced for more distant stars;
- 3. They are much narrower than stellar ones (o  $T \sim 100 \, \mathrm{K}$ ).



# Interstellar Gas (ISG)

- ullet Interstellar H is not observed in the visible spectrum: if T is low, Balmer's lines are too weak!
- The observed elements are Ca and Na, but also molecules: CH, CN, CH $^+$ . The latter imply a low gas density ( $n < 10^3\,{
  m cm}^{-3}$ ) and a low temperature:
  - Charged molecules like CH<sup>+</sup> neutralize quickly in laboratory conditions;
  - CH and CN are highly reactive.



## HI in ISG

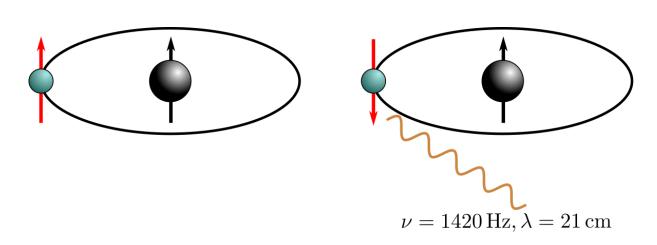
- It is reasonable to expect that H, even if not detectable in the visible spectrum, is the predominant component of the ISG. It can be revealed by measuring the 21 cm line.
- ullet This line is generated by the transition between the state of the HI atom with e/p spins parallel to the state with antiparallel spins. The two states have an energy difference of

$$\Delta E = 5.9 \times 10^{-6} \, \mathrm{eV},$$

and the transition probability is  $A=(11\,{
m Myr})^{-1}$  so that  $N=N_0e^{At}$  .



#### HI in ISG



The temperature associated with this radiation is

$$T_{
m 21\,cm} \sim rac{\Delta E}{k_B} = rac{5.9 imes 10^{-6}\,{
m eV}}{8.62 imes 10^{-5}\,{
m eV/K}} pprox 0.07\,{
m K}.$$

The CMB (2.7 K) is enough to populate the state with parallel spins!



## Facts about the 21 cm Line

The spin-parallel state  $\left(s=1\right)$  is such that

$$S=\sqrt{2}\hbar, \quad S_z=egin{cases} +\hbar/2,\ 0,\ -\hbar/2 \end{cases} ext{ (tripletto)},$$

while for the anti-parallel state (s=0)

$$S=0, \quad S_z=0 \quad \text{(singoletto)}.$$

If we assume that the gas is in thermal equilibrium and that the kinetic gas theory is valid (so we ignore the CMB and the fact that HI is not point-like), then we can use Maxwell's distribution:

$$rac{N_{
m tr}}{N_{
m sing}} = rac{g_{
m tr}}{g_{
m sing}} e^{-\Delta E/k_B T} = 3 e^{-\Delta E/k_B T}.$$

ullet But if  $k_BT\gg \Delta E$  , then

$$rac{N_{
m tr}}{N_{
m sing}} = 3e^{-\Delta E/k_BT} pprox 3.$$

• At the typical temperature of the Universe (≥ 2.7 K), there are three triplet atoms every singlet atom.



# Importance of the 21 cm Line

- The existence of this line was predicted in the '40 and revealed on March, 25th 1951 by Edwin Purcell's team (Harvard Univ., Nobel 1953).
- The characteristics of the line are:
  - 1. Visible both in emission and absorption;
  - 2. Insensitive to the presence of dust.



# Importance of the 21 cm Line

The 21-cm line has a wide range of applications:

- Fundamental for the study of gas in the ISM;
- Being insensitive to dust, it allows studying the structure of the Galaxy;
- Galaxy rotation and local motions can be reconstructed from Doppler measurements on the line;
- Study of ISM magnetic fields from the Zeeman effect on the line;
- ...and much more.



# Numerical Example

Suppose that a cloud of neutral H is located at a distance  $d=30\,\mathrm{pc}$ . The flux at 21 cm in emission, integrated over the solid angle, is

$$f = 4.5 \times 10^{-15} \, \mathrm{erg \, cm^{-2} \, s^{-1}}.$$

What is the mass of the hydrogen in the cloud?



# Solution

From the observed flux we can derive the total luminosity:

$$L_{21\,\mathrm{cm}} = 4\pi d^2 f = 4.85 imes 10^{26}\,\mathrm{erg\,s^{-1}}$$
 .



## Solution

We expect the following formula to hold:

$$L_{21\,\mathrm{cm}}pproxrac{3}{4}N_H\,A\,h
u,$$

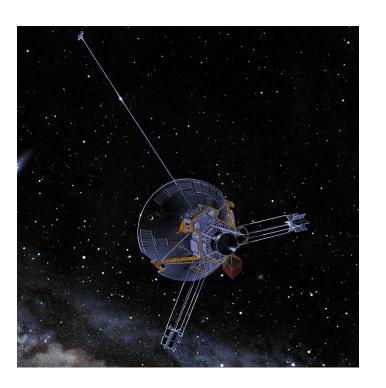
where  $A= au^{-1}=(11\,{
m Myr})^{-1}$  is the transition probability and the factor 3/4 takes into account the population in the two spin states. Therefore

$$N_Hpprox 2.4 imes 10^{58}, \qquad M_H=N_H imes m_ppprox 20\,M_\odot.$$

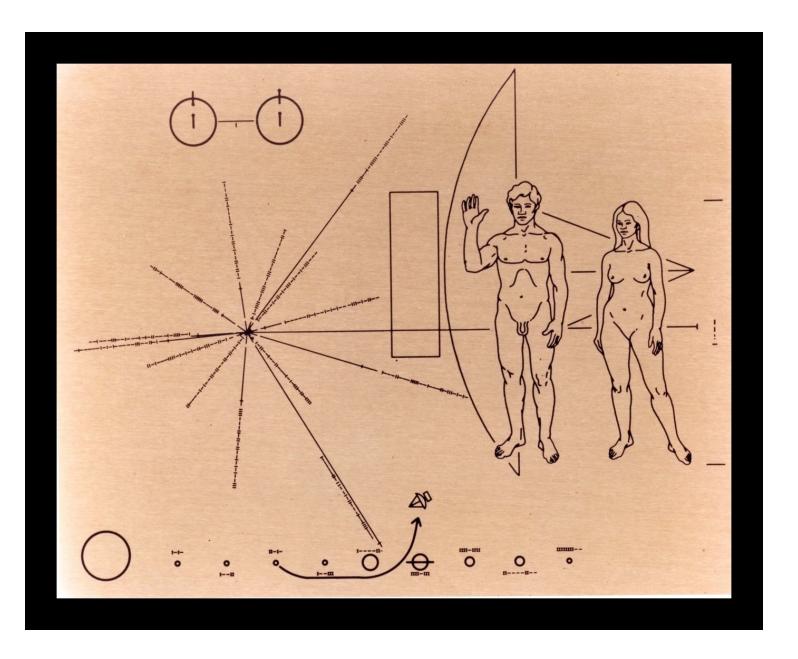


# Pioneer 10 (1972)

An interesting context in which the 21 cm line played an important role is the famous plaque installed on the Pioneer 10 probe, launched in 1972 by NASA to study Jupiter.







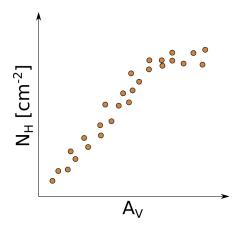


# HI in ISM

Quantity	Estimate
Temperature	100 K
Cloud size	10÷100 pc
HI density (cloud)	1÷10 cm <sup>-3</sup>
HI density (Galaxy)	$0.1  \text{cm}^{-3}$
Speed	$v_{ m rms} \sim \sqrt{k_B T/m_p} \sim 10^3{ m m/s}$



A correlation is observed between the HI column density (21 cm line) and dust (extinction measurements), therefore these components are mixed in the ISM.



The correlation ceases for high values of  $A_V$ . Why?

- 1. Does dust attenuate at 21 cm? No, insufficient n.
- 2. Do H<sub>2</sub> molecules form?



- The H<sub>2</sub> molecule is very difficult to detect because it does not emit the equivalent of the 21 cm line. Furthermore, it does not have a permanent dipole.
- Molecules like CO have a permanent dipole, and since the rotational energy is quantized,

$$E_r = rac{(I\omega)^2}{2I} = rac{L^2}{2I} = rac{\hbar^2 J(J+1)}{2I}.$$



- If there is a permanent dipole, the selection rule  $\Delta J = -1$  applies. A transition between rotational energy levels of CO therefore generates lines (v > 115 GHz).
- But this does not apply to  $H_2$ , which has only a weak quadrupole with selection rule  $\Delta J = -2$ .
- This generates weak emission around 10 μm, however covered by dust.
- It is easier to study the emission of other molecules, less abundant but with stronger lines (CO, CH, OH, CS, C<sub>3</sub>H<sub>2</sub>...).



- Does the loss of correlation between  $N_H$  and  $A_V$  tell us then that part of the ISM hydrogen is in molecular form?
- The densities involved would seem to advise against it: it is difficult to produce H<sub>2</sub> because (again!) of its symmetry.



- To join two H atoms together, it is necessary to bind them in an excited state, and then de-excite the system by radiating energy. But H<sub>2</sub> has no dipole moment, so it does not radiate!
- To produce H<sub>2</sub> it is first necessary that H<sup>-</sup> is formed:

$$\mathrm{H} + e^- 
ightarrow \mathrm{H}^- + h 
u, \ \mathrm{H}^- + \mathrm{H} 
ightarrow \mathrm{H}_2 + e^- + \mathrm{kinetic\ energy}$$

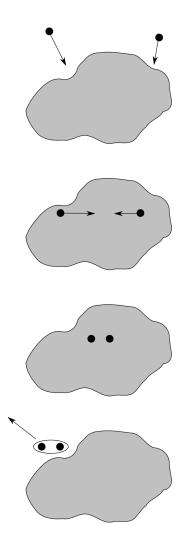
• But it is difficult to have H<sup>-</sup> in a cloud: it is slow to form and fast to destroy (by collisions with protons, photons or other positive ions).



# Dust and H<sub>2</sub> Molecules

- Dust can act as a catalyst. Nuclei are captured by grains and, after a random walk, they settle in sites from which they no longer move.
- Thus it is easier to make nuclei and electrons react with each other. To produce H<sub>2</sub>, the kinetic energy produced is 4.5 eV, sufficient to expel the molecule from the grain (and give it angular momentum...).





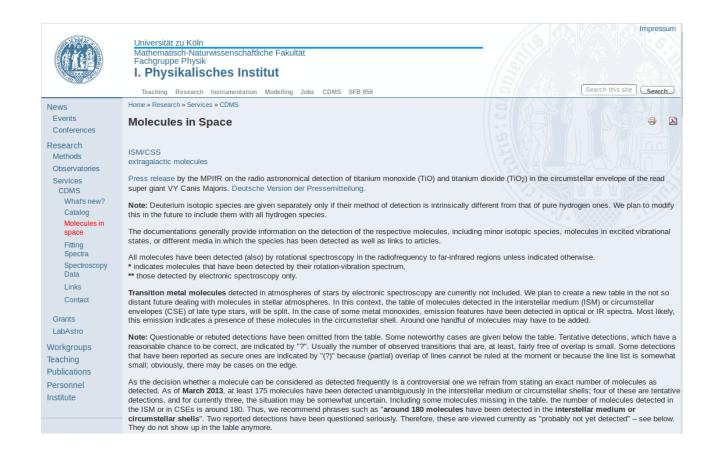


## Molecular Clouds

- In the interstellar medium (ISM), we can observe clouds composed of molecules.
- They are characterized by low temperatures (  $\sim 10\,\mathrm{K}$  ) and high densities (  $n\sim 10^3\,\mathrm{cm}^{-3}$  ).

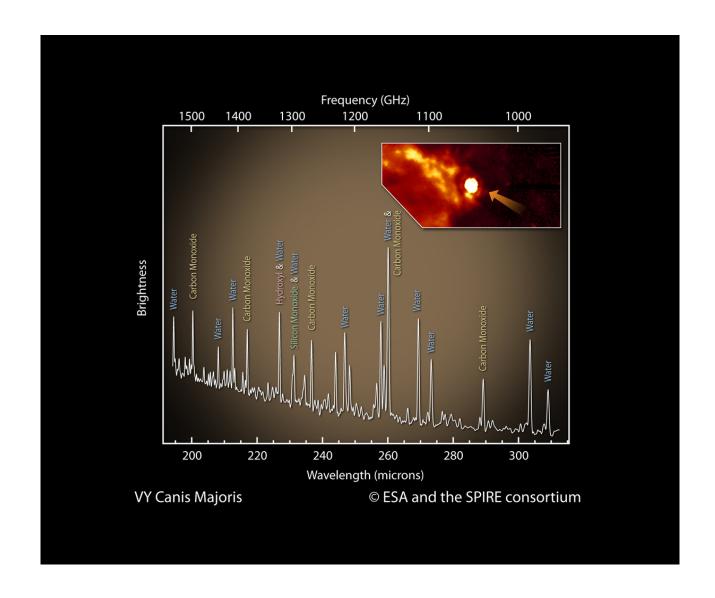


# Molecules found in the ISM



www.astro.uni-koeln.de/cdms/molecules





VY Canis Majoris (giant star,  $R\sim 2000\,R_\odot$ ) seen by Herschel



## What about HII?

- We have analyzed the presence in the galaxy of atomic hydrogen (HI) and molecular hydrogen (H<sub>2</sub>).
- The case of HII is equally interesting; however, we will discuss it in the context of star formation.





Under what conditions does a gas cloud induce the formation of a star?

Let's assume that the cloud is spherical and has uniform density. Gas and dust are present within it. For collapse to occur, the system needs to "de-virialize":

$$-U\gtrsim 2K, \ rac{3}{5}rac{GM^2}{R}\gtrsim 2rac{1}{2}rac{M}{m}k_BT.$$



But M and R are related to the density ho of the cloud (assumed constant):

$$M=rac{4}{3}\pi R^3
ho,$$

therefore

$$R>\sqrt{rac{15k_BT}{4\pi Gm
ho}}\equiv R_J'.$$

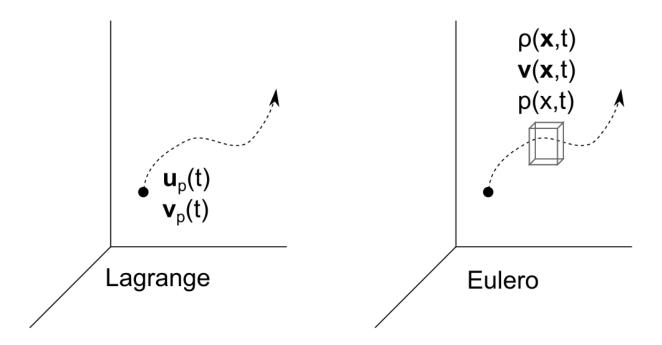
(We use the notation  $R_J^\prime$  because shortly we will derive the true value of  $R_J$  obtained by Jeans).



- The previous calculation is quite different from the one tackled by James Jeans (1877–1946), who did not use the virial theorem.
- We will now redo his calculations, and also highlight a logical problem in his treatment.
- Let's start by introducing the equations of fluid dynamics.



# Fluid Physics



- In the **Lagrangian** point of view, we describe the trajectory of the particle (analogously to Newton's laws).
- In the **Eulerian** point of view (the most convenient), we focus on the points of space.



# Newton's Equation

Since we know how to describe the motion of particles using Newtonian physics, we start from the Lagrangian point of view:

$$ec{F}_p = m\,ec{a}_p = m\,\dot{ec{v}}_p,$$

and express  $ec{v}_p$  in terms of Eulerian quantities:

$$ec{v}_p(t) = ec{v}ig(ec{u}_p(t),tig).$$



If we calculate the derivative of the product, we get

$$egin{aligned} \dot{ec{v}}_p &= rac{\mathrm{d}}{\mathrm{d}t} ec{v}ig(ec{u}_p(t),tig) = rac{\mathrm{d}}{\mathrm{d}t} ec{v}ig(u_{px}(t),u_{py}(t),u_{pz}(t),tig) = \ &= \partial_t ec{v} + (ec{v}\cdotec{
abla})ec{v}, \end{aligned}$$

where we exploit the fact that  $\partial_t ec{u}_p(t) = ec{v}_p(t) = ec{v}ig(ec{u}_p(t),tig)$  and that

$$(ec{v}\cdotec{
abla})ec{v} = egin{pmatrix} v_x\partial_xv_x + v_y\partial_yv_x + v_z\partial_zv_x \ v_x\partial_xv_y + v_y\partial_yv_y + v_z\partial_zv_y \ v_x\partial_xv_z + v_y\partial_yv_z + v_z\partial_zv_z \end{pmatrix}.$$



#### **Material Derivative**

• The *material derivative* is the expression

$$\dot{ec{v}}_p = \partial_t ec{v} + (ec{v} \cdot ec{
abla}) ec{v}$$

- It states that the change in velocity of a fluid particle can be caused by:
  - 1. a temporal variation of the field  $\vec{v}$  within the small volume element (term  $\partial_t \vec{v}$  );
  - 2. a velocity difference between the volume element where the particle is located at time t and the one it has "jumped" to at time  $t+\mathrm{d}t$  (term  $(\vec{v}\cdot\vec{\nabla})\vec{v}$ ).

- Let's now transform Newton's equation so that the Eulerian quantities ho and  $ec{v}$  appear, but by moving from the equation describing *one* particle to the one describing N particles.
- Therefore, we sum the Newton's equations for each of the N particles in a small volume element:

$$egin{aligned} \sum_{i=1}^N ec{F}_p^{(i)} &= \sum_{i=1}^N m^{(i)} \dot{ec{v}}_p(t) = \sum_{i=1}^N m^{(i)} ig(\partial_t ec{v} + (ec{v} \cdot ec{
abla}) ec{v}ig) &= \ &= ig(\partial_t ec{v} + (ec{v} \cdot ec{
abla}) ec{v}ig) \sum_{i=1}^N m^{(i)}. \end{aligned}$$



If we assume that the masses  $m^{(i)}$  of the particles are all identical, then

$$egin{aligned} \sum_{i=1}^N ec{F}_p^{(i)} &= \left(\sum_{i=1}^N m^{(i)}
ight) \left(\partial_t ec{v} + (ec{v} \cdot ec{
abla}) ec{v}
ight), \ ec{F}_{ ext{tot}} &= 
ho \, \mathrm{d}V ig(\partial_t ec{v} + (ec{v} \cdot ec{
abla}) ec{v}ig), \end{aligned}$$

with  ${
m d}V$  being the volume of the small element and  $ec{F}_{
m tot}$  the total force acting on the small volume; note that all internal action/reaction forces cancel out.



#### **Force Terms**

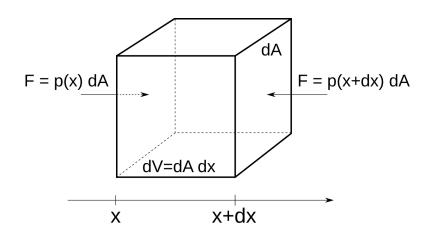
- ullet We now need to derive an expression for the term  $ec{F}_p$ . In the context of cloud collapse, there are two components:
  - 1. Pressure forces;
  - 2. Gravitational forces.

These are the same forces we considered in the derivation of the virial equation U=-2K.

Let's address them separately.



#### Pressure Forces



Let's consider only the force  $F_{
m pressure}$  exerted on the small volume along the x direction. If the forces are normal to the faces (perfect fluid), then

$$egin{aligned} F_{ ext{pressure}} &= ig( p(x) - p(x + \mathrm{d}x) ig) \, \mathrm{d}A \ &= -\partial_x p(x) \, \mathrm{d}x \, \mathrm{d}A = -\partial_x p(x) \, \mathrm{d}V. \end{aligned}$$



## Pressure forces

If we now consider motion in three dimensions instead of just along the x axis, the result generalizes trivially:

$$ec{F}_{
m pressure} = -ec{
abla} p \, \mathrm{d}V.$$



## **Gravitational force**

In the case of gravity, it is easy to express the force in terms of the potential  $\phi$ :

$$ec{F}_{
m grav} = - m \, ec{
abla} \phi,$$

where (Poisson's law)

$$abla^2 \phi = 4\pi G 
ho$$

and obviously  $m=
ho\,\mathrm{d}V$  .



#### Momentum conservation

The momentum conservation equation in the case of a cloud is therefore

$$ho\,\mathrm{d}V\,\left(\partial_tec{v}+(ec{v}\cdotec{
abla})ec{v}
ight)=ec{F}_{\mathrm{pressure}}+ec{F}_{\mathrm{grav}}=\ =-ec{
abla}p\,\mathrm{d}V-ec{
abla}\phi\,
ho\,\mathrm{d}V,$$

which can be rewritten as the system of 3 equations

$$\partial_t ec{v} + (ec{v} \cdot ec{
abla}) ec{v} = -rac{ec{
abla} p}{
ho} - ec{
abla} \phi$$

(a special case of the Navier-Stokes equations).



# Other equations

- With the previous vector equation and Gauss's law we have 4 equations but 6 unknowns  $(v_x, v_y, v_z, p, \rho, \phi)$ .
- We also use the mass conservation equation:

$$\dot{
ho} + \vec{
abla} \left( 
ho \, \vec{v} 
ight) = 0$$

and the relationship between pressure and density

$$p=
ho c_S^2,$$

where  $c_S$  is the speed of sound (for small oscillations and isothermality).



## Exercises

- ullet Derive an expression for the pressure p(h) of seawater as a function of depth h. Assume that the sea is at rest, that ho is constant, and that the force of gravity is F=mg.
- What pressure do you estimate at the bottom of the Mariana Trench ( $h=11\,\mathrm{km}$ )? (The measured value is  $\sim 1\,000\,\mathrm{bar}$ ).



## Exercises

• Do the same for the Earth's atmosphere. In this case you cannot assume that ho is constant: use the relationship  $p=c_S^2 
ho$ . The expected result is

$$p(h) = p_0 \exp(-h/h_0),$$

if h increases with height.

ullet Which value do you estimate for  $h_0$  for the Earth's atmosphere?