



Astrofisica Generale II — 5

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Interstellar Gas (ISG)



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In the 1930s, astronomers observed stellar spectra with weird optical absorption lines:

1. In binary systems, they do not show the Doppler effect;
2. They are more pronounced for more distant stars;
3. They are much narrower than stellar ones ($\rightarrow T \sim 100 \text{ K}$).



Interstellar Gas (ISG)

- Interstellar H is not observed in the visible spectrum: if T is low, Balmer's lines are too weak!
- The observed elements are Ca and Na, but also molecules: CH, CN, CH⁺. The latter imply a low gas density ($n < 10^3 \text{ cm}^{-3}$) and a low temperature:
 - Charged molecules like CH⁺ neutralize quickly in laboratory conditions;
 - CH and CN are highly reactive.



HI in ISG

- It is reasonable to expect that H, even if not detectable in the visible spectrum, is the predominant component of the ISG. It can be revealed by measuring the 21 cm line.
- This line is generated by the transition between the state of the HI atom with e/p spins parallel to the state with antiparallel spins. The two states have an energy difference of

$$\Delta E = 5.9 \times 10^{-6} \text{ eV},$$

and the transition probability is $A = (11 \text{ Myr})^{-1}$ so that $N = N_0 e^{-At}$.



Triplet/singlet states

The spin-parallel state ($s = 1$) is such that

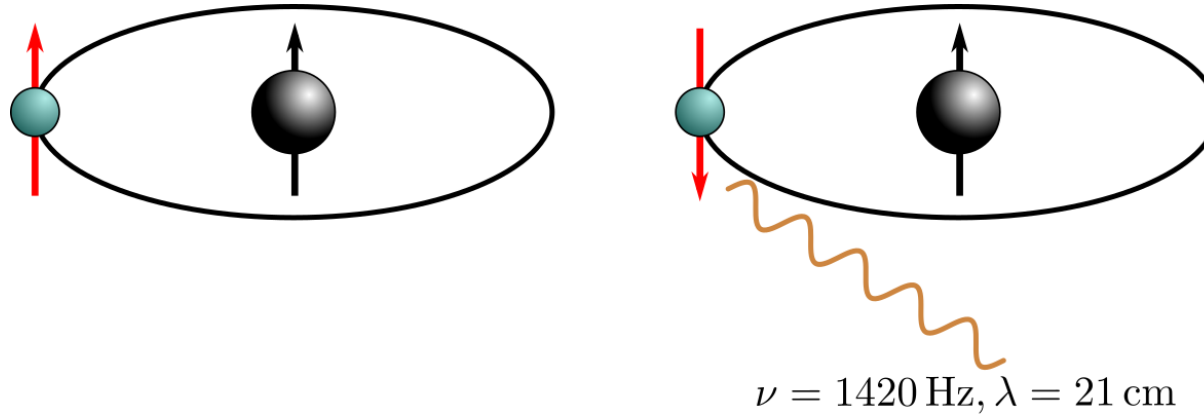
$$S = \sqrt{2}\hbar, \quad S_z = \begin{cases} +\hbar/2, \\ 0, \\ -\hbar/2 \end{cases} \quad (\text{triplet}),$$

while for the anti-parallel state ($s = 0$)

$$S = 0, \quad S_z = 0 \quad (\text{singlet}).$$



HI in ISG



The temperature associated with this radiation is

$$T_{21 \text{ cm}} \sim \frac{\Delta E}{k_B} = \frac{5.9 \times 10^{-6} \text{ eV}}{8.62 \times 10^{-5} \text{ eV/K}} \approx 0.07 \text{ K.}$$

The CMB (2.7 K) is enough to populate the state with parallel spins!



- If we assume that the gas is in thermal equilibrium and that the kinetic gas theory is valid (so we ignore the CMB and the fact that HI is not point-like), then we can use Maxwell's distribution:

$$\frac{N_{\text{tr}}}{N_{\text{sing}}} = \frac{g_{\text{tr}}}{g_{\text{sing}}} e^{-\Delta E/k_B T} = 3e^{-\Delta E/k_B T}.$$

- But if $k_B T \gg \Delta E$, then

$$\frac{N_{\text{tr}}}{N_{\text{sing}}} = 3e^{-\Delta E/k_B T} \approx 3.$$

- At the typical temperature of the Universe (≥ 2.7 K), there are three triplet atoms every singlet atom.



Importance of the 21 cm Line

- The existence of this line was predicted in the '40 and revealed on March, 25th 1951 by Edwin Purcell's team (Harvard Univ., Nobel 1953).
- The characteristics of the line are:
 1. Visible both in emission and absorption;
 2. Insensitive to the presence of dust.



Importance of the 21 cm Line

The 21-cm line has a wide range of applications:

- Fundamental for the study of gas in the ISM;
- Being insensitive to dust, it allows studying the structure of the Galaxy;
- Galaxy rotation and local motions can be reconstructed from Doppler measurements on the line;
- Study of ISM magnetic fields from the Zeeman effect on the line;
- ...and much more.



Numerical Example

Suppose that a cloud of neutral H is located at a distance $d = 30$ pc. The flux at 21 cm in emission, integrated over the solid angle, is

$$f = 4.5 \times 10^{-15} \text{ erg cm}^{-2} \text{ s}^{-1}.$$

What is the mass of the hydrogen in the cloud?



Solution

From the observed flux we can derive the total luminosity:

$$L_{21\text{ cm}} = 4\pi d^2 f = 4.85 \times 10^{26} \text{ erg s}^{-1}.$$



Solution

We expect the following formula to hold:

$$L_{21 \text{ cm}} \approx \frac{3}{4} N_H A h\nu,$$

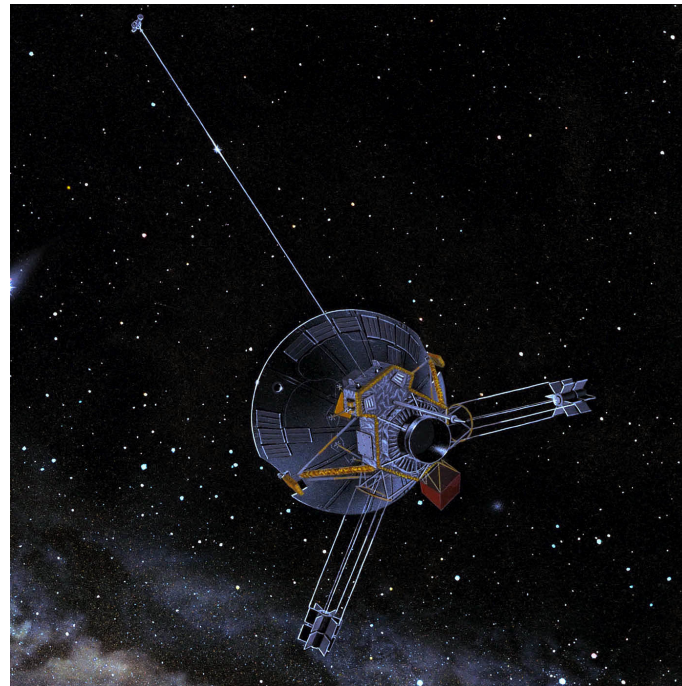
where $A = \tau^{-1} = (11 \text{ Myr})^{-1}$ is the transition probability and the factor $3/4$ takes into account the population in the two spin states. Therefore

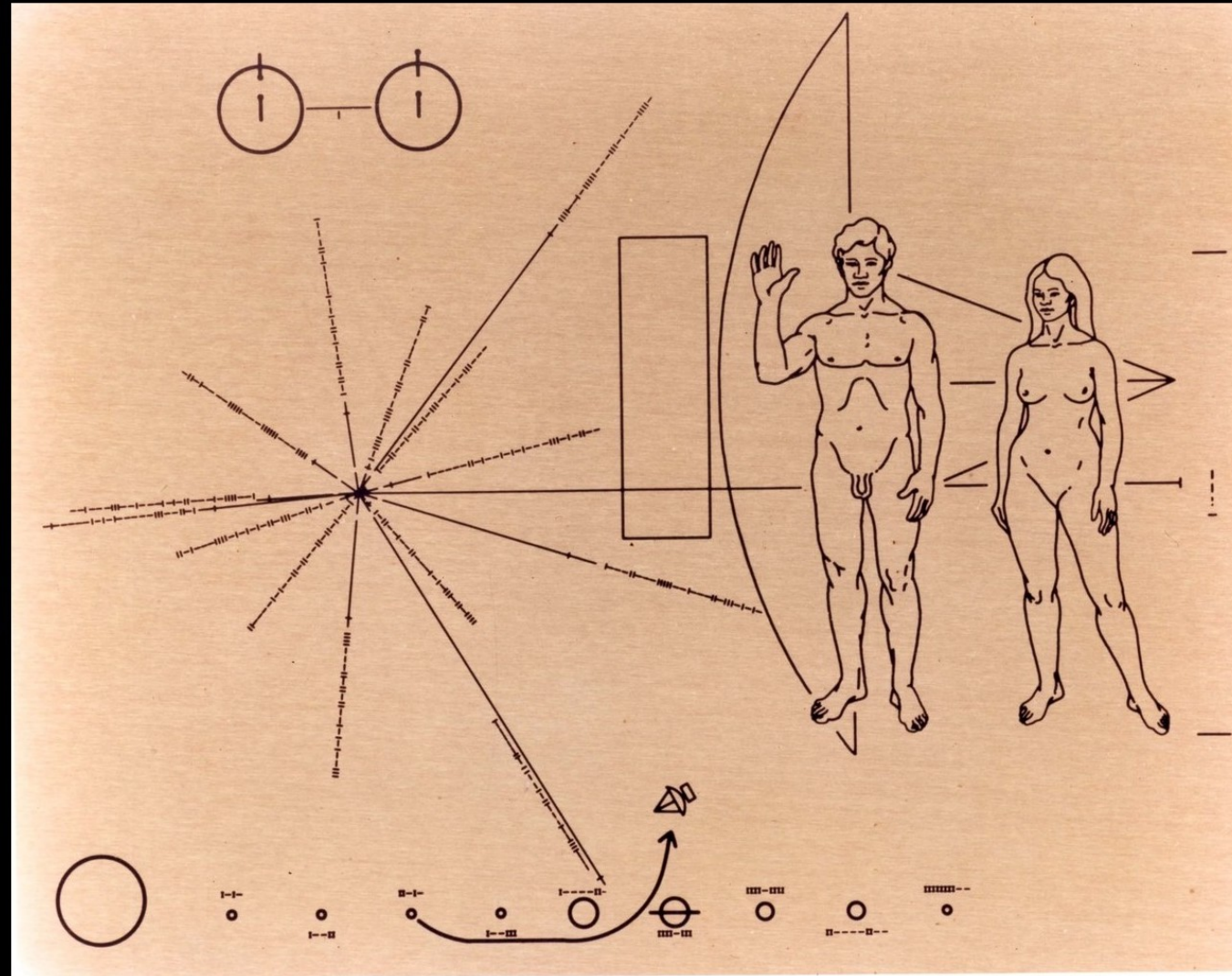
$$N_H \approx 2.4 \times 10^{58}, \quad M_H = N_H \times m_p \approx 20 M_{\odot}.$$



Pioneer 10 (1972)

An interesting context in which the 21 cm line played an important role is the famous plaque installed on the Pioneer 10 probe, launched in 1972 by NASA to study Jupiter.







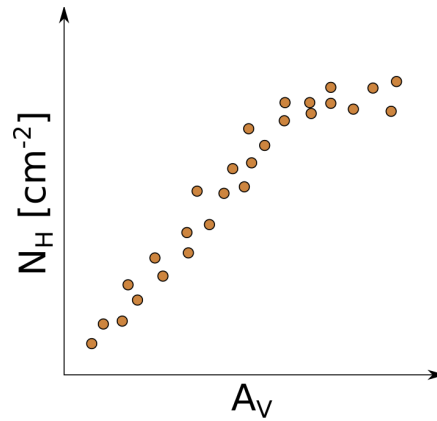
HI in ISM

Quantity	Estimate
Temperature	10÷100 K
Cloud size	10÷100 pc
HI density (cloud)	1÷10 cm ⁻³
HI density (Galaxy)	0.1 cm ⁻³
Speed	$v_{\text{rms}} \sim \sqrt{k_B T / m_p} \sim 10^3 \text{ m/s}$



Gas and Dust

- A correlation is observed between the HI column density (21 cm line) and dust (extinction measurements) → These components are mixed in the ISM.
- The correlation ceases for high values of A_V . Why?



1. Does dust attenuate at 21 cm? No, insufficient n .
2. Do H_2 molecules form?



Gas and Dust

- The H_2 molecule is very difficult to detect because it does not emit the equivalent of the 21 cm line. Furthermore, it does not have a permanent dipole.
- Molecules like CO have a permanent dipole, and since the rotational energy is quantized,

$$E_r = \frac{(I\omega)^2}{2I} = \frac{L^2}{2I} = \frac{\hbar^2 J(J+1)}{2I}.$$

- If there is a permanent dipole, the selection rule $\Delta J = -1$ applies. A transition between rotational energy levels of CO therefore generates lines ($\nu > 115$ GHz).



Gas and Dust

- However, H_2 has **no permanent dipole**, so it does not emit lines
- H_2 has a weak quadrupole with selection rule $\Delta J = -2$ that generates weak emission around $10\text{ }\mu\text{m}$ (covered by dust emission 😞)
- It's easier to study the emission of less abundant molecules with stronger lines (CO , CH , OH , CS , $\text{C}_3\text{H}_2\dots$).



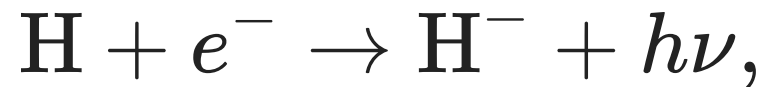
Gas and Dust

- Does the loss of correlation between N_H and A_V tell us then that part of the ISM hydrogen is in molecular form?
- The densities involved would seem to advise against it: it is difficult to produce H_2 because (again!) of its symmetry.



Gas and Dust

- To join two H atoms together, it is necessary to bind them in an excited state, and then de-excite the system by radiating energy. But H₂ **has no dipole moment**, so it does not radiate!
- To produce H₂ it is first necessary that H⁻ is formed:

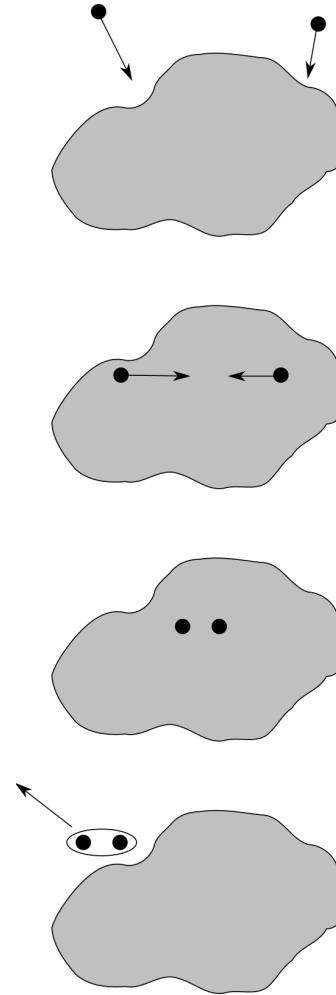


- But it is difficult to have H⁻ in a cloud: it is slow to form and fast to destroy (by collisions with protons, photons or other positive ions).



Dust and H₂ Molecules

- Dust can act as a catalyst. Nuclei are captured by grains and, after a random walk, they settle in sites from which they no longer move.
- Thus it is easier to make nuclei and electrons react with each other. To produce H₂, the kinetic energy produced is 4.5 eV, sufficient to expel the molecule from the grain (and give it angular momentum...).





Molecular Clouds

- In the interstellar medium (ISM), we can observe clouds composed of molecules.
- They are characterized by low temperatures ($\sim 10 \text{ K}$) and high densities ($n \sim 10^3 \text{ cm}^{-3}$).



Molecules found in the ISM



The screenshot shows the website of the I. Physikalisches Institut at the University of Cologne. The header includes the university's name, faculty, and a search bar. The left sidebar contains a navigation menu with links to News, Events, Conferences, Research, Methods, Observatories, Services, CDMS, What's new?, Catalog, Molecules in space (highlighted in red), Fitting, Spectra, Spectroscopy, Data, Links, Contact, Grants, LabAstro, Workgroups, Teaching, Publications, Personnel, and Institute. The main content area is titled "Molecules in Space" and contains information about the ISM/CSS extragalactic molecules project. It includes a press release link, a note about deuterium isotopic species, a description of the project's documentation, a list of molecules detected by rotational spectroscopy, and a note about transition metal molecules. The footer mentions the date March 2013 and the number of molecules detected (around 180).

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Molecules in Space

ISM/CSS
extragalactic molecules

Press release by the MPIfR on the radio astronomical detection of titanium monoxide (TiO) and titanium dioxide (TiO₂) in the circumstellar envelope of the red super giant VY Canis Majoris. [Deutsche Version der Pressemitteilung.](#)

Note: Deuterium isotopic species are given separately only if their method of detection is intrinsically different from that of pure hydrogen ones. We plan to modify this in the future to include them with all hydrogen species.

The documentations generally provide information on the detection of the respective molecules, including minor isotopic species, molecules in excited vibrational states, or different media in which the species has been detected as well as links to articles.

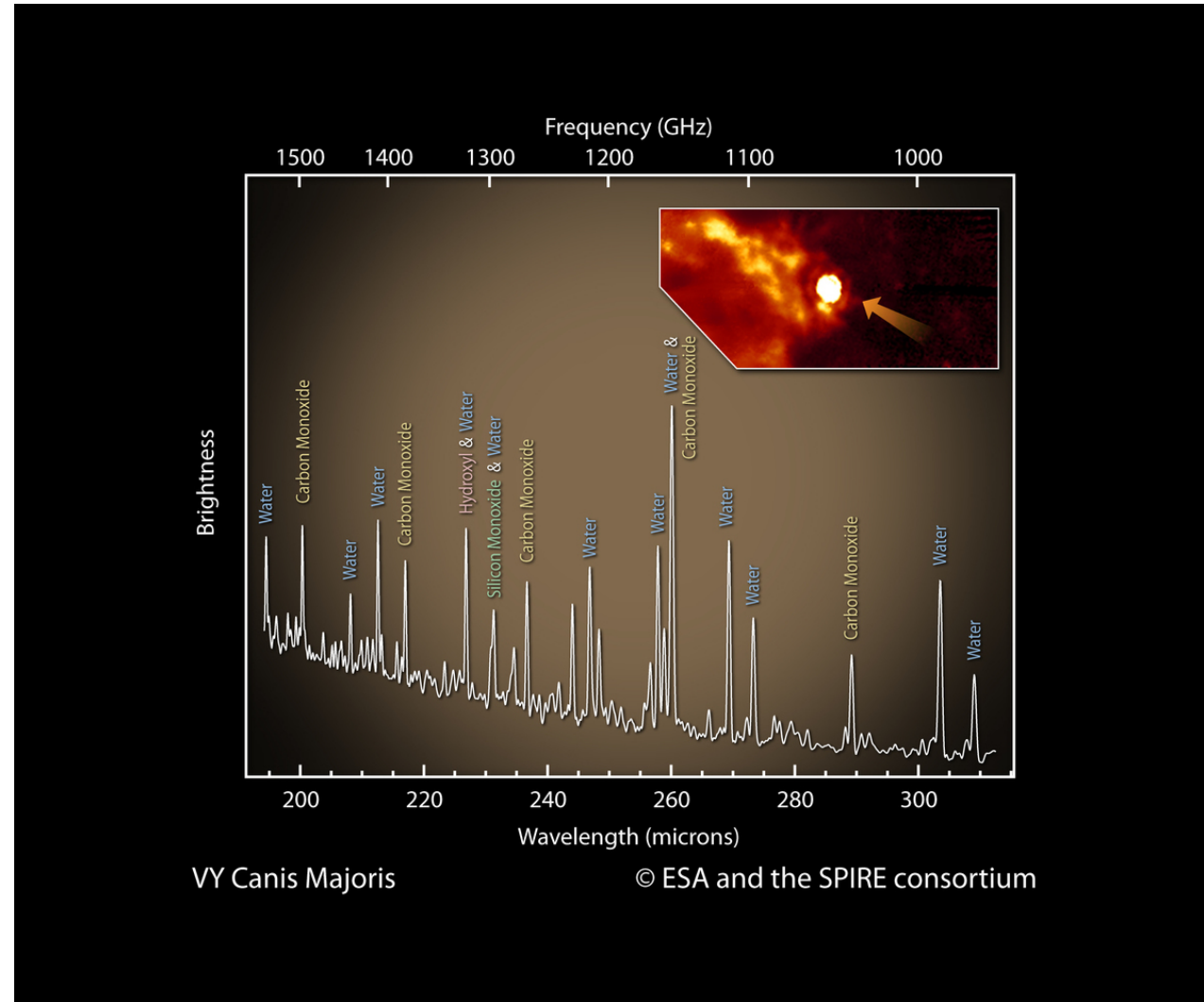
All molecules have been detected (also) by rotational spectroscopy in the radiofrequency to far-infrared regions unless indicated otherwise.
* indicates molecules that have been detected by their rotation-vibration spectrum,
** those detected by electronic spectroscopy only.

Transition metal molecules detected in atmospheres of stars by electronic spectroscopy are currently not included. We plan to create a new table in the not so distant future dealing with molecules in stellar atmospheres. In this context, the table of molecules detected in the interstellar medium (ISM) or circumstellar envelopes (CSE) of late type stars, will be split. In the case of some metal monoxides, emission features have been detected in optical or IR spectra. Most likely, this emission indicates a presence of these molecules in the circumstellar shell. Around one handful of molecules may have to be added.

Note: Questionable or rebutted detections have been omitted from the table. Some noteworthy cases are given below the table. Tentative detections, which have a reasonable chance to be correct, are indicated by "?". Usually the number of observed transitions that are, at least, fairly free of overlap is small. Some detections that have been reported as secure ones are indicated by "(?)" because (partial) overlap of lines cannot be ruled at the moment or because the line list is somewhat small; obviously, there may be cases on the edge.

As the decision whether a molecule can be considered as detected frequently is a controversial one we refrain from stating an exact number of molecules as detected. As of **March 2013**, at least 175 molecules have been detected unambiguously in the interstellar medium or circumstellar shells; four of these are tentative detections, and for currently three, the situation may be somewhat uncertain. Including some molecules missing in the table, the number of molecules detected in the ISM or in CSEs is around 180. Thus, we recommend phrases such as **"around 180 molecules** have been detected in the **interstellar medium or circumstellar shells"**. Two reported detections have been questioned seriously. Therefore, these are viewed currently as "probably not yet detected" – see below. They do not show up in the table anymore.

www.astro.uni-koeln.de/cdms/molecules



VY Canis Majoris (giant star, $R \sim 2000 R_{\odot}$) seen by Herschel



What about HII?

- We have analyzed the presence in the galaxy of atomic hydrogen (HI) and molecular hydrogen (H_2).
- The case of HII is equally interesting; however, we will discuss it in the context of star formation.



Star Formation



Star Formation

Under what conditions does a gas cloud induce the formation of a star?

Let's assume that the cloud is spherical and has uniform density. Gas and dust are present within it. For collapse to occur, the system needs to “de-virialize”:

$$-U \gtrsim 2K,$$
$$\frac{3}{5} \frac{GM^2}{R} \gtrsim 2 \frac{1}{2} \frac{M}{m} k_B T.$$



Star Formation

But M and R are related to the density ρ of the cloud (assumed constant):

$$M = \frac{4}{3}\pi R^3 \rho,$$

therefore

$$R > \sqrt{\frac{15k_B T}{4\pi G m \rho}} \equiv R'_J.$$

(We use the notation R'_J because shortly we will derive the true value of R_J obtained by Jeans).

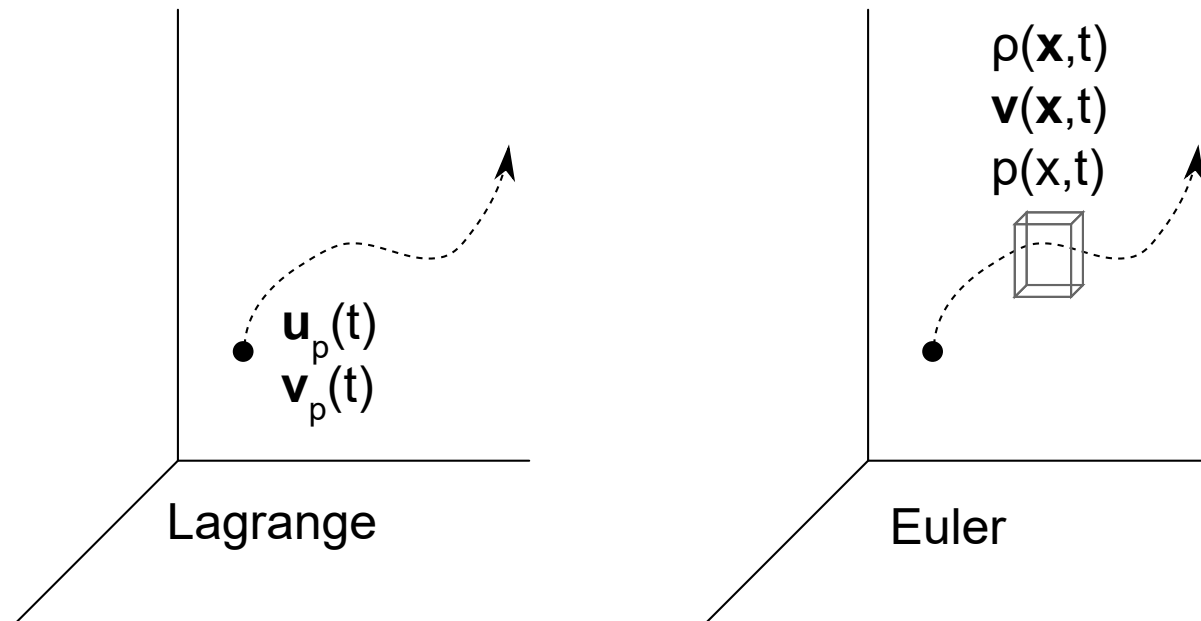


Star Formation

- The previous calculation is quite different from the one tackled by James Jeans (1877–1946), who did not use the virial theorem.
- We will now redo his calculations, repeating every passage...
- ...and we will re-make a mistake he made!
- Let's start by introducing the equations of fluid dynamics.



Fluid Physics



- In the **Lagrangian** point of view, we describe the trajectory of the particle (analogously to Newton's laws).
- In the **Eulerian** point of view (the most convenient), we focus on points in space and their neighborhood.



Newton's Equation

Since we know how to describe the motion of particles using Newtonian physics, we start from the Lagrangian point of view:

$$\vec{F}_p = m \vec{a}_p = m \dot{\vec{v}}_p,$$

and express \vec{v}_p in terms of Eulerian quantities:

$$\vec{v}_p(t) \approx \vec{v}(\vec{u}_p(t), t).$$



If we calculate the derivative of the product, we get

$$\begin{aligned}\dot{\vec{v}}_p &= \frac{d}{dt} \vec{v}(\vec{u}_p(t), t) = \frac{d}{dt} \vec{v}(u_{px}(t), u_{py}(t), u_{pz}(t), t) = \\ &= \partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v},\end{aligned}$$

where we exploit the fact that $\partial_t \vec{u}_p(t) = \vec{v}_p(t) \approx \vec{v}(\vec{u}_p(t), t)$ and that

$$(\vec{v} \cdot \vec{\nabla}) \vec{v} = \begin{pmatrix} v_x \partial_x v_x + v_y \partial_y v_x + v_z \partial_z v_x \\ v_x \partial_x v_y + v_y \partial_y v_y + v_z \partial_z v_y \\ v_x \partial_x v_z + v_y \partial_y v_z + v_z \partial_z v_z \end{pmatrix}.$$



Material Derivative

- The *material derivative* is the expression

$$\dot{\vec{v}}_p = \partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v}$$

- It states that the change in velocity of a fluid particle is caused by two terms:
 1. a variation in time of \vec{v} within the small volume element;
 2. a velocity difference between the volume element where the particle is located at time t and the one it has “jumped” to at time $t + dt$.



- Let's now turn the unknowns in Newton's equation into the Eulerian quantities ρ and \vec{v} appear. We are no longer interested in one particle, but in a collection of N particles.
- Therefore, we sum the N equations for those N particles that are in the same small volume element:

$$\begin{aligned}\sum_{i=1}^N \vec{F}_p^{(i)} &= \sum_{i=1}^N m^{(i)} \dot{\vec{v}}_p(t) = \sum_{i=1}^N m^{(i)} \left(\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) = \\ &= \left(\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) \sum_{i=1}^N m^{(i)}.\end{aligned}$$



We now substitute the masses $m^{(i)}$ with the density ρ :

$$\sum_{i=1}^N \vec{F}_p^{(i)} = \left(\sum_{i=1}^N m^{(i)} \right) (\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v}),$$

$$\vec{F}_{\text{tot}} = \rho dV (\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v}),$$

with dV being the volume of the small element and \vec{F}_{tot} the total force acting on the small volume; note that all internal action/reaction forces cancel out.



Force Terms

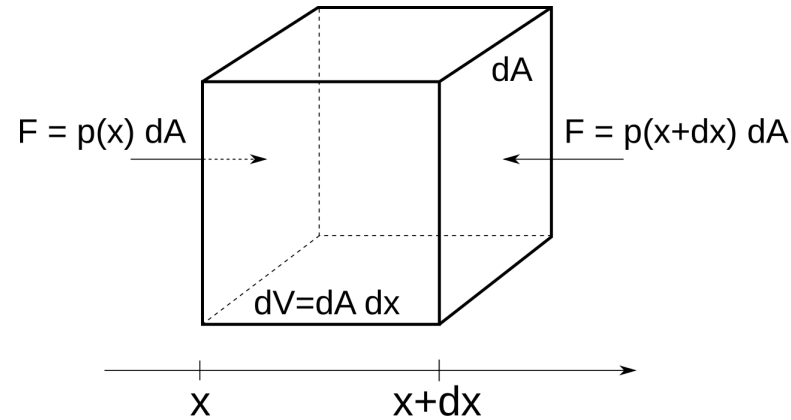
- We now need to derive an expression for the term \vec{F}_p . In the context of cloud collapse, there are two components:
 1. Pressure forces;
 2. Gravitational forces.

These are the same forces we considered in the derivation of the virial equation $U = -2K$.

- Let's address them separately.



Pressure Forces



Let's consider only the force F_{pressure} exerted on the small volume along the x direction. If the forces are normal to the faces (perfect fluid), then

$$\begin{aligned} F_{\text{pressure}} &= (p(x) - p(x + dx)) dA \\ &= -\partial_x p(x) dx dA = -\partial_x p(x) dV. \end{aligned}$$



Pressure forces

If we now consider motion in three dimensions instead of just along the x axis, the result generalizes trivially:

$$\vec{F}_{\text{pressure}} = -\vec{\nabla} p \, dV.$$



Gravitational force

In the case of gravity, it is easy to express the force in terms of the potential ϕ :

$$\vec{F}_{\text{grav}} = -m \vec{\nabla} \phi,$$

where (Poisson's law)

$$\nabla^2 \phi = 4\pi G \rho$$

and obviously $m = \rho \, dV$.



Momentum conservation

The momentum conservation equation in the case of a cloud is therefore

$$\begin{aligned}\rho \, dV \left(\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) &= \vec{F}_{\text{pressure}} + \vec{F}_{\text{grav}} = \\ &= -\vec{\nabla} p \, dV - \vec{\nabla} \phi \, \rho \, dV,\end{aligned}$$

which can be rewritten as the system of 3 equations

$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla} \phi$$

(a special case of the *Navier-Stokes equations*).



Other equations

- With the previous vector equation and Gauss's law we have 4 equations but 6 unknowns ($v_x, v_y, v_z, p, \rho, \phi$).
- We also use the mass conservation equation:

$$\dot{\rho} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

and the relationship between pressure and density

$$p = \rho c_S^2,$$

where c_S is the speed of sound (for small oscillations and isothermality).



Exercises

- Derive an expression for the pressure $p(h)$ of seawater as a function of depth h . Assume that the sea is at rest, that ρ is constant, and that the force of gravity is $F = mg$.
- What pressure do you estimate at the bottom of the Mariana Trench ($h = 11$ km)? (The measured value is $\sim 1\,000$ bar).



Exercises

- Do the same for the Earth's atmosphere. In this case you cannot assume that ρ is constant: use the relationship $p = c_S^2 \rho$. The expected result is

$$p(h) = p_0 \exp(-h/h_0),$$

if h increases with height.

- Which value do you estimate for h_0 for the Earth's atmosphere?