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GYROSCOPICS.

A Treatise on Gyrostatics and Rotational Motion. Theory and Applications. By Prof. Andrew Gray. Pp. xx + 530. (London: Macmillan and Co., Ltd., 1918.) Price 42s. net.

THE exhibition at the International Mathematical Congress at Cambridge in 1912, although unnoticed in the official record of the Proceedings, was attractive as a collection of scientific books on view of all the chief publishers in the world, and of apparatus designed for use in mathematical instruction, including a very complete assortment of calculating machines of all kinds.

But the foreign visitor was delighted chiefly to see and handle the gyrostats and apparatus, and so to clear up much of the obscurity in the mere description and diagrams of the "Treatise on Natural Philosophy" of Thomson and Tait.

The apparatus was designed, and explained, and shown at work in the skilful hands of Dr. James Gray, son of our author, engaged since in the development of the warlike applications; and we are promised a sequel devoted to this side of the subject of gyrostatics as soon as the seal of secrecy has been removed with the advent of peace.

Mention and description can then be made, too, of the peaceful applications of gyroscopic principles, such as to the design of the centrifuges employed for centrifugal and whirling operations in chemical and laundry work, to drain off the moisture in a saturated substance swiftly and with no internal disturbance. These were described in *Engineering* for February 7 last, where each centrifuge must be treated as a great spinning-top, upright as if asleep, requiring the upper end to be quite free in precession, and so actuated from the lower end of the axle, in this case by a Pelton wheel.

The gyro-compass is held over to the sequel, as involving the operation of secret processes; without it the navigation of a submarine could not have been possible. But a full description is given in chap. viii. of Schlick's sea gyroscope, with the theory designed to ensure a dry ship and easy roller in all weather.

Prof. Gray has succeeded, in the chair of natural philosophy at Glasgow University, to the gyrostatic apparatus of Lord Kelvin, his predecessor, and has added important developments of his own invention. As shown in the diagrams, these are of elaborate construction, and demand the aid of electric motive power to impart and maintain the high rate of revolutions required, and so will not be allowed far from the lecture-desk.

But Maxwell's opinion must be maintained that the real instruction of the student is derived from the crude apparatus made by his own hands, and that he learns most from his own failures.

So we venture to suggest to Prof. Gray the encouragement of his students in the use of such simple apparatus as that in his Fig. 30(*b*) on p. 128, where a bicycle wheel is shown as a cheap, efficient top, spun by hand, and no string or electric motor is required. If the ordinary 28-in. wheel is not considered large enough, it will cost little more to order one of double or three-fold diameter, as the delicate part of the hub and ball-bearings can serve for all, and is bought cheap when manufactured in large quantities. These can be handled and thrown about, and brandished, and so provide the muscular sensations on a large scale of gyroscopic domination. Any inventor's idea can be tested at once and an advantage followed up.

If the point of a top is free to wander about on the floor, either as a sharp tip or a rounded ball, the dynamical treatment is intractable in the present state of mathematical analysis.

The point must be kept still, and we avoid the hideous unreality of the "perfectly rough" of the text-book jargon by placing it, as in Fig. 30(*a*), in a small cup recess, the wheel spinning freely about the ball-bearings of the hub fixed on the stalk.

The top must then have uniaxial symmetry if the motion is to be expressible by the elliptic function, as explained in chap. xii.; and these functions appear created expressly to speak the language of such gyroscopic motion.

In the old Cambridge mathematical tradition, praised by Todhunter, it was considered of no intellectual merit to have seen and worked an experiment in Natural Philosophy and not to have grasped the idea by mere thinking.

Maxwell strove hard to destroy this tradition, and pointed out the superiority at Glasgow of Sir William Thomson's stimulating treatment of dynamics with experiments. Maxwell was given a chance of working out his ideas by the erection of the Cavendish Laboratory for his benefit, gift of the Chancellor, the Duke of Devonshire. But as Maxwell's inaugural lecture was delivered to bare walls, the Chancellor desired to make his gift complete by presenting an appropriate collection of apparatus. Such an order could not be given out at once in those days, and the demands extended over a few years, during which some busybodies, self-styled business men, were always worrying Maxwell to make his final demand and declare the Cavendish Laboratory complete; and as Maxwell was then approaching his fatal illness he was too weak to protest, leaving his successor, Lord Rayleigh, the inheritance of a large establishment with no endowment for upkeep and progress.

The tradition there of research has been chiefly electrical, so that the interests of dynamics have not been studied equally, and, to judge from the ordinary text-books in use, the old Victorian tradition still survives, copied from one to the other, and not looking up from the page at the great developments taking place around, a great contrast to Prof. Gray's treatise before us.

The elliptic function solution is restricted to the top of uniaxial symmetry. If the top is taken to be a body of any shape, as may be imitated with the screws of the Maxwell top, the analytical complexity in chap. xvii. defied a Weierstrass, who handed his difficulties over to the young Kowalevski, to break her teeth over the problem.

The ardent spirit is not deterred, but, on the contrary, rather stimulated, to tackle a question declared intractable; so Prof. Gray gives a *résumé* in chap. xvii. of the progress made so far by other daring mathematicians—Russians for the most part—although we miss a figure and description of the Maxwell top, to be placed on the table in front and twirled by a finger and thumb.

The spherical pendulum of chap. xv. was early to receive attention as a problem in mere particle dynamics, realised in swinging a plummet about at the end of a thread. This is a case of gyroscopic-top motion where the component angular momentum (A.M.) about the axle is zero, and is realised in the apparatus of Fig. 30(b) by projecting the wheel without rotation. But this limitation makes the motion very uninteresting analytically, except as illustrating a solution of a Lamé equation of the second order. The simple case of holding out the axle horizontal, and projecting it horizontally without any rotation of the wheel, is of interest as giving a state of motion that has a simple analytical solution, which may be written down here:

$$\sin\theta\cos(\psi - ht) = \sqrt{(\sec\theta_3 - \cos\theta_3)}\sqrt{(\cos\theta)},$$
$$\sin\theta\sin(\psi - ht) = \sqrt{(\cos\theta_3 - \cos\theta . \cos\theta + \sec\theta_3)},$$

where $2h$ denotes the precession when the axle is horizontal, and θ_3 is the extreme angle of the axle with the downward vertical, to which the axle sinks and then rises up again to the horizontal.

This can serve as a penultimate case where the spherical pendulum is whirled round swiftly, apparently in a horizontal circle, as with the lariat or bola, as on p. 302, contrasted with swift whirling in a vertical circle, penultimate case of pendulum motion, and an extreme contrast to small plane oscillation near the vertical.

Lagrange came to grief over the small conical oscillations of the spherical pendulum (*cf.* § 5, p. 302), yet he could have saved himself and detected his error but for the self-imposed restraint of excluding the diagram from his "Mécanique analytique." So it is curious to find the same fashion coming again in the modern school of pure analytical treatment, of doing away with an appeal to the visual sense of a geometrical figure.

In swift rotation about an axis in the neighbourhood of a principal axis, as the axis of figure of a symmetrical top, the instantaneous axis does not wander far from the principal axis, and the axis of A.M. keeps close by also, even when the body, like the top, is acted on continuously by a force or couple which causes the A.M. vector to move.

The kindergarten explanation of top motion, in considering only the rotation about the axis, can then be made more exact, when it is assumed that the divergence of the axis of A.M. and angular velocity from the axis of figure is always small, so that one may be used indiscriminately for the other.

In this way, by calling CR the A.M. above the axis of figure, and $gMh\sin\theta$ the couple of gravity on the top when the axis points up at an angle θ with the upward vertical, the simple formula is obtained for μ, the precession:

$$\mu CR\sin\theta = gMh\sin\theta, \quad \mu = \frac{gMh}{CR},$$

provided θ is not too small.

Poinsot applied the same principle in his treatment of precession and nutation ("Connaissance des temps," 1858), assuming the divergence of the axis of rotation and of A.M. from the axis of figure of the earth as insensible; otherwise we should see the stars dancing about. The treatment here in chap. x. could be simplified in Poinsot's method. The Glasgow problem on p. 13 of the calculation of the diameter of the earth's axis at the pole may be cited as a justification of Poinsot's assumption.

It was a mathematical genius who changed in precession to the reckoning in $\frac{A, C}{C-A} = 304, 305$, or some say 305, 306, instead of the usual reciprocals in small decimals, indistinguishable numerically. And we venture to put in a plea for the sidereal day as the unit of time in these measurements, and not the solar year, thus making $R = 2\pi$ for the earth.

The effect of precession is to shorten the year about twenty minutes, and thus the period is 26,000 years of a complete revolution of the equinox through the stars. The classical scholar may be encouraged to take up the study of Astronomy when he hears that stray references to the stars by Homer are a guide to us in assigning limits to the age in which he lived and wrote. Astronomy was a much more living, actual interest in the days before clock and watch was so plentiful.

G. GREENHILL.

A PHYSIOLOGIST'S CONTRIBUTION TO WAR SURGERY.

Intravenous Injection in Wound Shock. Being the Oliver-Sharpey Lectures delivered before the Royal College of Physicians of London in May, 1918. By Prof. W. M. Bayliss. Pp. xi+172. (London: Longmans, Green, and Co., 1918.) Price 9*s.* net.

THE war has brought into touch with directly practical problems many whose interests, before its outbreak, lay in fields of investigation which were popularly regarded as purely academic and remote from contact with everyday needs. In no department of research has the value of "pure" science been more finely vindicated than in that of physiology; and the gain to both physiology and practical medicine from this closer alliance of theory and application has been the subject of general remark. There could scarcely be a better example of this recent tendency than Prof. Bayliss's book on the treatment of "wound shock," which embodies, with much added detail and illustration, the substance of his