Universität Stuttgart

Geodätisches Institut

Parameterschätzung Übung 6

Ausarbeitung im Studiengang

Geodäsie und Geoinformatik

an der Universität Stuttgart

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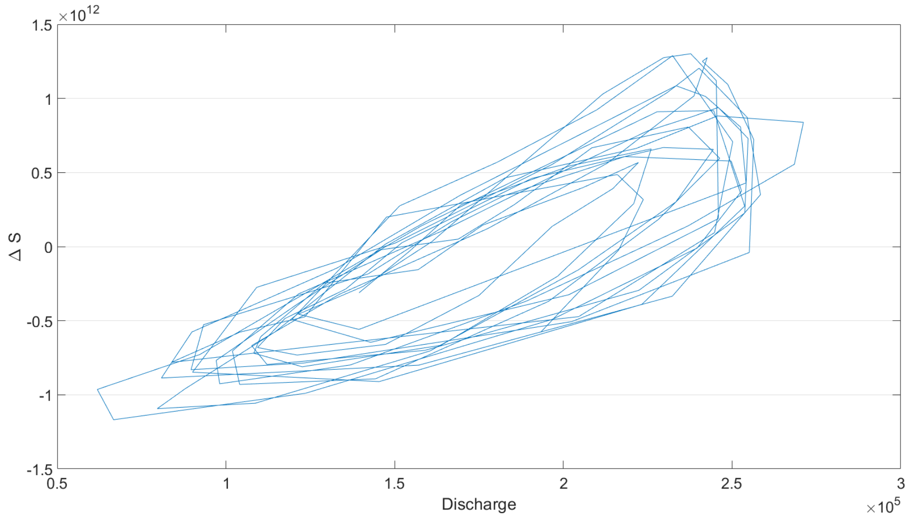
Chapter 1

Report

1.1 a

The Relationships between Discharge and ΔS is shown in Figure 1.1.



(a)

Figure 1.1:

1.2 b

$$\frac{dR}{dt} + \frac{R}{\tau} = \omega_n^2 S$$

1.3 c

$$\frac{R_t - R_{t-1}}{\Delta t} + \frac{R_t}{\tau} = \omega_n^2 S_t$$

If we take Δt as 1 unit.

$$
\begin{aligned}
R_t - R_{t-1} + \frac{R_t}{\tau} &= \omega_n^2 S_t \\
\left(1 + \frac{1}{\tau}\right) R_t &= R_{t-1} + \omega_n^2 S_t \\
R_t &= \frac{\tau}{\tau+1} R_{t-1} + \frac{\tau\omega_n^2}{\tau+1} S_t
\end{aligned}
$$

1.4 d

After solve this ODE:

$$
\begin{aligned}
R(t_{n+1}) &= R(t_n)e^{\frac{\Delta t}{\tau}} + \bar{S}(t_n) \cdot \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) \\
R(t_{n+1}) &= R(t_n)e^{\frac{\Delta t}{\tau}} + (S_0 + \Delta S(t_n)) \cdot \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1)
\end{aligned}
$$

1.5 Part 2

before from CSR, after JPL

$$
d \begin{bmatrix} \Delta S \\ R \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ \omega^2 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \Delta S \\ R \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ \omega^2 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ P \\ ET \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta S_{measured} \\ R_{measured} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta S_{predict} \\ R_{predict} \end{bmatrix}
$$

which means

$$
\begin{bmatrix} \Delta S_{n+1} \\ R_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) & e^{\frac{\Delta t}{\tau}} \end{bmatrix} \begin{bmatrix} \Delta S_n \\ R_n \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) & 0 & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ P \\ ET \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta S_{measured} \\ R_{measured} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta S_{predict} \\ R_{predict} \end{bmatrix}
$$

Variance for R, I have R from 1968.

$$
\sigma^2_{R_{January}} = \frac{1}{n-1} \sum (R_{i,January} - \bar{R}_{January})^2
$$