



Parameterschätzung Übung 6



Ausarbeitung im Studiengang
Geodäsie und Geoinformatik
an der Universität Stuttgart

Ziqing Yu, 3218051

Stuttgart, July 12, 2021

Betreuer: Dr.-Ing. Mohammad Tourian
Universität Stuttgart

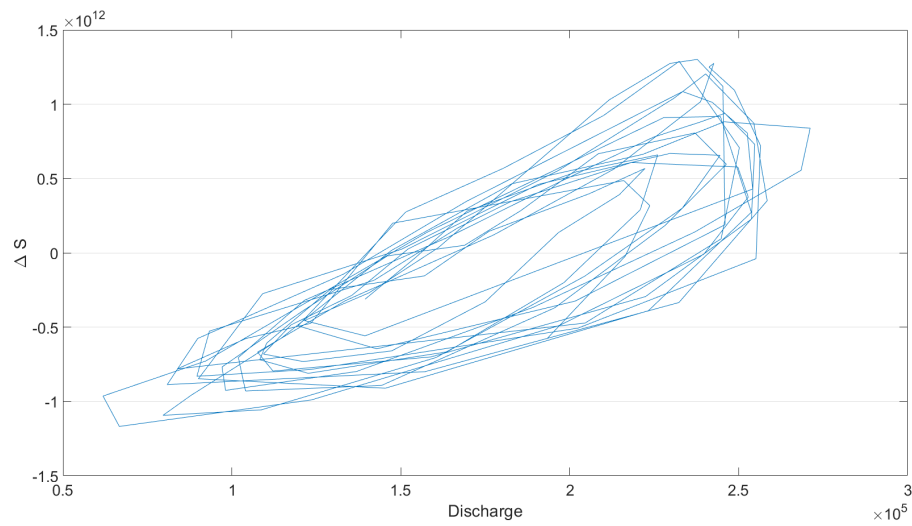
Dr. Karim Douch
Universität Stuttgart

Chapter 1

Report

1.1 a

The Relationships between Discharge and ΔS is shown in Figure 1.1.



(a)

Figure 1.1:

1.2 b

$$\frac{dS}{dt} = P - ET - R$$

$$\frac{dR}{dt} + \frac{R}{\tau} = \omega_n^2(\Delta S + S_0)$$

1.3 c

$$\frac{R_t - R_{t-1}}{\Delta t} + \frac{R_t}{\tau} = \omega_n^2 S_t$$

If we take Δt as 1 unit.

$$\begin{aligned} R_t - R_{t-1} + \frac{R_t}{\tau} &= \omega_n^2 S_t \\ \left(1 + \frac{1}{\tau}\right) R_t &= R_{t-1} + \omega_n^2 S_t \\ R_t &= \frac{\tau}{\tau + 1} R_{t-1} + \frac{\tau \omega_n^2}{\tau + 1} S_t \end{aligned}$$

1.4 d

After solve this ODE:

$$\begin{aligned} R(t_{n+1}) &= R(t_n) e^{\frac{\Delta t}{\tau}} + \bar{S}(t_n) \cdot \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) \\ R(t_{n+1}) &= R(t_n) e^{\frac{\Delta t}{\tau}} + (S_0 + \Delta S(t_n)) \cdot \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) \end{aligned}$$

1.5 Part 2

before from CSR, after JPL

$$\begin{aligned} d \begin{bmatrix} \Delta S \\ R \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ \omega^2 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \Delta S \\ R \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ \omega^2 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ P \\ ET \end{bmatrix} \\ \begin{bmatrix} \Delta S_{measured} \\ R_{measured} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta S_{predict} \\ R_{predict} \end{bmatrix} \end{aligned}$$

which means

$$\Delta S(t_{n+1}) = \Delta S(t_n) + P(t_n) - ET(t_n) - R(t_n)$$

$$\begin{aligned} \begin{bmatrix} \Delta S_{n+1} \\ R_{n+1} \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) & e^{\frac{\Delta t}{\tau}} \end{bmatrix} \begin{bmatrix} \Delta S_n \\ R_n \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) & 0 & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ P \\ ET \end{bmatrix} \\ \begin{bmatrix} \Delta S_{measured} \\ R_{measured} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta S_{predict} \\ R_{predict} \end{bmatrix} \\ R_{measured} &= R_{predict} \end{aligned}$$

Variance for R , I have R from 1968.

$$\sigma_{\bar{R}_{January}}^2 = \frac{1}{n-1} \sum (R_{i,January} - \bar{R}_{January})^2$$

$$\begin{bmatrix} 0 & 1 & -1 \\ \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1) & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_{S_0} \\ \sigma_P \\ \sigma_{ET} \end{bmatrix}$$