$h = at + b$

1 SVD

$$\boldsymbol{p} = \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \tag{1}$$

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{h} - \bar{h}, \boldsymbol{t} - \bar{t} \end{bmatrix} \tag{2}$$

Ziel: $min(\boldsymbol{Mp})^2 \rightarrow min(\boldsymbol{Mp})^t(\boldsymbol{Mp}) \rightarrow min(\boldsymbol{p^t M^t M p})$

$\boldsymbol{M^t M}$ ist normal Matrix $\rightarrow svd(\boldsymbol{M^t M}) = \boldsymbol{USU^t}$

$min((\boldsymbol{p^t U})\boldsymbol{S}(\boldsymbol{p^t U})^t) \rightarrow \lambda_1(\boldsymbol{u_1^t p})^2 + \lambda_2(\boldsymbol{u_2^t p})^2$

Annehmen $(\boldsymbol{u_1^t p}) = \cos(\alpha)$ und $(\boldsymbol{u_2^t p}) = \sin(\alpha)$

$\cos^2(\alpha) = s \rightarrow min((\lambda_1 - \lambda_2)s + \lambda_2) \rightarrow s = 1$

$\boldsymbol{p} = \boldsymbol{u_1}$: Richtung bekannt -> a bekannt -> Mittelwert einsetzen -> b berechnet

2 GMH

$$\boldsymbol{H} = \begin{bmatrix} t_1 & 1 \\ t_2 & 1 \\ \vdots & \vdots \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = (\boldsymbol{H^t H})^{-1}\boldsymbol{H^t h} \tag{4}$$

$$f = h - at - b \tag{5}$$

$$\boldsymbol{A} = \begin{bmatrix} -t_1 & -1 \\ -t_2 & -1 \\ \vdots & \vdots \end{bmatrix} \tag{6}$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & -a & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & -a & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{7}$$

3 Experiment

$$\boldsymbol{h} = [7, 7, 11, 11, 15, 16, 19]^t \tag{8}$$
$$\boldsymbol{t} = [3, 4, 5, 6, 7, 8, 9]^t \tag{9}$$

svd: a = 2.1458; b = -0.5892

ohne: a = 2.0714; b = -0.1429

mit $\boldsymbol{P} = diag([1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6, 7])$: a = 2.0711; b = - 0.1395

$\phi = \arctan(a) \rightarrow \boldsymbol{p} = [\cos(\phi)\,\sin(\phi)]$
$\boldsymbol{M} = [\boldsymbol{h} - b, \boldsymbol{t}]$
$e = (\boldsymbol{Mp})^t(\boldsymbol{Mp})$

$e_{svd} = 1564.807;$
$e_{ohne} = 1485.715;$
$e_{mit} = 1485.715$