$$\hat{x} = (A^T\Sigma_y^{-1}A + \Sigma_x)^{-1}(A^T\Sigma_y^{-1}y + \Sigma_x^{-1}\mu_0)$$

$\boxed{\Sigma_y}$

1 mm $\rightarrow$ $\sigma = 3$ mm

$A = [\ ]$

$$\sigma^2 = \boxed{\frac{\sum (y-\bar{y})^2}{N-1}} \qquad \boxed{\frac{e^Te}{\frac{m-n}{N}}}$$

$$\hat{x} = (A^T\Sigma_y^{-1}A)^{-1}A^T\Sigma_y^{-1}\boxed{y} = \bar{y}$$

$$\boxed{\hat{\sigma}^2 = \frac{e^TPe}{m-n}}$$

$$P = \sigma_0^{-2}\Sigma_y^{-1}$$

$\sigma_0 = 1$

$\hat{\sigma}_0$

$$\Rightarrow P = \boxed{\hat{\sigma}_0^{-2}}\Sigma_y^{-1}$$

$$Q_x = (A^TPA)^{-1}$$

$$\boxed{\hat{Q}_{\hat{x}} = \hat{\sigma}_0^2 Q_{\hat{x}}}$$

σ_B

Conjugate priors

A density func is called a conjugate prior, if it leads after being multiplied by the likelihood to a posterior density function which belongs to the same family of distrib

prior $\propto$ liklihood $\propto$ posterior

Normal

$\boxed{\text{Normal-Gamma}}$

Normal-Gamma Distribution

x be an $n\times 1$ random vector

τ a random variable

let the conditional density function $\boxed{p(x|\mu, \tau^{-1}V)}$ $\rightarrow$ Normal distribution

$\boxed{\tau}$ known $\rightarrow$

$N(\mu, \tau^{-1}V)$

$\tau \sim G(b, p)$ slope; an inverse scale paramt

$p(\tau|b,p) \sim G(b,p)$

The joint density function $\boxed{p(x, \tau|\mu, V, b, p)}$

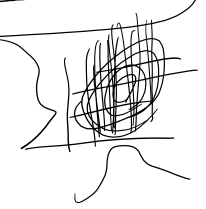
$$= p(x|\mu, \tau^{-1}V)\,p(\tau|b,p) \sim NG(\mu, V, b, p)$$

$$p(x,\tau|\mu,V,b,p) = (2\pi)^{-n/2}(\det V)^{-1/2}\, b^p (\Gamma(p))^{-1}\, \tau^{n/2+p-1} \exp\left\{-\frac{\tau}{2}\left[2b + (x-\mu)^T V^{-1}(x-\mu)\right]\right\}$$

$$b>0,\ p>0,\ 0<\tau<\infty,\ -\infty<x_i<\infty$$

$$\Gamma(p) = (p-1)!$$

$$\Gamma(p+1/2) = \frac{(2p-1)(2p-3)\ldots 5\times 3\times 1}{2^p}\sqrt{\pi}$$



marginal distribution for x

$p(x) \sim t(\mu, bV/p, 2p)$ $\rightarrow$ D

marginal distribution τ

$$p(\tau) \sim G(b, p) = \frac{b^p}{\Gamma(p)}\tau^{p-1}\exp\{-b\tau\} \qquad b>0\ \ p>0,\ \ 0<\tau<\infty$$

$$E(\tau) = p/b$$

$$D(\tau) = p/b^2$$

$$\boxed{\sigma_B = \tau^{-1}}$$

$$y = Ax + e$$

$\boxed{\Sigma_x}$

$$x \sim \boxed{N}(\mu_0, \Sigma_x)$$

$$p(y|x, \sigma_B^2) \sim N(A\hat{x}, \boxed{\sigma_B^2}\Sigma_y)$$

$$p(y|\mu, \Sigma_y) = \frac{1}{(2\pi)^{n/2}(\det \sigma_B^2\Sigma_y)^{1/2}}\, e^{-1/2\{(y-\mu)^T\Sigma_y^{-1}(y-\mu)\}}$$

$\det(cJ) = c^n \det(J)$ for $n\times n$ J matrix

$$\tau = \frac{1}{\sigma_B^2}$$

likelihood

$$p(y|\mu, \tau) = (2\pi)^{-n/2}(\det \Sigma_y^{-1})^{1/2}\,\tau^{n/2}\exp\left[-\frac{\tau}{2}(y-Ax)^T\Sigma_y^{-1}(y-Ax)\right]$$

prior

$$p(x, \tau) \sim NG(\mu_0, \Sigma_x, b_0, p_0)$$

posterior

$$\boxed{\sigma_B = ?}$$