Universität Stuttgart

Geodätisches Institut

Parameterschätzung Übung 4

Ausarbeitung im Studiengang

Geodäsie und Geoinformatik

an der Universität Stuttgart

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Kapitel 1

Ausarbeitung

1.1 a

There are 110 heads in 200 tosses, the frequency is 55%, which means it is possible that the coin is unfair.

1.2 b

Bayes'theorem:

$$prob(H|\{results\}) = \frac{prob(\{results\}|H)prob(H)}{prob(\{results\})}$$

- likelihood func: $prob(\{results\}|H)$
- prior func: $prob(H)$

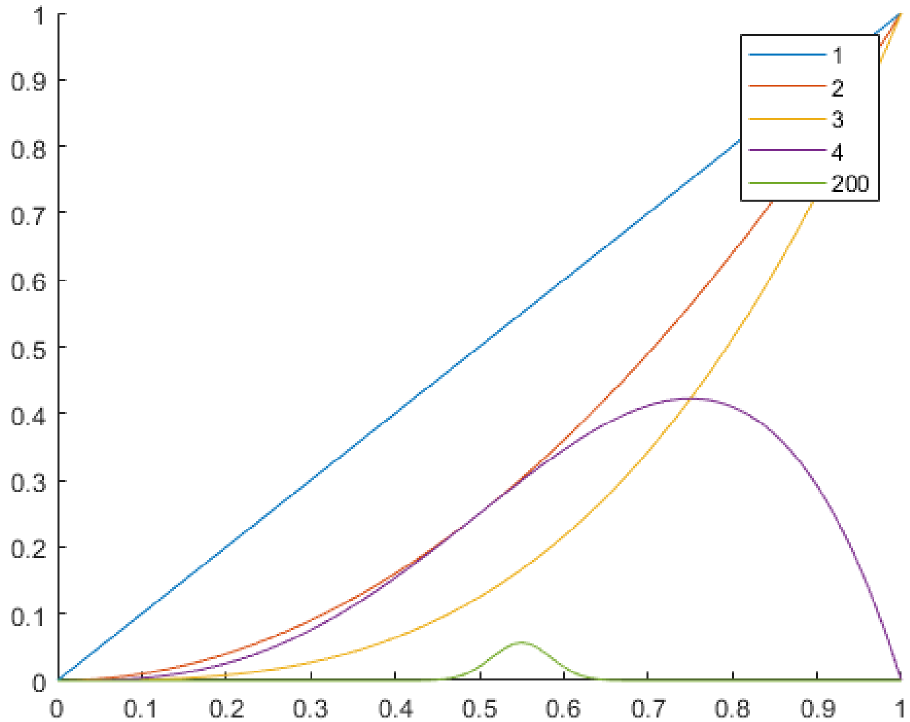
1.3 c

Münzenwurf gehört zur Binomialverteilung.

$$prob(\{results\}|H) = \begin{bmatrix} N \\ R \end{bmatrix} \cdot H^R(1-H)^(N-R) = \frac{N!}{R!(N-R)!}H^R(1-H)^{(N-R)}$$

1.4 d & e

The posterior pdf is in Abbildung 1.1. because the first tosses are head, so when N = 1 3, the maximum value of pdf is when $H = 1$. The fourth toss is, so the pdf returns 0 for $H = 1$, the max pdf ist at $H = 0.75$. When we have 200 tosses, there is more possibility, the maximal value is at $H = 0.55$.



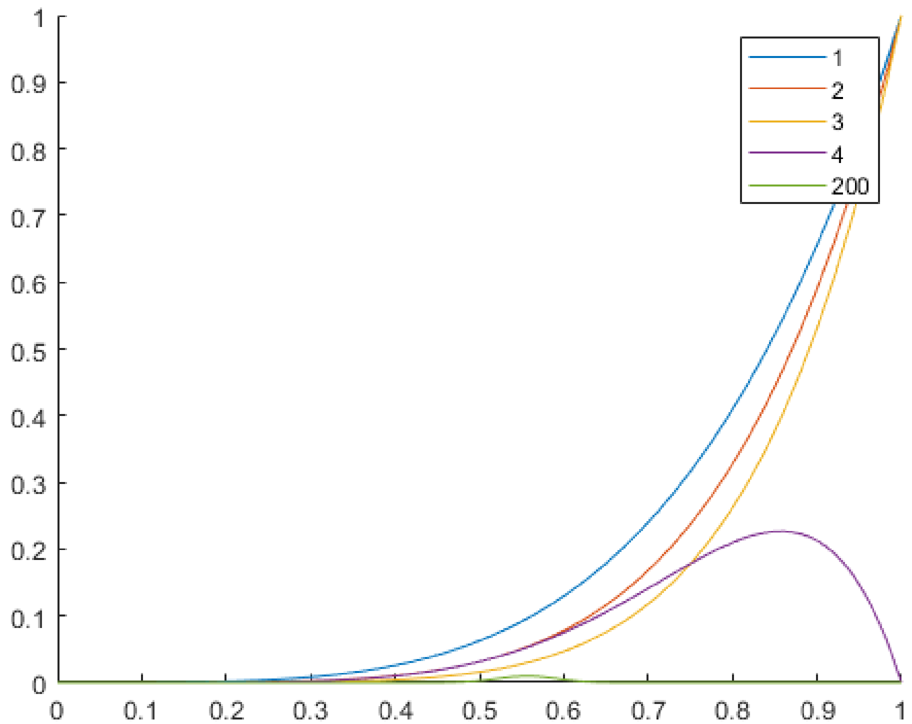
(a) pdf

Abbildung 1.1: *PDF*

1.5 f

$$
\begin{aligned}
P_N &= \frac{\frac{N!}{R!(N-R)!} H^R (1-H)^{(N-R)} \cdot prob(H)}{prob(\{results\})} \\
P_{N+1} &= \frac{\frac{(N+1)!}{(R)!(N+1-R)!} H^R (1-H)^{(N+1-R)} \cdot prob(H)}{prob(\{results\})} \\
\frac{P_{N+1}}{P_N} &= \frac{(N+1)(1-H)}{N+1-R}(1-H) \\
P_{N+1} &= \frac{(N+1)(1-H)}{N+1-R}(1-H) \cdot P_N
\end{aligned}
$$

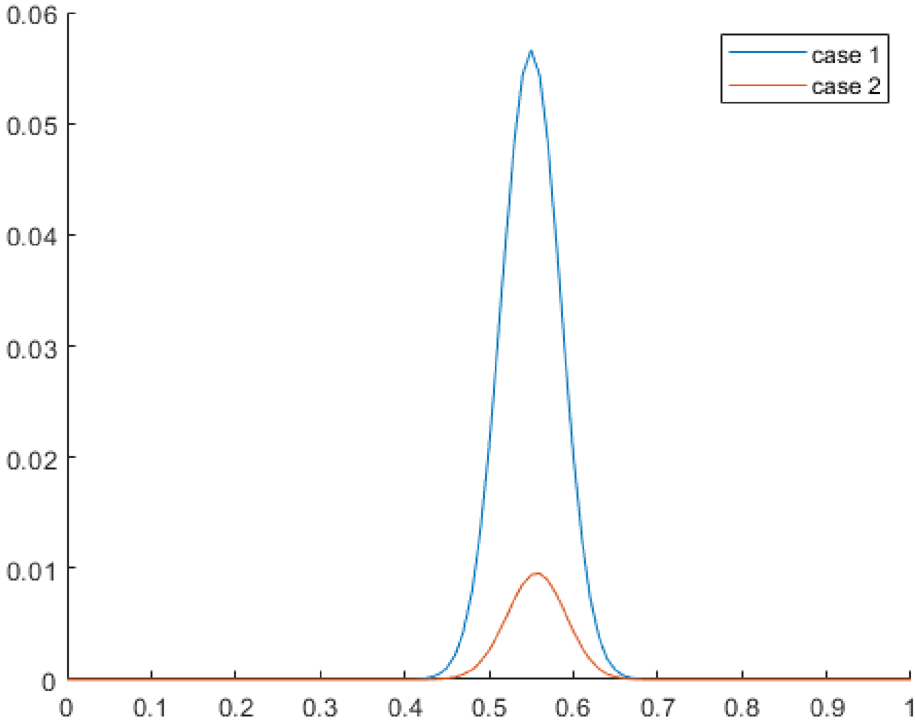
1.6 g



(a)

Abbildung 1.2: $3H^2$

In this case, the coin is tend to be more unfair compared to (e) Abbildung 1.2, the height of the peak is also lowerAbbildung 1.3. (This distribution is not exactly correct because of the accuracy limit of Matlab).



(a)

Abbildung 1.3: *N = 200*

1.7 Maximum-Liklihood-Methode

$$prob(\{results\}|H) = \frac{N!}{R!(N-R)!}H^R(1-H)^{(N-R)}$$

$$\begin{aligned}\log \mathcal{L}(H) =& \log \frac{N!}{R!(N-R)!} + \log H^R + \log(1-H)^{(N-R)} \\ =& R\log H + (N-R)\log(1-H)\end{aligned}$$

Um die maximale Werte zu berechnen:

$$\frac{\partial \log \mathcal{L}(H)}{\partial H} = R \cdot \frac{1}{H} - (N-R)\frac{1}{1-H} = \frac{R-HN}{H(1-H)}$$

When $R = HN$, also $H = \frac{R}{N}$, Likelihood function is maximum.