21.04.2021

- Least squares A, B, GH ✓✓ ①
- Kalman filter, EKF, UKF ✓✓ ⑤
- Bayes schätzung ✓ ④
- Least Squares Collocation ✗ ②
- probability theory ✓ ③
- Machine Learning ✗ ⑥
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Topics

- GH
- LSC
- probability theory
- Bayes estimation
- Kalman filter
- Machine Learning

Form

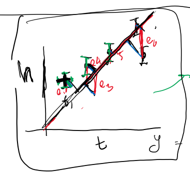
- inter-active lecture
  - basics and main parts come from Tourian
  - a discussion point will remain in every lecture
  - Mr. Yu will work on the open point
  - in the week after in the first half an hour the open point is presented by Mr. Yu and will be discussed.

Gauss-Helmert Method (GHM)

$\boxed{f(y,x) \doteq 0}$

$y_{m\times 1}$ Vector of observation

$x_{n\times 1}$ " " unknowns



$\boxed{y = Ax + e}$

$y = at + b$

$y = \begin{bmatrix} t_1 & 1 \\ \vdots & \vdots \\ t_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

$\qquad\quad A \qquad\quad x$

Least squares error

$t, y \qquad e^T e \to \min$

$\boxed{Q_y} \qquad e_1, \ldots, e_n$

$\boxed{Q_t}$

$\boxed{e^T e \to \min}$

$\boxed{Q_x, Q_y}$

$\hat{a}, \hat{b}$

$[y \quad t]$

$h - at - b = 0$

observation type

(t, y)

$\frac{\partial f}{\partial y} \quad m \times km$

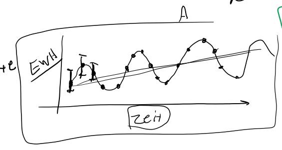
$\boxed{y = [t \quad h]}$

$\boxed{x = [a \quad b]}$

To linearize we have

$\tilde{x} = x_0 + \Delta x$

$\tilde{y} = y + e = \underbrace{y - y_0}_{\Delta y} + y_0 + e = y_0 + \Delta y + e$



Tp: $x = x_0$
$\quad y = y_0$

$f(\tilde{y}, \tilde{x}) = f(y_0 + \Delta y + e, x_0 + \Delta x)$

$f(y,x) = \underbrace{f(y,x)\big|_0}_{W_0} + \underbrace{\frac{\partial f}{\partial y}(y,x)\Big|_0}_{B^T}(\Delta y + e) + \underbrace{\frac{\partial f(y,x)}{\partial x}\Big|_0}_{A}\Delta x$

$= W_0 + B^T(\Delta y + e) + A\Delta x$

$= \underbrace{W_0 + B^T\Delta y}_{W} + B^T e + A\Delta x$

$\boxed{W + B^T e + A\Delta x = 0}$

$\boxed{P = \begin{bmatrix} Q_y^{-1} & 0 \\ 0 & Q_x^{-1} \end{bmatrix}}$

$L(\Delta x, e, \lambda) = \frac{1}{2} e^T P e + \lambda^T (W + A\Delta x + B^T e) \longrightarrow \min$

$\frac{\partial L}{\partial e}(\hat{e}, \hat{\Delta x}, \hat{\lambda}) = P\hat{e} + B\hat{\lambda} = 0$

$\frac{\partial L}{\partial \Delta x}(\hat{e}, \hat{\Delta x}, \hat{\lambda}) = A^T\hat{\lambda} = 0$

$\frac{\partial L}{\partial \lambda}(\hat{e}, \hat{\Delta x}, \hat{\lambda}) = (B^T\hat{e} + A\hat{\Delta x} + W) = 0$

$\hat{e} = -P^{-1}B\hat{\lambda}$

$A^T\hat{\lambda} = 0$

$-B^T P^{-1} B\hat{\lambda} + A\hat{\Delta x} + W = 0$

$\begin{cases} B^T P^{-1} B\hat{\lambda} - A\hat{\Delta x} - W = 0 \\ -A^T\hat{\lambda} = 0 \end{cases}$

$\begin{bmatrix} B^T P^{-1} B & -A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} \hat{\lambda} \\ \hat{\Delta x} \end{bmatrix} = \begin{bmatrix} W \\ 0 \end{bmatrix}$

$\hat{\Delta x} = -\left[A^T (B^T P^{-1} B)^{-1} A\right]^{-1} A^T (B^T P^{-1} B)^{-1} W$

$\hat{\lambda} = (B^T P^{-1} B)^{-1} \left\{ W - A\left[A^T (B^T P^{-1} B)^{-1} A\right]^{-1} A^T (B^T P^{-1} B)^{-1} W \right\}$

$\hat{e} = -B P^{-1} (B^T P^{-1} B)^{-1} \left\{ W - A\left[A^T (B^T P^{-1} B)^{-1} A\right]^{-1} A^T (B^T P^{-1} B)^{-1} W \right\}$

Matlb

$\boxed{y_0 = y}$ $\quad x_0 =$

while $|\hat{\Delta x}| > \varepsilon$

$\quad \boxed{\Delta y} = y - y_0$

$\quad A = \frac{\partial f(x,y)}{\partial x}\Big|_0 \quad m \times n$

$\quad B^T = \frac{\partial f(x,y)}{\partial y}\Big|_0 \quad m \times km$ → type of observation

$\quad W = W_0 + B^T\Delta y \quad m \times 1$

$\quad W_0 = f(x,y)\big|_0$

$\quad \underbrace{\begin{bmatrix} B^T P^{-1} B & -A \\ -A^T & 0 \end{bmatrix}}_{J_1} \underbrace{\begin{bmatrix} W \\ 0 \end{bmatrix}}_{J_2}$

$\quad L = J_1 \backslash J_2$

$\quad \hat{\lambda} = L(1:m)$

$\quad \hat{\Delta x} = L(m+1:\mathrm{End})$

$\quad \hat{x} = x_0 + \hat{\Delta x}$

$\quad \hat{e} = -P^{-1}B\hat{\lambda}$

$\quad y_0 = \boxed{y + \hat{e}}$

end

$W + B^T e + A\Delta x = 0$

$\quad m \times km \qquad m \times n \quad n \times 1$

$P: km \times km$

$Q = \begin{bmatrix} \boxed{\sigma_{t_1}^2} & & & & \\ & \sigma_{h_1}^2 & & & \\ & & \sigma_{t_2}^2 & & \\ & & & \sigma_{h_2}^2 & \\ & & & & \ddots \end{bmatrix}$

$P = Q^{-1}$

$y = \begin{bmatrix} h \\ t \end{bmatrix} \qquad h - at - b = 0$

$x = \begin{bmatrix} a \\ b \end{bmatrix}$

$f(h_i, t_i, a, b) = f_i(h_i^0, t_i^0, a^0, b^0) + \frac{\partial f}{\partial h_i}\Big|_0 (h_i - h_i^0 + e_{y_i}) + \frac{\partial f}{\partial t_i}\Big|_0 (t_i - t_i^0 + e_{t_i})$

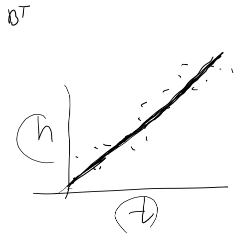
$+ \frac{\partial f}{\partial a}\Big|_0 (a - a^0) + \frac{\partial f}{\partial b}\Big|_0 (b - b^0)$

$= \underbrace{f_i(h_i^0, t_i^0, a^0, b^0) + \begin{bmatrix} \frac{\partial f_i}{\partial h_i}\Big|_0 & \frac{\partial f_i}{\partial t_i}\Big|_0 \end{bmatrix} \begin{bmatrix} \Delta h_i \\ \Delta t_i \end{bmatrix}}_{W}$

$\underbrace{\begin{bmatrix} \frac{\partial f}{\partial h_i}\Big|_0 & \frac{\partial f_i}{\partial t_i}\Big|_0 \end{bmatrix}}_{B^T} \underbrace{\begin{bmatrix} e_{hi} \\ e_{ti} \end{bmatrix}}_{e} + \underbrace{\begin{bmatrix} \frac{\partial f}{\partial a}\Big|_0 & \frac{\partial f}{\partial b}\Big|_0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix}}_{\Delta x}$

$B^T = \begin{bmatrix} x & x & 0 & \cdots & & \\ 0 & 0 & x & x & \cdots & \\ 0 & 0 & 0 & 0 & x & x & 0 & \cdots \end{bmatrix}$

Singular value decomposition



GH

a, b

mit Fehler