

## Ausgewählte Kapitel der Parameterschätzung — Übung 3

Ausgabedatum: 21. Mai 2021

Abgabetermin: 1. Juni 2021, 18 Uhr

### Densification of the crustal horizontal velocity field over Europe by Least-Squares Collocation

The objective of this exercise is to estimate the field of crustal horizontal velocity on the nodes of a regular grid covering Europe. Such a dense grid is required to compute the strain field and determine the intensity of the relative movements and deformation in a given region. The velocity of the crustal movement at a given point is typically determined from a multi-year observation of a GNSS receiver. However, due to the uneven distribution of observation sites (see figure 1), it is necessary to adjust and interpolate consistently the observations over the queried points. We suggest here to use Least-Squares Collocation (LSC) to achieve such a task.

Following the notation of Moritz, we formulate the problem as follows:

$$\mathbf{Y} = \mathbf{A}\mathbf{x} + \mathbf{S}' + \mathbf{e} \quad (1)$$

where  $\mathbf{Y}$  is the observation vector containing the horizontal components of the crustal velocity for the different receivers considered in this lab and  $\mathbf{A}\mathbf{x}$  encodes to the deterministic part of the signal, also called "trend". For this problem, it corresponds to the velocity of the plate regarded as a rigid body. In such a case, it can be shown that the movement of a rigid body on a sphere is fully described by a rotation about an axis passing through the centre of the sphere that is,  $\mathbf{V}_{det}^h = (\mathbf{R} \times \boldsymbol{\omega})^h$  where  $\mathbf{R}$  is the receiver position and  $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)^T$  is an unknown rotation vector.

In the file `GNSS_VelocityField.txt`, you will find a table describing the position (X, Y, Z) of each receiver in the first 3 columns and the velocity components ( $V_x$ ,  $V_y$ ,  $V_z$ ) in column 4 to 6. Positions and velocity are expressed in the ERTF 2014 (which is an Earth-centred, Earth-fixed reference frame) and the units are m and m/yr, respectively. The data are derived from continuous observations during the period spanning from January 1996 to February 2021

- Compute the geocentric coordinates of each points
- Rotate the velocity vector from the ERTF 2014 to the local ENU frame for each point.
- Determine the matrix  $\mathbf{A}$

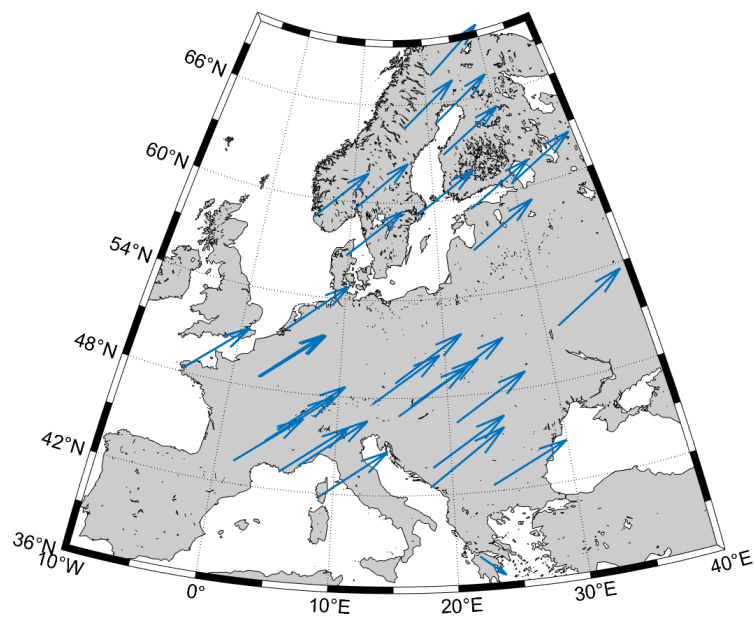


Figure 1: Horizontal velocity field as determined at various GNSS station across Europe