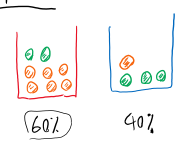
Ausgewählte Kapitel der Parameterschätzung

02.06.2021

probability theory



$B \swarrow^{r}_{b}$ $F \swarrow^{a}_{o}$

$p(B=r) = \frac{6}{10}$ $p(B=b) = \frac{4}{10}$

1. what is the overall probability that the selection procedure we pick an apple?
2. Given that we have chosen an orange, what is the probability that the box we chose is blue?

$p(F=a|B=r) = \frac{1}{4}$
$p(F=o|B=r) = \frac{3}{4}$] 1

$p(F=a|B=b) = \frac{3}{4}$
$p(F=o|B=b) = \frac{1}{4}$] 1

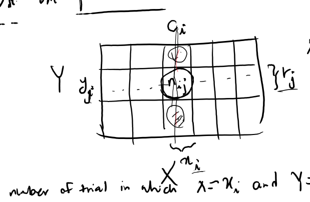
$$p(F=a) = \sum_B p(F,B)$$
$$= \sum_B p(F|B_i)\,p(B_i)$$
$$= p(F=a|B=r)\,p(B=r) + p(F=a|B=b)\,p(B=b)$$
$$\frac{1}{4} \times \frac{6}{10} + \frac{3}{4} \times \frac{4}{10} = \frac{9}{20}$$

$$\boxed{p(F=o) = \frac{11}{20}}$$

$$p(B=b|F=o) = \frac{p(F=o|B=b)\,p(B=b)}{p(F=o)} = \frac{1/4 \times 4/10}{\frac{11}{20}}$$

Sum rule and product rule

X
Y



X can take x_i, $i=1,\dots M$
Y can take y_j, $j=1\dots L$

$\boxed{N}$

the number of trial in which $X=x_i$ and $Y=y_j$ is n_{ij}

$p(X=x_i, Y=y_j)$ = joint probability
$$= \frac{n_{ij}}{N}$$

$c_i = \sum_j n_{ij}$

marginal probability $\rightarrow \boxed{p(X=x_i)} = \frac{c_i}{N}$
$$= \sum_{j=1}^{L} p(X=x_i, Y=y_j)$$

$\rightarrow$ sum rule $\boxed{p(x) = \sum_y p(x,y)}$

$$P(Y=y_j|X=x_i) = \frac{n_{ij}}{c_i}$$

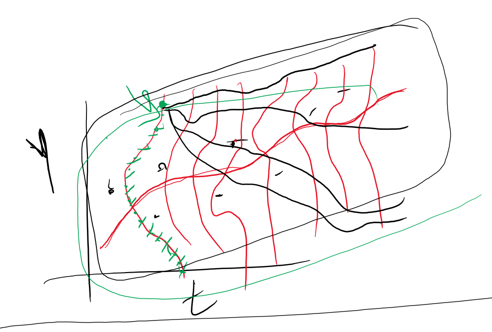
$$p(X=x_i, Y=y_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N}$$

product rule $\quad p(X=x_i, Y=y_j) = p(Y=y_j|X=x_i) \cdot p(X=x_i)$

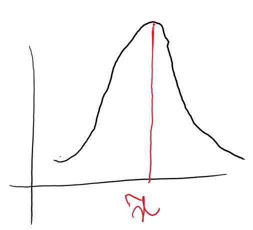
$$p(x) = \sum_Y p(x,y)$$
$$p(x,y) = p(y|x)\,p(x)$$

$$p(y|x) = \frac{p(x,y)}{p(x)} = \frac{p(y,x)}{p(x)} = \frac{p(x|y)\,p(y)}{p(x)}$$

$$\boxed{p(y|x) = \frac{p(x|y)\,p(y)}{p(x)}}$$ Bayes Theorem



$$y = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \dots + a_m t^M$$



posterior, likelihood function

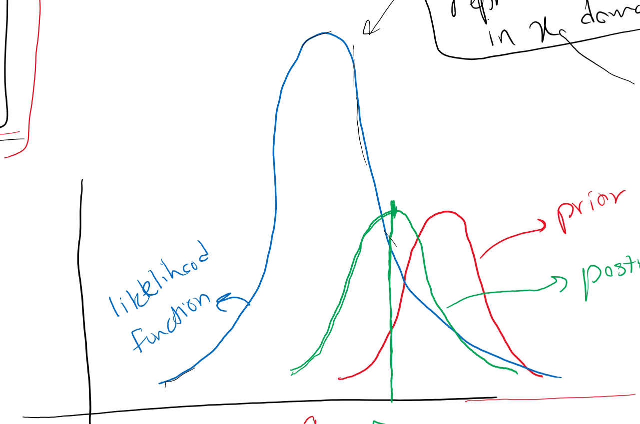
$$\boxed{p(x|y)} = \frac{\boxed{p(y|x)}\,\boxed{p(x)}}{p(y)} \rightarrow \boxed{\text{prior}}$$

$$x = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_M \end{bmatrix}$$

$a_0 \ \Sigma_{a_0}$
$a_1 \ \Sigma_{a_1}$
$\vdots$

$$p(y) = \int p(y|x)\,p(x)\,dx$$

$p(y|x)$ represented in x domain



Maximum a posteriori Solution = MAP

$$\frac{\mu_0, \Sigma_x}{\Sigma_y}$$

$$\hat{x} = (A^T P A + \Sigma_x^{-1})^{-1} (A^T P y + \Sigma_x^{-1} \mu_0)$$

λ $\qquad \lambda = \frac{\Sigma_y}{\Sigma_x}$