Ausgewählte Kapitel der Parameterschätzung
07.07.2021

## Discrete Kalman Filter

$$\begin{cases} x_t = \Phi x_{t-1} + e_t \\ y_t = H_t x_t + v_t \end{cases}$$

$$E(e_t v_t^T) = 0$$

$$D\{e_t\} = Q_t \qquad D\{v_t\} = \boxed{\sigma^2} \Sigma_{y_t}$$

Prediction step

time update
TU

$$\boxed{t,t} \quad \boxed{t,t-1}$$

$$\boxed{\hat{x}_{t,t-1}} = \Phi_{t,t-1} \boxed{\hat{x}_{t-1,t-1}}$$

$$\sigma^2 \Sigma_{x_{t,t-1}} = \sigma^2 \Phi_{t,t-1} \Sigma_{x_{t-1,t-1}} \Phi_{t,t-1}^T + Q_{t-1}$$

$$\Sigma_{x_{0,0}} = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \end{bmatrix}$$

Estimation step

measurement update
MU

$$\hat{x}_{t,t} = \hat{x}_{t,t-1} + F_t (y_t - H_t \hat{x}_{t,t-1})$$

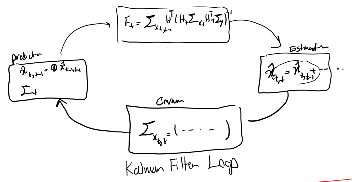
$$F_t = \Sigma_{x_{t,t-1}} H_t^T (H_t \Sigma_{x_{t,t-1}} H_t^T + \Sigma_{y_t})^{-1}$$

$$\Sigma_{x_{t,t}} = (I - F_t H_t) \boxed{\hat{\sigma}_t^2} \Sigma_{x_{t,t-1}}$$

$$D(x_t | y_1 \dots y_t) = \boxed{\hat{\sigma}^2} \Sigma_{x_{t,t}}$$

$$\hat{\sigma}_t^2 = (n = \dots )$$

Q: $\hat{\sigma}_t^2$ should the $\hat{\sigma}_t^2$ be used in the loop of Kalman filter or only to scale the estimate covariance at each epoch?




Kalman Filter Loop

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$$\boxed{x_t} = \Phi x_{t-1} + e_t$$

State Vector

$$\begin{bmatrix} r_{x_0} \\ r_{y_0} \\ r_{z_0} \\ v_x \\ v_y \\ v_z \\ a_x \\ a_y \\ a_z \\ \alpha \\ \kappa \\ \theta \end{bmatrix}_t = \begin{bmatrix} 0 & 0 & 0 & \Delta t & \dots \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix}_{t-1}$$

$$r_{x_t} = \boxed{\quad}$$

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## Extended Kalman Filter

EKF

$$\begin{cases} x_t = f(x_{t-1}, u_t) + e_t \\ y_t = h(x_t) + v_t \end{cases}$$

$f(\cdot)$, $h(\cdot)$ are known funct

$$x_t = x_t^* + \Delta x_t$$

$$\begin{cases} x_t^* + \Delta x_t = f(x_{t-1}^* + \Delta x_{t-1}, u_{t-1}) + e \\ y_t = h(x_t^* + \Delta x_t) + v_t \end{cases}$$

$$\to \underbrace{f(x_{t-1}^*, u_{t-1})}_{x_{t-1}^*} + \frac{\partial f}{\partial x}(x_{t-1}, u_{t-1}) \Delta x_{t-1}$$

$\Delta x$ is small

$$\begin{cases} \Delta x_t = \Phi_{t,t-1} \Delta x_{t-1} + e_t \\ y_t - h(x_t^*) = H_t \Delta x_t + v_t \end{cases}$$

$$\Phi_{t,t-1} = \left. \frac{\partial f(x_{t-1}^*, u_{t-1})}{\partial x} \right|_{Tp = x^*}$$

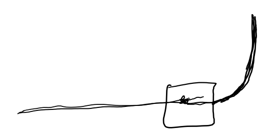
$$H_t = \left. \frac{\partial h(x)}{\partial x} \right|_{Tp = x^*}$$

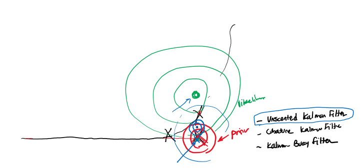
$$\boxed{\hat{\Delta x}_{t,t}} = \hat{\Delta x}_{t,t-1} + F_t (y_t - h(x_t^*) - H_t \hat{\Delta x}_{t,t-1})$$

$$x_{t,t}^* + \hat{\Delta x}_{t,t} = x_{t,t}^* + \hat{\Delta x}_{t,t-1} + F_t [y_t - h(\hat{x}_{t,t-1})]$$

$$\Longrightarrow \hat{x}_{t,t} = \hat{x}_{t,t-1} + \boxed{F_t} [y_t - h(\hat{x}_{t,t-1})]$$

$$F_t = \Sigma_{x_{t,t-1}} H_t^T (H_t \Sigma_{x_{t,t-1}} H_t^T + \Sigma_{y_t})^{-1}$$



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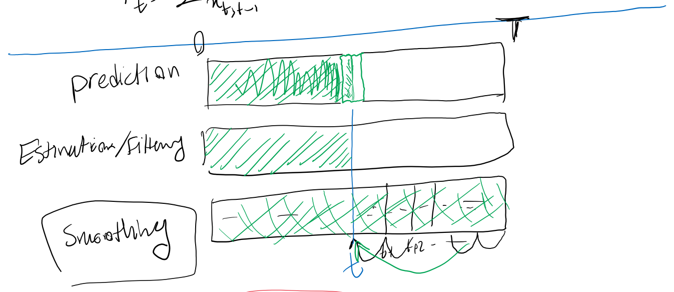
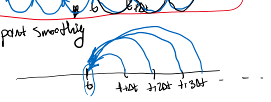


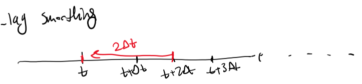

1) fixed interval smoothing

RTS method


2) fixed point smoothing




3) fixed-lag smoothing




Rauch, Tung, Striebel (RTS)

$$\hat{x}_{t,t}^s = \hat{x}_{t,t} + A_t [(\hat{x}_{t+1,t+1}^s - \hat{x}_{t+1,t})]$$

$$\boxed{A_t = \Sigma_{x_{t,t}} \Phi_{t+1,t}^T \Sigma_{x_{t+1,t}}^{-1}}$$

$$\Sigma_{x_{t,t}^s} = \Sigma_{x_{t,t}} + A_t [\Sigma_{x_{t+1,t+1}^s} - \Sigma_{x_{t+1,t}}] A_t^T$$