Universität Stuttgart
Geodätisches Institut

Parameterschätzung Übung 6

Ausarbeitung im Studiengang
Geodäsie und Geoinformatik
an der Universität Stuttgart

Ziqing Yu, 3218051

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Betreuer: Dr.-Ing. Mohammad Tourian
Universität Stuttgart

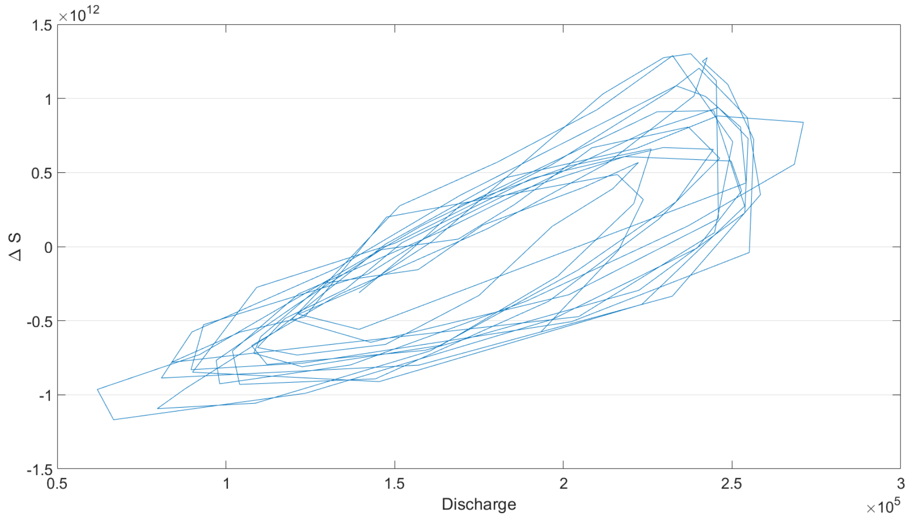
Dr. Karim Douch
Universität Stuttgart

Chapter 1

Report

1.1 a

The Relationships between Discharge and ΔS is shown in Figure 1.1.



(a)

Figure 1.1:

1.2 b

$$\frac{dR}{dt} + \frac{R}{\tau} = \omega_n^2 S$$

1.3 c

$$\frac{R_t - R_{t-1}}{\Delta t} + \frac{R_t}{\tau} = \omega_n^2 S_t$$

If we take Δt as 1 unit.

$$
\begin{aligned}
R_t - R_{t-1} + \frac{R_t}{\tau} &= \omega_n^2 S_t \\
\left(1 + \frac{1}{\tau}\right) R_t &= R_{t-1} + \omega_n^2 S_t \\
R_t &= \frac{\tau}{\tau+1} R_{t-1} + \frac{\tau\omega_n^2}{\tau+1} S_t
\end{aligned}
$$

1.4 d

$$
\begin{aligned}
R_t =& \frac{\tau}{\tau+1} R_{t-1} + \frac{\tau\omega_n^2}{\tau+1} S_t \\
=& \frac{\tau}{\tau+1} R_{t-1} + \frac{\tau\omega_n^2}{\tau+1} S_0 + \frac{\tau\omega_n^2}{\tau+1} \sum_{i=1}^{t} \Delta S_i \\
=& \alpha R_{t-1} + \beta + \gamma \sum_{i=1}^{t} \Delta S_i
\end{aligned}
$$

Thus we have:

$$
\begin{aligned}
S_0 &= \frac{\beta}{\gamma} \\
\omega_0 &= \sqrt{\frac{\gamma}{\alpha}} \\
\tau &= \frac{\alpha}{1-\alpha}
\end{aligned}
$$

The 3 parameters can be calculated using least square since we have discharge and ΔS data.

$$R(t) = R(t-1)e^{\frac{\Delta t}{\tau}} + e^{\frac{\Delta t}{\tau}}\omega^2 \int S(t)e^{\frac{-\Delta t}{\tau}}dt$$
$$R(t) = R(t-1)e^{\frac{\Delta t}{\tau}} + \omega^2 \left[\frac{S(t-1)+S(t)}{2} \cdot \Delta t\right]$$

$$\omega^2\tau(e^{\frac{\Delta t}{\tau}} - 1)$$
$$R(t_{n+1}) = R(t_n)e^{\frac{\Delta t}{\tau}} + \bar{S}(t_n) \cdot \omega^2\tau(e^{\frac{\Delta t}{\tau}} - 1)$$
$$R(t_{n+1}) = R(t_n)e^{\frac{\Delta t}{\tau}} + (S_0 + \Delta S(t_n)) \cdot \omega^2\tau(e^{\frac{\Delta t}{\tau}} - 1)$$