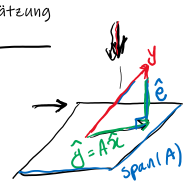
Ausgewählte Kapitel der Parameterschätzung
28.04.2021

LS $\quad y = Ax + e$



$$\boxed{\hat{x} = (A^TA)^{-1}A^Ty}$$

$$\hat{y} = A\hat{x} = \underbrace{A(A^TA)^{-1}A^T}_{P_A}y$$

$$\hat{y} = P_A y$$

$$\hat{e} = y - \hat{y} = (I - P_A)y = P_A^{\perp} y$$

$$\boxed{I - A(A^TA)^{-1}A^T} \to \boxed{U_\perp U_\perp^T}$$

SVD

= svd(A,'econ')

$$\boxed{A = \tilde{U}\Sigma V^T}$$

$$\hat{x} = (A^TA)^{-1}A^Ty = (V\Sigma^T\underbrace{\tilde{U}^T\tilde{U}}_{I}\Sigma V^T)^{-1}V\Sigma^T\tilde{U}^Ty$$

$\Sigma^T = \Sigma$

$$= (V\Sigma^2V^T)^{-1}V\Sigma\tilde{U}^Ty$$

$$= V\Sigma^{-2}\underbrace{V^TV}_{I}\Sigma\tilde{U}^Ty$$

$$= V\Sigma^{-2}\Sigma\tilde{U}^Ty$$

$$\hat{x} = \underbrace{\boxed{V\Sigma^{-1}\tilde{U}^T}}_{\text{pseudo inverse of } A}y$$

$$\hat{x} = \boxed{(A^TA)^{-1}A^T}y$$

$$[(A^TA)^{-1}A^T]A = (A^TA)^{-1}A^TA = I$$

$$V\Sigma^{-1}\tilde{U}^T\underbrace{\tilde{U}\Sigma V^T}_{A} = V\Sigma^{-1}\Sigma V^T = VV^T = I$$

$(ABC)^{-1}$

$C^{-1}B^{-1}A^{-1}$

A, B, C invertible

$V^{-1} = V^T$

$$\Sigma = \begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 \ddots & \\ 0 & & \sigma_n \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} 1/\sigma_1 & & 0 \\ & 1/\sigma_2 & \\ 0 & & 1/\sigma_n \end{bmatrix}$$

$\boxed{A}$

$[U, S, V]$ = svd(A,'econ')

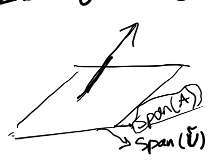
temp = $\boxed{V * S \, U^T}$ – pinv(A) =

pinv(A)

$$\hat{y} = A[(A^TA)^{-1}A^T]y = P_A y$$

$$= \tilde{U}\Sigma V^T[V\Sigma^{-1}\tilde{U}^T]y = \tilde{U}\Sigma\Sigma^{-1}\tilde{U}^Ty = \tilde{U}\tilde{U}^Ty$$

$$P_A = \tilde{U}\tilde{U}^T$$



$$\hat{e} = y - \hat{y} = (I - \tilde{U}\tilde{U}^T)y = P_A^{\perp}y$$

$$\boxed{P_A^{\perp} = I - \tilde{U}\tilde{U}^T}$$

$$U = [\tilde{U} \,\vdots\, U^{\perp}]$$

$$\boxed{U^TU = UU^T} = I = [\tilde{U} \,\vdots\, U^{\perp}]\begin{bmatrix} \tilde{U}^T \\ U^{\perp T} \end{bmatrix}$$

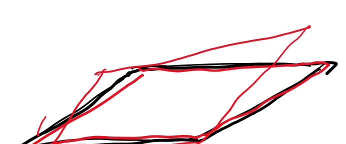
$$I = \tilde{U}\tilde{U}^T + U_\perp U_\perp^T \Rightarrow \boxed{P_A^{\perp} = U_\perp U_\perp^T}$$

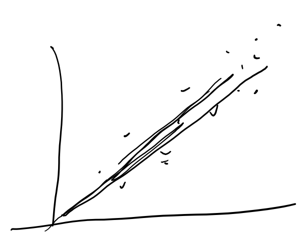
$$P_A^{\perp} = I - P_A$$

$\boxed{A}$, B

Van Huffel and Zhan, 1993

LS





$$\hat{x}_{TLS} = (A^TA - \sigma I)^{-1}A^Ty$$

$$\underset{m \times (n+1)}{\boxed{[A ; y]}} = U\Sigma V^T$$

$$E_a = \boxed{[E_A ; \hat{e}]} = [A ; y] - [\hat{A} ; \hat{y}] = U(:,end)\,S(end,end)\,V(:,end)^T$$

$m \times (n+1)$

Matlab:

$$\hat{e} = E_a(:,end)$$

$$\Rightarrow \hat{y} = y - \hat{e}$$