Ausgewählte Kapitel der Parameterschätzung — Übung 3

Ausgabedatum: 21. Mai 2021 Abgabetermin: 8. Juni 2021, 18 Uhr

Densification of the crustal horizontal velocity field over Europe by Least-Squares Collocation

The objective of this exercise is to estimate the field of crustal horizontal velocity on the nodes of a regular grid covering Europe. A dense grid can be required to compute the strain field and determine the intensity of the deformation in a given region. It is also necessary to implement the "no net rotation condition" when estimating the ITRF. The velocity of the crustal movement at a given location is typically determined from a multi-year observation of a GNSS receiver. However, due to the uneven distribution of observation sites (see figure 2), it is necessary to adjust and interpolate consistently the observations over the queried points. We suggest here to use Least-Sqaures Collocation (LSC) to achieve such a task.

Following the notation of Moritz, we formulate the problem as follows:

$$\mathbf{Y} = \mathbf{A}\mathbf{x} + \mathbf{S}' + \mathbf{e} \tag{1}$$

where $\mathbf{Y}$ is the observation vector containing the **horizontal components of the crustal velocity** for the different receivers considered in this lab and $\mathbf{Ax}$ carries the deterministic part of the signal, also called "trend". For this problem, the trend corresponds to the mean horizontal velocity field of the plate regarded as a perfectly rigid body. To model this deterministic movement, we use the Euler's rotation theorem, which states that the motion of a rigid plate on a sphere is described by a rotation about an axis passing though the centre of the sphere that is, for any point P on the plate surface the horizontal velocity vector $\mathbf{V}(P) = \omega \times \mathbf{OP}$ where $\mathbf{OP}$ is the receiver position and $\omega = (\omega_x, \omega_y, \omega_z)^\top$ is an unknown rotation vector. The intersection of the rotation axis with the Earth's surface in the direction of a positive rotation is called the **Euler's pole**.

In the file `EPN_data.mat`, you will find a table describing the position (P_x, P_y, P_z) of 254 receivers in the first 3 colums and the corresponding velocity components (V_x, V_y, V_z) in columns 4 to 6. Positions and velocities are expressed in the ERTF 2014 (which is an Earth-centred, Earth-fixed reference frame) and the units are m and m/yr, respectively. The data are derived from continuous observations during the period spanning from January 1996 to February 2021.

a) Compute the geocentric coordinates of each points.

b) Determine the vector of observations $\mathbf{Y}$ (508×1) consisting of the concatenation of the east- and north-component (denoted respectively u and v) of the velocity vector at each point. This is achieved by rotating the velocity vector from the ERTF 2014 to the local ENU (East-North-Up) frame (see Fig. 1). Plot the horizontal velocity field using the `MATLAB` function `quiverm`. Compute also the observation error variance-covariance matrix assuming that the standard-deviation of the measurement error for each component (V_x, V_y, V_z) is 0.1mm/yr.

c) Determine accordingly the matrix $\mathbf{A}$ relating the unknown vector ω to $\mathbf{Y}$.

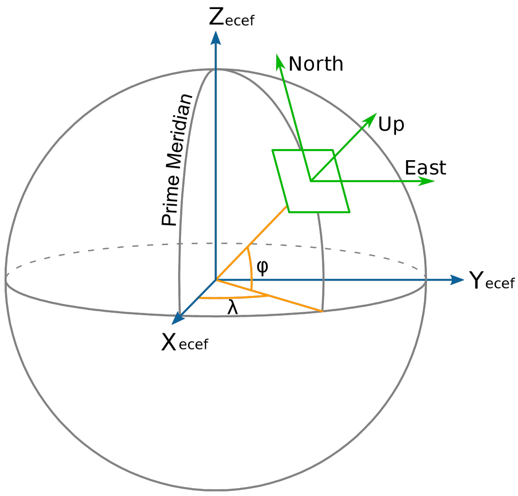

Figure 1: ECEF and local geodetic coordinate system ENU (source *Wikipedia*).

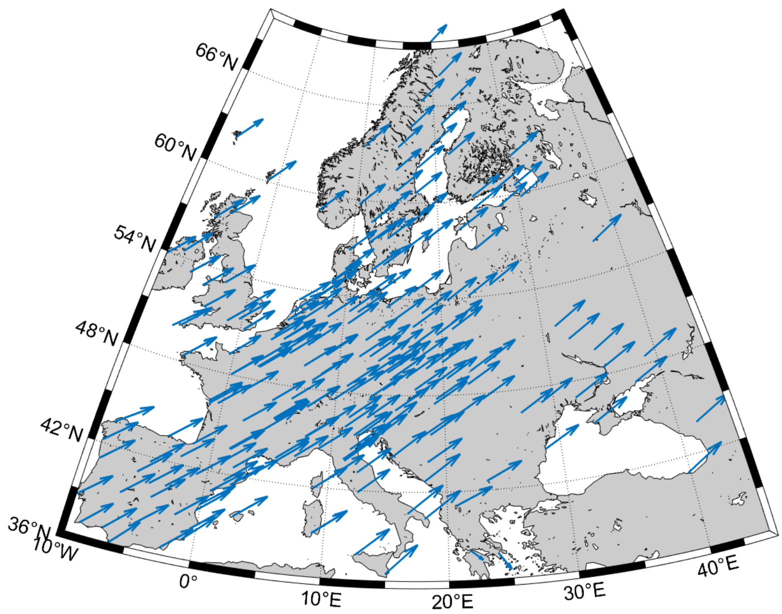

Figure 2: Horizontal velocity field as determined at various GNSS stations across Europe.

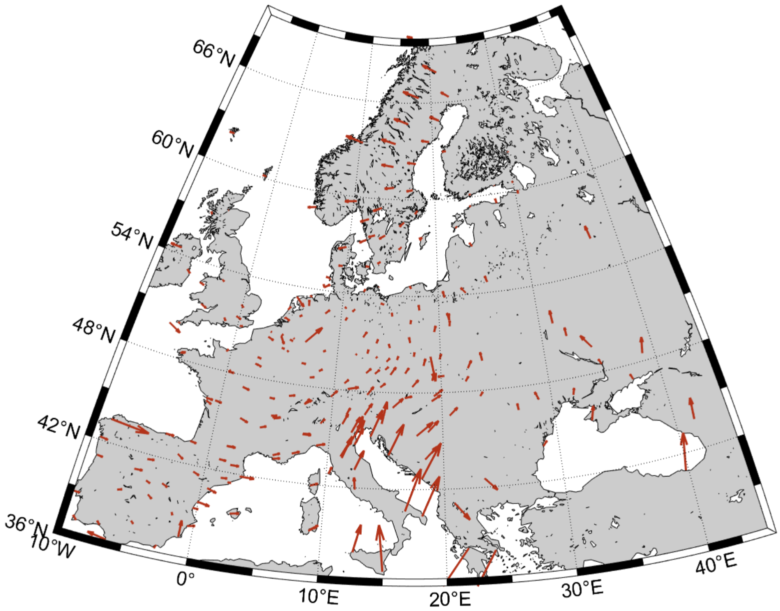

Figure 3: Horizontal velocity field after removing the estimated rigid body rotation.

d) Use least-squares adjustment to estimate the vector ω and plot the residual horizontal velocity field. What are the coordinates of the Euler's pole? According to you, does the residual field still contain a deterministic signal?

e) Now we would like to interpolate the field with LSC over a regular grid spanning from longitude 10° West to 20° East and latitude 40° North to 60° North with a sampling of 1°. Recall the hypotheses concerning the statistical properties of the different element of equation (1).

f) For both horizontal components, the signal covariance function $K(d)$ is supposed to be isotropic that is, it only depends on the spherical distance d between 2 points. The function $K(d)$ is estimated to be well approximated by

$$K(d) = \frac{K_0}{1 + (\frac{d}{a})^2} \tag{2}$$

where $a = 150\text{km}$ and $K_0 = 1.36\text{mm}^2/\text{yr}^2$. Furthermore, we assume that the 2 horizontal components are not correlated, In other words, for 2 points P_i and P_j:

$$\mathrm{E}[u_{P_i} u_{P_j}] = \mathrm{E}[v_{P_i} v_{P_j}] = K(d_{P_i P_j}) \quad \text{but} \quad \mathrm{E}[u_{P_i} v_{P_j}] = 0 \tag{3}$$

Apply the LSC algorithm to interpolate the field on the grid. You can use the `MATLAB` function `distance` to compute the distance between 2 points on the sphere or a reference ellipsoid.

g) plot the interpolated field.