Ausgewählte Kapitel der Parameterschätzung — Übung 6

Ausgabedatum: 06. Juli 2021 Abgabetermin: 21. Juli 2021, 18 Uhr

Closure of the water mass balance over the Òbidos watershed

The objective of this lab is to refine in a consistent manner the observations of the different components of the water budget over the Òbidos watershed and estimate missing data. Òbidos is a Brazilian town located in the downstream region of the Amazon river, approximately 800 km upstream from the river outlet. There, a stream gauge has been recording the river flow daily since 1968. The corresponding watershed, which defines the sub-drainage basin of the Amazon river up to this point, represents an area $\approx 4,680,000 km^2$, which corresponds to 79.4% of the whole Amazon basin. When dealing with water balance over a drainage basin, the fundamental equation is the water mass conservation, which is given by:

$$\frac{dS}{dt} = P - ET - R \tag{1}$$

where S represent the total volume of water stored in the surface and sub-surface of the watershed, R is the discharge at the outlet and P and ET are respectively the aggregated precipitation and evapotranspiration. Note that R, P and ET are fluxes! In the exercise file on ILIAS, you will find

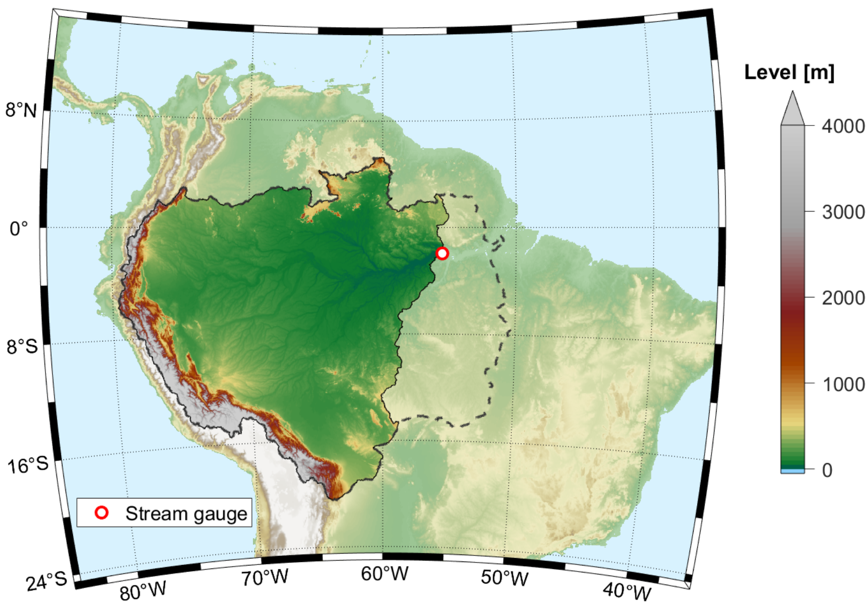

Figure 1: Map of the Òbidos watershed (bright colours) with the location of the stream gauge. The dashed line delineates the full Amazon river drainage basin.

the different observations collected and that wil be used in this lab: the Terrestrial Water Storage

Anomaly (TWSA) ΔS derived from GRACE measurement, the discharge time series at the Òbidos stream gauge, and the monthly precipitation and evapotranspiration across the whole watershed, estimated from an ensemble mean. Note that no error characterization is provided for the discharge. Furtheremore, keep in mind that some data are missing for some epoch and the units may be different.

0.1 Part I: Identification of a storage-discharge dynamic system

In this subsection, we assume that the discharge at Òbidos is governed by a law of the form $\dot{R} = f(R, S_D)$ where f is a priori unknown, $S_D = S_0 + \Delta S$ is the TDWS (Total Drainable Water Storage) and S_0 is an unknown constant. The objective of this section is to estimate the parameters of a conceptual model that approximates f.

a) To get an idea of what the function f can look like, first plot the observed discharge R as a function of ΔS. Describe and try to interprete what you observe (hysteresis cycle, nonlinearities etc.).

b) A dynamic system exhibiting a hysteresis cycle can actually be represented by a linear model as simple as a 1^{st} order ODE of the form

$$\frac{dR}{dt} + \frac{R}{\tau} = \omega_n^2 S \tag{2}$$

where τ, ω_n^2 are the model parameters to estimate. How can you interprete physically the parameter τ?

c) Equation (2) is a continuous-time equation while we have discrete data at our disposal. Suggest a way to discretize the ODE to a form adapted to the sampling of the data. The discretized equation should take the form of a 1^{st} order autoregressive model with exogeneous input (ARX)

$$R(n+1) = \alpha R(n) + \beta S(n) \tag{3}$$

where α and β are constants. Can you relates the parameters of the continuous model to the ones of the discrete mode?

d) Estimate the parameters of your model, including the term S_0. Justify the choice of the method you used for the estimation and if possible, provide the standard deviation of the parameters estimates.

e) (Optional) Examine the histogram of the observation residuals. Is the linear model really appropriate? Can you think of a simple nonlinearity to add to equation (3) to refine the modelling? If yes, estimate this new model.

0.2 Part II: Data filtering and smoothing

The objective is now to combine the different data set within a Bayesian framework in order to refine the estimation of the time series R and ΔS.

1. We can confidently assume that $S = S_D$ in the case of the Òbidos watershed. Rewrite equations (1) and (2) into a state space representation where the state is given by the vector $\mathbf{x} = (R\ \Delta S)^\top$, the input by the vector $\mathbf{u} = (S_0\ P\ ET)^\top$ and the observation $\mathbf{y}$ is either R, ΔS or both, depending on data availability and your choice.

2. Discretize the previous continuous-time state-space representation to the correct sampling rate. You can use for that the MATLAB function `c2d`.

3. You should now have a discrete state-space representation of the form:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{w}(k) \\ \mathbf{y}(k) &= \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{v}(k) \end{aligned} \tag{4}$$

   where $\mathbf{w}$ and $\mathbf{v}$ are respectively random process and observation noise. Examine the condition on the matrix $C(k)$, which allows to apply the Kalman filter to the system (4). Recall in this framework what is the prior, the likelihood function and the posterior estimate.

4. Apply the Kalman filter during the period spanning from January 2003 to March 2020. In particular, estimate the missing values in the time series of ΔS.