Universität Stuttgart

Geodätisches Institut

Parameterschätzung Übung 6

Ausarbeitung im Studiengang

Geodäsie und Geoinformatik

an der Universität Stuttgart

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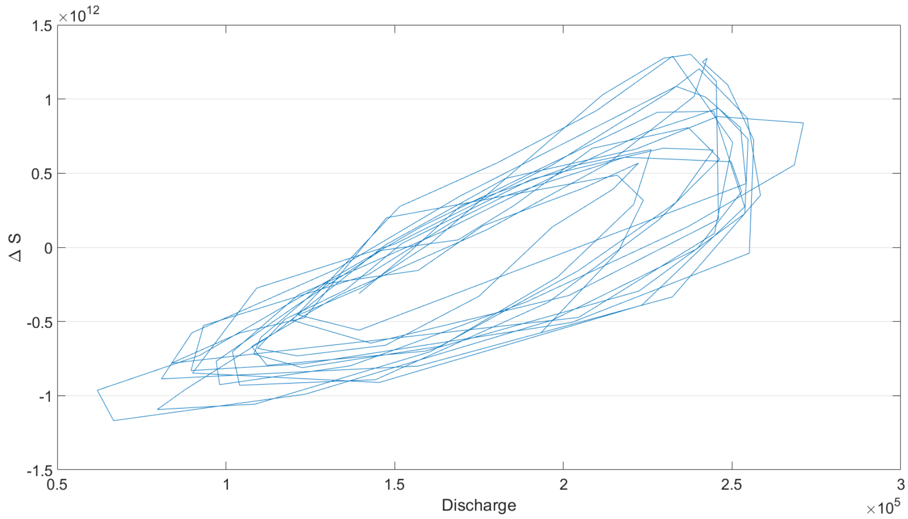
Chapter 1

Report

1.1 a

The Relationships between Discharge and ΔS is shown in Figure 1.1. This indicates a linear



(a)

Figure 1.1: *relationship between discharge and total water storage anomaly, the units are* m^3 *in y axis and* m^3/s *in x axis*

relationship between discharge and time shifted storage anomaly.

1.2 b

$$\frac{dR}{dt} + \frac{R}{\tau} = \omega_n^2(\Delta S + S_0)$$

where τ is a hydraulic time constant

1.3 c

If we take Δt as 1 unit, this ODE can be discretized.

$$R(t_{n+1}) = R(t_n)e^{\frac{\Delta t}{\tau}} + \bar{S}(t_n) \cdot \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1)$$
$$R(t_{n+1}) = R(t_n)e^{\frac{\Delta t}{\tau}} + (S_0 + \Delta S(t_n)) \cdot \omega^2 \tau (e^{\frac{\Delta t}{\tau}} - 1)$$
$$R(t_{n+1}) = \alpha R(t_n) + \beta \Delta S(t_n) + \gamma$$

1.4 d

Using least square or total least square, α, β, γ can be calculated and so are S_0, ω and τ. In this job, $\tau = 0.3912\,\text{month}^{-1}$, $\omega = 0.2048\,\text{month}^{-1}$ and $S_0 = 2.18 \cdot 10^{12}\,\text{m}^3$ which are not accurate.

1.5 Part 2

The GRACE data are from JPL from 2003 to 2020 while the precipitation and evapotranspiration from 2003 to 2019.

$$d\begin{bmatrix}\Delta S \\ R\end{bmatrix} = \begin{bmatrix}0 & -1 \\ \omega^2 & -\frac{1}{\tau}\end{bmatrix}\begin{bmatrix}\Delta S \\ R\end{bmatrix} + \begin{bmatrix}0 & 1 & -1 \\ \omega^2 & 0 & 0\end{bmatrix}\begin{bmatrix}S_0 \\ P \\ ET\end{bmatrix}$$
$$\begin{bmatrix}\Delta S_{measured} \\ R_{measured}\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}\Delta S_{estimate} \\ R_{estimate}\end{bmatrix}$$

Or $R_{measured} = R_{estimate}$ when there is no TWSA data.

$$d\begin{bmatrix}\Delta S \\ R\end{bmatrix} = A\begin{bmatrix}\Delta S \\ R\end{bmatrix} + B\begin{bmatrix}S_0 \\ P \\ ET\end{bmatrix}$$
$$\begin{bmatrix}\Delta S_{measured} \\ R_{measured}\end{bmatrix} = C\begin{bmatrix}\Delta S_{predict} \\ R_{predict}\end{bmatrix}$$

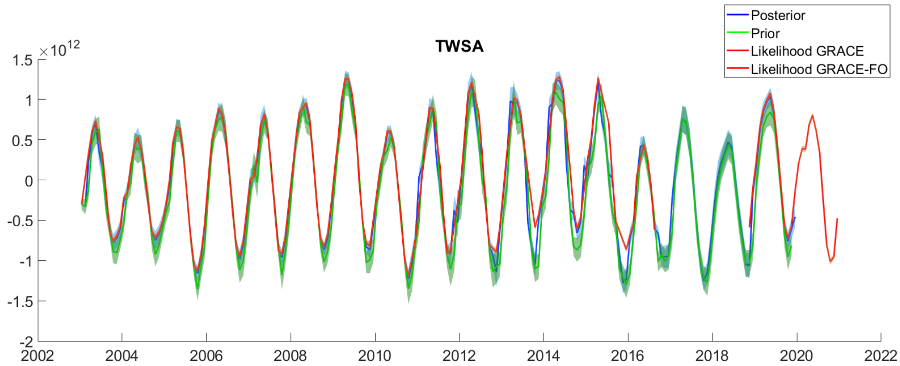
To discretize this state model:

$$\boldsymbol{A}_d = e^{\boldsymbol{A}\Delta t}$$
$$\boldsymbol{B}_d = \boldsymbol{A}^{-1}\left(\boldsymbol{A}_d - \boldsymbol{I}\right)\boldsymbol{B}$$
$$\boldsymbol{C}_d = \boldsymbol{C}$$

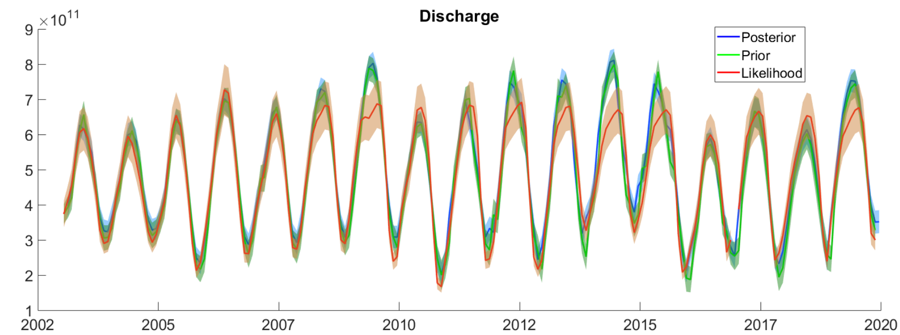
So that

$$\boldsymbol{x}_{n+1} = \boldsymbol{A}_d\boldsymbol{x}_n + \boldsymbol{B}_d\boldsymbol{u}$$
$$\boldsymbol{y}_n = \boldsymbol{C}_d\boldsymbol{x}_n$$

Using given parameter $\tau = 1.201\,\frac{1}{\text{month}}$, $\omega = 0.4787\,\frac{1}{\text{month}}$ and $S_0 = 1.82 \cdot 10^{12}\,\text{m}^3$. The variance for The precipitation, evapotranspiration and total water storage anomaly are known, the standard deviation of discharge will be 10% of the discharge using the rule of thumb. A Kalman filter can then be implemented, the result are shown in Figure 1.2



(a)



(b)

Figure 1.2: *Kalman filter result*