Exercise 6: Legendre functions and spherical harmonics

Name, First Name	
Matriculation number	
k-value	

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Task 1: Prepare figures for fully normalized zonal, tesseral and sectorial Legendre functions $\bar{P}_{lm}(\cos\theta)$ and spherical harmonics $\bar{Y}_{lm}(\theta,\lambda) = \bar{P}_{lm}(\cos\theta)\cos m\lambda$ of degree $l = 10$ within $\theta \in [0^\circ\ 180^\circ]$ using both Rodrigues-Ferrers and recursive formulas. How many zero crossings do the fully normalized Legendre functions $\bar{P}_{lm}(\cos\theta)$ contain dependent on degree l and order m? Compare results from the two aforementioned formulas. How many zero crossings do the fully normalized spherical harmonics $Y_{lm}(\theta,\lambda)$ contain in North-South direction and East-West direction dependent on degree l and order m?

Task 2: Consider a Legendre polynomial in $\cos\psi_{PQ}$, in which the ψ_{PQ} is the spherical distance between point P and Q. The addition separates the composite angle argument into contributions from the point P and Q individually

$$P_l(\cos\psi_{PQ}) = \frac{1}{2l+1}\sum_{m=0}^{l}\bar{P}_{lm}(\cos\theta_P)\bar{P}_{lm}(\cos\theta_Q)\{\cos m\lambda_P\cos m\lambda_Q + \sin m\lambda_P \sin m\lambda_Q\}$$

For all P and Q in a same meridian ($\lambda_P = \lambda_Q$), we have

$$P_l(\cos\psi_{PQ}) = \frac{1}{2l+1}\sum_{m=0}^{l}\bar{P}_{lm}(\cos\theta_P)\bar{P}_{lm}(\cos\theta_Q)$$

For $\theta_P = 90^\circ$ and $\theta_Q \in [0^\circ\ 90^\circ]$ display the difference between the right and left hand side of above equation for different ψ and for different degree l varying from 0 to 100.

Task 3: When $\theta_P = \theta_Q = \theta$ then we have

$$P_l(1) = 1 = \frac{1}{2l+1}\sum_{m=0}^{l}\bar{P}^2_{lm}(\cos\theta)$$

For $\theta = [0^\circ\ 180^\circ]$ display the right hand side of above equation for different degree l varying from 0 to 100. Do you get 1 for all degree and θ?

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Spherical harmonics series expansion, EGM96

The gravitational potential V in the exterior (mass-free) domain is determined by means of a spherical harmonics series expansion as

$$V(\lambda,\theta,r) = \frac{GM}{R}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^{l+1}\sum_{m=0}^{l}\bar{P}_{l,m}(\cos\theta)(\bar{c}_{l,m}\cos m\lambda + \bar{s}_{l,m}\sin m\lambda)$$

Various models with coefficients $\bar{c}_{lm}$ and $\bar{s}_{lm}$ exist, which have been estimated for instance from the analysis of terrestrial or satellite gravity data. One of these models is the EGM96 (Earth Gravity Model 1996) of the NASA.

Task 4: Determine the gravity and gravitational potential W and V at a point P with the following spherical coordinates by applying the EGM96 (available at ILIAS)

$$\begin{aligned}\lambda &= (10+k)^\circ\\ \theta &= (42+k)^\circ\\ r &= 6379\,245.458\ [\text{m}]\end{aligned}$$

Constants:
$R = 6378\,136.300\,\text{m}$, $GM = 3.986004415\cdot 10^{14}\,\text{m}^3\text{s}^{-2}$, $\omega = 7.292115\cdot 10^{-5}\,\text{s}^{-5}$