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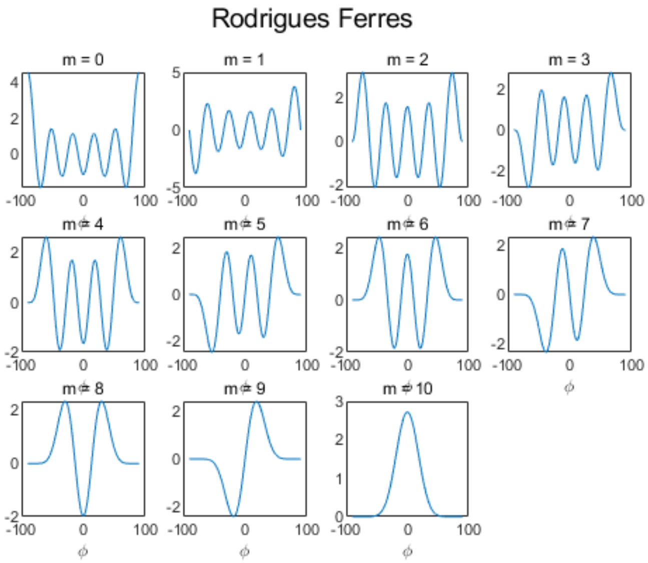
1 Task 1

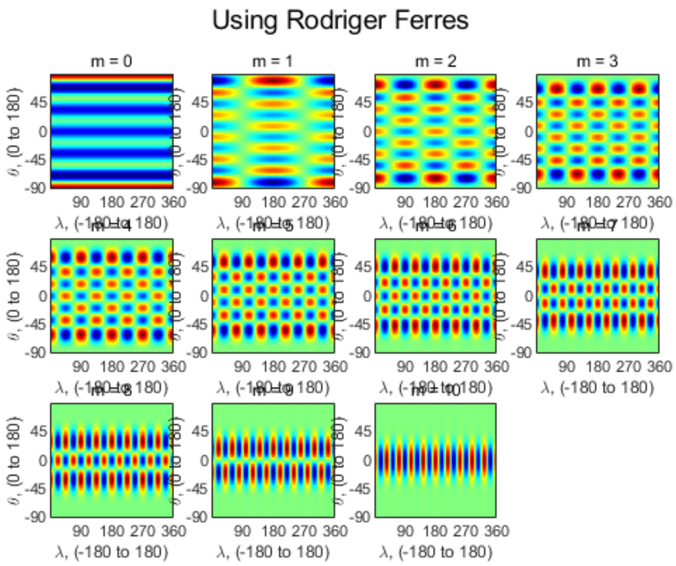
1.1 Rodrigues-Ferrers

Mit Rodrigues-Ferrers Verfahren kann man so rechnen:

$$\bar{P}_{l,m}(t) \begin{cases} \sqrt{2l+1}P_{l,m}(t) & m = 0 \\ \sqrt{2(2l+1)\frac{(l-m)!}{(l+m)!}}P_{l,m}(t) & m > 0 \end{cases}$$



a Legendre Funktion



b Spherical Harmonics

Es ist deutlich zu sehen, die Nulldurchgänge gleich $l - m$ in erste Bild. Die Nulldurchgänge sind gleich $l - m$ in Nord-Süd Richtung und gleich $2m$ in Ost-West

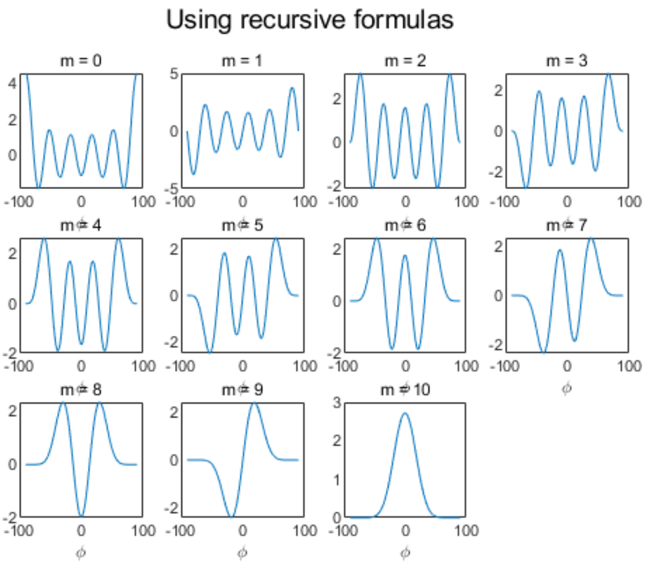
1.2 Rekursiv

Die Formeln von Rekursive Verfahren stellen darunter:

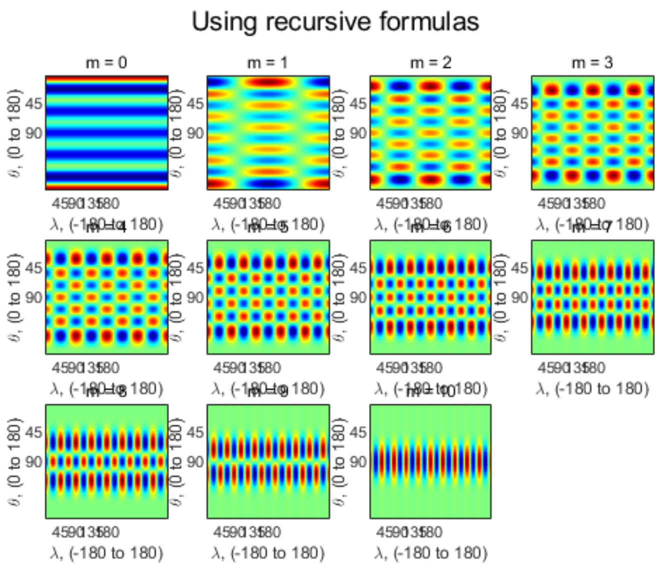
$$Initial: \bar{P}_{0,0}(t) = 1 \quad \bar{P}_{1,0}(t) = \sqrt{3}t \quad \bar{P}_{1,1}(t) = \sqrt{3(1-t^2)}$$

$$Sectorial: \bar{P}_{l,l}(t) = \sqrt{\frac{2l+1}{2l}}\sqrt{1-t^2}\bar{P}_{l-1,l-1}(t)$$

$$Othre: \bar{P}_{l,m}(t) = \sqrt{\frac{2l+1}{(l+m)(l-m)}}\{\sqrt{2l-1}t\bar{P}_{l-1,m}(t) - \sqrt{\frac{(l-1+m)(l-1-m)}{2l-3}}\bar{P}_{l-1,m}(t)\}$$



c Legendre Funktion



d Spherical Harmonics

Die Ergebnisse sind gleich wie mit Rodrigues-Ferrers Verfahren

2 Task 2

Theoretisch gibt es eine Gleichung:

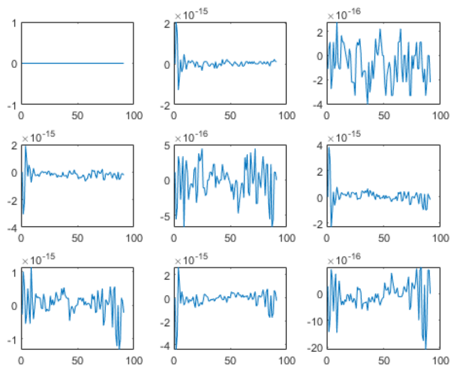
$$P_l(\cos\Phi_{PQ}) = \frac{1}{2l+1}\sum_{m=0}^{l} P_{lm}(\cos\theta_P)P_{lm}(\cos(\theta_Q))\{\cos m\lambda_P \cos m\lambda_Q + \sin m\lambda_P \sin m\lambda_Q\}$$

Für P und Q mit gleichen Länge:

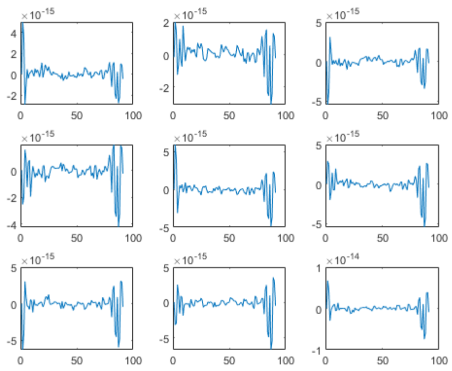
$$P_l(\cos\Phi_{PQ}) = \frac{1}{2l+1}\sum_{m=0}^{1} \bar{P}_{lm}(\cos\theta_P)\bar{P}_{lm}(\cos\theta_Q)$$

Für $\theta_P = 90°$ und $\theta_Q = [0°90°]$ können wir linke Seite und Rechte Seite vergleichen. Linke Seite rechnen wir mit dem Verfahren von Rodriger-Ferrers, rechte Seite rechnen wir mit dem rekursivem Verfahren. Die Unterschiede von $l = 0$ bis 100

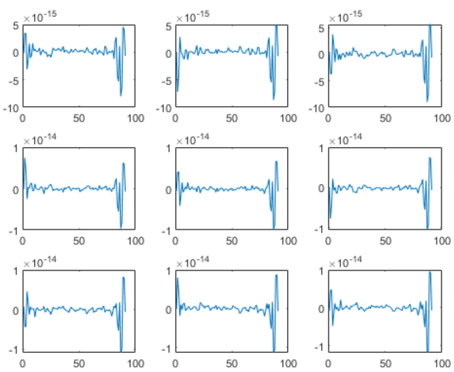
Die Unterschied erhöht sich mit größere Grad l, obwohl die Unterschied zwischen beiden Seite



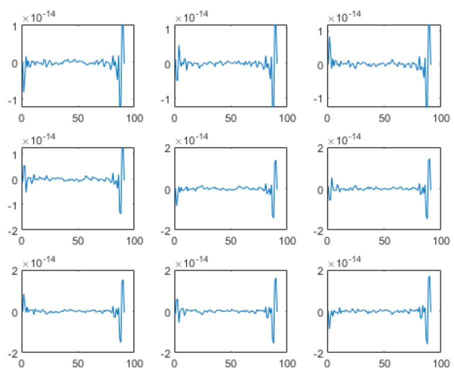
e l = 0 bis 8



f l = 9 bis 17

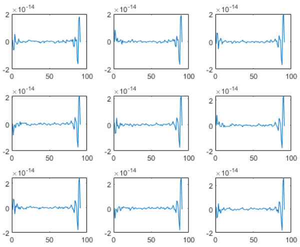


g l = 18 bis 26

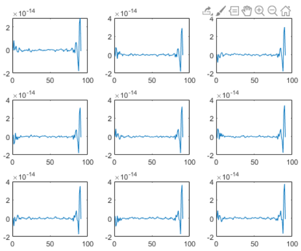


h l = 27 bis 35

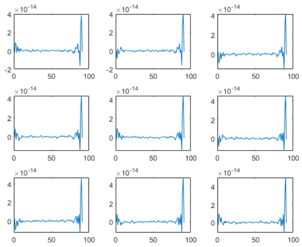
sehr klein sind.



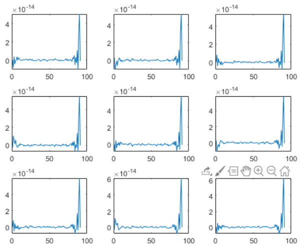
i l = 36 bis 44



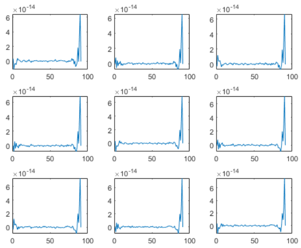
j l = 45 bis 53



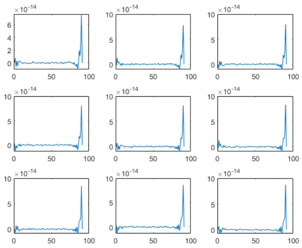
k l = 54 bis 62



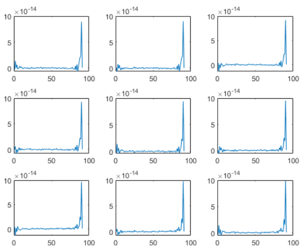
l l = 63 bis 71



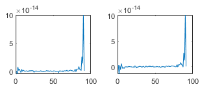
m l = 72 bis 80



n l = 81 bis 89



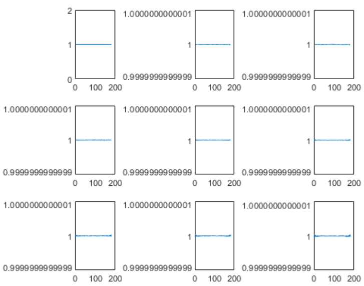
o l = 90 bis 98



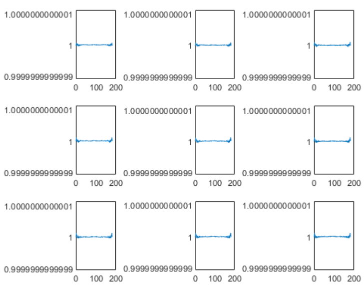
p l = 99 bis 101

3 task 3

Wenn $\theta_P = \theta_Q = [0°180°]$, sind $P_l(1) = 1 = \frac{1}{2l+1}\sum_{m=0}^{l}\bar{P}_{lm}^2(\cos\theta)$, mit gleichem Verfahren wie in task 2 können wir finden, dass für alle Grad $\frac{1}{2l+1}\sum_{m=0}^{l}\bar{P}_{lm}^2(\cos\theta) = 1$ sind.



q l = 0 bis 8



r l = 9 bis 17

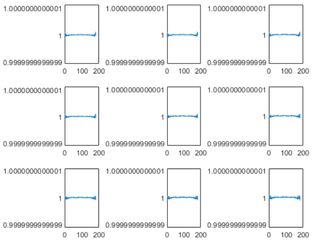
4 Task 4

Die Koordinaten von P: $\lambda = 11°$, $\theta = 43°$, $r = 6379245.458$, Die Gravitation und Schwerkraft werden in dieser Aufgabe berechnet.(hier ist $l = 36$)

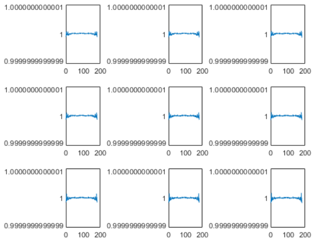
$$V(\lambda, \theta, r) = \frac{GM}{R} \sum_{l=0}^{37} (\frac{R}{r})^{l+1} \sum_{m=0}^{l} \bar{P}_{l,m}(\cos\theta)(\bar{c}_{l,m} \cos m\lambda + \bar{s}_{l,m} \sin m\lambda) = 62463927.101$$

$$V_c(\lambda, \theta, r) = \frac{1}{2}\omega^2 r^2 \sin^2\theta = 50324.829$$

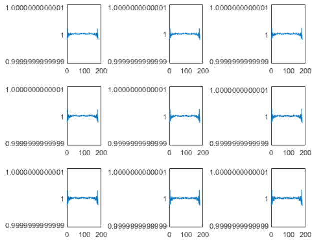
$$W = V + V_c = 62514251.930$$



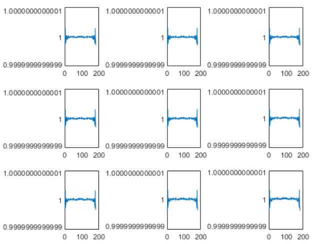
s l = 18 bis 26



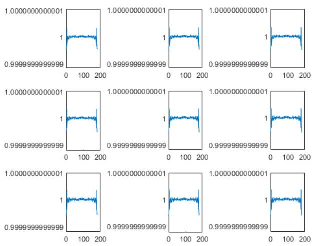
t l = 27 bis 35



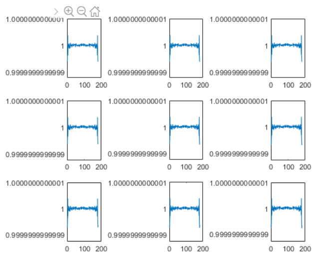
u l = 36 bis 44



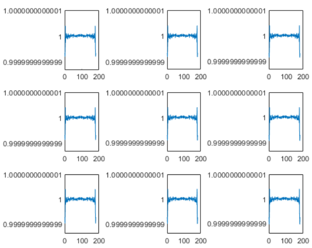
v l = 45 bis 53



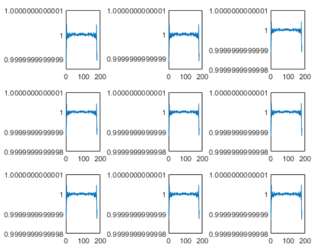
w l = 54 bis 62



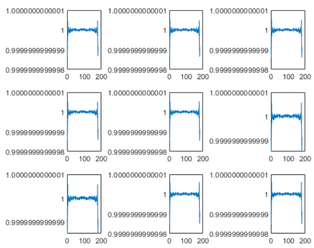
x l = 63 bis 71



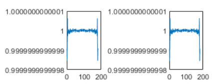
y l = 72 bis 80



z l = 81 bis 89



aa l = 90 bis 98



ab l = 99 bis 101