An Effective Photoplethysmography Denosing Method based on Diffusion Probabilistic Model

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APPENDIXES

A. Coefficient Estimation

To optimize the likelihood of the conditional diffusion process, we employ the evidence lower bound (ELBO) criterion. Following modifications to the derivations in [1], the ELBO is defined as:

$$\begin{aligned}\mathrm{ELBO} = -\mathbb{E}_q\Bigg[&\mathrm{KL}\left(q_{\mathrm{cdiff}}(x_T|x_0,g)\|p_{\mathrm{latent}}(x_T|g)\right)\\&+\sum_{t=2}^{T}\mathrm{KL}\left(q_{\mathrm{cdiff}}(x_{t-1}|x_t,x_0,g)\|p_\theta(x_{t-1}|x_t,g)\right)\\&-\log p_\theta(x_0|x_1,g)\Bigg].\end{aligned} \tag{1}$$

The transition distribution $q_{\mathrm{cdiff}}(x_t|x_{t-1},g)$ is given by:

$$\begin{aligned}q_{\mathrm{cdiff}}(x_t|x_{t-1},g) = \mathcal{N}\Bigg(&x_t; \frac{1-m_t}{1-m_{t-1}}\sqrt{\alpha_t}x_{t-1}\\&+\left(m_t-\frac{1-m_t}{1-m_{t-1}}m_{t-1}\right)\sqrt{\bar{\alpha}_t}g,\\&\delta_{t|t-1}I\Bigg),\end{aligned} \tag{2}$$

where the variance term $\delta_{t|t-1}$ satisfies:

$$\delta_{t|t-1} = \delta_t - \left(\frac{1-m_t}{1-m_{t-1}}\right)^2 \alpha_t\delta_{t-1}. \tag{3}$$

Using Bayes' theorem and the Markov property, the posterior distribution $q_{\mathrm{cdiff}}(x_{t-1}|x_t,x_0,g)$ is derived as:

$$\begin{aligned}q_{\mathrm{cdiff}}(x_{t-1}|x_t,x_0,g) = \mathcal{N}\Bigg(&x_{t-1}; \frac{1-m_t}{1-m_{t-1}}\frac{\delta_{t-1}}{\delta_t}\sqrt{\alpha_t}x_t\\&+(1-m_{t-1})\frac{\delta_{t|t-1}}{\delta_t}\sqrt{\bar{\alpha}_{t-1}}x_0\\&+\left(m_{t-1}\delta_t-\frac{m_t(1-m_t)}{1-m_{t-1}}\alpha_t\delta_{t-1}\right)\\&\cdot\frac{\sqrt{\bar{\alpha}_{t-1}}}{\delta_t}g, \delta_t' I\Bigg),\end{aligned} \tag{4}$$

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where δ_t', the variance of $q_{\mathrm{cdiff}}(x_{t-1}|x_t,x_0,g)$, is defined as:

$$\delta_t' = \frac{\delta_{t|t-1}\cdot\delta_t}{\delta_{t-1}}. \tag{5}$$

The coefficients c_{xt}, c_{gt}, and $c_{\epsilon t}$ in Eq.(**??**) $\mu_\theta(x_t,g,t)$ are computed as follows:

$$\begin{aligned}c_{xt} =&\frac{1-m_t}{1-m_{t-1}}\frac{\delta_{t-1}}{\delta_t}\sqrt{\alpha_t}\\&+(1-m_{t-1})\frac{\delta_{t|t-1}}{\delta_t}\frac{1}{\sqrt{\alpha_t}},\end{aligned} \tag{6}$$

$$c_{gt} = \left(m_{t-1}\delta_t-\frac{m_t(1-m_t)}{1-m_{t-1}}\alpha_t\delta_{t-1}\right)\frac{\sqrt{\bar{\alpha}_{t-1}}}{\delta_t}, \tag{7}$$

$$c_{\epsilon t} = (1-m_{t-1})\frac{\delta_{t|t-1}}{\delta_t}\sqrt{\frac{1-\bar{\alpha}_t}{\alpha_t}}. \tag{8}$$

Then the simplified training objective becomes Eq.(**??**). The simplified training objective aligns with these coefficients to optimize the reverse process for denoising.

B. Preprocessing Procedure for PPG signals

Below is the pseudocode for the preprocessing procedure of PPG signals.

Algorithm 1 Preprocessing Procedure for PPG Signals

Input: clean data $D_{\text{clean}}(c_1, c_2, c_3, a)$ and noisy data $D_{\text{noisy}}(c_1, c_2, c_3, a)$, where c_1, c_2, c_3 are PPG channels and a is the ambient channel.

Step 1: Align Clean and Noisy Data
Compute start and end time index:
$i_{\text{start}} \leftarrow \max(\min(D_{\text{clean}}[\text{t}]), \min(D_{\text{noisy}}[\text{t}]))$
$i_{\text{end}} \leftarrow \min(\max(D_{\text{clean}}[\text{t}]), \max(D_{\text{noisy}}[\text{t}]))$
Truncate both datasets:
$D_{\text{clean}} \leftarrow D_{\text{clean}}[i_{\text{start}} : i_{\text{end}}]$
$D_{\text{noisy}} \leftarrow D_{\text{noisy}}[i_{\text{start}} : i_{\text{end}}]$

Step 2: Resample Clean and Noisy Data
Resample clean data using noisy timestamps:
$D_{\text{clean}}[\text{t}] \leftarrow \text{interpolate}(D_{\text{clean}}[\text{t}], D_{\text{noisy}}[\text{t}])$

Step 3: Subtract Ambient Channel
For $c \in \{c_1, c_2, c_3\}$:
$D_{\text{clean}}[c] \leftarrow D_{\text{clean}}[c] - D_{\text{clean}}[a]$
$D_{\text{noisy}}[c] \leftarrow D_{\text{noisy}}[c] - D_{\text{noisy}}[a]$

Step 4: Extract Segments (T_w: window size; T_s: step size)
Initialize total segment numbers:
$N \leftarrow \lfloor \text{len}(D)/T_s \rfloor$, $j \leftarrow 0$
while $j < N$:
Compute $i_{\text{start}} \leftarrow j \cdot T_s$, $i_{\text{end}} \leftarrow i_{\text{start}} + T_w$
if $i_{\text{end}} > \text{len}(D)$: **break**
Extract $s_{\text{clean}} \leftarrow D_{\text{clean}}[c][i_{\text{start}} : i_{\text{end}}]$
Extract $s_{\text{noisy}} \leftarrow D_{\text{noisy}}[c][i_{\text{start}} : i_{\text{end}}]$
$j \leftarrow j + 1$

Step 5: Normalize and Bandpass Filter
For each s_{clean} and s_{noisy}:
Normalize $s \leftarrow s/\max(s)$
Apply bandpass filter $s \leftarrow \text{BandPassFilter}(s)$

Step 6: Segment Quality Control
For $(s_{\text{clean}}, s_{\text{noisy}})$:
Compute PRR for both.
if PRR(s_{clean}) $> 5\%$ or PRR(s_{clean}) $>$ PRR(s_{noisy}) or PRR(s_{noisy}) $< 5\%$:
Discard $(s_{\text{clean}}, s_{\text{noisy}})$

Return: Preprocessed dataset with valid, filtered segments.

C. Complexity comparison with traditional methods

Traditional methods are advantageous due to their low computational requirements and practicality, with runtime evaluations typically based on an 8-second PPG frame. We evaluated the inference time of our proposed model on a 30-second PPG frame, which required 0.12 seconds for processing. To facilitate comparison with prior traditional methods, this was scaled to an 8-second PPG input, resulting in an inference time of approximately 32 ms. This performance aligns closely with the results reported by [2] (32 ms) and [3] (28 ms), while being slightly slower than [4] (8.5 ms). The results demonstrate that our model achieving competitive performance even against traditional methods with minimal latency, making our model a viable solution for practical deployment.

References

[1] J. Ho, A. Jain, and P. Abbeel, "Denoising diffusion probabilistic models," *Advances in neural information processing systems*, vol. 33, pp. 6840–6851, 2020.

[2] M. B. Mashhadi, E. Asadi, M. Eskandari, S. Kiani, and F. Marvasti, "Heart rate tracking using wrist-type photoplethysmographic (ppg) signals during physical exercise with simultaneous accelerometry," *IEEE Signal Processing Letters*, vol. 23, no. 2, pp. 227–231, 2015.

[3] M. B. Mashhadi, M. Farhadi, M. Essalat, and F. Marvasti, "Low complexity heart rate measurement from wearable wrist-type photoplethysmographic sensors robust to motion artifacts," in *2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2018, pp. 921–924.

[4] A. Temko, "Accurate heart rate monitoring during physical exercises using ppg," *IEEE Transactions on Biomedical Engineering*, vol. 64, no. 9, pp. 2016–2024, 2017.