Abstract

Designs are good finite subsets that approximate a given space. On various spaces, we have a Fisher type lower bound on the sizes of designs of strength t with even t. When t is odd, ad hoc methods are used to get the corresponding Fisher type lower bound. These bounds are known for decades, while no major improvements are made.

In this paper, we give a new lower bound on the sizes of designs in a very general setup covering both combinatorial designs and geometric designs. On Johnson association schemes and real unit spheres, our new bound matches the Fisher type lower bound, and hence gives a unified proof of the Fisher type bound for both even and odd t. On some union or product of association schemes, our new bound is better than the Fisher type bound, hence improves the known result. On unitary groups and projective unitary groups, our new bound is worse than the Fisher type bound, but we can use the fact that two bounds are different to prove the nonexistence of certain "tight" designs.

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