A rational design is a design consisting of rational points. Let $\mathcal{Z}$ be a good space on which we could define designs. We assume a condition on the integrals of polynomials over $\mathcal{Z}$, which is a necessary condition for the existence of rational designs, and assume that $\mathcal{Z}$ has enough rational points, which is a necessary condition in many cases. We prove that, under these two assumptions, if $\mathcal{Z}$ is "algebraically path-connected", then for every natural number t, there exist rational t-designs on $\mathcal{Z}$, and we give an asymptotic lower bound on the number of rational t-designs.

In particular, the unit open interval satisfies the requirements, hence there exist rational interval t-designs. Although the real unit sphere is not "algebraically path-connected", we can still prove that there exist "almost-rational" spherical t-designs. More precisely, there are spherical t-designs in which for each point of the design, all but the first coordinate of the point are rational numbers.

All the results in this paper are effective, namely, in theory we are able to give an effective upper bound on the sizes of t-designs we found. The possible sizes of rational t-design are discussed in the end. We show for each natural number $0 \leq t \leq 21$ an explicit n_t such that for all natural number $n \geq n_t$, there exist rational interval t-designs of size n.