Abstract

A rational design is a design consisting of rational points. Let $\mathcal{Z}$ be a good space on which we could define designs. We assume a condition on the integrals of polynomials over $\mathcal{Z}$, which is a necessary condition for the existence of rational designs, and assume that $\mathcal{Z}$ has enough rational points, which is a necessary condition in many cases. We prove that, under these two assumptions, if $\mathcal{Z}$ is "algebraically path-connected", then for every natural number t and all sufficiently large integer n, there exist rational t-designs on $\mathcal{Z}$ of size n. Moreover, we give an asymptotic lower bound on the number of rational t-designs of n.

In particular, the unit open interval satisfies the requirements, hence there exist rational interval t-designs of all sufficiently large sizes. In this case, the asymptotic lower bound we give is actually an asymptotic formula. The real unit is not "algebraically path-connected". We only prove that there exist "almost-rational" spherical t-designs. More precisely, there are spherical t-designs in which for each point of the design, all but the first coordinate of the point are rational numbers.

All the results in this paper are effective, namely, in theory we are able to give an explicit upper bound on the sizes of t-designs we found, and an explicit upper bound on smallest n_t that ensures the existence of rational t-designs of size $n \geq n_t$. For interval designs and $0 \leq t \leq 23$, we give explicit bounds at the end of the paper.