# ECE3330

# Control Systems

# Lab#1

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# Introduction

In this lab, a system consisting of a 18W Maxon DC motor equipped with an optical encoder counter were studied. Data were gathered by using DAQ card and Quanser's QuaRC software. The data were then compared with the theoretical value calculated from pre-lab to validate the equations describes the behavior of DC motor.

# Objective

The objective of this lab is to model a DC motor using the provided first order equation:

$$\frac{d}{dt}\omega_m(t) + \alpha\omega_m(t) = \alpha K V_a(t)$$

The unknown parameters of a and K are needed to be calculated by following to be measured parameters:

1. Input voltage to motor terminal (Va)
2. The motor angular velocity (wa)
3. The motor current (Ia)

# Pre-laboratory

The pre-laboratory can be found in Appendix A.

## First Approach: Static Relations

In this experiment, the resistance and the current of a DC motor were studied by holding the motor at its stall torque and applying various constant DC voltages. The biased current was measured by setting the input voltage to 0. The measured biased current value is included in Table 1. The measured current values and their corresponding input voltage is included in Table 2.

Table 1: Biased Current

| Input voltage Va [V] | Biased Current Ibias[A] |
| --- | --- |
| 0 | 0.0021 |

Table 2: Motor Resistance Experimental Results

| Input voltage Va[V] | Measured Current Imeas[A] | Corrected for Bias Ia [A] | Resistance Ra[Ω] |
| --- | --- | --- | --- |
| -5 | -0.4731 | -0.4734 | 10.56 |
| -4 | -0.4195 | -0.4216 | 9.487 |
| -3 | -0.3 | -0.3021 | 9.93 |
| -2 | -0.1684 | -0.1705 | 11.73 |
| -1 | -0.0798 | -0.0819 | 12.21 |
| 1 | 0.0797 | 0.0776 | 12.86 |
| 2 | 0.178 | 0.1756 | 11.37 |
| 3 | 0.3137 | 0.3116 | 9.63 |
| 4 | 0.4104 | 0.4083 | 9.8 |

| | | | |
|---|---|---|---|
| 5 | 0.4826 | 0.4805 | 10.4 |
| Average Resistance[Ω] | | | 10.8 |

The following relation describes a linear relationship between changes in input voltage and current as determined from Pre-lab.

$$R_a = \frac{V_a}{i_a}$$

From this equation, a range of input voltages (Va) from -5 to 5 volts were tested and the corresponding current levels (ia) was recorded in Table 2. From these values, the internal resistance could be calculated. After these values were averaged, the final internal resistance was found to be 10.8 Ω. The internal resistance value was 1.88%.

Error calculation:

$$\text{Error Percentage} = \frac{|10.6 - 10.8|}{10.6} = 1.88\%$$

The error percentage is quite small; the error could result by the manufacturing error of the DAQ board or the malfunction of DC motor.

Following this experiment, another experiment where the motor shaft was free to spin was conducted. The same DC voltages were applied. The angular speed and current of the DC motor were recorded. The results are shown in Table 3 below.

Table 3: Back EMF Constant Experimental Results

| Input Voltage $V_a[V]$ | Measured Speed [rad/s] $\omega_m\ [\frac{rad}{s}]$ | Measure Current $i_{meas}\ [A]$ | Corrected for bias $i_a[\ A]$ | Back EMF [Vs/rad] $k_b[\frac{V_s}{rad}]$ |
|---|---|---|---|---|
| -5 | -101.2 | -0.006 | -0.0081 | 0.0485 |
| -4 | -80.08 | -0.005 | -0.0071 | 0.049 |
| -3 | -59.7 | -0.002 | -0.0041 | 0.0492 |
| -2 | -39.5 | -0.0008 | -0.0029 | 0.0498 |
| -1 | -19.03 | -0.0002 | -0.0023 | 0.0514 |
| 1 | 18.9 | -0.011 | 0.009 | 0.0476 |
| 2 | 39.6 | -0.006 | 0.0038 | 0.0495 |
| 3 | 59.94 | -0.008 | 0.00558 | 0.049 |
| 4 | 80.19 | -0.003 | 0.0006 | 0.0497 |
| 5 | 100.7 | -0.0065 | 0.0044 | 0.0498 |
| Average Back EMF Constant [ Vs / rad] | | | | 0.0493 |

For each input voltage, a motor back-EMF constant, kb , was calculated by the following relation:

$$k_b = \frac{V_a R_a i_a}{\omega_m}$$

The results were recorded in column 5 of Table 3. Back EMF is the motor's internal resistance to spinning due to changing electric fields within the motor.

The average back EMF was calculated to be 0.0493 Vs/rad. This was close to the theoretical value of 0.0502 Vs/rad. The percentage error was determined to be approximately 0.4%.

Error calculation:

$$\text{Error Percentage} = \frac{|0.0502 - 0.0493|}{0.0502} = 1.79\%$$

The theoretical and experimental back-EMF constants are within 10% error. The small discrepancy can be attributed to the small errors in reading the empirical current and angular speed values from the DAQ.

## Motor Transfer Function

Following this experiment, the motor transfer function could be determined. The motor transfer function is calculated below:

Calculating the pole:

$$\alpha = \frac{BR_a + k_b^2}{J_{eq}R_a}$$

$$\alpha = \frac{0.0493^2}{(2.21\text{x } 10^{-5})(10.8)}$$

$$\alpha = 10.183$$

Calculating the gain:

$$K = \frac{k_b}{J_{eq}R_a}$$

$$K = \frac{0.0493}{(2.21\text{x } 10^{-5})(10.8)}$$

$$K = 206.55$$

General transfer function:

$$\frac{\omega_m(s)}{V_a(s)} = \frac{K}{s+\propto}$$

Experimental transfer function:

$$H(s) = \frac{\omega_m(s)}{V_a(s)} = \frac{206.55}{s + 10.183}$$

Calculated pre-lab theoretical transfer function:

$$\frac{\omega_m(s)}{V_a(s)} = \frac{214.57}{s + 10.77}$$

The calculated pre-lab theoretical transfer function is very close to experimental transfer function. Both the experimental gain and pole is smaller than the theoretical values. The error of K was calculated to be 3.736% and the error for $\propto$ was calculated at 5.45%. The reason why these constants had this error was because the errors in kb and Rb carried through.

$$\text{Error Percentage} = \frac{|214.57 - 206.55|}{214.57} = 3.736\%$$

$$\text{Error Percentage} = \frac{|10.77 - 10.183|}{10.77} = 5.45\%$$

## Second Approach: Experimental Determination of System Dynamics

The purpose of this experiment was to examine a more graphical approach to find the transfer function of the motor. Using this method, a constant input is chosen. After a stable system reaches equilibrium, the input is then changed to a new level and the output data is recorded.

To determine the transfer function for the motor, the values of K and $\alpha$ were computed.

From the laboratory notes:

$$K = \frac{\widehat{K}}{\tau}$$

$$\alpha = \frac{1}{\tau}$$

Where,

$$\widehat{K} = \frac{\Delta y}{\Delta u}$$

Based on the graph:

$$\Delta y = 39.13$$

$$\tau = 0.102s$$

Calculating the gain:

$$\widehat{K} = \frac{\Delta y}{\Delta u} = \frac{39.13}{2} = 19.565$$

$$K = \frac{\widehat{K}}{\tau}$$

$$K = \frac{19.565}{0.102\ s}$$

$$K = 191.8$$

Calculating the pole:

$$\alpha = \frac{1}{\tau}$$

$$\alpha = \frac{1}{0.102s}$$

$$\alpha = 9.8$$

Therefore, the transfer function is:

$$H(s) = \frac{\omega_m(s)}{V_a(s)} = \frac{191.8}{s + 9.8}$$

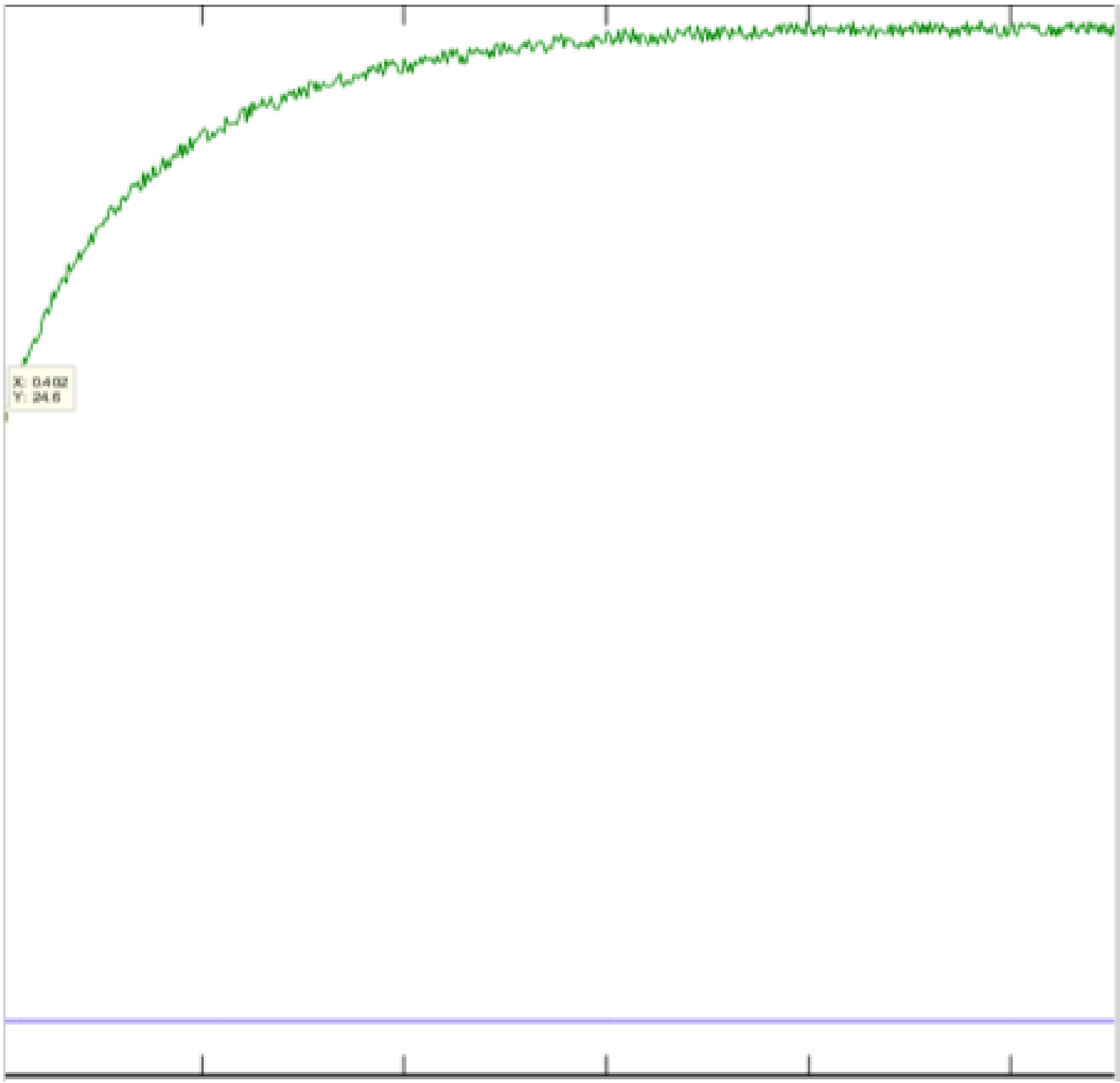
The theoretical transfer function of the motor is:

$$\frac{\omega_m(s)}{V_a(s)} = \frac{214.57}{s + 10.77}$$

$$\text{Error Percentage} = \frac{|214.57 - 191.8|}{214.57} = 10.6\%$$

$$\text{Error Percentage} = \frac{|10.77 - 9.8|}{10.77} = 9\%$$

The calculated pre-lab theoretical transfer function is far from experimental transfer function. Both the experimental gain and pole is smaller than the theoretical values. The error of K was calculated to be 10.6% and the error for $\propto$ was calculated at 9%. The reason why these constants had this error was because human error.



# Model Validation

The purpose of this section is to verify the accuracy of the system models presented in earlier sections. The transfer functions were validated by applying both a square and saw tooth waveforms.

Figure 1: Square Step Function Comparison for Model 1

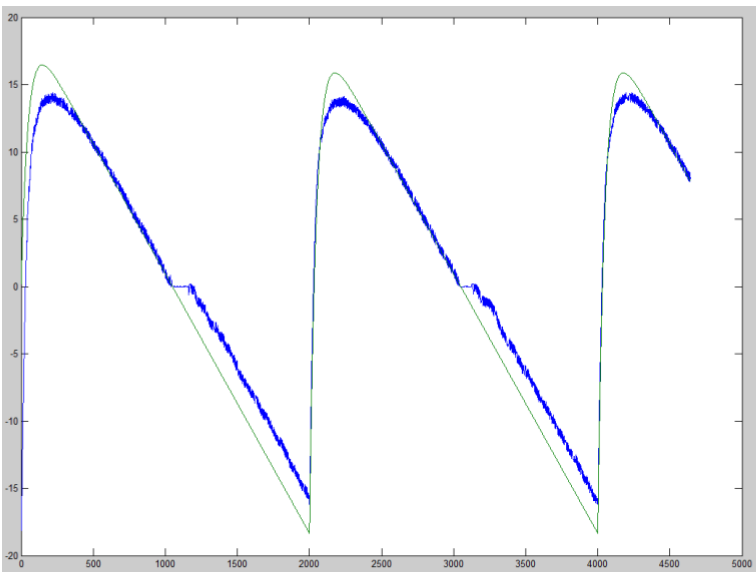



Figure 2: Sawtooth Input Comparison for Model 1

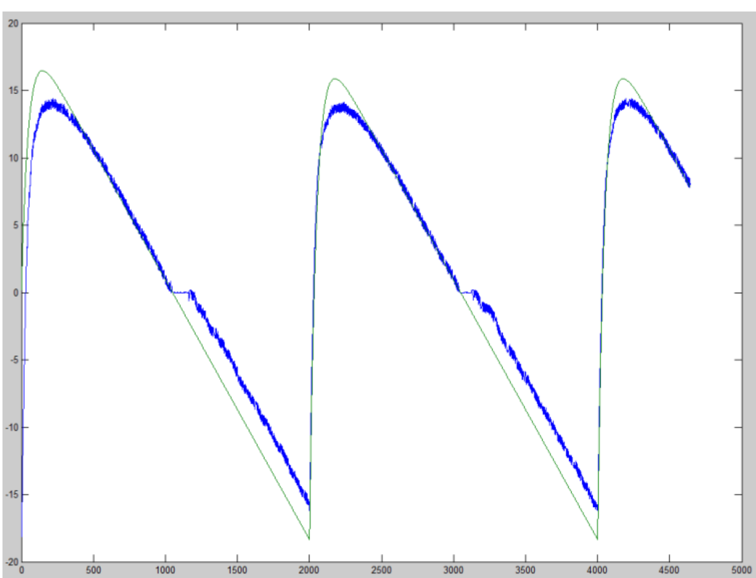


Figure 3: Sawtooth Input Comparison for Model 2

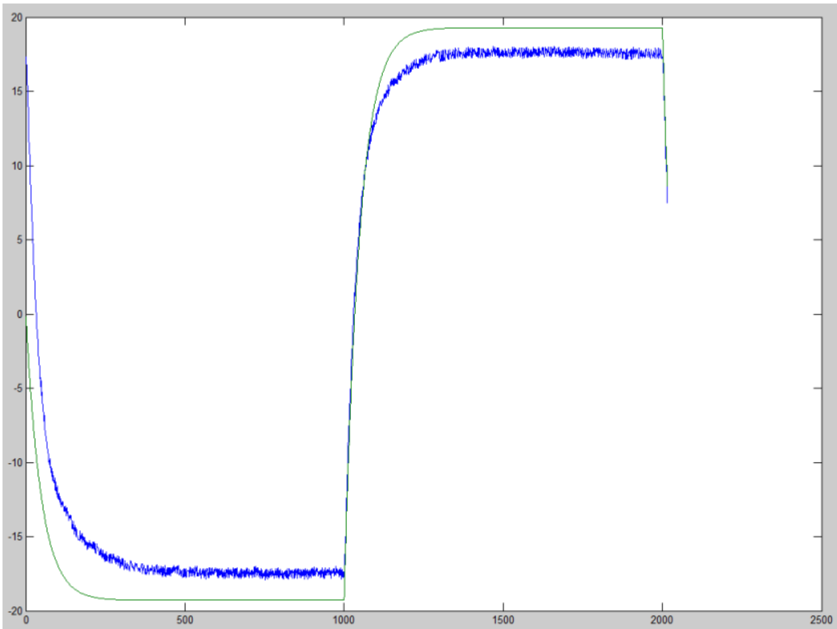
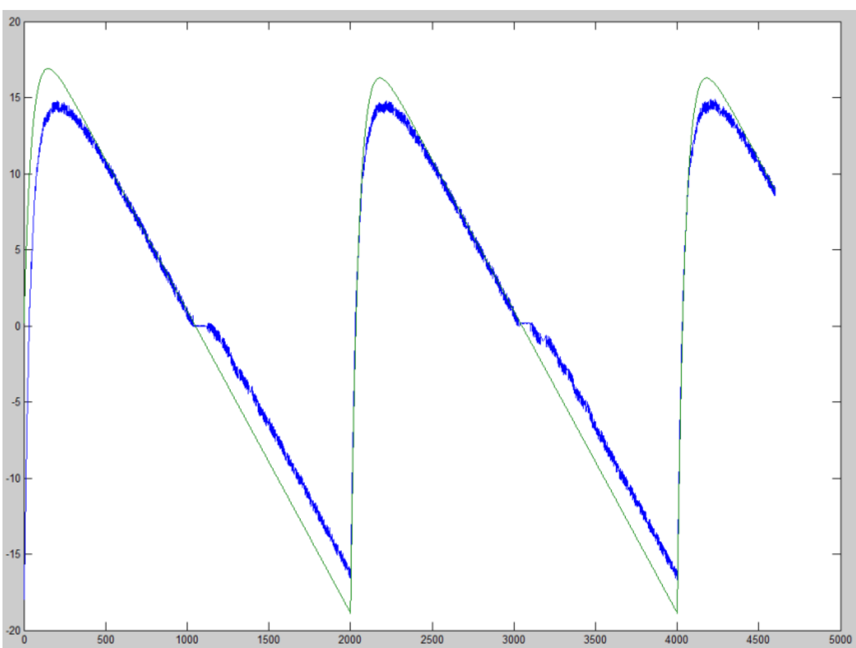



Figure 4: Square Step Function Comparison for Model 2



# Conclusion

In this lab, a DC motor system was examined and the transfer function of the system was evaluated in two ways. Overall speaking the lab was quite successful and the data collected are accurate and reasonable.

In the first approach, two different approach were used to calculate the motor resistance and torque constant, which are hold motor shaft still and motor free to spin, the current value and motor speed were measured from -5V to 5V, and the average resistance and EMF constant were calculated. The transfer function was determined to be $\mathrm{H(s)} = \frac{\omega_{\mathrm{m}}(\mathrm{s})}{\mathrm{V_a(s)}} = \frac{206.55}{\mathrm{s}+10.183}$.

In the second approach, the transient response graph of the system was examined and the time constant was measured, the transfer function was determined to be $\mathrm{H(s)} = \frac{\omega_{\mathrm{m}}(\mathrm{s})}{\mathrm{V_a(s)}} = \frac{191.8}{\mathrm{s}+9.8}$.

Both models were validated to examine the accuracy of the obtained transfer functions. This was achieved through applied square and sawtooth input functions. From the graphs, transfer response closely followed the response of the actual motor response for both square and sawtooth input functions.

Appendix I

Pre-laboratory Assignments. S.Q.

① While holding the motor shaft stationary (stall torque mode, $\omega_m = 0$)

For equation ③ $V_a = R_a i_a + k_b \omega_m$

$$\therefore V_a = R_a i_a$$

$$R_a = \frac{V_a}{i_a}$$

② While the motor is free to spin = No load $\tau_m = 0$

From ③ $V_a \approx R_a i_a + k_b \omega_m$

$$k_b \omega_m = V_a - R_a i_a$$

$$k_b = \frac{V_a - R_a i_a}{\omega_m}$$

③

$\tau_m - B\omega_m = J_{eq} \frac{d}{dt}\omega_m$ ④ $\Rightarrow k_b i_a - B\omega_m = J_{eq}\frac{d}{dt}\omega_m$ ①

$\tau_m = k_b i_a$

$V_a \approx R_a i_a + k_b \omega_m$ ① $\Rightarrow i_a = \frac{V_a}{R_a} - \frac{k_b}{R_a}\omega_m$ ②

② → ①

$$k_b\left(\frac{V_a}{R_a} - \frac{k_b}{R_a}\omega_m\right) - B\omega_m = J_{eq}\frac{d}{dt}\omega_m$$

$$J_{eq}\frac{d}{dt}\omega_m + \frac{k_b^2}{R_a}\omega_m + B\omega_m = \frac{k_b V_a}{R_a}$$

$$\left(J_{eq}\cdot s + \frac{k_b^2}{R_a} + B\right)\omega_m(s) = \frac{k_b}{R_a}V_a(s)$$

$$\frac{\omega_m(s)}{V_a(s)} = \frac{\frac{k_b}{R_a}}{J_{eq}\cdot s + \frac{k_b^2}{R_a} + B}$$

$\therefore$ ⑧ is verifyed

④ $J_{eq} = J_m + J_L = J_m + \frac{1}{2} M_L r_L^2$

$$= 1.16\times10^{-6} + \frac{1}{2}(0.068)(0.0248)^2$$
$$= 2.2071\times10^{-5}\ kg\cdot m^2$$

⑤

$$k = \frac{k_b}{J_{eq}R_a} = \frac{0.0502}{2.2071\times10^{-5}\times10.6} = 214.57$$

$$\alpha = \frac{B.R_a + k_b^2}{J_{eq}R_a} = \frac{0 + (0.0502)^2}{2.2071\times10^{-5}\cdot(10.6)} = 10.77$$

$$\frac{W_m(s)}{V_a(s)} = \frac{k}{s+\alpha} = \frac{214.57}{s+10.77}$$

⑥ $W_m = \frac{d}{dt}\theta_m$ $\quad \Rightarrow W_m(s) = s\theta_m(s) + \theta_m(0^-)$

$$\frac{\theta_m(s)}{W_m(s)} = \frac{1}{s} \qquad \frac{\theta_m(s)}{W_m(s)}\cdot\frac{W_m(s)}{V_m(s)} = \frac{\theta_m(s)}{V_m(s)} = \frac{k}{s+\alpha}\cdot\frac{1}{s} = \frac{1}{s}\cdot\frac{214.57}{s+10.77}$$

⑦ $V_a(t) = 2u(t)$ $\quad L \rightarrow V_a(s) = \frac{2}{s}$

$$W_m(s) = \frac{k}{s+\alpha}\cdot V_a(s) = \frac{214.57}{s+10.77}\cdot\frac{2}{s} = \frac{k_1}{s} + \frac{k_2}{s+10.77}$$

$$k_1 = \frac{2(214.57)}{s+10.77}\Big|_{s=0} \qquad k_1 = 39.85$$

$$k_2 = \frac{214.57\cdot2}{s}\Big|_{s=-10.77} \qquad k_2 = -39.85$$

$$W_m(s) = \frac{39.85}{s} - \frac{39.85}{s+10.77}$$

$$W_m(t) = 39.85 - 39.85e^{-10.77t}.$$