ECE 3330
Lab #4

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Introduction

In the previous lab, various different optimization methods such as lead compensator were examined and evaluated, the time-domain analysis and design of closed-loop control systems were performed. In this lab, a frequency-domain analysis and design of closed-loop control systems will be performed.
Data were gathered using DAQ board and Simulink software. The data were also compared with the calculated pre-lab theoretical values.

Pre-lab assignment

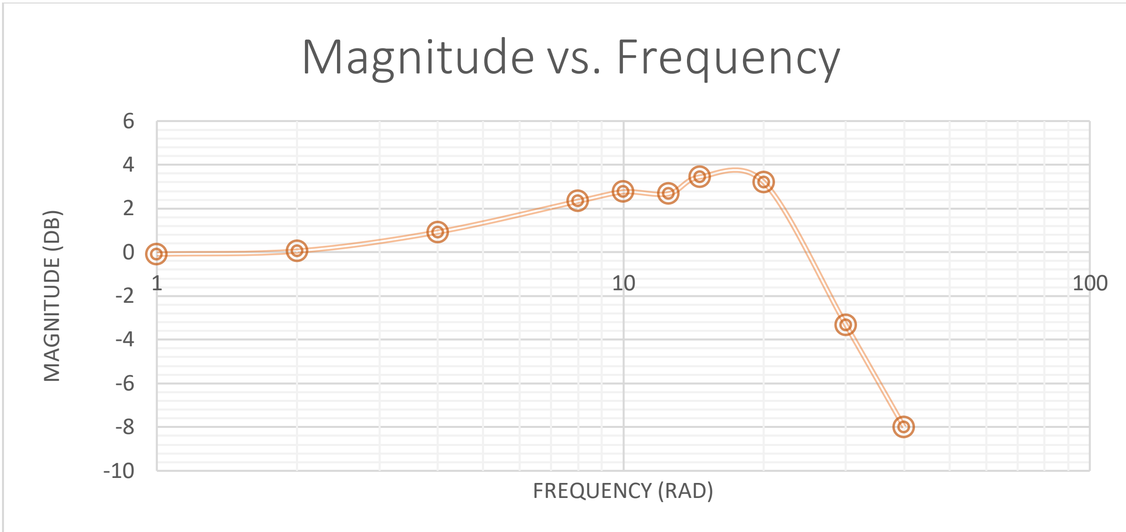
The pre-lab assignment can be found in Appendix A

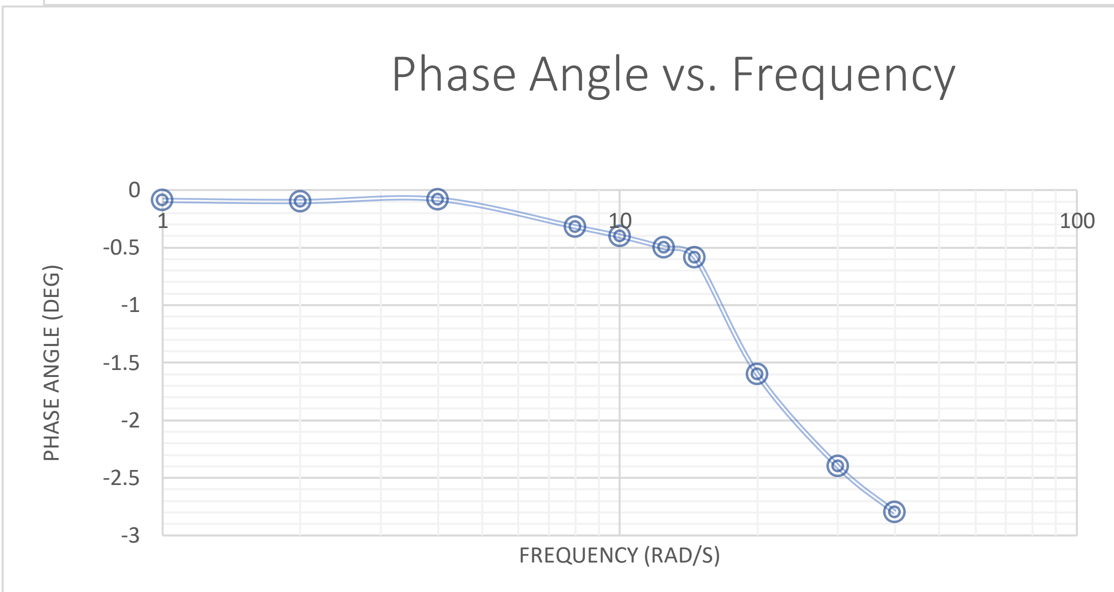
Frequency Response of the DCMCT System

The pulse generator was set to sine wave, 5 rad of amplitude and 1 rad/s of frequency. After 5 minutes set up time for MATLAB and Simulink, the simulation was performed, and the data were recorded using MATLAB.

Frequency of Input ω [rad/s]	Amplitude of Input Mi [rad]	Amplitude of Output Mo [rad]	Time Difference between Peaks Δt [sec]	Magnitude Response M(ω) [dB]	Phase Response φ(ω) [rad]
1	5	4.95	0.09	-0.087296108	-0.09
2	5	5.03	0.05	0.051959614	-0.1
4	5	5.56	0.02	0.922095745	-0.08
8	5	6.54	0.04	2.33215488	-0.32
10	5	6.87	0.04	2.759734654	-0.4
12.5	5	6.8	0.04	2.670778167	-0.5
14.6	5	7.43	0.04	3.440376188	-0.584
20	5	7.23	0.08	3.203365859	-1.6
30	5	3.41	0.08	-3.324312507	-2.4
40	5	1.99	0.07	-8.002338559	-2.8

Table 1: Frequency Response of DCMCT System





With the increasing frequency of input and same amplitude of input, the amplitude of output increased until it reached a frequency of 14.6 rad/s, after that, the amplitude of output decreased as the frequency increases, the amplitude dropped from a peak of 7.795 rad/s to 1.786 rad/s. The time difference between peaks follows the reverse pattern as the amplitude of output, however it reached the lowest point with the frequency of 12.5 rad/s instead if 14.6 rad/s, this might due to the error in measurement. The Magnitude Response follows the same pattern as the amplitude of output, started from a negative value of -0.108 dB, reached peak of 3.890 dB at 14.6 rad/s of input frequency. And rapidly dropped to -8.782 dB as the frequency increased to 40.0 rad/s. Nevertheless, the phase response decreases as the frequency of input increases, it dropped from -0.087 rad to -3.00 rad, the rate of decrease is very steady and proportional.

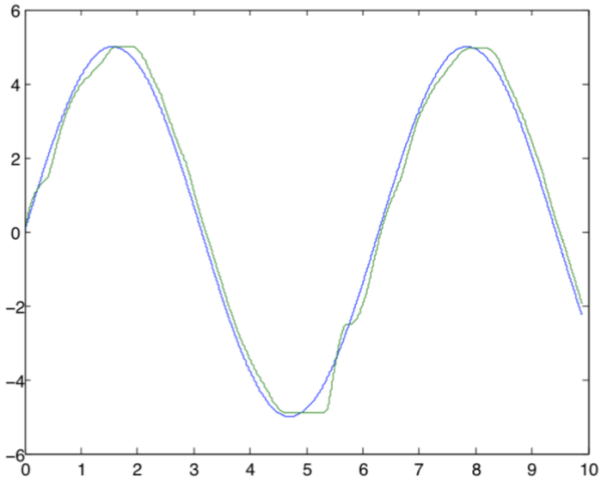


Figure 1: Frequency response for ω=1

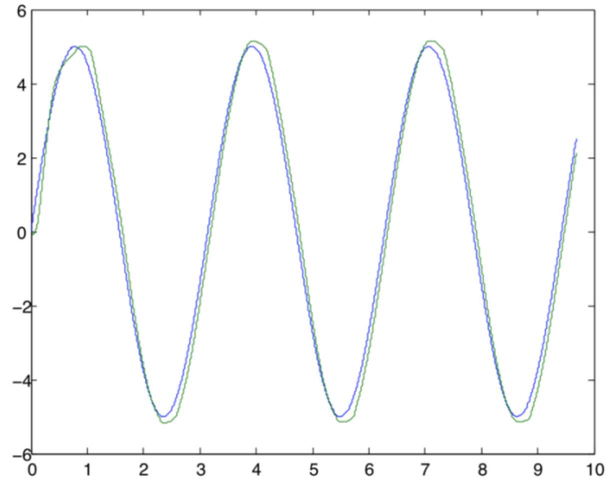


Figure 2: Frequency response for ω=2

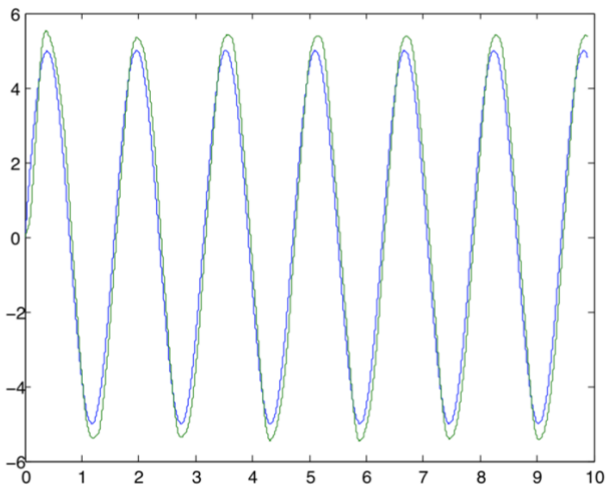


Figure 3: Frequency response for ω=4

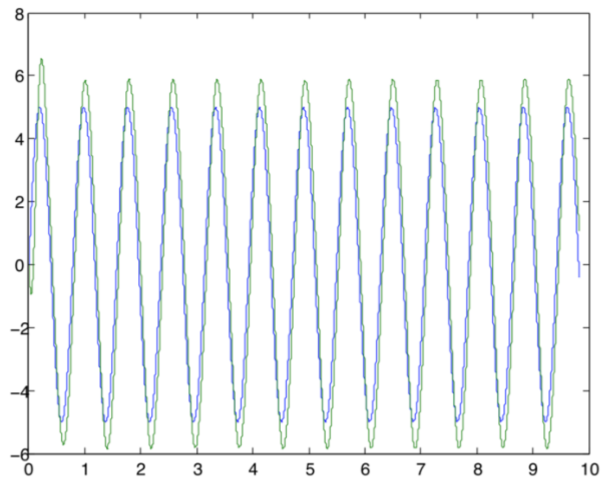


Figure 4: Frequency response for ω=8

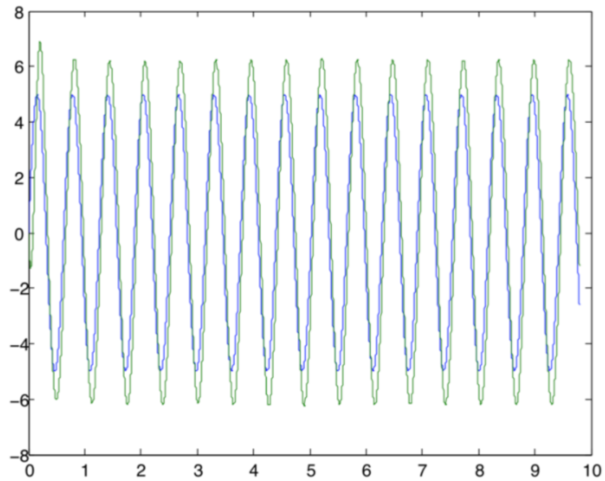


Figure 5: Frequency response for ω=10

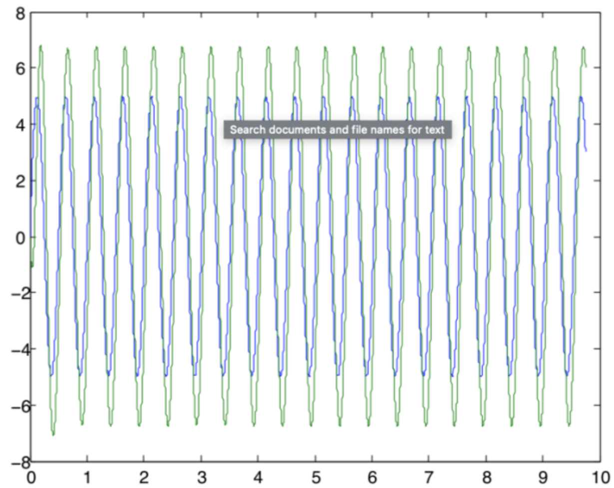


Figure 6: Frequency response for ω=12.5

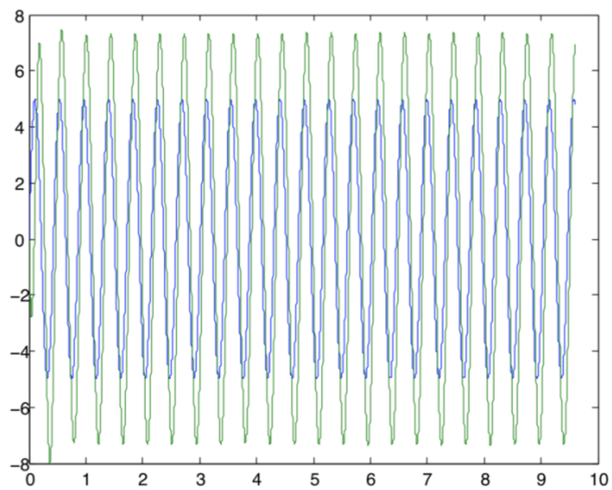


Figure 7: Frequency response for ω=14.6

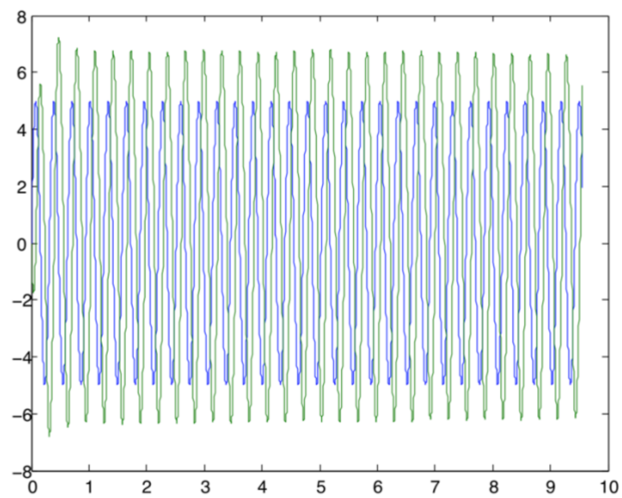


Figure 8: Frequency response for ω=20

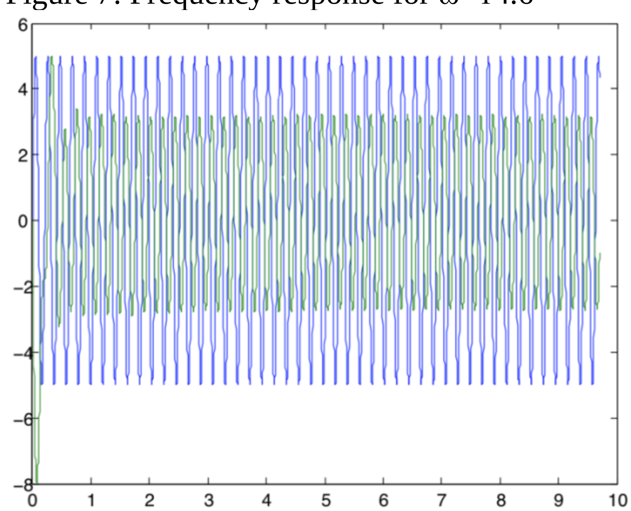


Figure 9: Frequency response for ω=30

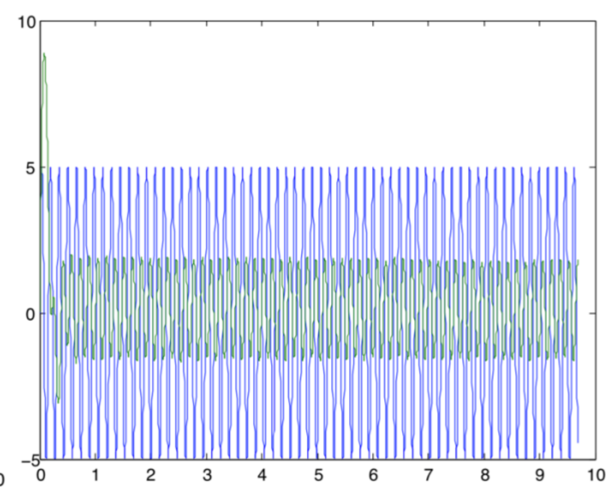


Figure 10: Frequency response for ω=40

As shown above, the graphs tend to 'squeeze' as the input frequency increases, when ω=1, the period is approximately 6.3s, and when ω=40, the period decreased to roughly 0.12s. in addition, as the input frequency arise from 1 to 20, the green portion of the graph (output amplitude)started to expand, however from 20 to 40, the blue portion of the graph expanded dramatically.

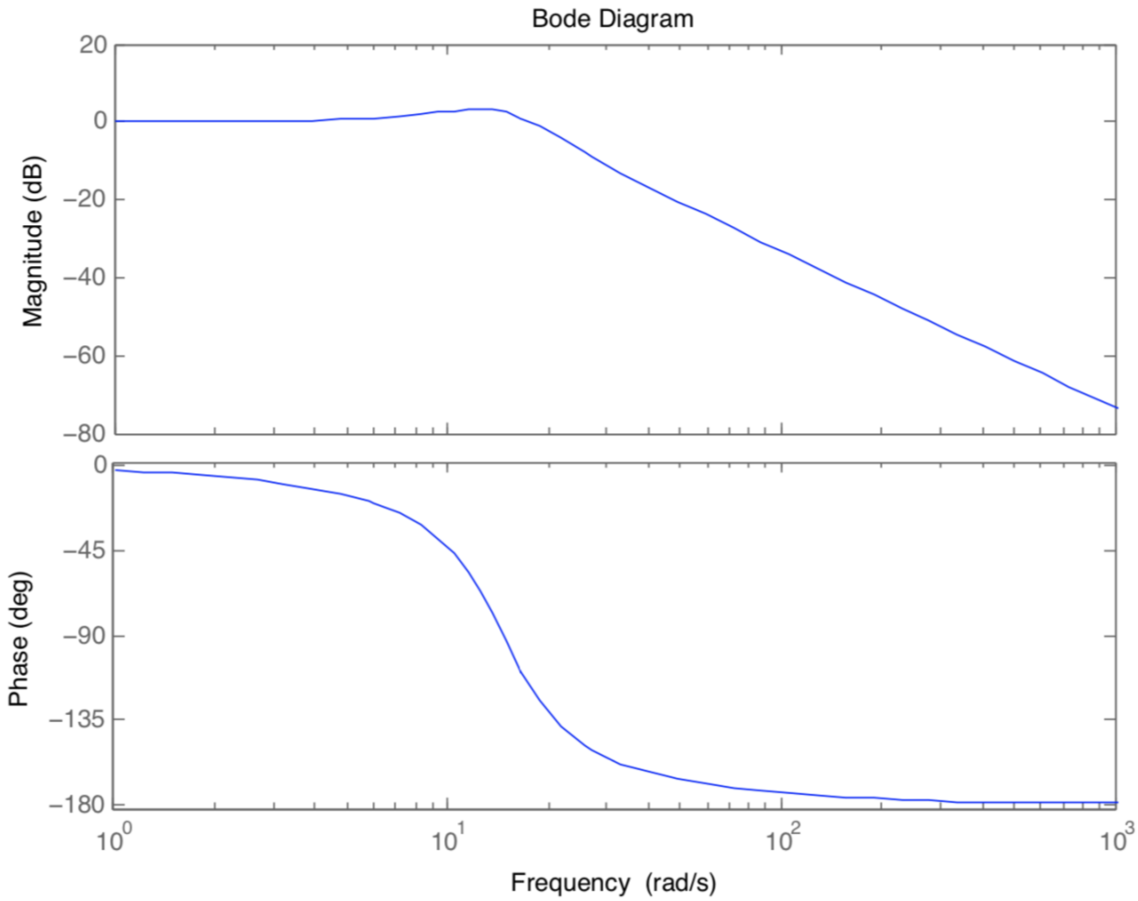


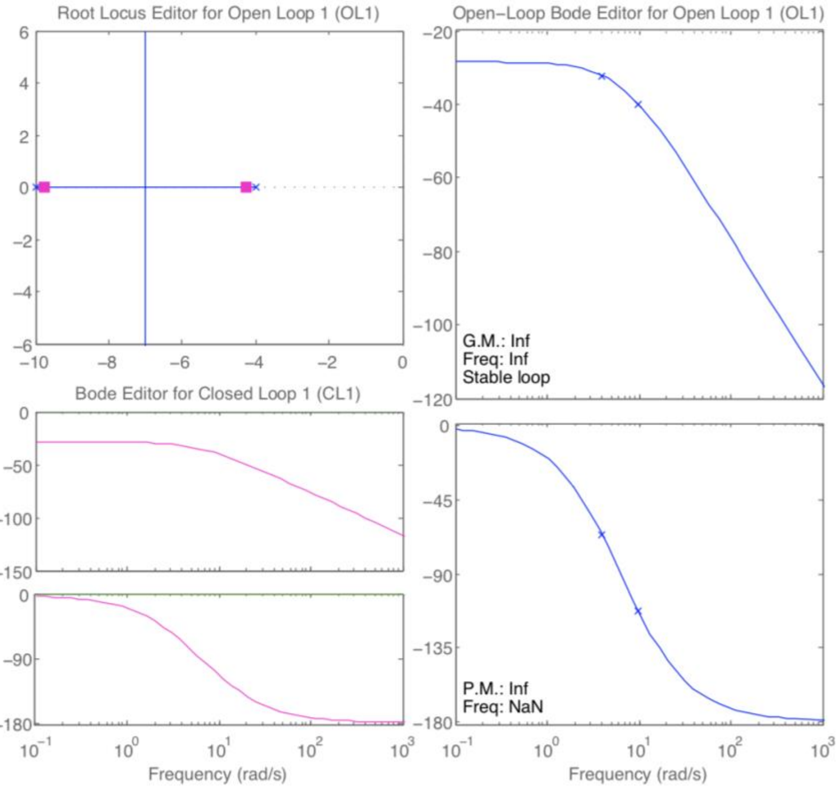
Figure 11: Bode Plot for the system

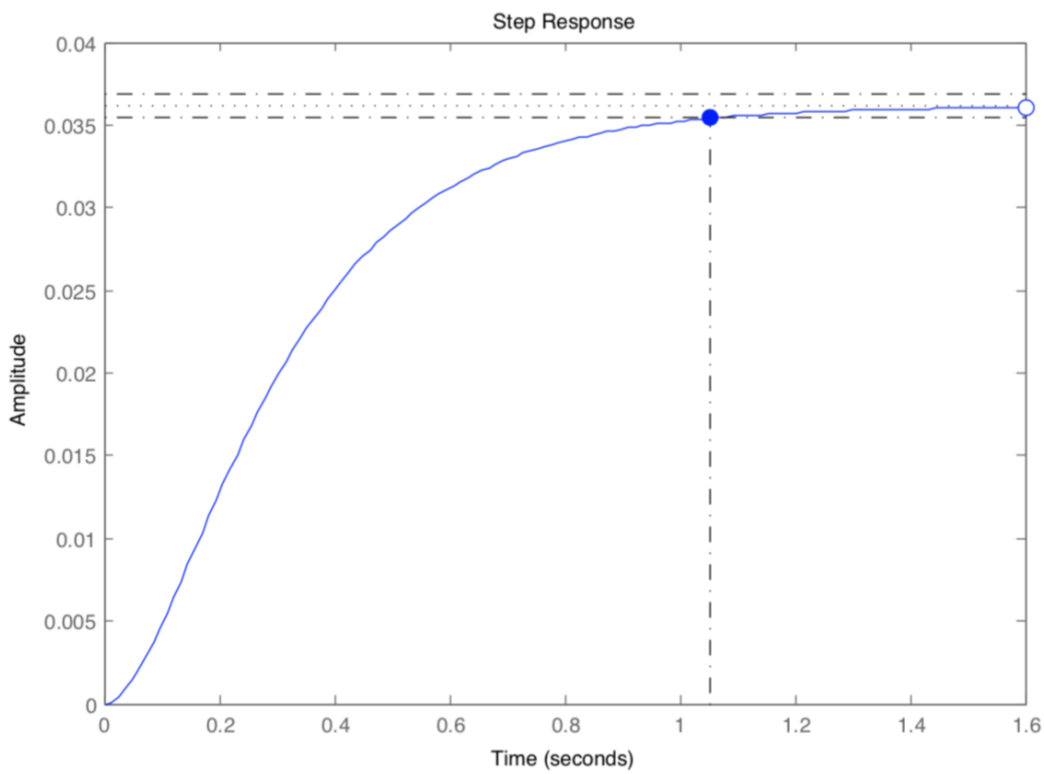
Initially the plot started at 0dB, and it slowly increased a small peak of approximately 5 dB, then experienced a steady, fast decrease from 5 dB to 75 dB. In the phase diagram, the plot also started at 0dB then steadily decreased to -45 degrees, then experienced a sharp decrease from -45 degrees to -150 degrees in the period of 10^1 to 10^2, finally a slow and steady decrease was introduced from -150 degrees to -180 degrees. This is because when ω is small, the gain will be 0, and the graph reached peak when ω=ωn, and constantly decrease afterwards.

Controller Design Using Bode Diagram Techniques

In this section, analysis was done by using bode diagram, by using SISOTOOL, the experiment data were recorded and bode plot graphs were captured.

	Results of the Pre-Lab						Results of Bode Plot									LTI Viewer	
K	ζ	ωn	ωr	Mr	Mn	OS	Ts	ωn	ωr	Mr	Mn	ωbw	ζ	OS	Ts	OS	Ts
10	10	7.42	-	-	-16.8	-	0.94	7.42	-	-	-16.8	-	0.94	0.02%	0.56	0.91%	0.7
50	50	10.71	3.81	-3.61	-6.07	-	0.67	10.71	5.902281	-3.61	-6.07	-	0.67	5.87%	0.55	6.60%	0.56
250	250	20.38	17.96	2.93	2.37	27.7	0.32	20.8	18.67008	2.93	2.37	27.7	0.33	34.61%	0.57	31.60%	0.54
750	750	34.09	32.72	7.62	7.42	51.0	0.19	34.82	33.284438	7.62	7.42	51	0.20	50.93%	0.56	51.52%	0.5


Root Locus Editor for Open Loop 1 (OL1)
Open−Loop Bode Editor for Open Loop 1 (OL1)
G.M.: Inf
Freq: Inf
Stable loop
Bode Editor for Closed Loop 1 (CL1)
P.M.: Inf
Freq: NaN
Frequency (rad/s)
Frequency (rad/s)


Step Response
Amplitude
Time (seconds)

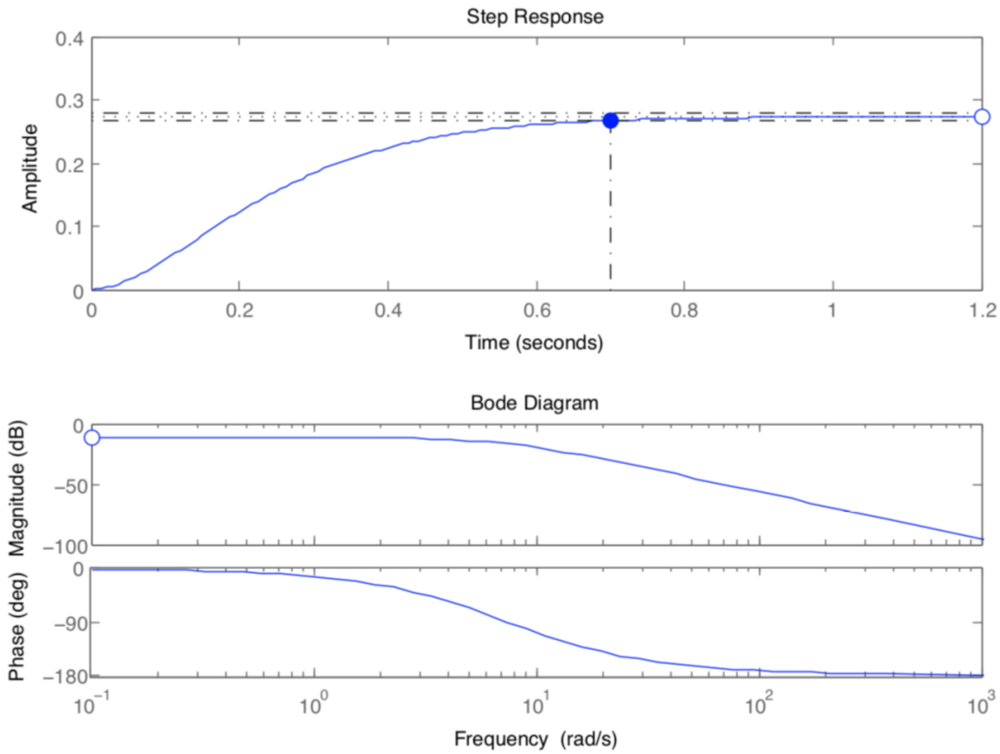


Figure 14: Step Response and Bode Diagram when K=10

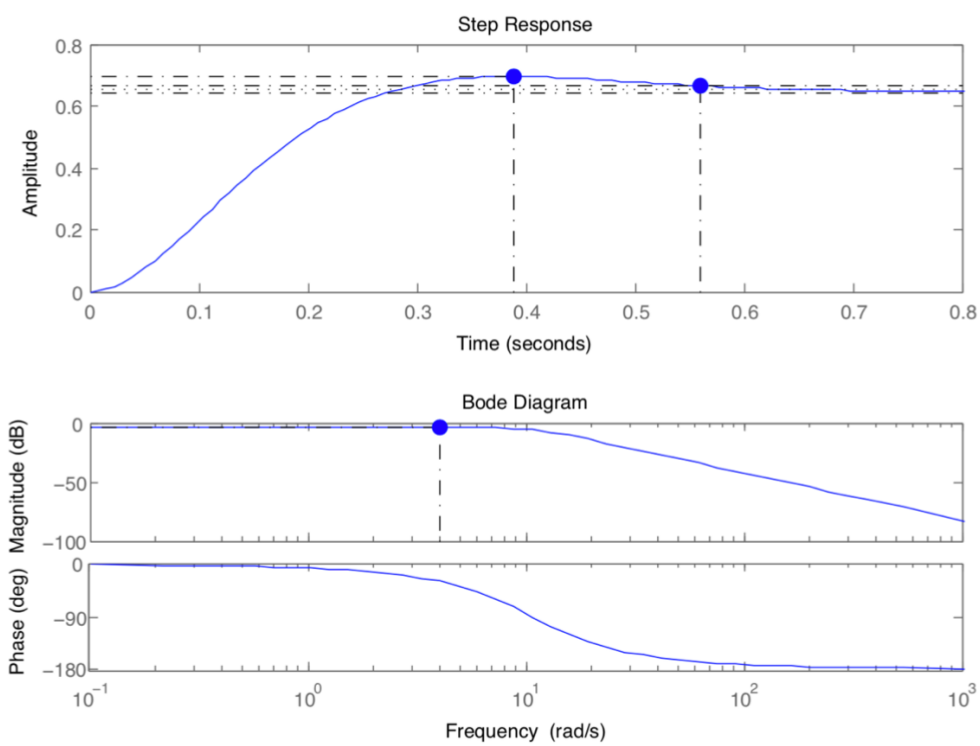


Figure 15: Step Response and Bode Diagram when K=50

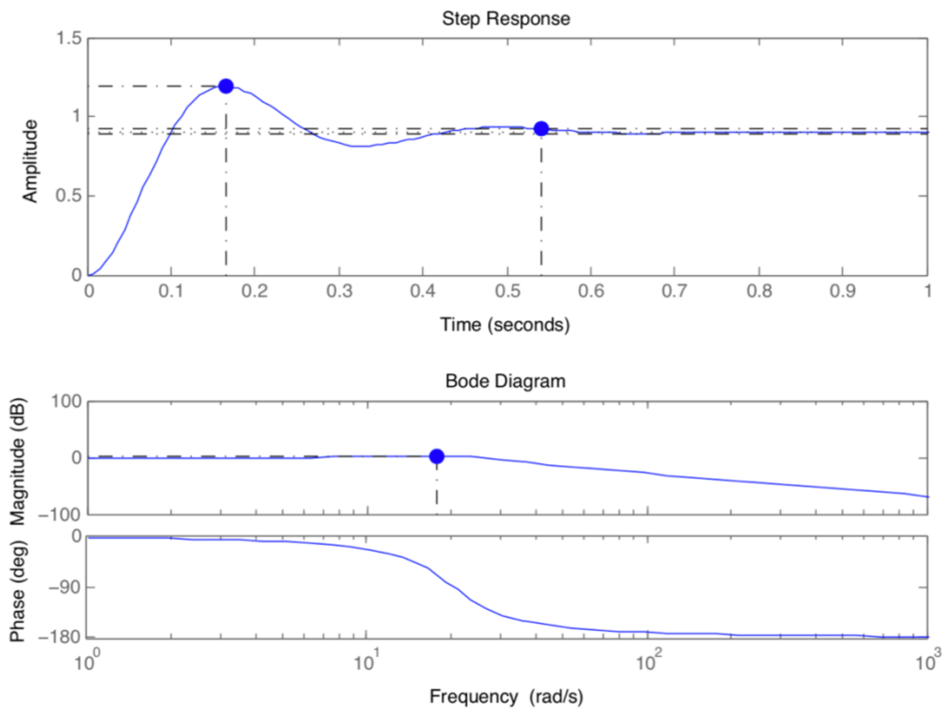


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Figure 16: Step Response and Bode Diagram when K=250

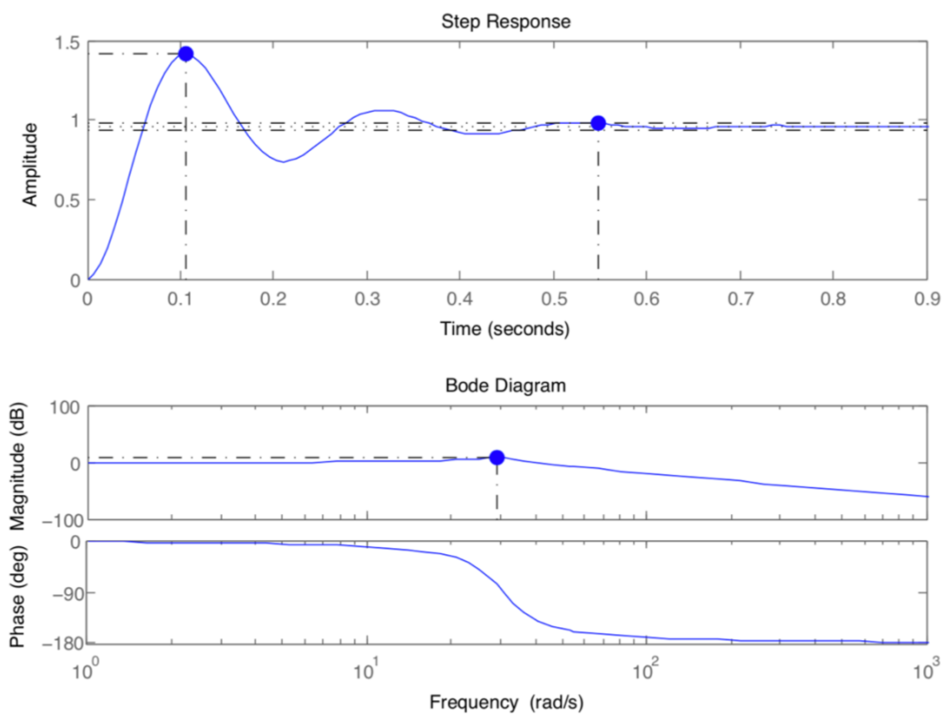

Figure 17: Step Response and Bode Diagram when K=600

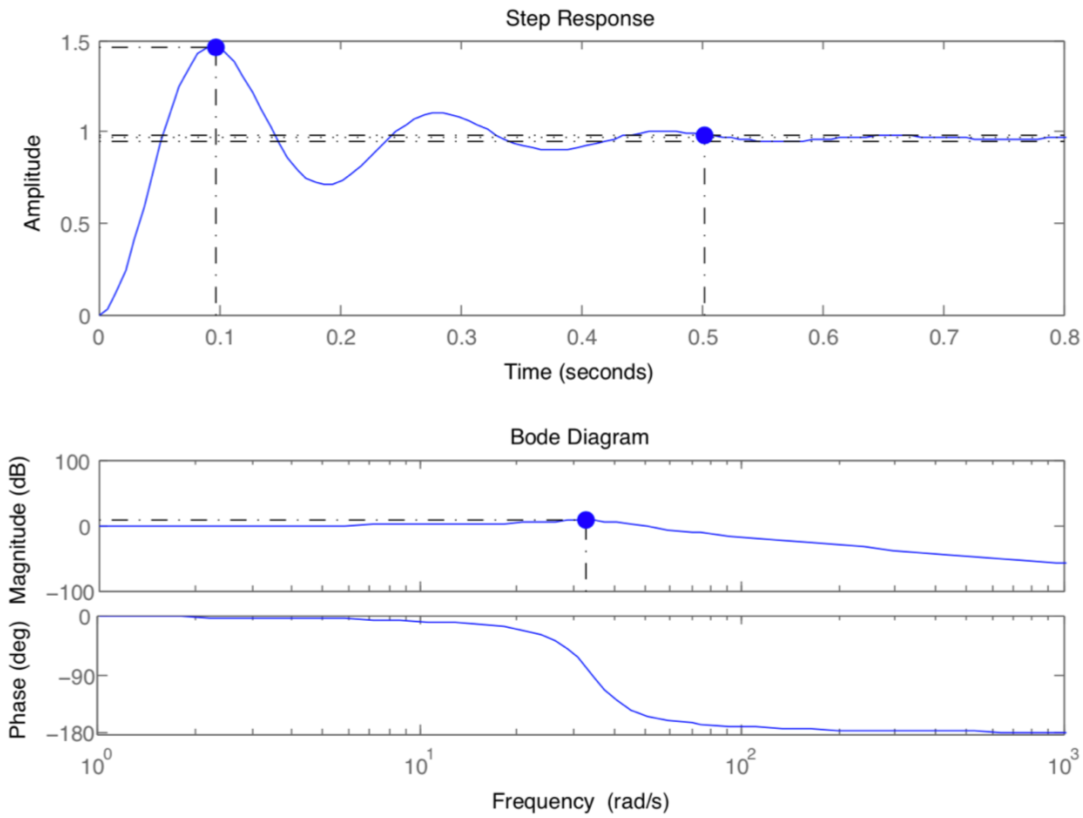

Figure 18: Step Response and Bode Diagram when K=750

The above graphs indicate that as K value increases, the gain plots frequencies being to change, when K=10 and K=50, the step response is a typical overdamped system, and when K=250 ,600 and 750, the step response became an underdamped system. And while the K value becomes larger, the system natural frequency also increases. For the underdamped systems, which are the cases K=250, 600 and 750, as the magnitude reached peak, the phase will accurately reach -90 degrees, this is because arctan(infinity) = -90 degrees which is expected. For the overdamped systems, which are the cases K=10, the peak in magnitude does not correspond to -90 degrees in phase, this is because peak only exist when damping ratio is between 0 ~ 1/√ 2. For case K=50, there is a small peak that cannot be distinguished without magnification of the graph or could due to the measurement error in the experimental values.

Conclusion

The effect of controlled system changing frequency were accurately demonstrated in this lab. In frequency analysis, the relevance of system gains and phase lag/lead to the input frequency and reference signal were demonstrated, while the frequency is too high, the gain will be very small, and the phase will have too much lag. Possible sources of error can result from noise from measurements and human error when reading from graphs.

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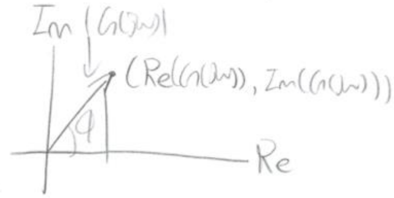
①

$$G(s)\Big|_{s=j\omega} = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\xi j\left(\frac{\omega}{\omega_n}\right) + 1}$$

$$G(j\omega) = \frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) + 2\xi j\frac{\omega}{\omega_n}} \cdot \frac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) - j2\xi\frac{\omega}{\omega_n}}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) - 2\xi j\frac{\omega}{\omega_n}}$$

$$= \frac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) - 2j\xi\frac{\omega}{\omega_n}}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 - \left(2\xi\frac{\omega}{\omega_n}\right)^2} \qquad Im(G(j\omega)) = \frac{-2\frac{\omega}{\omega_n}\xi}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) + \left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

$$Re(G(j\omega)) = \frac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}$$



$$\rightarrow |G(j\omega)| = \sqrt{Re(G(j\omega))^2 + Im(G(j\omega))^2} = \sqrt{\left(\frac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}\right)^2 + \left(\frac{-2\frac{\omega}{\omega_n}\xi}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}\right)^2}$$

$$= \sqrt{\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$$dB(G(j\omega)) = 20\log_{10}|G(j\omega)| = 20\log_{10}\left(\sqrt{\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}\right)$$

$$= -20\log_{10}\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}$$

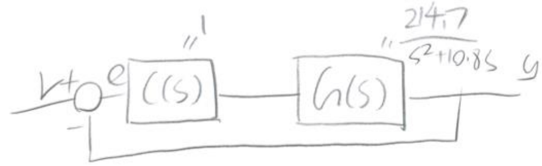
$$\varphi(j\omega) = \operatorname{atan2}\left[\frac{Im(G(j\omega))}{Re(G(j\omega))}\right] = \tan^{-1}\left[\frac{\dfrac{2\xi\left(\frac{\omega}{\omega_n}\right)}{\cancel{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}}{\dfrac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}{\cancel{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}}\right]$$

$$\varphi(j\omega) = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

%

MK

②



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{214.7}{s^2+10.8s}}{1+\frac{214.7}{s^2+10.8s}}$$

$$\frac{Y(s)}{R(s)} = \frac{214.7}{s^2+10.8s+214.7} = \frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2} \rightarrow \omega_0 = 14.6$$

$\& \ \zeta = 0.369$

$$M(\omega) = -20\log_{10}\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}$$

$$= -20\log_{10}\sqrt{\left(1-\left(\frac{\omega}{14.6}\right)^2\right)^2 + \left(2\cdot 0.369\left(\frac{\omega}{14.6}\right)\right)^2}$$

and $\varphi(\omega) = -\tan^{-1}\left[\frac{2(0.369)\left(\frac{\omega}{14.65}\right)}{1-\left(\frac{\omega}{14.65}\right)^2}\right]$

③ case① $\omega \ll \omega_n$ case② $\omega = \omega_n$

$M(\omega) = -20\log_{10}\sqrt{1^2+0^2} = 0$ $M(\omega) = -20\log_{10}\sqrt{0^2+(2(0.369))^2} = 2.638$

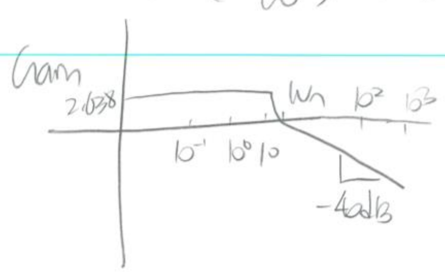
$\varphi(\omega) = \tan^{-1}\left(\frac{0}{1}\right) = 0$ $\varphi(\omega) = -\tan^{-1}\left(\frac{0.738}{0}\right) = -90^\circ$

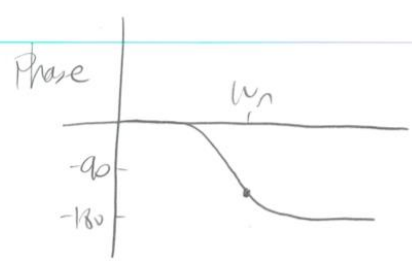
case③ $\omega \gg \omega_n$

$$M(\omega) = -20\log_{10}\sqrt{\left(\frac{\omega}{14.65}\right)^4 + 0.738^2\left(\frac{\omega}{14.65}\right)^2} = -40\log_{10}\left(\frac{\omega}{14.65}\right)$$

$$\varphi(\omega) = -\tan^{-1}\left(\frac{2(0.389)\left(\frac{\omega}{14.65}\right)}{1-\left(\frac{\omega}{14.65}\right)^2}\right) = -\tan^{-1}\left(\frac{2(0.389)\left(\frac{\omega}{14.65}\right)}{\left(\frac{\omega}{14.65}\right)^2}\right)$$

$$= -\tan^{-1}\left(\frac{14.65}{\omega}\right) \Rightarrow \text{towards } -180$$





③a MK

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+C(s)G(s)} = \frac{\frac{k(1.5)}{s^2+14s+40.02}}{1+\frac{k(1.5)}{s^2+14s+40.02}} = \frac{k(1.5)}{s^2+14s+40.02+1.5k} = G_u(s)$$

$$\Rightarrow \omega_n = \sqrt{40.02+1.5k} \quad \zeta = \frac{14}{2\sqrt{40.02+1.5k}}$$

$$\therefore M(\omega) = 20\log_{10}\left(\sqrt{\frac{1}{\left(1-\left(\frac{\omega}{\sqrt{40.02+1.5k}}\right)^2\right)^2 + \left(2\left(\frac{7}{\sqrt{40.02+1.5k}}\right)\left(\frac{\omega}{\sqrt{40.02+1.5k}}\right)\right)^2}}\right)$$

and $$\varphi_u(\omega) = \tan^{-1}\left(\frac{2\left(\frac{7}{\sqrt{40.02+1.5k}}\right)\left(\frac{\omega}{\sqrt{40.02+1.5k}}\right)}{1-\left(\frac{\omega}{\sqrt{40.02+1.5k}}\right)^2}\right)$$

① For $\omega \ll \omega_n$

$M_u(\omega) = -20\log_{10}\sqrt{1} = 0$

$\varphi_u(\omega) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

② for $\omega \gg \omega_n$

$M_u(\omega) = 20\log_{10}\sqrt{\frac{1}{\omega^4}}$

$M_u(\omega) = -40\log_{10}(\omega)$

$\varphi_u(\omega) = \tan^{-1}(0) = -180°$

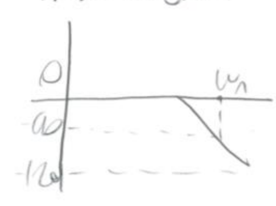
③b ③ For $\omega_n = \omega$

$M_u(\omega) = -40\log\left(\sqrt{\frac{1.4}{\sqrt{40.02+1.5k}}}\right)$

$\varphi_u(\omega) = -90$

Gain Margin

Phase Diagram



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(3c) $\zeta = \frac{7}{\sqrt{40.02 + 1.5k}}$ $\omega_n = \sqrt{40.02 + 1.5k}$

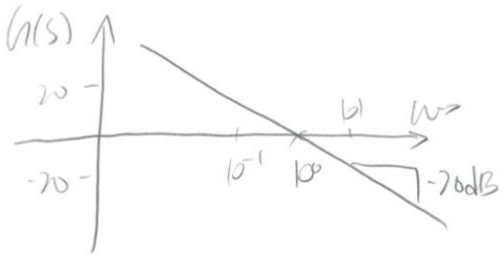
$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$ $M_r = 20\log\left(\frac{1}{2\zeta\sqrt{1-\zeta^2}}\right)$ $M_n = -40\log\sqrt{\frac{14}{\sqrt{40.02 + 1.5k}}}$ $\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

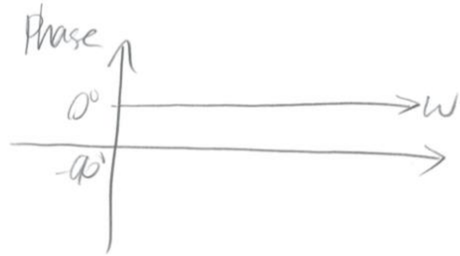
$T_s = \frac{4}{\zeta\omega_n}$

K	ζ	ω_n	ω_r	M_r	M_n	OS	T_s
10	0.943	7.417	\	4.04	-5.517	0.013	0.571
50	0.652	10.72	4.126	0.097	-2.315	6.68	0.571
250	0.343	20.37	17.805	3.804	3.258	31.68	0.571
750	0.205	34.13	32.665	7.927	7.741	51.77	0.571

(4) $G(s) = \frac{1}{s} \rightarrow G(j\omega) = \frac{1}{j\omega} = -j/\omega$ $|G(j\omega)| = 20\log_{10}\left(\frac{1}{\omega}\right) = -20\log_{10}(\omega)$

$\varphi(\omega) = -\tan^{-1}\left(\frac{1/\omega}{0}\right) = -90°$





The effect of $G(s) = \frac{1}{s}$ is it caused the phase of the system to shift -90°. Also for log unit increase w, the system gain varies by -20dB meaning that for higher value of w more gain occurs. This is essentially the same as taking the integral of some sinusoidal input, this would cause the integrated signal to be 90° lagging the input and scaled by 1/f

$\sin\omega t \rightarrow \boxed{\int dt} \rightarrow -\frac{1}{\omega}\cos\omega t$

⑤ $G(s) = \frac{s+4.3}{s+2.8} \to G(jw) = \frac{jw+4.3}{jw+4.8}$ $G(jw) = \frac{4.3(\frac{jw}{4.3}+1)}{28(\frac{jw}{28}+1)}$ MK

$\to w$ is small

$G(jw) = \frac{4.3}{28} = 0.153$ $|G(jw)| = 20\log_{10}(0.153) = -16.27$

$\varphi(jw) = -\tan^{-1}(0.153) = 0$

$\to w$ is large

$G(-jw) = 1$ $|G(-jw)| = 20\log_{10}(1) = 0$

$\varphi(jw) = -\tan^{-1}(1) = 0.78$