ECE3330

Lab#2

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Introduction

In this lab, a feedback loop and a motor control were added to the control system of the previous lab, to increase the accuracy of the motor. This is because by using the feedback loop, the system can compare the current state with the desired state and remove any offsets. The motor controller allows the adjustment of the system response. In this lab, the position and speed of the motor were controlled with either a negative or positive feedback loop. The empirical data was then compared with the theoretical data to test for accuracy.

Objective

The objective for this lab was to understand the function of a closed-loop control to a system, how the controller can affect a system. Also, manipulating controllers to meet specific design requirement was also examined.

Pre-laboratory Assignment

The pre-lab assignment can be found in the Appendix A.

Experiment 1: Speed Control

For this experiment, the proportional integrator (PI-controller) was programmed using the transfer function. This type of controller is ideal when the noise of a system needs to me minimized in the absence of a differentiator. The general form of the PI controller was determined to be:

$$C(t) = \frac{k_1 s + K_2}{s}$$

Where K1 is the proportional constant and K2 is the integrator constant. The proportional part of the controller changes the duty cycle of the generated signal. The proportional controller has a quicker reaction to larger errors and the integral controller dominated the gain when there are longer periods of time with a small error. The integrator component of the controller adjusts the output. The proportional controller has a quicker reaction to larger errors and the integral controller dominated the gain when there are longer periods of time with a small error. It does this by integrating the error over a set timeframe and multiplying the result by an integrating constant. These two components of the controller, therefore, complement each other.

Using the general form of the controller the following transfer can be found:

$$H(s) = \frac{\omega_m(s)}{R(S)} = \frac{k_b(sK_1 + K_2)}{sR_a\left(sJ_{eq} + \frac{K_b^2}{R_a}\right) + K_b(sK_1 + K_2)}$$

From this equation, the response of the system can be controled by subbing in different values of K1 and K2 The following four transfer functions were tested:

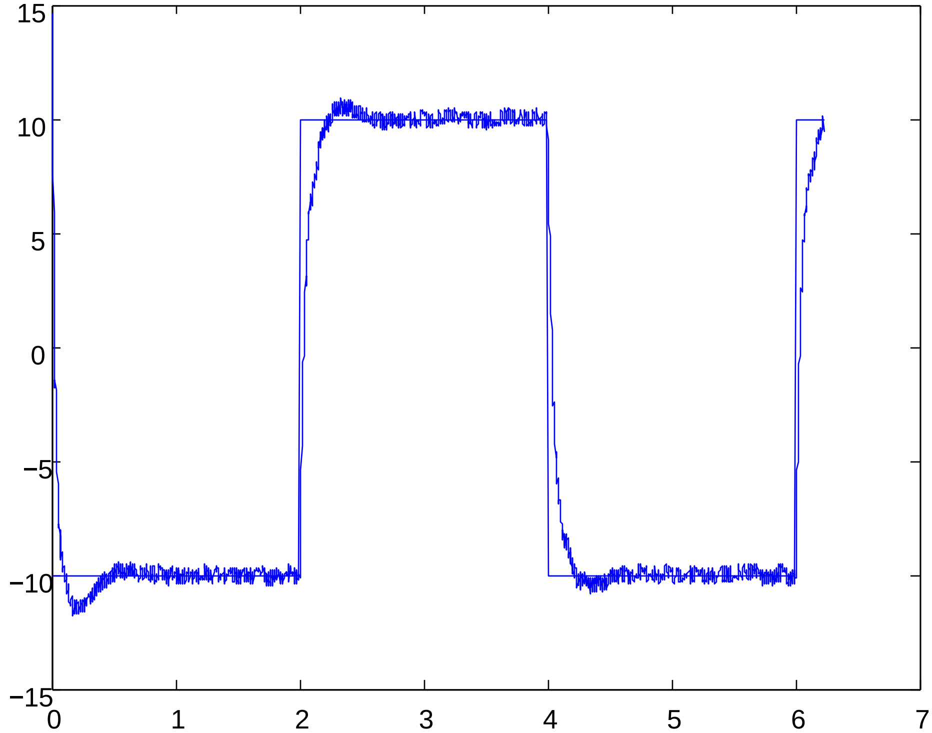
1. For Ts ≈ 0.25, K1 = 0.0985, K2 = 1.20
2. For Tp ≈ 0.2, K1 = 0.0295, K2 = 1.49
3. K1 = 0.2, K2 = 0
4. K1 = 2, K2 = 0

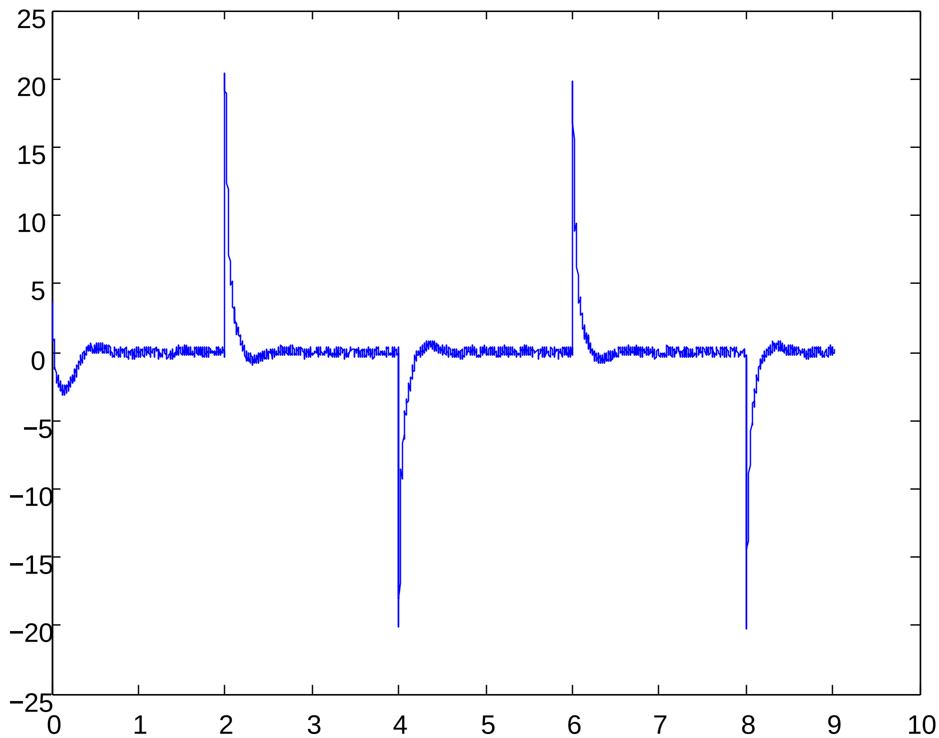
For each trial, a graph showing the input, output, and a graph showing the error signals were plotted and generated using MATLab. From each of the graphs the settling time, overshoot, and steady-state error were calculated, which can be seen in the table below:

	Settling time	Percent overshoot	Steady state error
Ts = 0.25			
Tp = 0.20			
K1 = 0.2, K2 = 0			
K1 = 2, K2 = 0			

Graph 1: Ts = 0.25 s

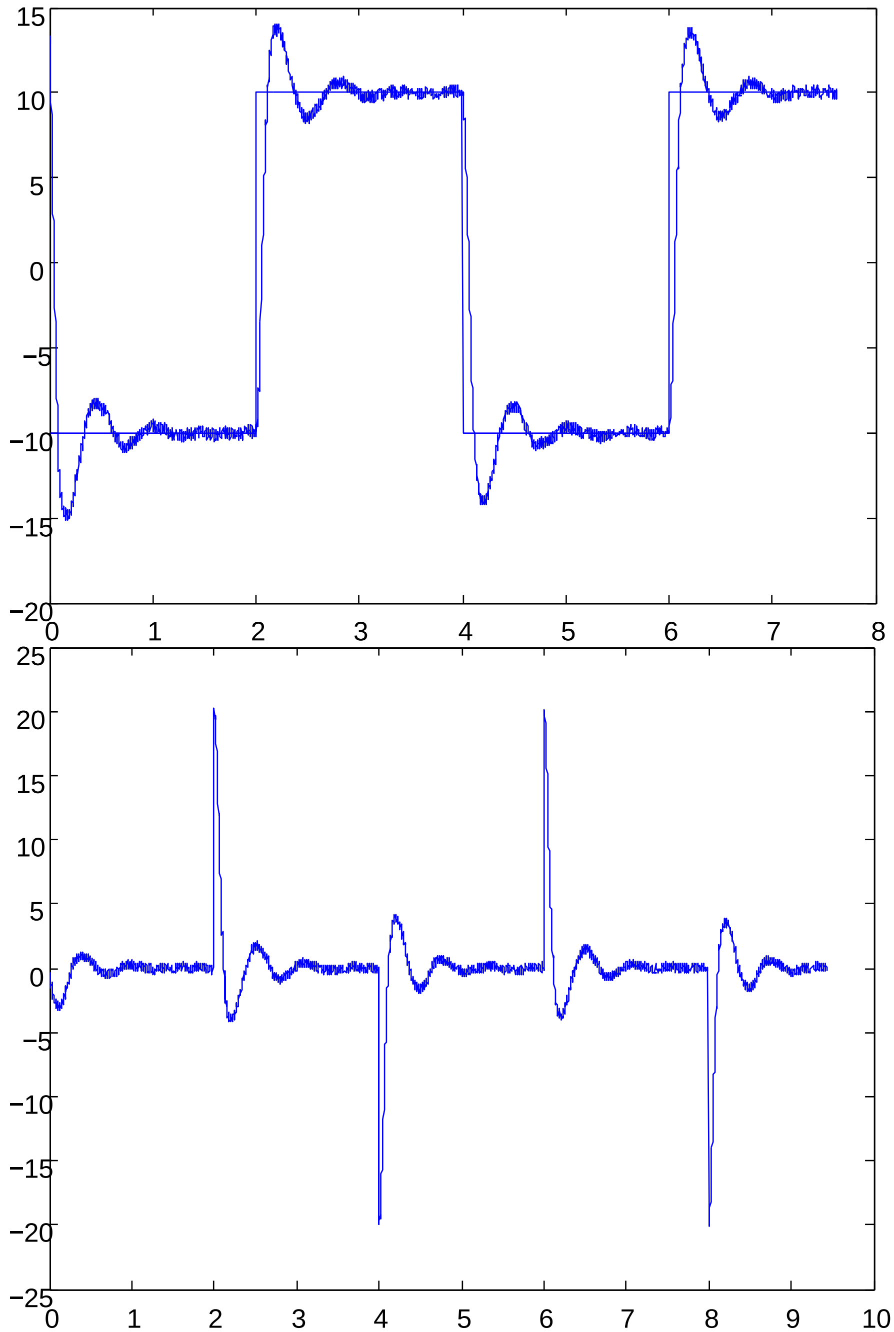
In the first graph, there was a small amount of overshoot even though the controller was set to be critically damped. Most of this error can be attributed to feedback noise and therefore can be mitigated. The settling time was found to be exactly 0.25 seconds which was expected.





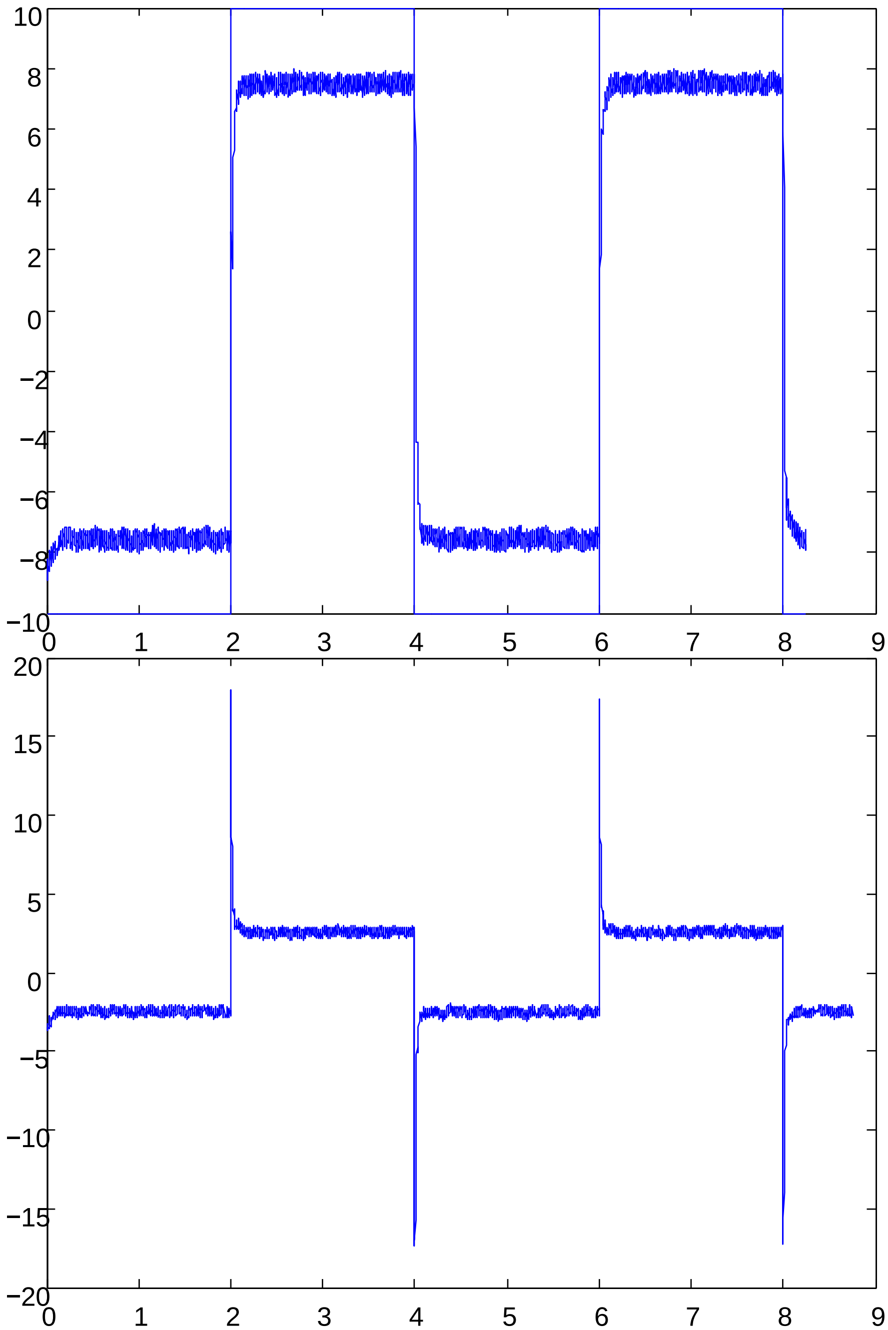
Graph 2: Tp = 0.2 s

The second graph shows and underdamped response. In this experiment, a 40% overshoot for the system was found which. The settling time was found to be 0.33 second. This was above the expected value 0.2 seconds and 18% for the settling time and the percent overshoot. Both parameters were not expected. The error could have been caused by rounding error on the prelab calculations and manufacturing error on the data acquisition software.



Graph 3: K1 = 0.2 and K2 = 0

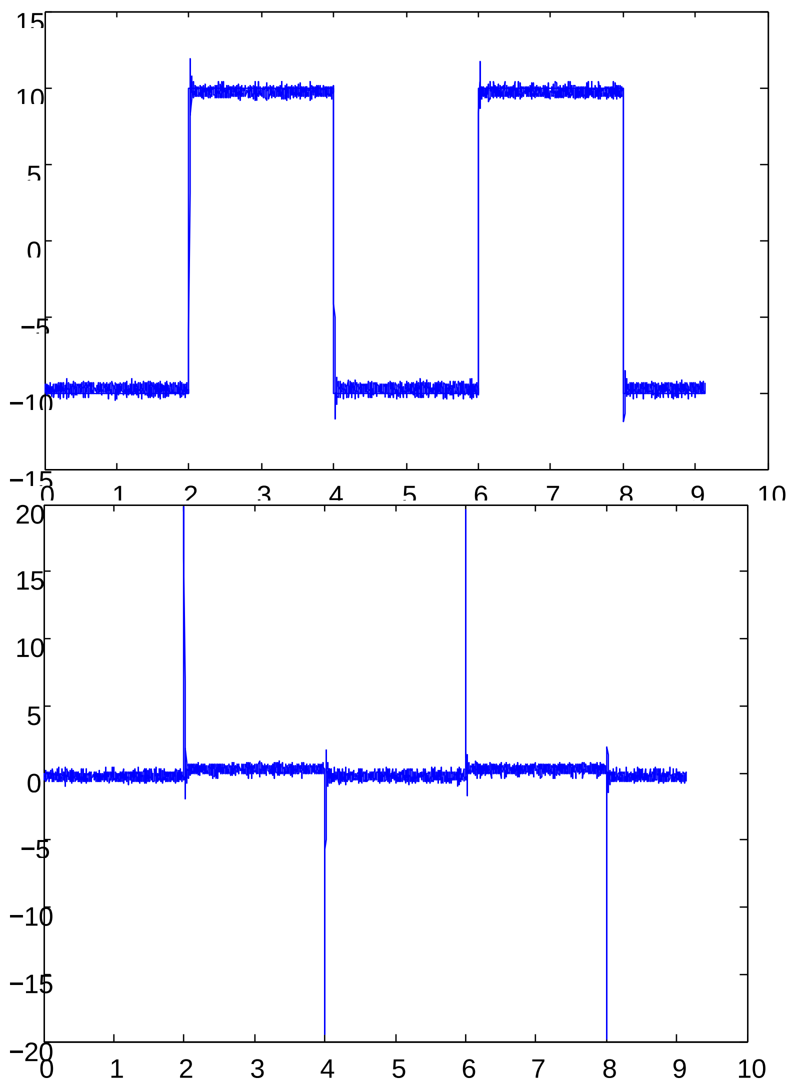
The third graph, the steady state error increased to ± 2 when K2 is set to zero and K1 is set to 0.2. The steady state error exhibited the pattern of being positive when the input is positive and negative then the input is negative. This transfer function exhibits the highest amount of error out of all 4 plots. Also, when the step input suddenly changes the output slightly lags which is when the error become the highest.



Graph 4: K1 = 2 and K2 = 0

For the last graph, the steady state error increased to ± 0.3 when K2 is set to zero and K1 is increased to 2. This significantly reduced the steady-state error when compared to the last graph. There was slightly more lag in this response as seen by the increased amount of error when the input suddenly changes. Similarly, to the last plot, the steady state error exhibited the pattern of being positive when the input is positive and negative then the input is negative.

When K2 is equal to zero the transfer function is a proportional controller. This was demonstrated in graphs 3 and 4 were both the steady state errors were non-zero. When K2 was equal to a non-zero value the transfer, function turns into a proportional-integrator controller. This was demonstrated in graphs 1 and 2. Although there was no steady state error there was overshoot (although negligible for the 1st graph). Therefore, integrator controllers should be used to correct the steady state error as experienced by proportional controllers.



Experiment 2: Position Control

The general form of the controller was determined to be:

$$C(t) = K_1$$

Where K1 is the proportional constant. Thus, the controller is a proportional controller unlike the proportional-integrator controller seen for the speed control experiment.

Using the general form of the controller the following transfer can be found:

$$H(s) = \frac{\omega_m(s)}{R(S)} = \frac{k_b K_1}{R_a\, J_{eq} s + K_b^2 + k_b K_1}$$

From this equation, the response of the system can be controlled by subbing in different values for K. The following four values for K were tested:

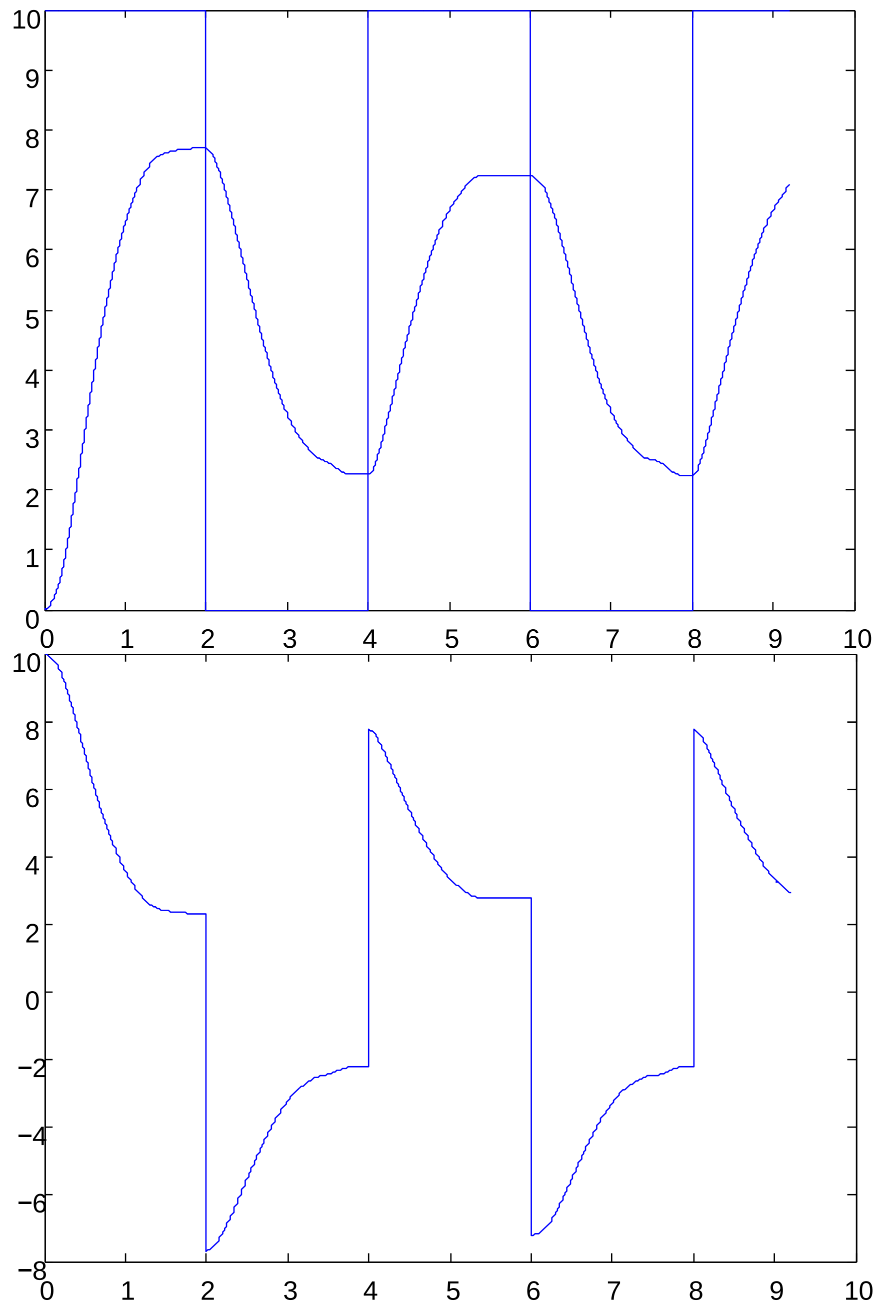
1. K = 0.05
2. K = 0.135
3. K = 0.30
4. K = 1.00

For each trial, a graph showing the input, output, and error signals were plotted and generated using MATLab. From each of the graphs, varoius properties were calculated, which can be seen in table below:

			Pre-Lab Calculations			Experimental Results			
K1	ζ	ωm (rad/s)	%OS (%)	Tp (s)	Ts (s)	%OS (%)	Tp (s)	Ts (s)	Tr (s)
0.050	1.644	3.276	--	--	0.743	--	--	1.484	0.478
0.135	1.006	5.35	~ 0	--	0.743	3.5%	0.582	0.334	0.314
0.300	0.671	8.024	0.058	0.528	0.743	12%	0.342	0.680	0.150
1.000	0.368	14.65	0.289	0.231	0.743	34%	0.182	0.664	0.078

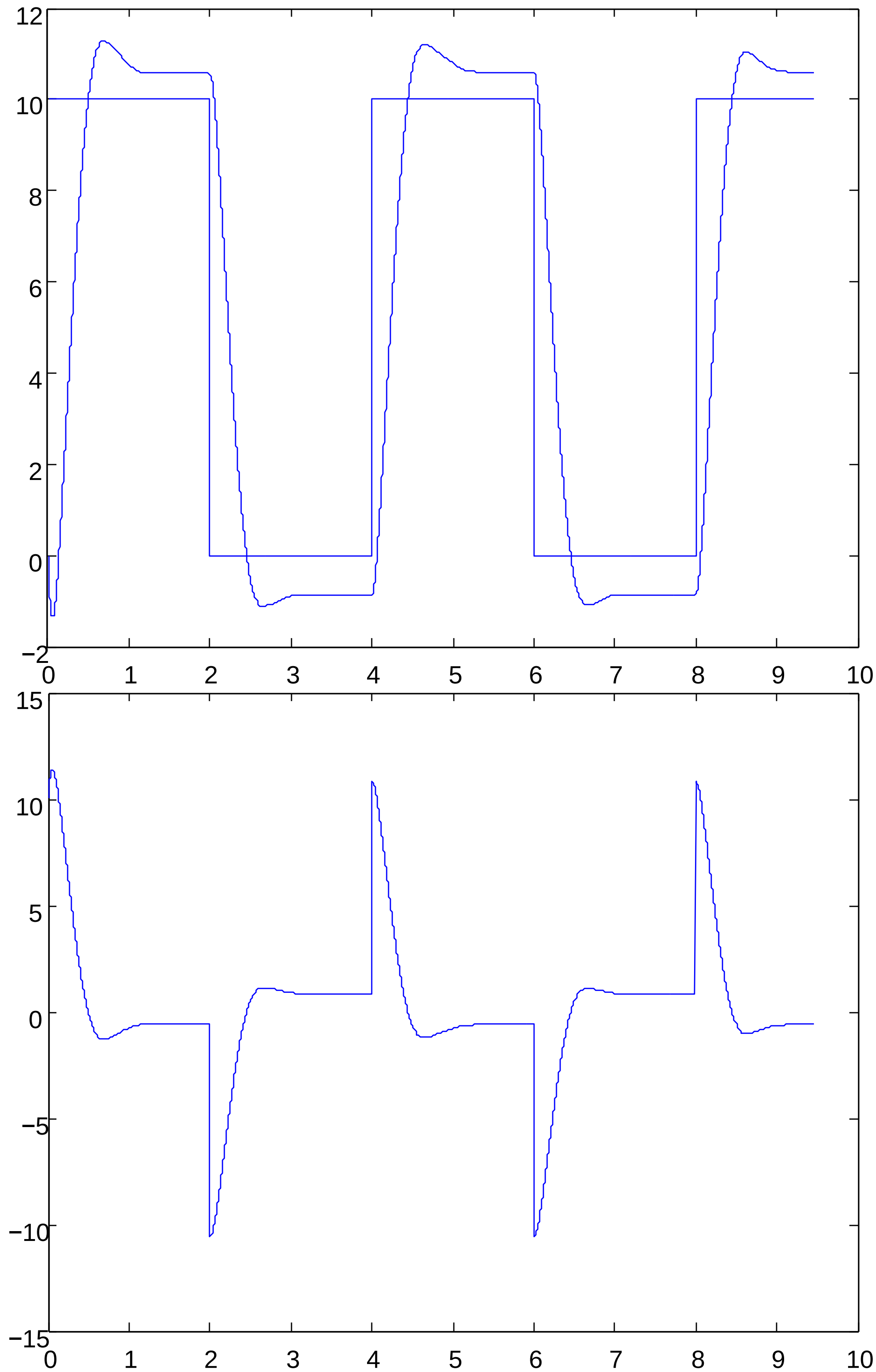
Graph 5: K = 0.05

In the prelab calculation, the system was found to be overdamped. The experimental results matched this. The theoretical settling time was found to be 0.743 seconds. The experimental settling time was found be a significant amount higher at 2 seconds. This error is large but can be attributed for various reasons. The equations used to calculate the system properties were only approximations and were only valid for small values of zeta. In the actual experiment, the value of zeta was large.



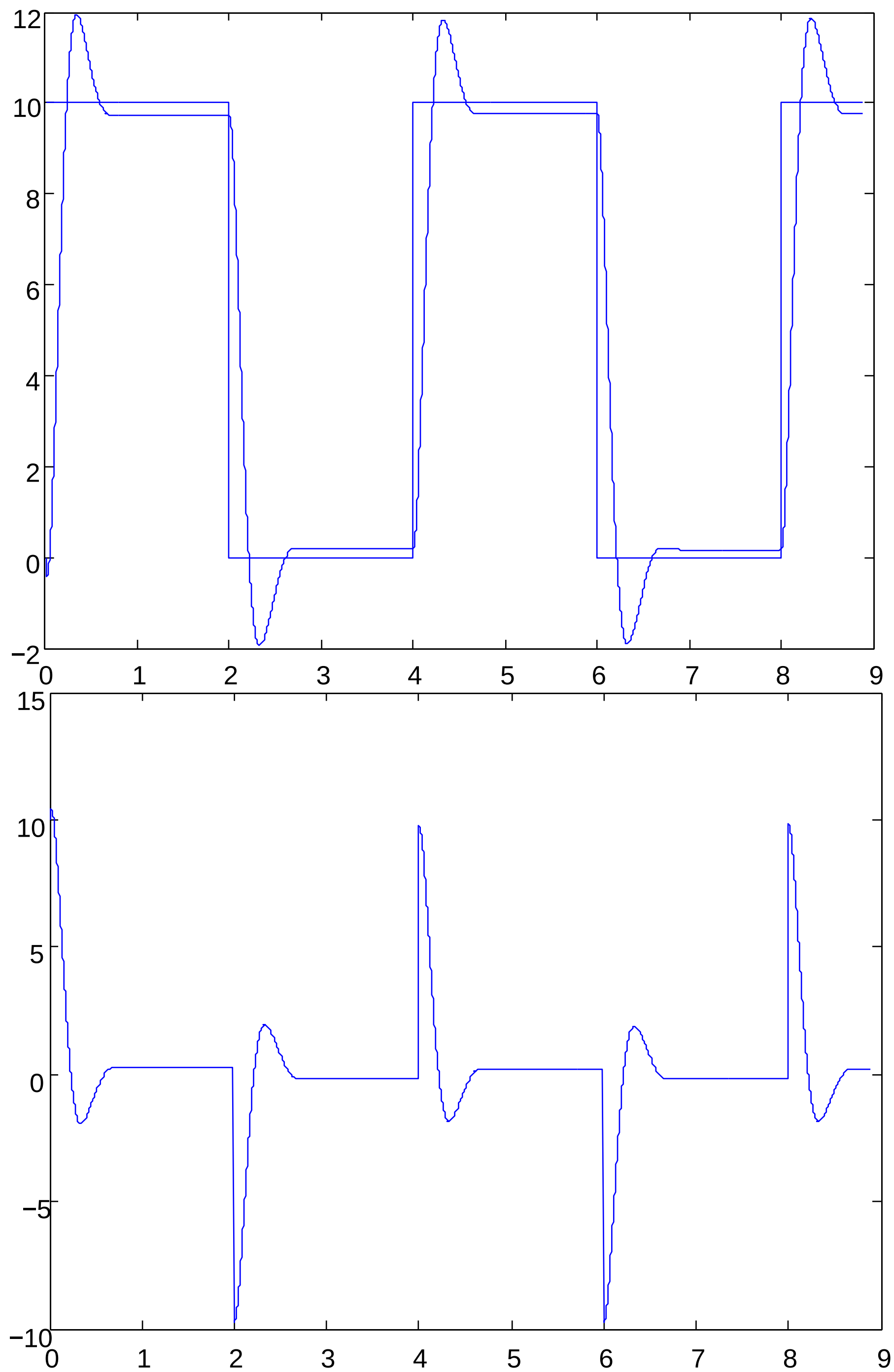
Graph 6: K = 0.135

The output of the system produced a critically damped response. The experimental settling time was found to be 0.4 seconds. The error was calculated to be 53.8% which is lower than the error seen in the previous experiment.



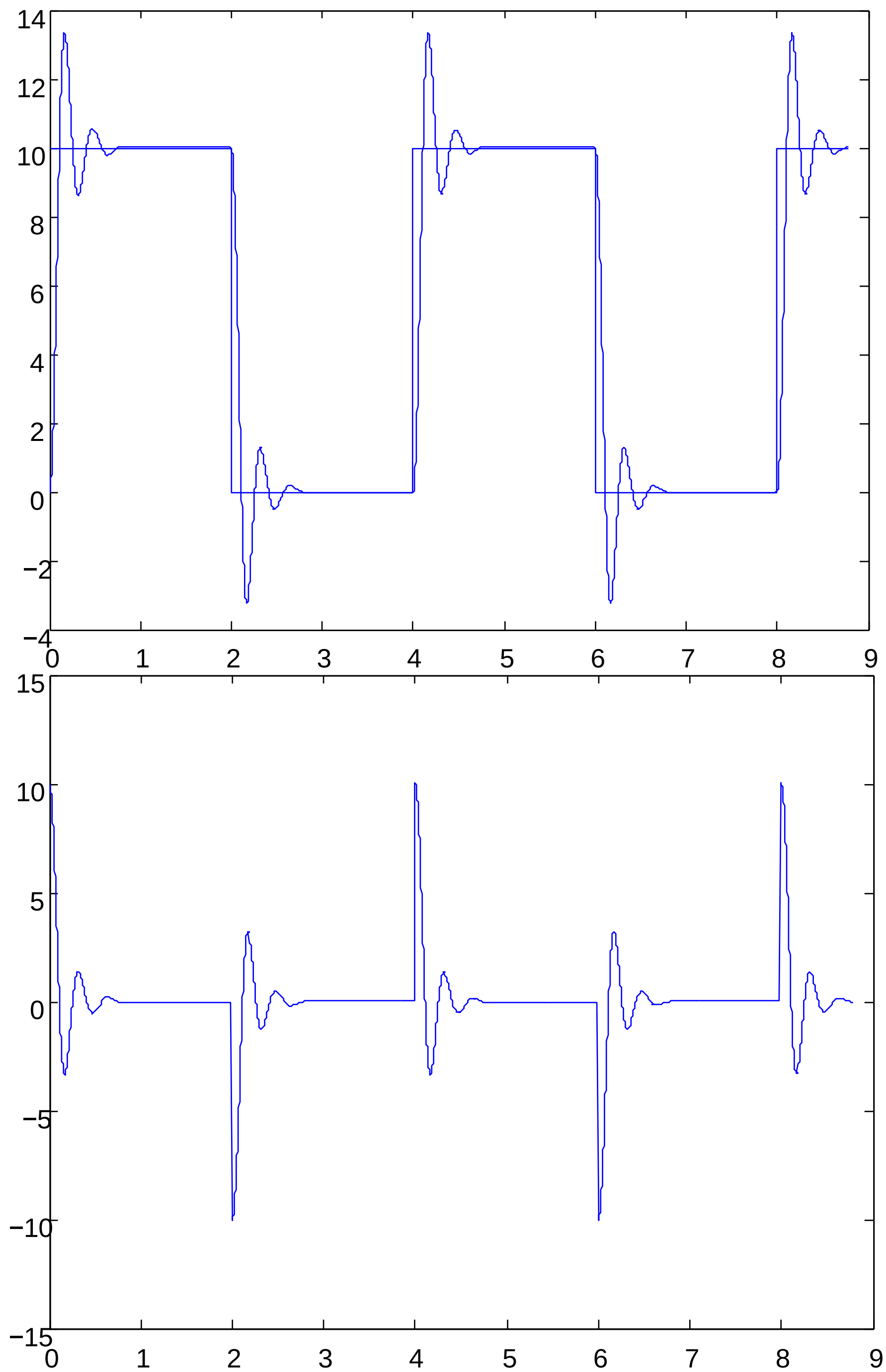
Graph 7: K = 0.3

The output of the system produced an underdamped response. The calculated experimental percent overshoot was 12%. The theoretical overshoot was calculated to be 6.4%. This represents a 5.8% error between the theoretical and experimental value. The peak time for the experiment was measured at 0.342 seconds which is 35% off the calculated value of 0.528 seconds. The settling time for the experiment was found to be 0.680 seconds. The percent error was found to be 12%



Graph 8: K = 1

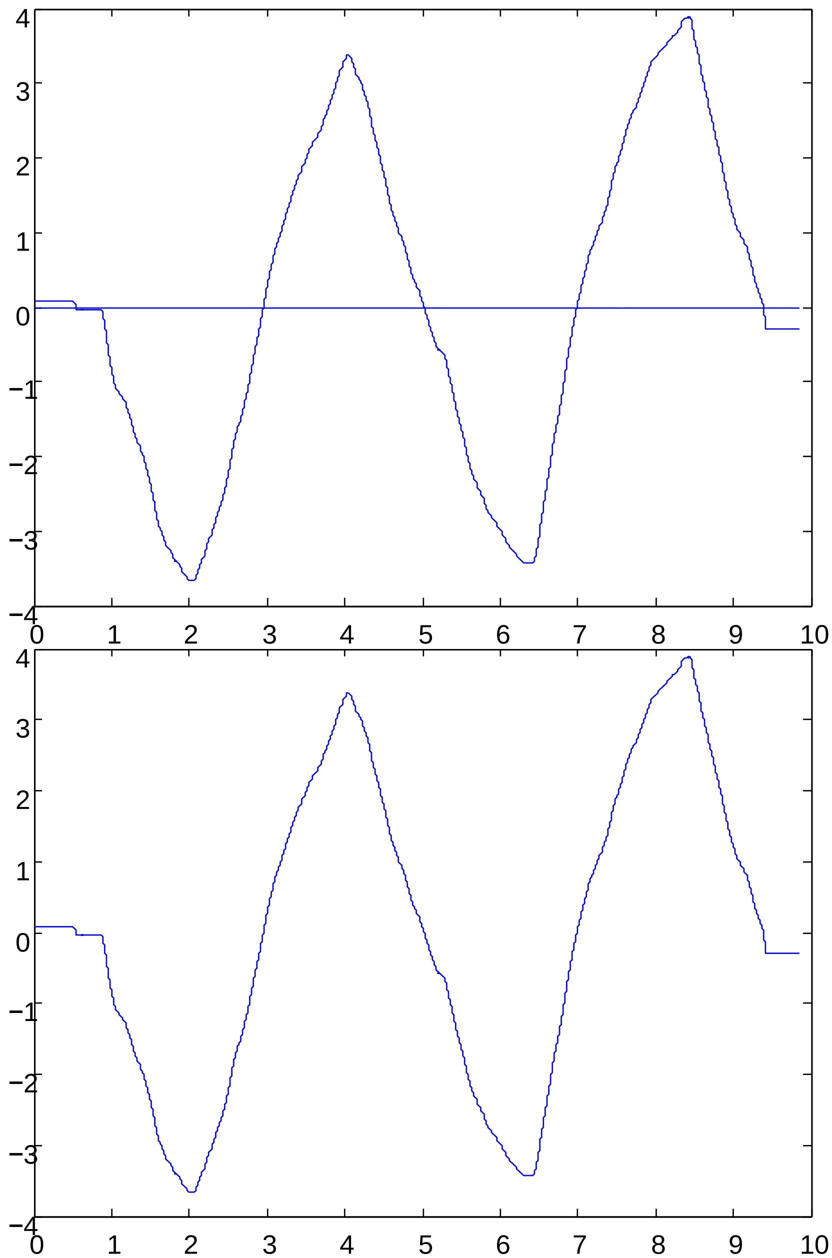
The output of the system produced an underdamped response which was expected. The calculated experimental percent overshoot was 34%. The theoretical overshoot was calculated to be 29.6%. This represents a 4.4% error between the theoretical and experimental value. The peak time for the experiment was measured at 0.182 which is 78.8% off the calculated value of 0.231 seconds. The settling time for the experiment was found to be 0.664 seconds. The percent error was found to be 12%



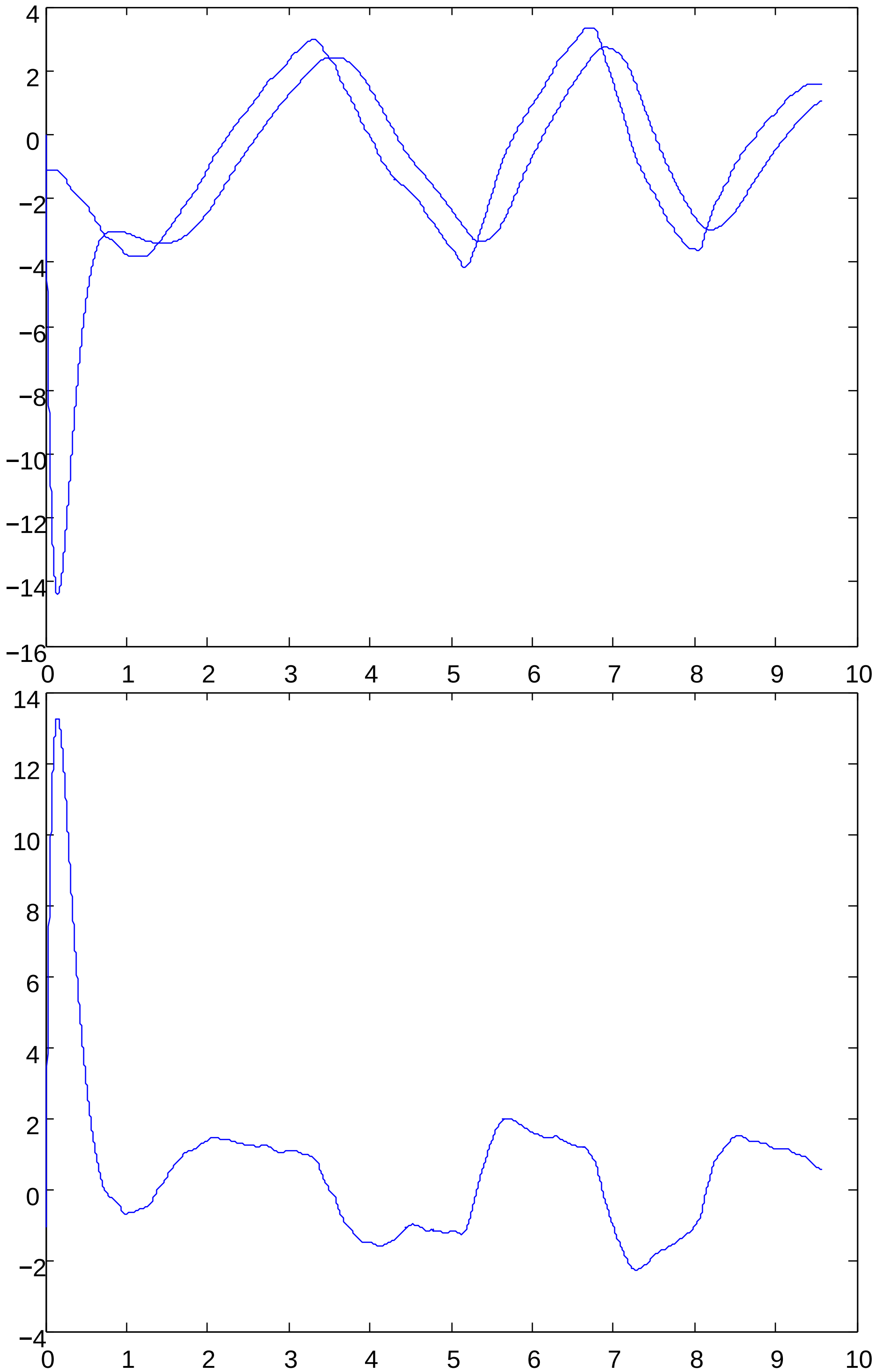
Experiment 3: Teleoperation

For the last experiment, the motor's angular position was controlled by a potentiometer rather than a square wave input. The potentiometer was changed by hand and motor's response to these changes was recorded. Various tests were conducted by altering the gain of the system. The tests were done to determine the ideal gain to minimize the motor's lag compared to the input.

Graph 9: K = 0.01



Graph 10: K = 0.1



4
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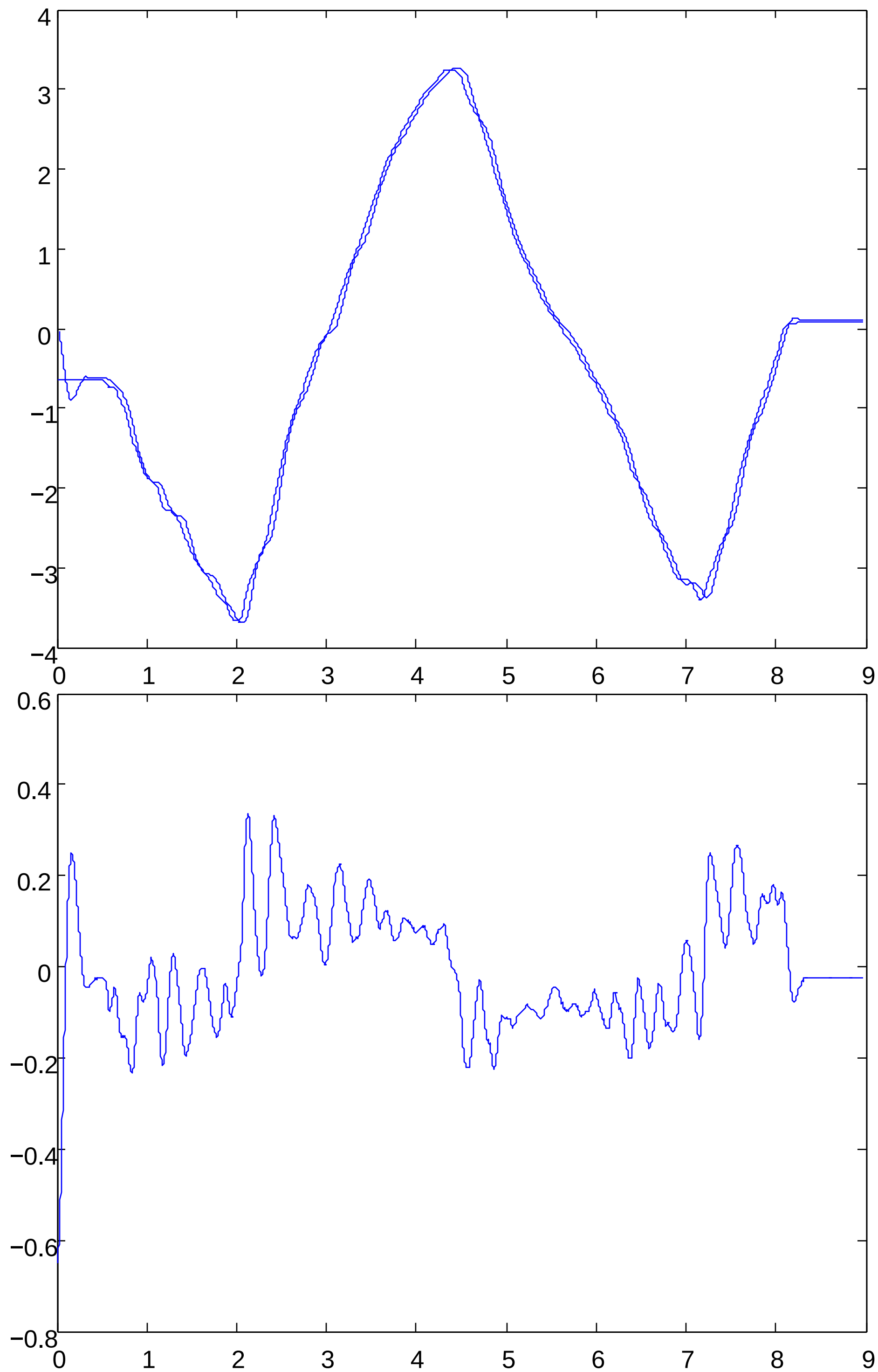
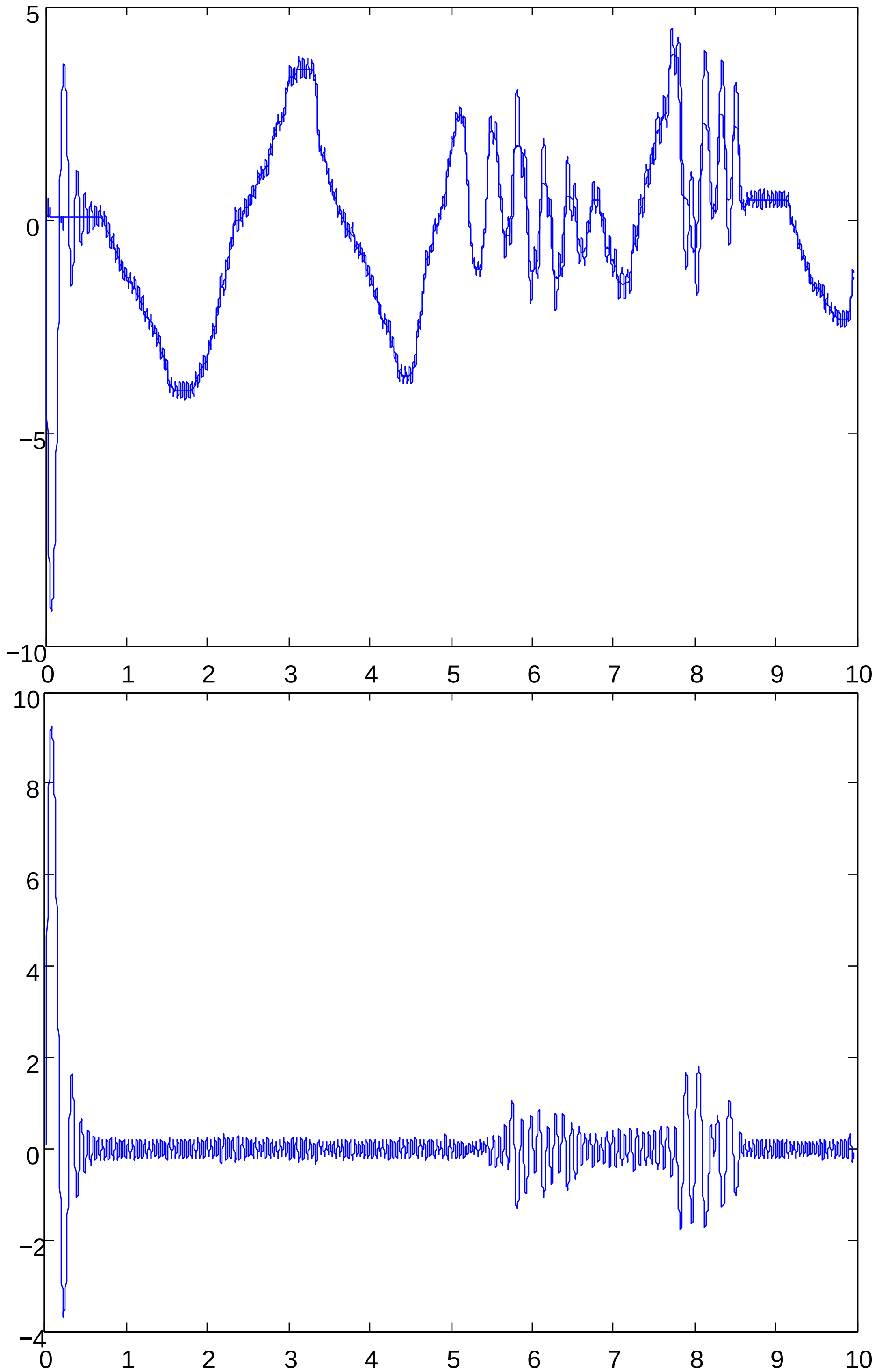
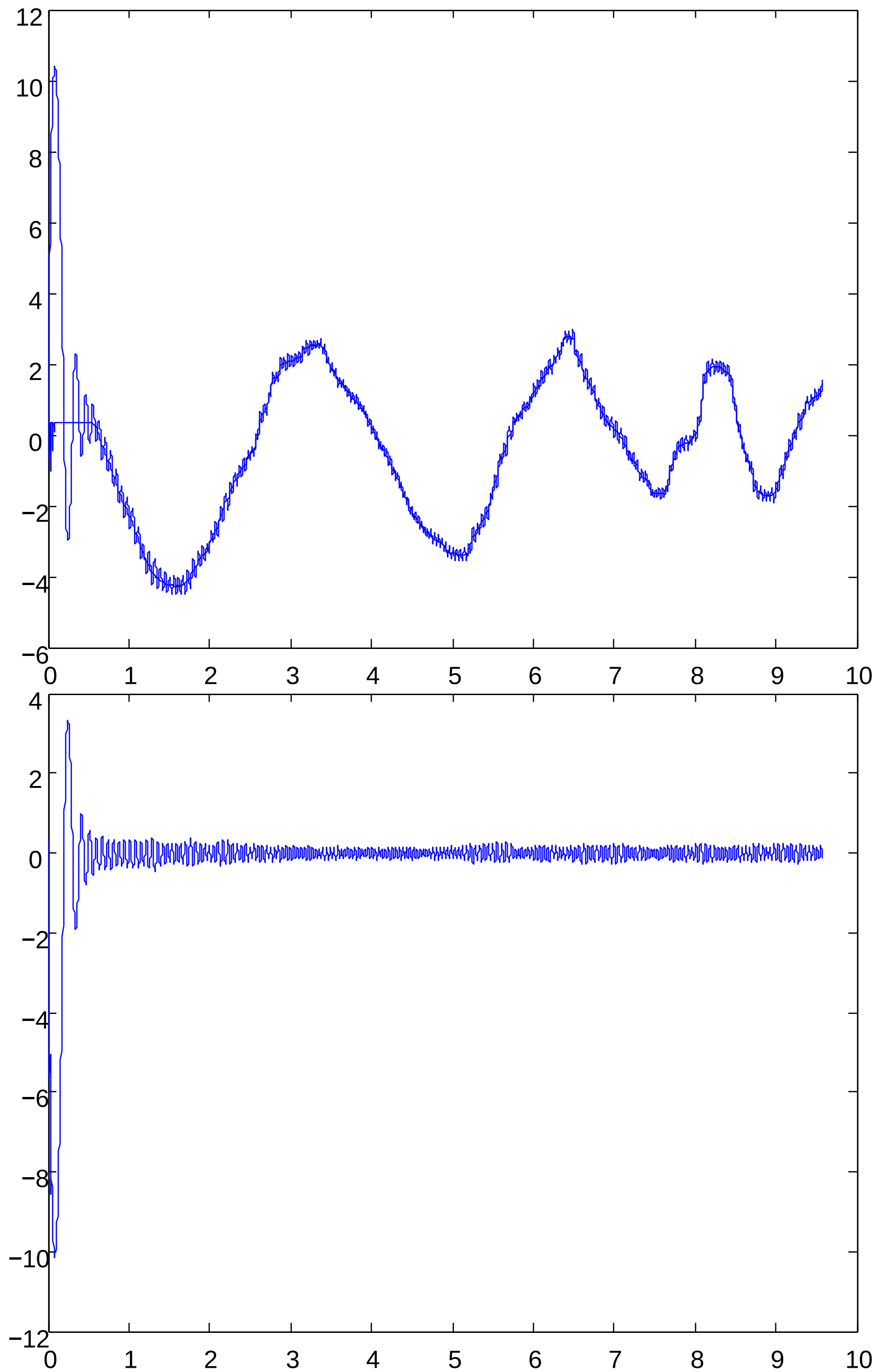
Graph 11: K = 1



Graph 12: K = 100



Graph 12: K = 1000



Conclusion

In this laboratory, it was found that PI controllers are more accurately model a motor's speed than only using a P controller. It was also found that using a proportional gain leads to offset. Using an integrator helps to reduce the error. High adjustment gain values were found to cause the system to overcorrect itself resulting in oscillations. In this experiment, the motor's position was controlled using a P controller. High gains experienced lower final offsets than lower gains. Also, lower gains did not oscillate as much. When the system isn't predictable it was found that high gains were favoured.

Appendix 1

S.P. ~~Jeo~~

① $e = R(s) - W_m(s)$

$e_1 C(s) = (R(s) - W_m(s))k_1 = u$ $\Rightarrow$ $W_m(s) = G(s)u = G(s)\cdot(R(s) - W_m(s))k_1$

$G(s) = \frac{W_m(s)}{(R(s) - W_m(s))k_1} = \frac{W_m(s)}{R(s)} \cdot \frac{1}{k_1 - \frac{W_m(s)}{R(s)}k_1}$

$G(s)(k_1)\left(1 - \frac{W_m(s)}{R(s)}\right) = \frac{W_m(s)}{R(s)} = k_1 G(s) - k_1 G(s)\frac{W_m(s)}{R(s)} = \frac{W_m(s)}{R(s)}$

$k_1 G(s) = \frac{W_m(s)}{R(s)}(1 + k_1 \cdot G(s)) \Rightarrow \frac{k_1 G(s)}{1 + k_1 G(s)} = \frac{W_m(s)}{R(s)}$

$$\frac{W_m(s)}{R(s)} = \frac{k_1 k_b}{k_1 k_b + R_a\left(J_{eq}s + B + \frac{k_b^2}{R_a}\right)}$$

② $G(s) = \frac{k_1 k_b}{k_1 k_b + R_a J_{eq} s + R_a B + k_b^2}$

$G(s)\cdot\frac{1}{s} = \frac{k_1 k_b}{s(k_1 k_b + R_a J_{eq} s + R_a B + k_b^2)} = \frac{k_1 k_b}{R_a J_{eq}} \cdot \frac{1}{s\left(s + \left(\frac{k_1 k_b + R_a B + k_b^2}{J_{eq} R_a}\right)\right)}$

$\Downarrow$

$W_m(s)$

$W_m(s) = \frac{k_1 k_b}{R_a J_{eq}} \cdot \frac{1}{s} \cdot \frac{1}{s + \left(\frac{k_1 k_b + R_a B + k_b^2}{J_{eq} R_a}\right)}$

$\downarrow L^{-1}$

$$\frac{k_1 k_b}{R_a J_{eq}}\left[\frac{1 - e^{-t\left(\frac{k_1 k_b + B R_a + k_b^2}{J_{eq} R_a}\right)}}{\frac{k_1 k_b + B R_a + k_b^2}{J_{eq} R_a}}\right]$$

$W_{mss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{k_1 k_b}{J_{eq} R_a s + k_1 k_b + k_b^2 +}$

$W_{mss} = \frac{k_1 k_b}{k_1 k_b + B R_a + k_b^2}$

③ $e_{ss} = 1 - W_{mss} = 1 - \frac{k_1 k_b}{k_1 k_b + B R_a + k_b^2}$

For ideal steady-state response, the tracking error would be 0, so W_{mss} should be 1, to get a value that very close to 1, a large value of k is desireable

④ S.P. ~~Sto~~

$e = R(s) - W_{in}(s)$

$e \cdot C(s) = \left(\frac{k_1 s + k_2}{s}\right)(R(s) - W_{in}(s)) = u$

$G_{wr}(s) \cdot u = W_m(s) \rightarrow G_{wr}(s) = \dfrac{W_m(s)}{\left[\frac{k_1 s + k_2}{s}\right](R(s) - W_m(s))}$

$$G(s) = \frac{W_m(s)}{R(s)} \cdot \frac{1}{\left[\frac{k_1 s + k_2}{s}\right]\left(1 - \frac{W_m(s)}{R(s)}\right)}$$

$$G(s)\left(\frac{k_1 s + k_2}{s}\right)\left(1 - \frac{W_m(s)}{R(s)}\right) = \frac{W_m(s)}{R(s)}$$

$$\frac{k_1 s + k_2}{s} G(s) = \frac{W_m(s)}{R(s)}\left[1 + \frac{k_1 s + k_2}{s} G(s)\right]$$

$$\frac{W_m(s)}{R(s)} = \frac{G(s)}{G(s)} \cdot \frac{k_1 s + k_2}{k_1 s + k_2 + s(G(s)^{-1})}$$

$$\frac{W_m(s)}{R(s)} = \frac{k_1 s + k_2}{k_1 s + k_2 + s\left(\frac{R_a}{k_b} \cdot \left(J_{eq} s + \left(B + \frac{k_b^2}{R_a}\right)\right)\right)} = \frac{k_1 s + k_2}{k_1 s + k_2 + s^2 \frac{R_a J_{eq}}{k_b} + \frac{B R_a}{k_b} + k_b}$$

⑤

$$W_{mss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{\cancel{k_1 s} + k_2}{\cancel{s^2 \frac{R_a J_{eq}}{k_b} + s\left(k_1 + \frac{B R_a}{k_b} + k_b\right)} + k_2} = \frac{k_2}{k_2} = 1$$

$e = 1 - W_{mss} = 0 \rightarrow$ The error will be 0 nomatter what.

This is an improvement because there will be no error and no adjustment should be made.

$$G(s) = \frac{k_b k_1 s + k_2 k_b}{s^2 + \left(\frac{k_1 k_b + B R_a + k_b^2}{J_{eq} R_a}\right) s + \frac{k_2 k_b}{J_{eq} R_a}}$$

⑥ S.P. ~~ㅈㅁ~~

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + \left(\frac{k_1 k_b + BR_a + k_b^2}{J_{eq}R_a}\right)s + \frac{k_2 k_b}{J_{eq}R_a}$$

$$2\zeta\omega_n = \frac{k_1 k_b + BR_a + k_b^2}{J_{eq}R_a}$$

$$\omega_n^2 = \frac{k_2 k_b}{J_{eq}R_a} \rightarrow k_2 = \frac{\omega_n^2 \cdot J_{eq}R_a}{k_b}$$

$$k_1 = \frac{2\zeta\omega_n \cdot J_{eq}R_a - BR_a - k_b^2}{k_b}$$

⑦ $$k_2 = \frac{\omega_n^2 (22.07\times10^{-6})(10.6)}{0.0502} = 4.66\times10^{-3}\omega_n^2$$

$$k_1 = \frac{2\zeta\omega_n (22.07\times10^{-6})(10.6) - (0.0502)^2}{0.0502} = 9.32\times10^{-3}\zeta\omega_n - 0.0502$$

$$T_s = \frac{4}{\zeta\omega_n} = 0.25 \Rightarrow \text{let } \zeta = 1 \quad \omega_n = 16$$

$k_2 = 1.192$

$k_1 = 0.09932$

⑧ $OS\% = 18\%$

$$\ln 0.18 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \rightarrow (\ln 0.18)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2}$$

$\zeta = 0.479$

$$T_p = 0.2 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{0.2\sqrt{1-(0.479)^2}}$$

$\omega_n = 17.89$

$k_2 = 1.491$

$k_1 = 0.02967$

S.P.

(a) $e = R(s) - \theta_m(s)$

$e \cdot C(s) = (R(s) - \theta_m(s)) \cdot k_1 = u$

$\theta_m(s) = G(s)u = G(s) \cdot (R(s) - \theta_m(s))k_1$

$\frac{\theta_m(s)}{(R(s) - \theta_m(s))k_1} = G(s) \rightarrow \frac{\theta_m(s)}{R(s)} \cdot \frac{1}{k_1 - \frac{\theta_m(s)}{R(s)}k_1} = G(s)$

$\frac{\theta_m(s)}{R(s)} = G(s) \cdot k_1 \left(1 - \frac{\theta_m(s)}{R(s)}\right)$

$$\frac{\theta_m(s)}{R(s)} = \frac{k_1}{k_1 + \left(\frac{k_b}{R_a} \cdot \frac{1}{s(J_{eq}s + (B + \frac{k_b^2}{R_a}))}\right)^{-1}}$$

$$\frac{\theta_m(s)}{R(s)} = \frac{k_1 k_b}{k_1 k_b + R_a J_{eq} s^2 + R_a(B + \frac{k_b^2}{R_a})s}$$

(b) $$\frac{\theta_m(s)}{R(s)} = \frac{k_1(0.0502)}{0.0502k_1 + ((10.6)(22.07 \times 10^{-6})s^2 + \left(\frac{(0.0502)^2}{10.6}\right)s}$$

@ $k_1 = 0.05$

$$\frac{\theta_m(s)}{R(s)} = \frac{(2.51 \times 10^{-3}) / (10.6)(22.07 \times 10^{-6})}{s^2 + \left(\frac{k_b^2}{(10.6)(22.07 \times 10^{-6})}\right)s + \frac{2.51 \times 10^{-3}}{(10.6)(22.07 \times 10^{-6})}}$$

$$\frac{\theta_m(s)}{R(s)} = \frac{10.73}{s^2 + 10.77 + 10.73}$$

@ $k_1 = 0.135$

$$\frac{\theta_m(s)}{R(s)} = \frac{28.97}{s^2 + 10.77 + 28.97}$$

@ $k = 0.3$

$$\frac{\theta_m(s)}{R(s)} = \frac{64.37}{s^2 + 10.77 + 64.37}$$

@ $k_1 = 1$

$$\frac{\theta_m(s)}{R(s)} = \frac{214.5}{s^2 + 10.77 + 214.5}$$

(11) S.P. ~~Sts~~

k_1	ζ	ω	%OS	T_p	T_s
0.05	1.644	3.276	/	/	0.743
0.135	1.006	5.35	/	/	0.743
0.3	0.671	8.024	0.058	0.528	0.743
1	0.368	14.65	0.284	0.231	0.743

k_1	%OS	T_p	T_s	T_r
0.05				
0.135				
0.3				
1				