Lab#3

ECE3330

Ziqin Shang

250890594

## Introduction

In the previous lab, the control system with a feedback loop and a motor control in order to decrease the error were studied. However, in real life, some control systems are hard to simulate in laboratory. In this lab, a large chemical plant was taken into consideration, which is extremely expensive and risky to test physically, but we can test the control system virtually by using the MATLAB tools such as SISO Design Tool, LTI Viewer, and Graphical Tuning Window. Those tools can be used to test any given single-input, single-output system. For this lab, a chemical system using an actuator were simulated in MATLAB. A step input was implemented to obtain the system responses.

## Objective

In this lab the chemical process system will be tested using the SISO tool in MATLAB.

The learn the following windows in the SISO Design Tools in MATLAB:

1. Control and Estimation Tool Manager
2. Graphical Tuning Window
3. LTI Viewer

The software will be used to:

1. Analyze the behaviour of a system
2. Achieve specific system requirements by manipulating the compensator gain
3. Eliminate Steady-State Error by adding an integrator controller

## Pre-laboratory Assignment

Appendix A contains the pre-laboratory assignment.

# Controlling Temperature of a Chemical Process

For this experiment, a chemical system was simulated. The purpose of the system is to regulate the temperature of a chemical process. The chemical system consisted of an actuator and a valve, a controller, a heat process block, and a feedback loop.

The following individual components of the transfer function were used:

$$C(s) = K \qquad G_{av}(s) = 1 \qquad G_{chp}(s) = \frac{0.7}{s^2 + 1.7s + 0.25}$$

Where K is the controller gain constant that can be manipulated to move the poles and zeros of a system.

# Analyzing the behavior of a system – Part 1

The following transfer function was defined in MATLAB for the Gchp transfer function:

```
>> G_chp = tf([0.7],[1  1.7  0.25])
```

Once the system was setup in MATLAB, various value of the controller gain could be tested. To verify the results in the prelab the same values of k were used (k = 0, 0.5, 0.675, 2, 6). For each value of k an open root locus plot of each system was plotted. The root locus plots verified the prelab calculations.

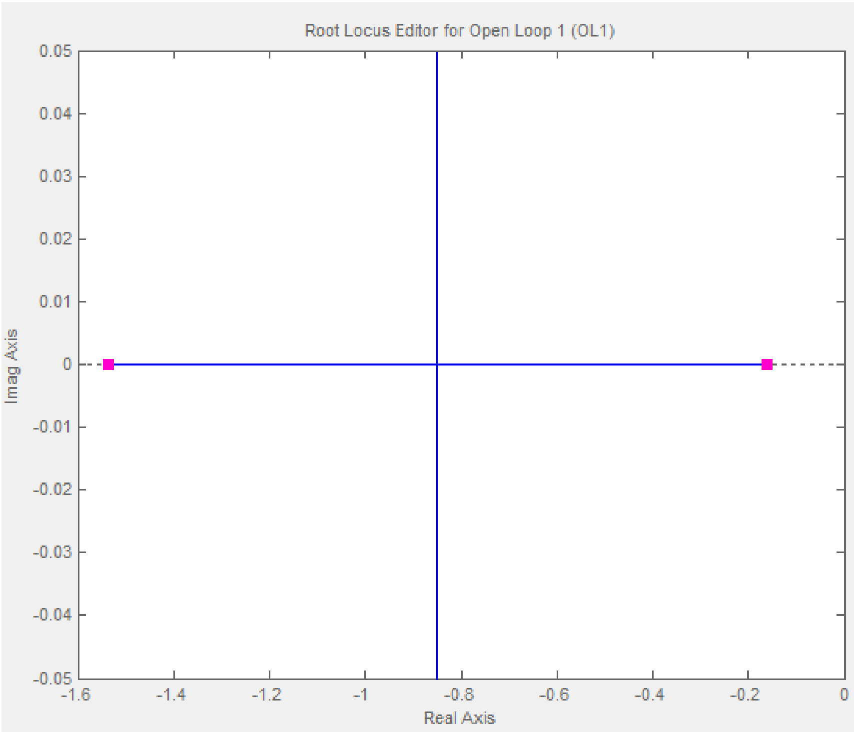
By changing the compensator gain, the type of system response could be changed from underdamped, critically damped, and overdamped. For values of K less than 0.675, the system is overdamped. At K = 0.675, the system is critically damped. Lastly, the system was found to be underdamped for values of K above 0.675. It was found that the poles were bounded to -0.85 and move horizontally along the real axis as the system becomes more damped. Once the system is critically damped at (0,0), the poles were bounded to the vertical axis and move vertically as the system becomes more underdamped. All values of k resulted in a stable response assuming k is positive. This means that there were no poles located in the positive real region.

Afterwards, the response of each system was tested with an input step function. Peak time, overshoot, steady state value, setline time and rise time were found for each value of k.

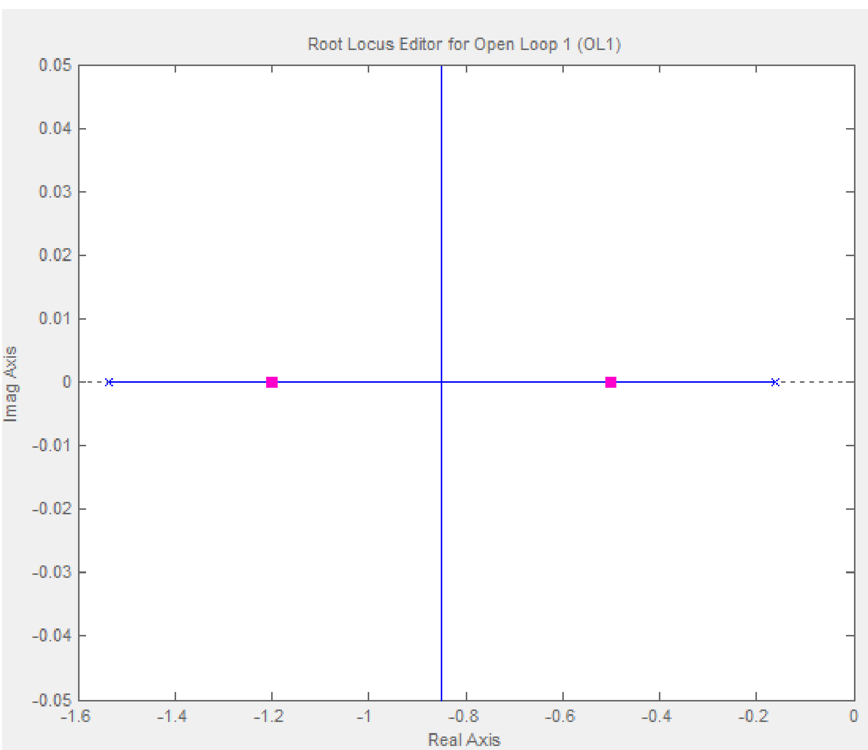
| K | Rise Time $T_r$ (s) | Overshoot (%) | Settling Time $T_s$ (s) | Steady State Value |
|---|---|---|---|---|
| 0.5 | 4.97 | N/A | 8.9 | 0.583 |
| 0.675 | 3.95 | N/A | 6.86 | 0.654 |
| 2 | 1.57 | 6.25 | 4.68 | 0.848 |
| 6 | 0.697 | 25.1 | 3.99 | 0.944 |

As the value of K increased, the roots were closer to each other on the real axis of the plane. At 0.68, the closed loops poles were at the same location on the graph. Above 0.675, the poles started moving away from each other vertically. Any value for K below -0.375 resulted in an unstable system.

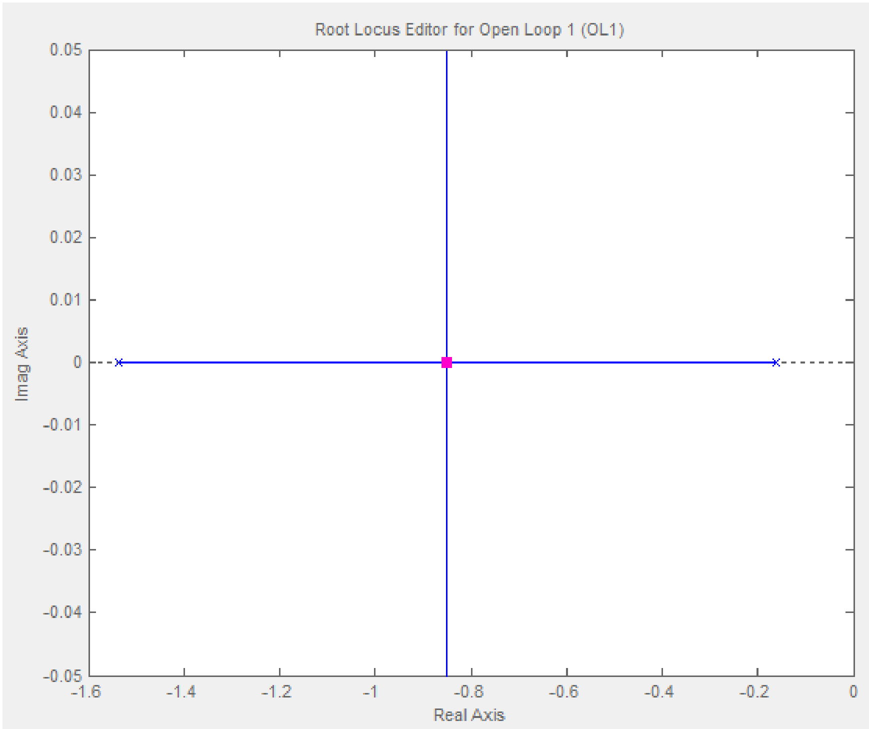
The overshoot values were very similar for both the experimental and pre-laboratory results. The steady state values for the laboratory and pre-laboratory are identical. Thus, the final value theorem is a valid method for calculating the steady state value. There was a significant error between the experimental and pre-laboratory calculations for the settling results. In the prelab the settling time was expected to remain at 4.71s for all values of k meaning that K was independent of the value of K. In the lab, the settling time was found to be inversely proportional to K.



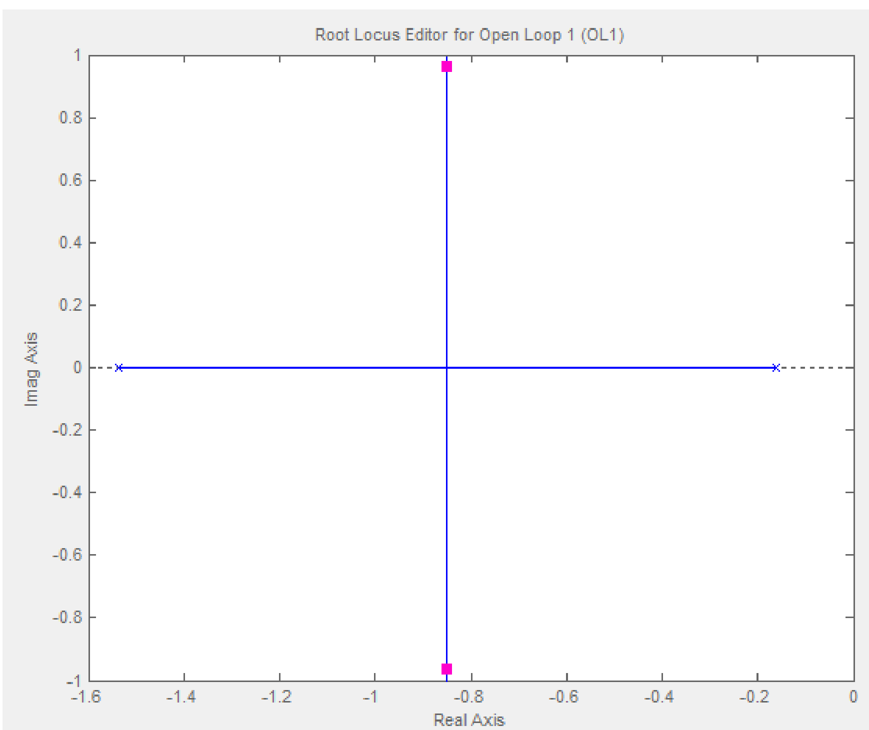

Graph 1: Root Locus of system K = 0



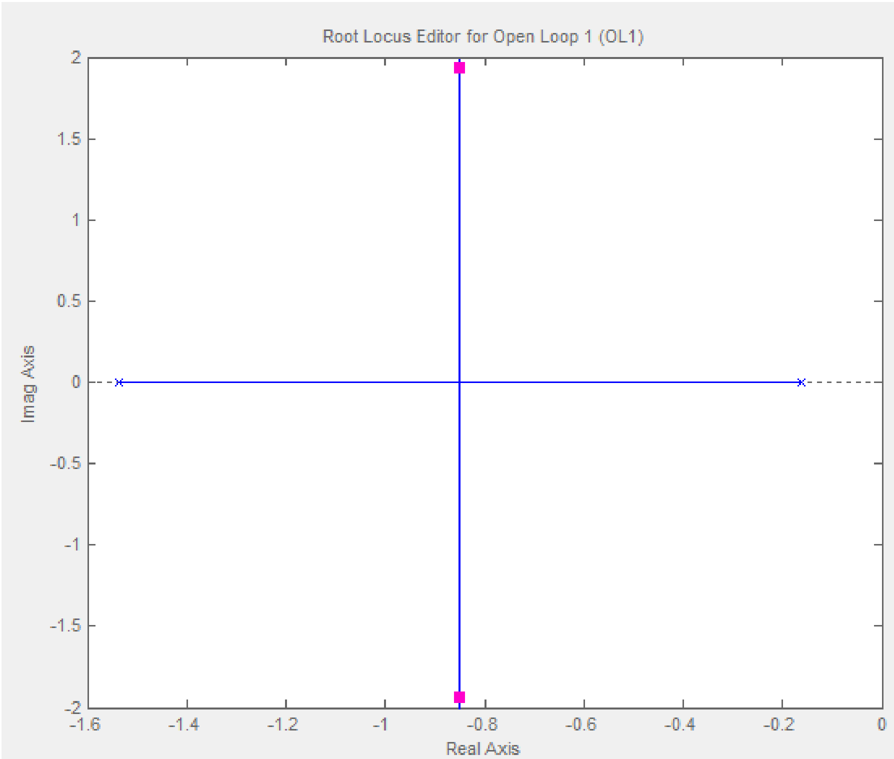

Graph 2: Root Locus of system K = 0.5



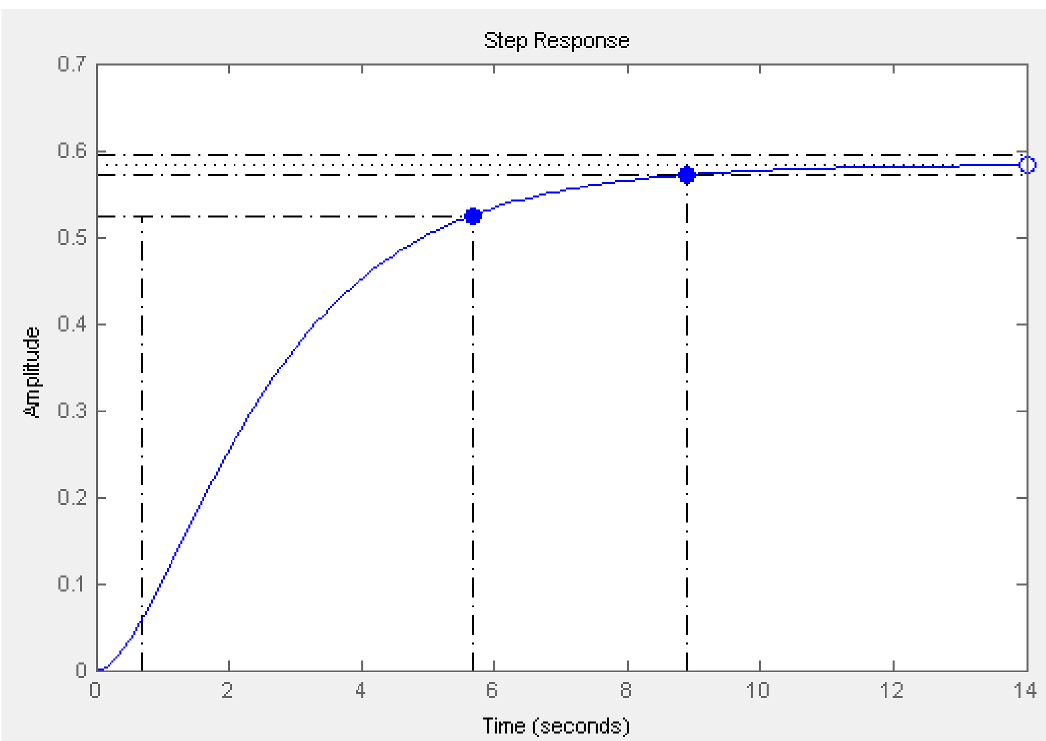

Graph 3: Root Locus of system K = 0.675



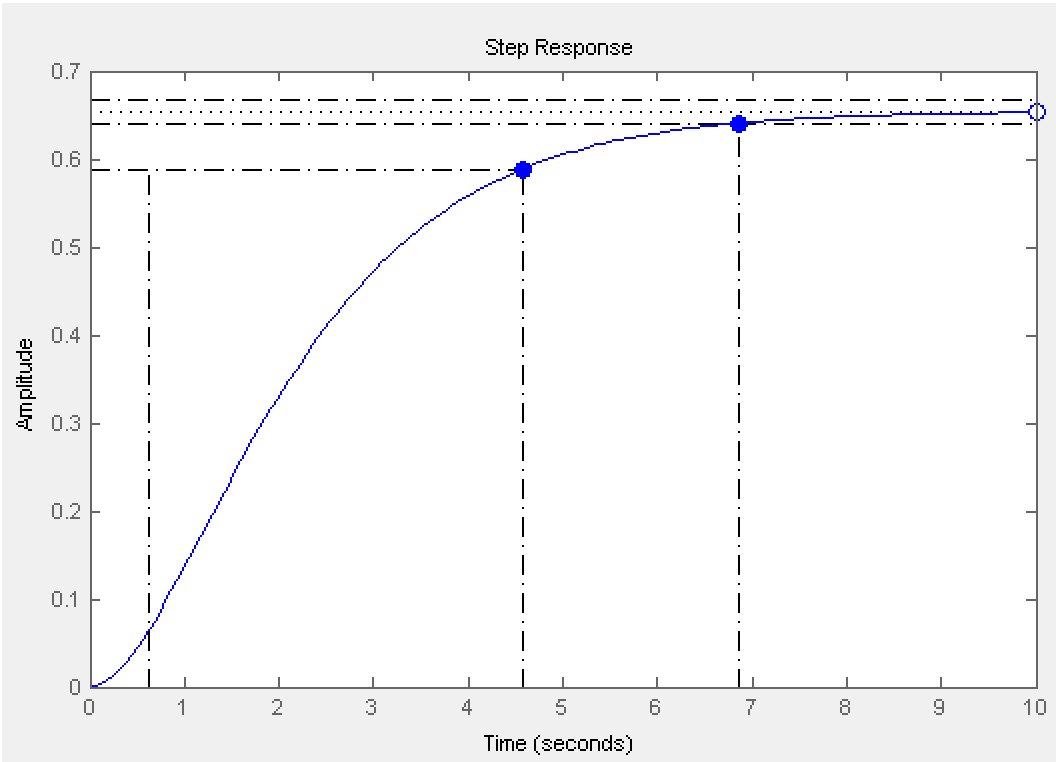

Graph 4: Root Locus of system K = 2



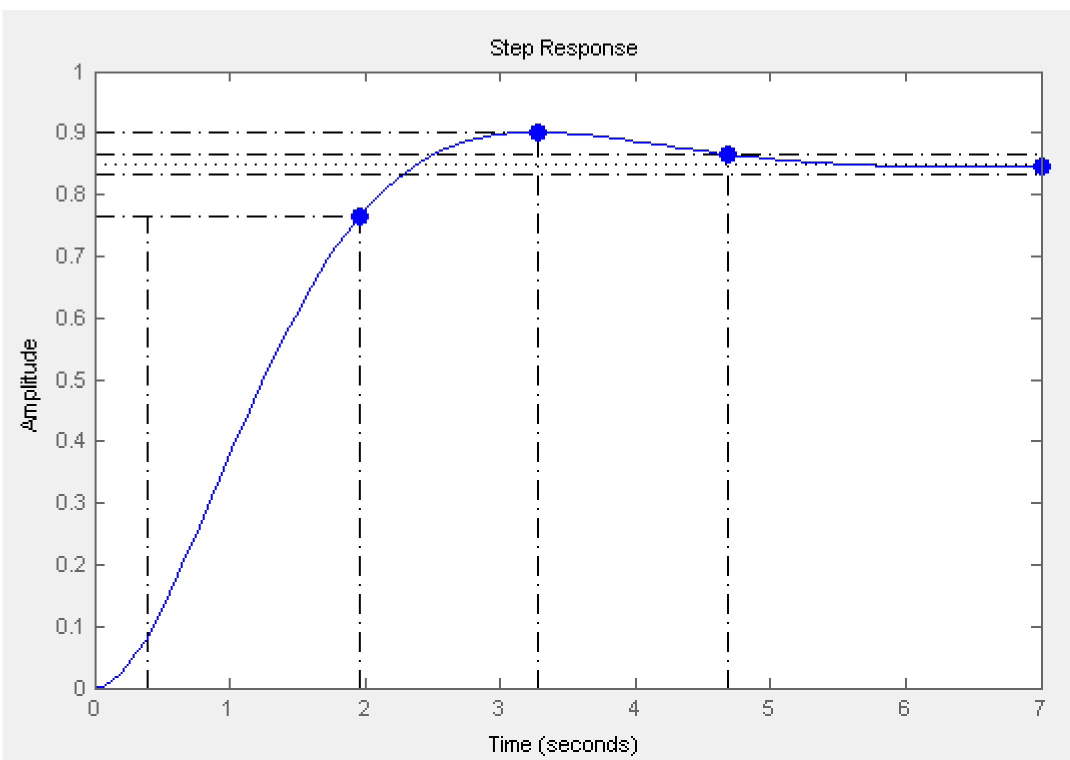

Graph 5: Root Locus of system K = 6



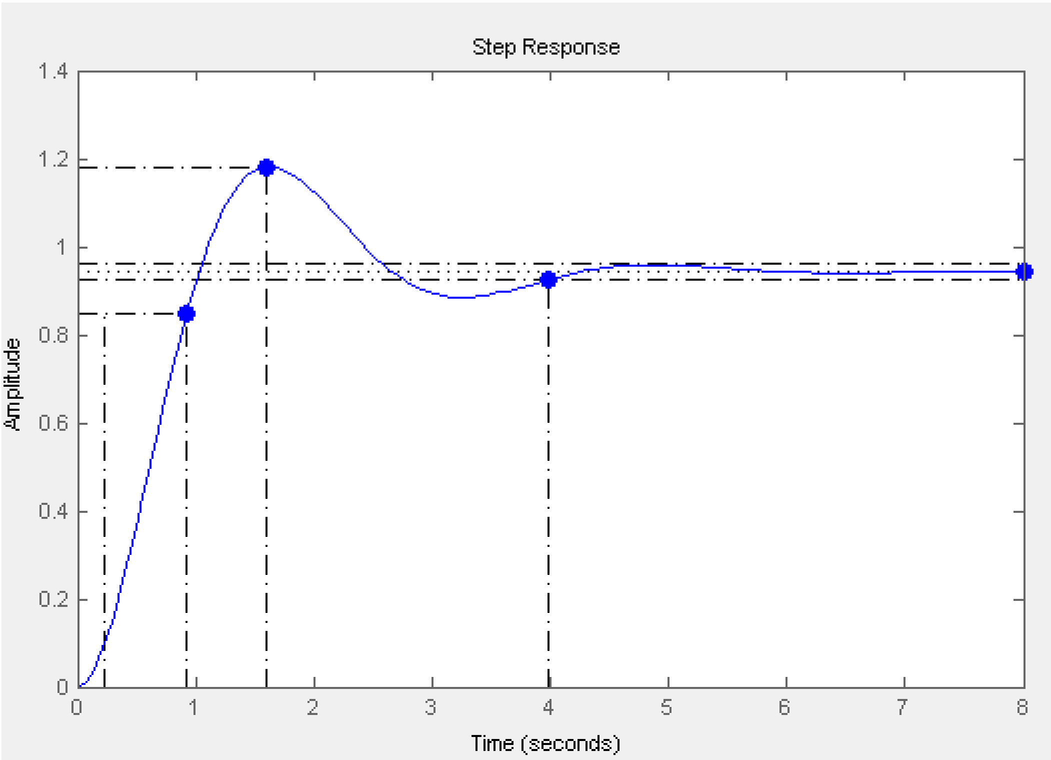

Graph 6: System response of system K = 0.5




Graph 7: System response of system K = 0.675




Graph 8: System response of system K = 2




Graph 9: System response of system K = 6

## Analyzing the behavior of a system – Part 2

The following individual components of the transfer function were updated to:

$$C(s) = K \qquad G_{av}(s) = \frac{1}{s+0.4} \qquad G_{chp}(s) = \frac{0.7}{s^2+1.7s+0.25}$$

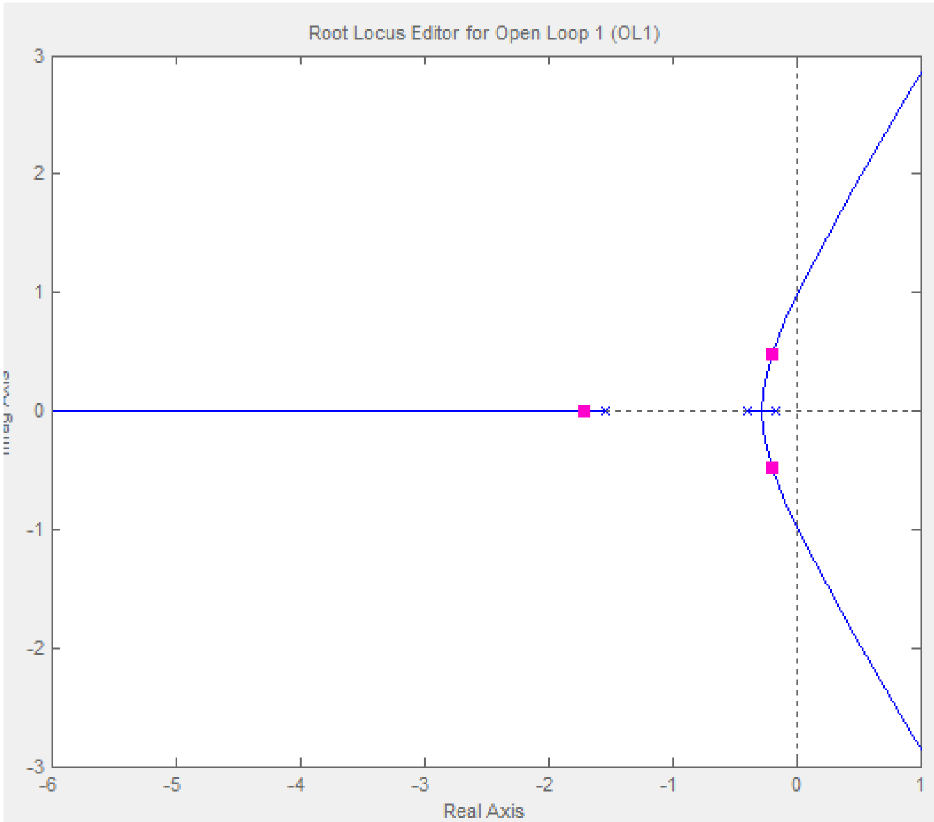
Figure 4: Transfer function values

In this section, the actuator and valve were included in the design. The following transfer function was defined in MATLAB for the Gchp transfer function:

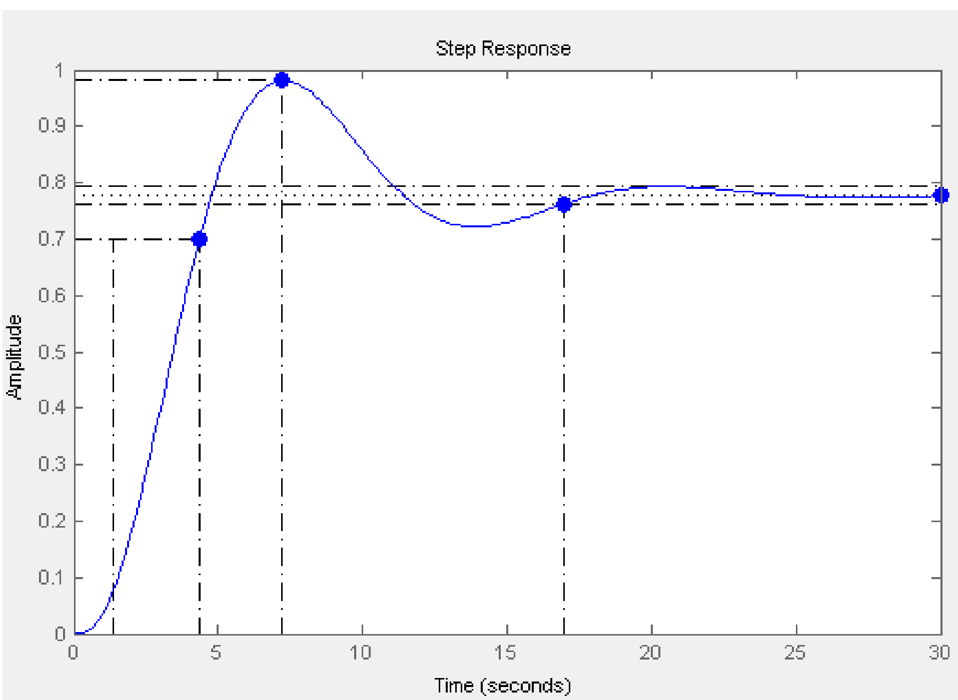
```
>> G_av = tf( [1 ], [1  0.4] )          .
>> G_chp = tf([0.7], [1  1.7  0.25])
>> G_plant = series (G av, G chp)
```

With the addition of the actuator/valve, a third pole is added to the system.  As a result, now the system can become unstable for certain values of K. This is because now the poles can be found on the positive region of the graph as well as the negative one. The root locus graph was plotted. From the graphs, it was determined that the system would remain stable for values of K less than 0.675. It was also found that as K increase, rise time and percent overshoot increases, while the steady state error decreases.

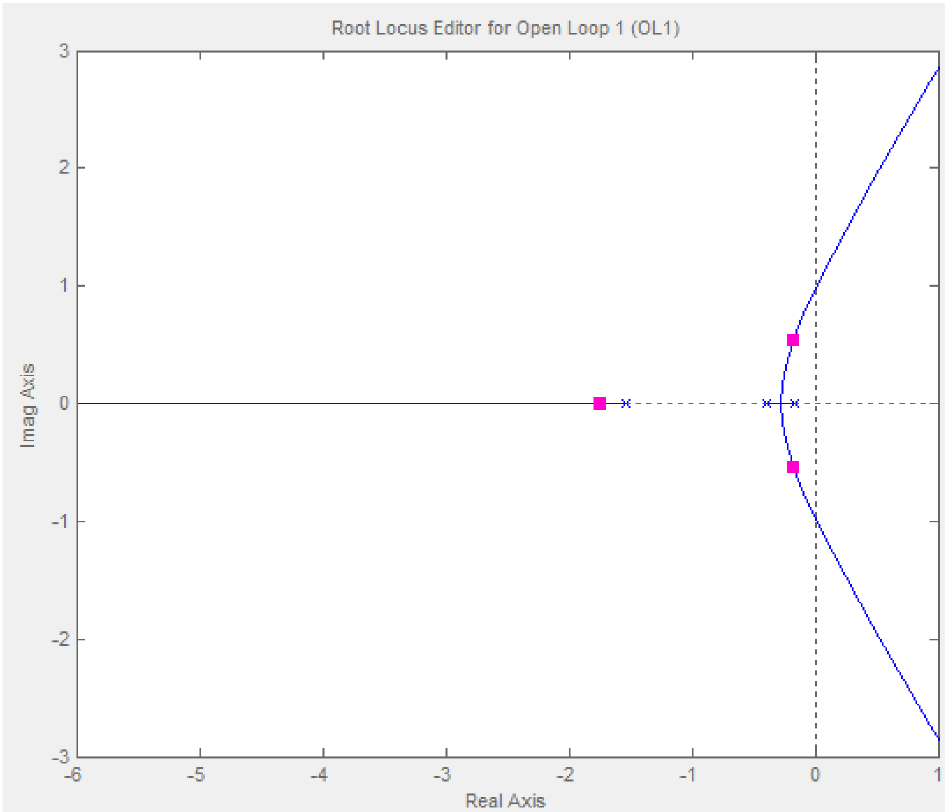
In the next section of the lab, various design constraints on the system were set. The first desired system requirement was an overshoot less than 10%. The next design requirement was a settling time of less than 12 seconds. The root locus graph was then updated with these requirements. If the poles laid in the yellow shaded region of the graph, then the requirements were not meet. The overshoot design restriction causes a sideways V to appear. The settling time requirement causes a vertical line to be plotted. For this type of controller, it was possible to satisfy the requirement of having an overshoot less than 10% as long as the value of K stayed below 0.241. However, it was impossible to have a settling time of less than 12 seconds for any value of K. In order to achieve a smaller settling time, the controller must be modified as seen in the next section of the lab.



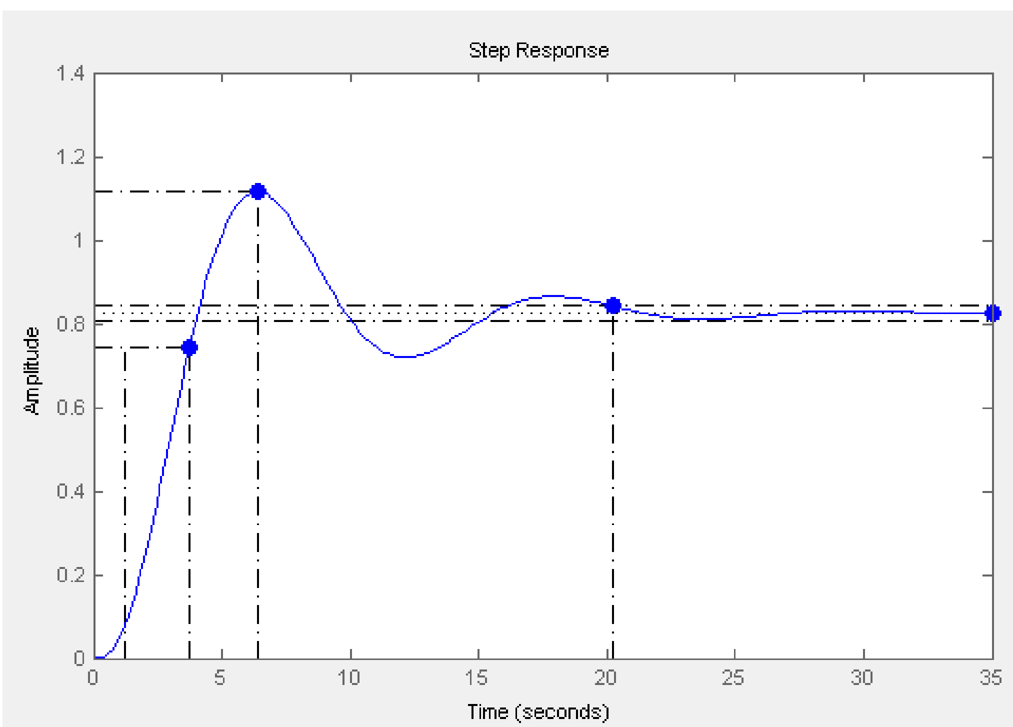

Graph 10: Root Locus of system K = 0.5



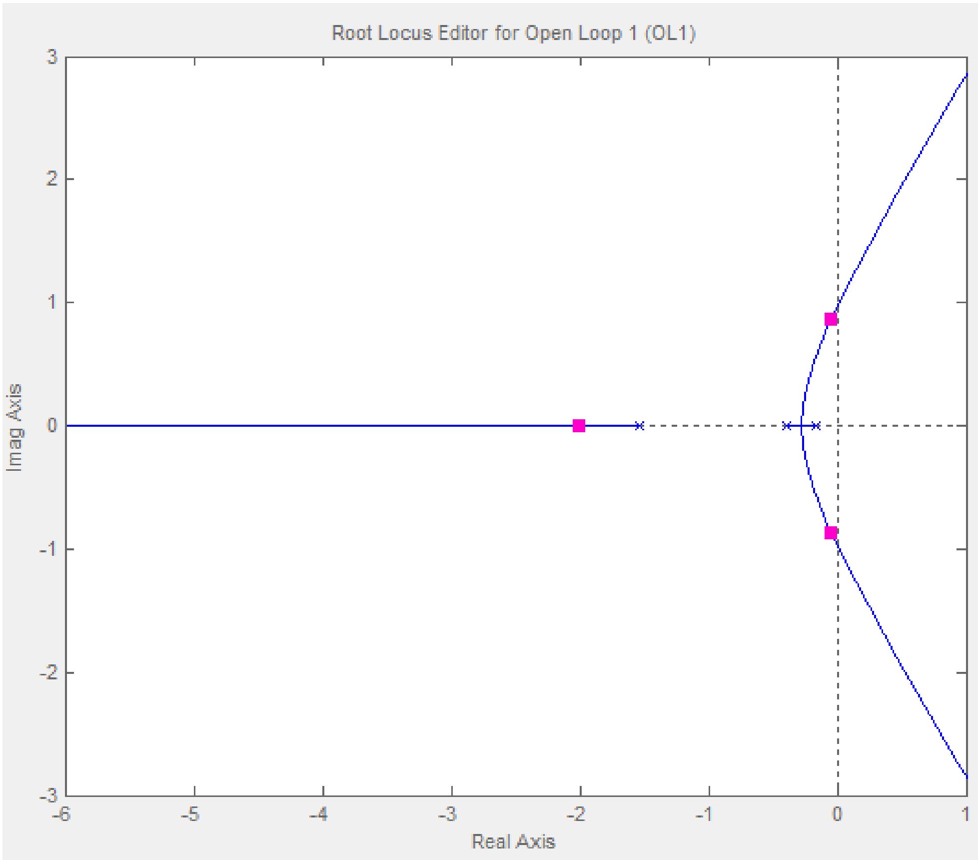

Graph 11: System response of system K = 0.5



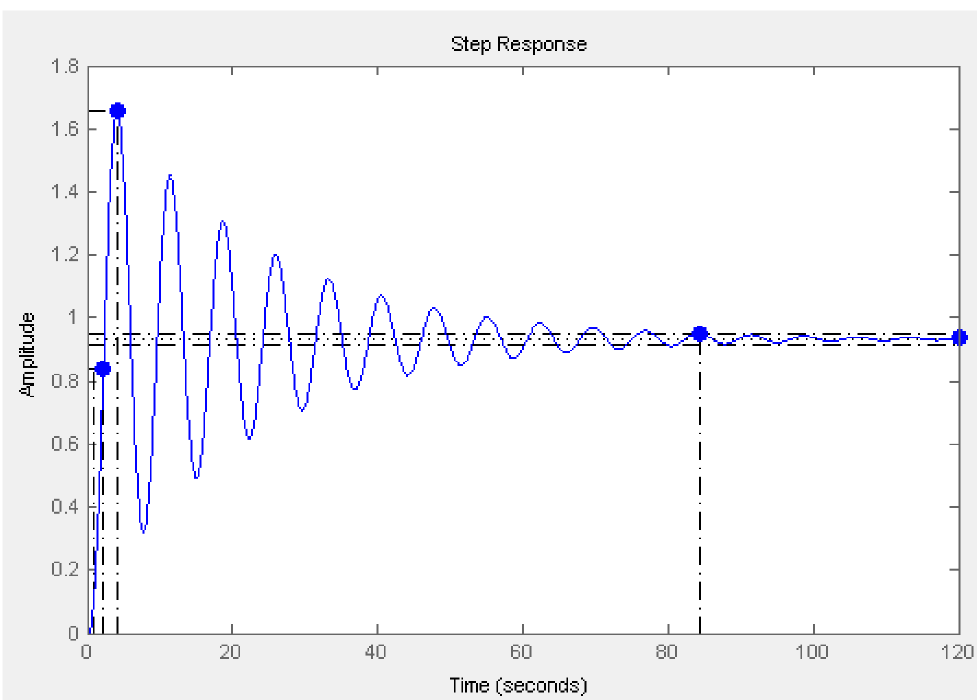

Graph 12: Root Locus of system K = 0.675



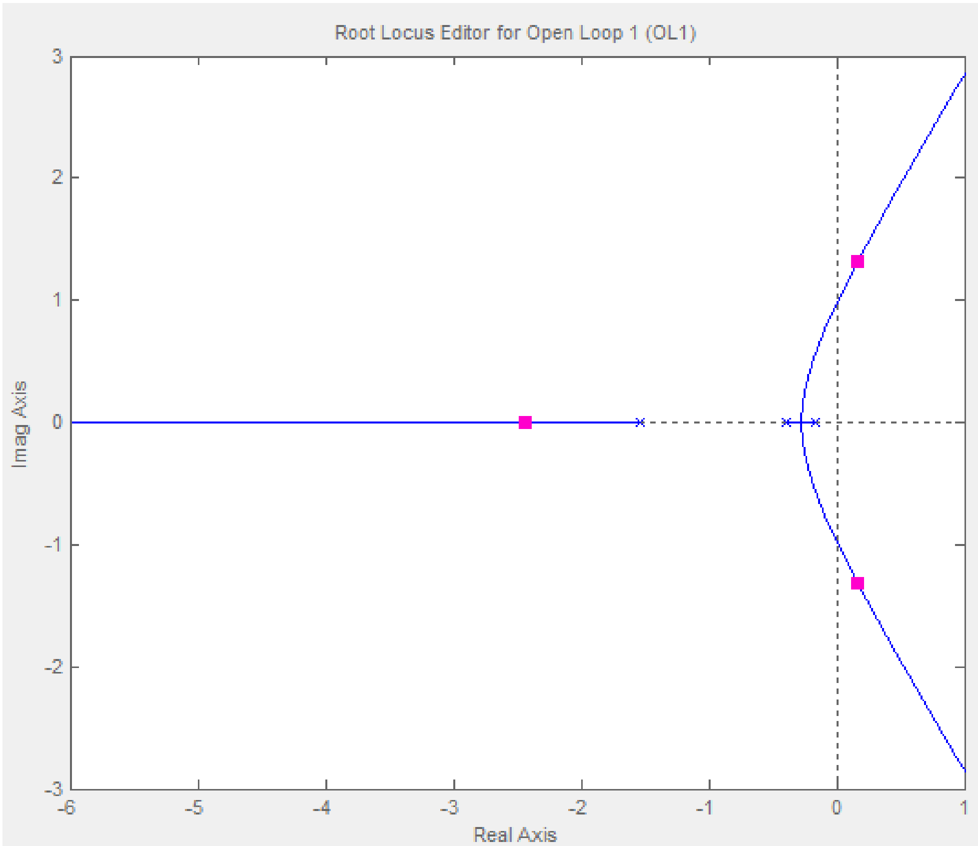

Graph 13: System response of system K = 0.675



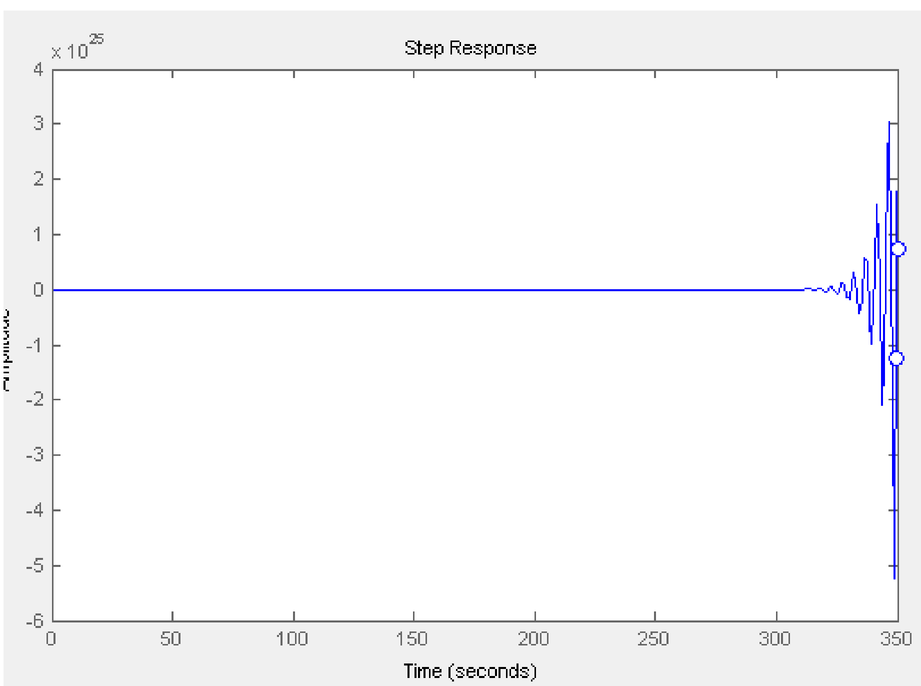

Graph 14: Root Locus of system K = 2




Graph 15: System response of system K = 2




Graph 16: Root Locus of system K = 6



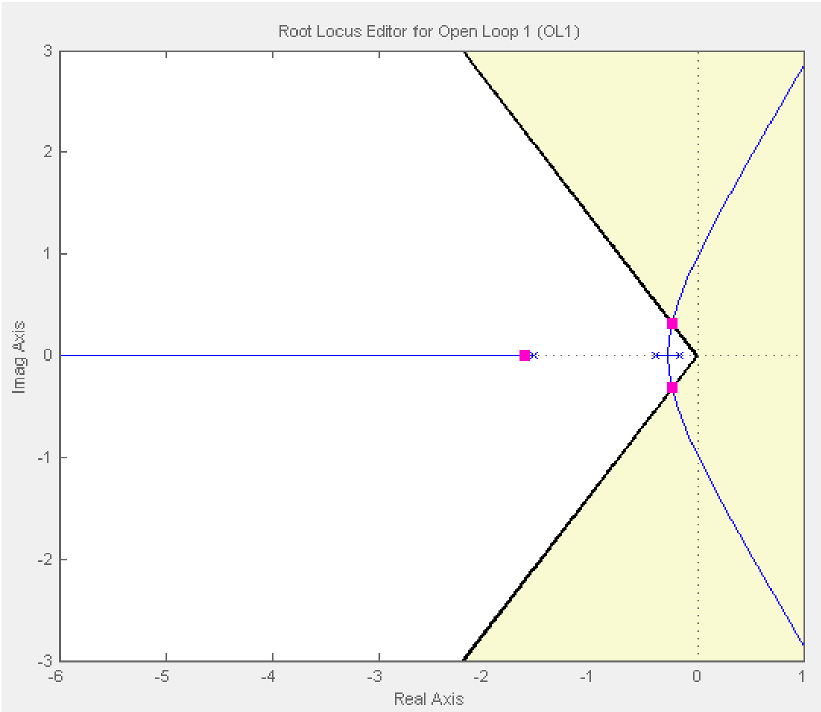

Graph 17: System response of system K = 6

## Achieving Specific System Requirements

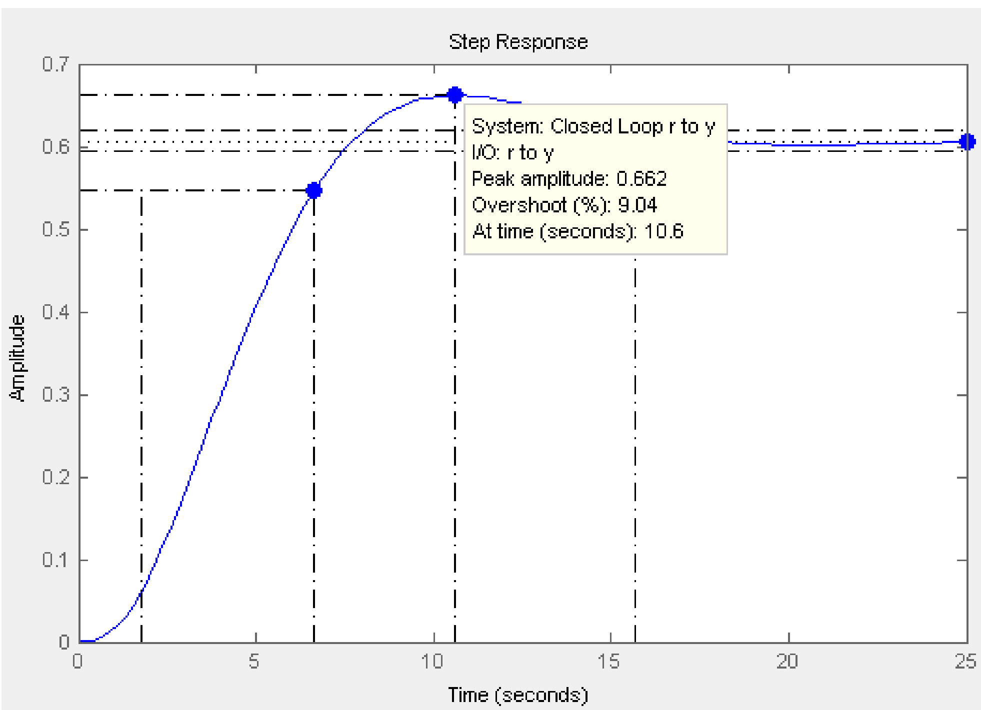
In this section, a new controller was design to achieve the design requires of a percent overshoot less than 10% and a settling time below 12 seconds. This was accomplished by using a Lead Compensator. The following lead compensator was used:

$$C(s) = \frac{s + 0.476}{s + 20}$$

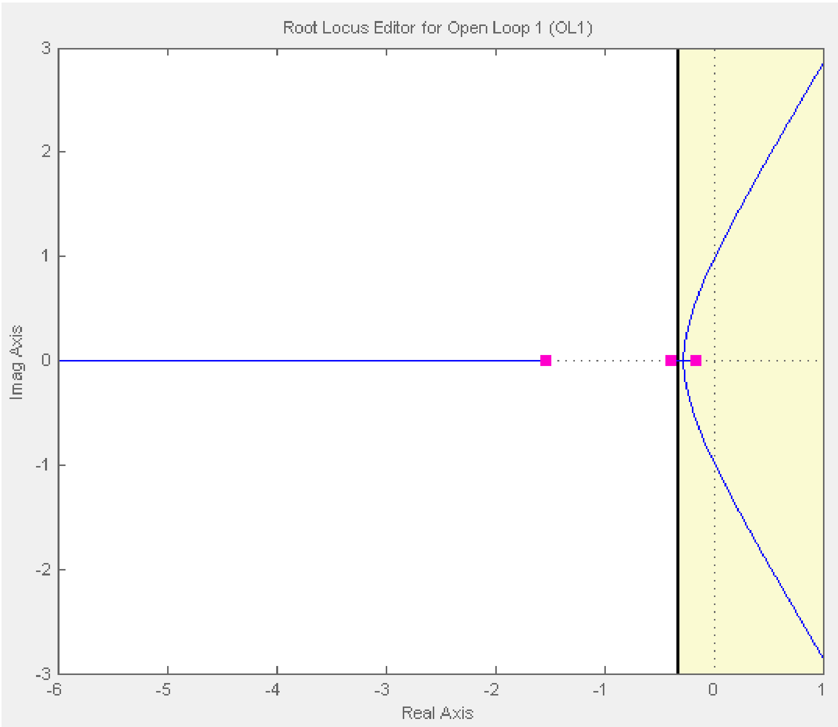
Using the above function, the range of poles shifted left on the root locus graph. As a result, this created a range of values for K that could satisfy both design requirements. The range of values for K that satisfy an overshoot less than 10% and a settling time below 12s was found to be between 0.125 and 1.04. However, it was impossible to satisfy of reducing the steady state error to less than 0.1. When the poles were moved as close together as possible while still in the white region, the steady state error was greater than 0.1. In order to remove the steady-state error, the controller must be once again redesigned as seen in the next section.



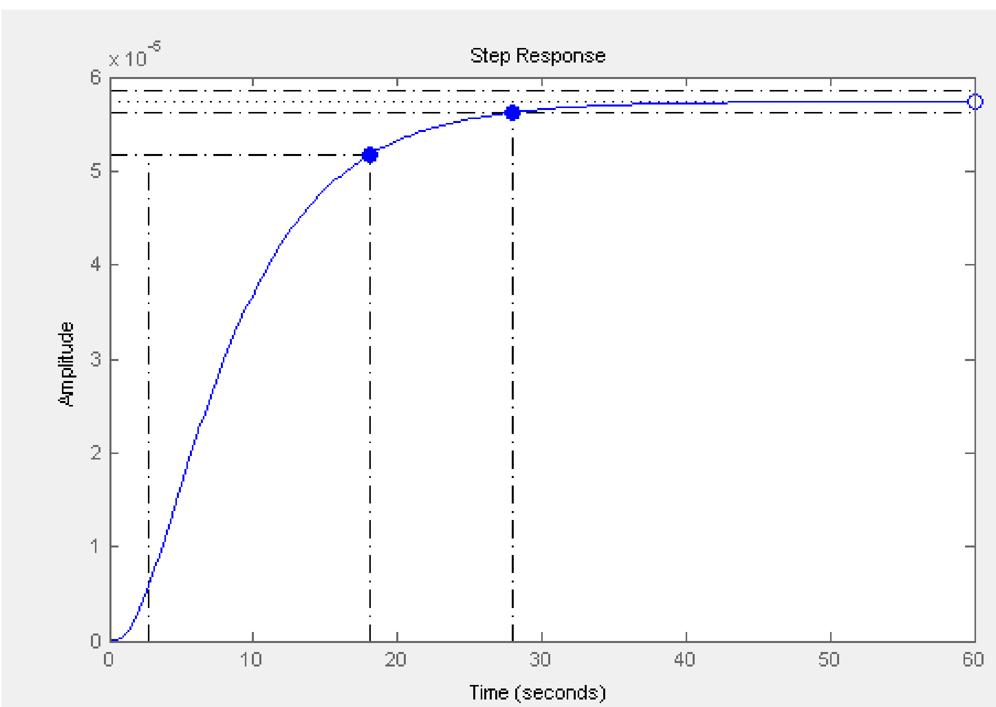

Graph 18: Root Locus of 10% overshoot



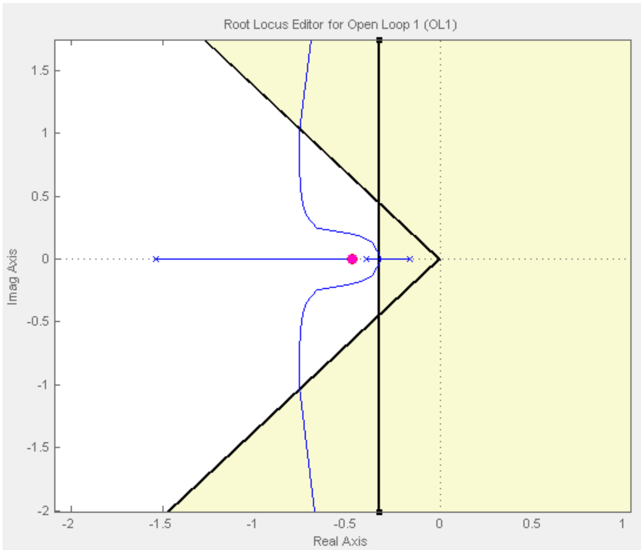

Graph 19: System response of 10% overshoot



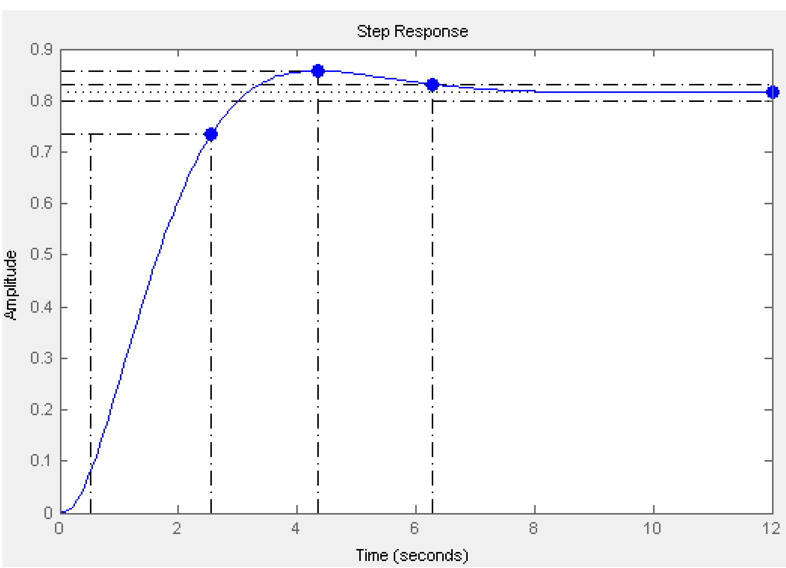

Graph 20: Root Locus of 12s TS




Graph 21: System response of 12 TS




Graph 22: Lead compensator root locus



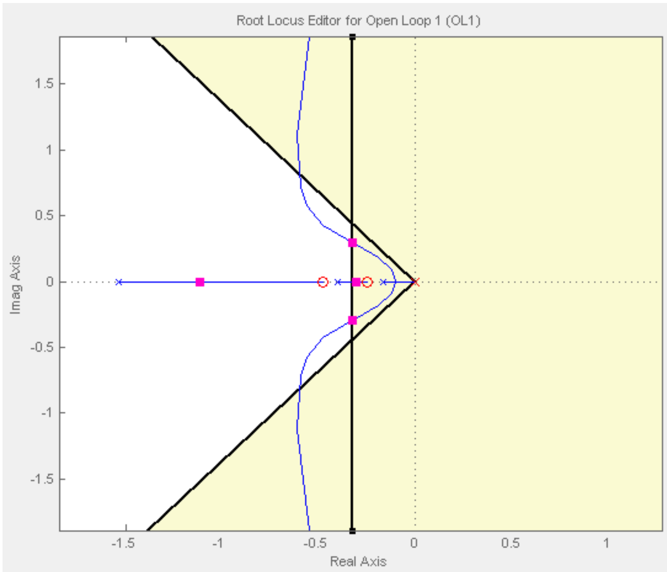

Graph 22: Lead compensator system response

## Eliminating Steady-State Error

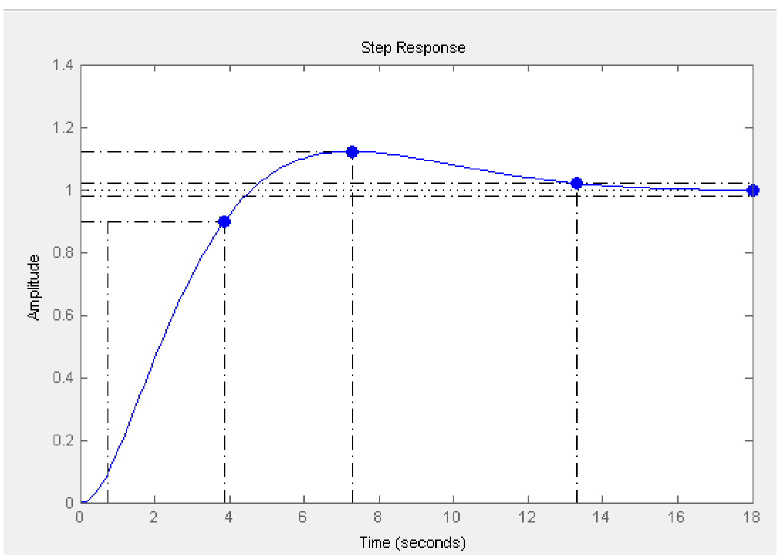
To eliminate the error from the system, an integrator controller was used. This was implemented by adding a pole at 0 and a zero at -0.25. The following integrator function was used:

$$C(s) = \frac{(s+0.25)(s+0.476)}{s(s+20)}$$

The addition of the integrator made it possible to remove the steady state error. This was verified by the pre-laboratory calculations. However, removing the steady state error caused the poles to move towards the right making it impossible to satisfy both previous requirement since it was impossible to place all four closed-loop poles within the constraints (white region). It was found that only the percent overshoot constraint could be satisfied and not a settling time less than 12 seconds. Thus, integrators are ideal for removing steady state errors but performance in other areas maybe be sacrificed.




Graph 23: integrator root locus




Graph 24: integrator system response

## Conclusion

For this lab, a SISO system was examined. Using MATLAB and the SISO Design Tool the out output of a system can be tested without physically running the system. This is ideal for large scale operations like a temperature controlled chemical process system. The purpose of the lab was to create a chemical process system and analyse the behaviour of a system, achieve specific system requirements, and eliminate the steady-state error.

In the first section, the system was analyzed to determine the accuracy of the predicted response based on the prelab calculations. The predicted responses proved to be accurate as almost all values and were within 1% of the MATLAB values. Next, the transfer function was updated to include an actuator and a valve. It was found that as K increased, the rise time would increase, the percent overshoot would increase, and the steady state error would decrease. Once the root locus was plotted, the controller was tested if it could meet certain system requirements. These requirements included an overshoot less than 10% and a settling time less than 12seconds. It was determined that the controller could achieve an require overshoot for values of K between 0.124 and 1.04.

To achieve the system requirements a lead compensator controller was added to the system. Using a lead compensator, the two previous system requirements were satisfied, the root locus was shifted to the left into the desire region. It was found that K must be above 0.124 and below 1.04 to satisfy the two achievements. Lastly, another design requirement of having a steady state error less than 0.1 was added to the system. However, this was not possible since the lowest the steady state error possible was only 0.11.

To eliminate the steady state error seen in the previous experiment, an integrator controller must be added. While the steady state error was removed with this addition, a side effect was the system unable to satisfy the previous system requirements.

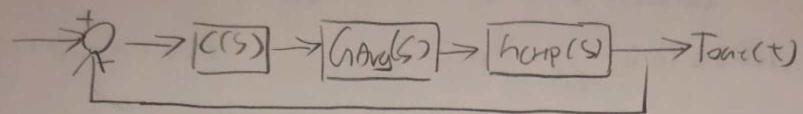
In conclusion, the lab demonstrated how MATLAB could be used to test all control systems and the ease of doing so. Certain elements of control systems such as Lead Compensators and integrators were demonstrated as to teach the use of each.

## Appendix A

S. P.

# 3330 Lab #3

1a)




$C(S)=k \quad G_{avg}(S)=1 \quad G_{CAPS}(S)=\frac{0.7}{S^2+1.7S+0.75}$

$G(S)=C(S)\,G_{avg}(S)\,G_{caps}(S)$

$G(S)=\frac{k\cdot 1\cdot 0.7}{S^2+1.7S+0.25}=\frac{0.7k}{S^2+1.7S+0.25}$

$\frac{T_{out}(S)}{T_d(S)}=\frac{0.7k}{S^2+1.7S+0.25}\cdot\left(\frac{S^2+1.7S+0.25}{S^2+1.7S+0.25+0.7k}\right)$

$=\frac{0.7k}{S^2+1.7S+0.25+1.7k}$

$\rightarrow k=0$

$\frac{T_{out}(S)}{T_d(S)}=0$

$\rightarrow k=0.5$

$\frac{T_{out}(S)}{T_d(S)}=\frac{0.35}{S^2+1.7S+0.6}\cdot\frac{(10)}{(10)}$

$=\frac{3.5}{10S^2+17S+6}=\frac{3.5}{10S^2+17S+6}=\frac{3.5}{(2S+1)(5S+6)}$

$\rightarrow S=-0.5$

$S=-1.2$

0.22

S.P. ~~Sta~~

@ $k = 0.675$

$$\frac{T_{at}(s)}{T_d(s)} = \frac{0.4725}{s^2+1.7s+0.7225}$$

$$s_{1,2} = -0.85$$

@ $k = 2$

$$\frac{T_{at}(s)}{T_d(s)} = \frac{1.4}{s^2+1.7s+1.65}$$

$$s_{1,2} = -0.85 \pm 0.983j$$

@ $k=6$ $\frac{T_{at}}{T_d(s)} = \frac{4.2}{s^2+1.7s+4.5}$

$$s_{1,2} = -0.85 \pm 1.951j$$

→ find unstable values of k using R-H

$$\begin{array}{c|cc} s^2 & 1 & 0.25+0.7k \\ s^1 & 1.7 & 0 \\ s^0 & 0.25+0.7k & 0 \end{array}$$

→ unstable when

$0.25+0.7k < 0$

$0.7k < 0.25$

$k < -0.357$

→ critically damped

$2\zeta\omega_n = 1.7$

$\omega_n^2 = 0.25+0.7k$

$\omega_n = \sqrt{0.25+0.7k}$

$$\zeta = \frac{0.85}{\sqrt{0.25+0.7k}}$$ → $\zeta=1$ for crit damped

$$A \; \frac{0.85}{\sqrt{0.25+0.7k}} \qquad k = 0.675$$

→ over damped range for k and underdamped.

$k = 0.675$ is the critically

$k < 0.675$ is overdamped

$k > 0.675$ is underdamped

→ Finding region of stability →

S.P. ~~Sto~~

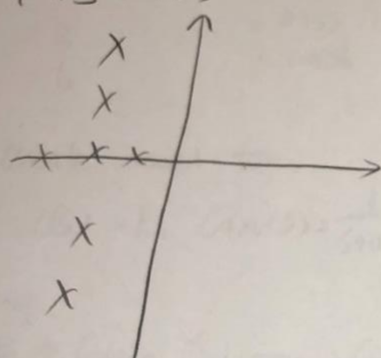
· R-H table

| | | |
|---|---|---|
| $s^2$ | 1 | $0.25+0.7k$ |
| $s^1$ | 1.7 | 0 |
| $s^0$ | $0.25+0.7k$ | 0 |

→ for stability the number of sign changes in the first column is 0 $\therefore 0.25+0.7k \geq 0$

$0.7k \geq -0.25 \rightarrow k \geq -0.357$ for the system to be stable

→ plotting locations



$a_2$) determine the values of $\zeta$ $w_n$ and $T_s$.

$$\zeta = \frac{0.85}{\sqrt{0.25+0.7k}} \quad W_n = \sqrt{0.25+0.7k} \quad \%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$$

$T_s = 4/\zeta W_n$

@ $k=0$ $\zeta=1.7$ $W_n=0.5$ $\%OS=N/A$ $T_s=4.706s$

@ $k=0.5$ $\zeta=1.097$ $W_n=0.775$ $\%OS=N/A$ $T_s=4.706s$

@ $k=0.675$ $\zeta=1$ $W_n=0.85$ $\%OS=N/A$ $T_s=4.706$

@ $k=2$ $\zeta=0.661$ $W_n=1.285$ $\%OS=$ ~~NA~~ $6.27\%$ $T_s=4.706$

@ $k=6$ $\zeta=0.403$ $W_n=2.110$ $\%OS=25.07\%$ $T_s=4.704s$

a step input using FVT

$$T_{act}(s) = \left(\frac{1}{s}\right)\left(\frac{0.7k}{s^2+1.7s+0.25+0.7k}\right)$$

$$T_{ss} = \lim_{s\to 0} s\frac{1}{s}\left(\frac{0.7k}{s^2+1.7s+0.25+0.7k}\right) = \frac{0.7k}{0.25+0.7k}$$

$$e_{ss} = 1 - T_{ss} \to e_{ss} = 1 - \frac{0.7k}{0.25+0.7k}$$

| | | |
|---|---|---|
| $k = 0$ | $e_{ss} = 1$ | |
| $= 0.5$ | $= 0.417$ | |
| $= 0.675$ | $= 0.346$ | |
| $= 2$ | $= 0.152$ | |
| $= 6$ | $= 0.056$ | |

→ By increasing $k$ the steady state error will approach to 0, due to the 0.25 term in the denominator the error will always be non-zero

b1) Create model for

$$C(s) = k, \quad G_{ov}(s) = \frac{1}{s+0.4} \quad \text{and} \quad G_{LHP} = \frac{0.7}{s^2+1.7+0.25}$$

→ R-H

$$TF \to G(s) = C(s)\,G_{av}(s)\,G_{OH}(s) = k \cdot 0.7\left(\frac{1}{s+0.4}\right)\left(\frac{1}{s^2+1.7+0.25}\right)$$

$$G(s) = \frac{0.7k}{(s+0.4)(s^2+1.7s+0.25)} \to G(s) = s^3 + 2.1s^2 + 0.93s + 0.1$$

$$\frac{T_{act}(s)}{T_{ds}(s)} = \frac{0.7k}{s^3+2.1s^2+0.93s+0.1+0.7k}$$

RH

| | | |
|---|---|---|
| $s^3$ | $1$ | $0.93$ |
| $s^2$ | $2.1$ | $0.1 + 0.7k$ |
| $s^1$ | $-0.333k + 0.882$ | $0$ |
| $s^0$ | $-2.331k^2 + 0.589k + 0.0882$ | $0$ |

S. P. [illegible]

$a_1$) $-0.333k + 0.887 > 0$

$k \le 2.64$

$a_2$) $-2.331k^2 + 0.584 + 0.887 > 0$

$-0.406 \le k \le 0.357$

$a_2$ condition satisfies $a_1$ conditions

$\therefore$ Range of stability is $-0.106 \le k \le 2.649$

$b_2$) Determine steady state and error of TF

$$T_{OL} = \frac{1}{s}\left(\frac{0.7k}{s^3+2.1s^2+0.93s+0.1+0.7k}\right)$$

$$T_{ss} = \lim_{s\to 0} s \cdot \tfrac{1}{s}\, T_{OL}(s)$$

$$= \frac{0.7k}{0.1+0.7k} \qquad e_{ss} = 1 - \frac{0.7k}{0.1+0.7k}$$

$k=0 \quad e=1 \qquad k=0.675 \quad e=0.175$

$k=0.5 \quad e=0.222 \qquad k=2 \quad e=0.067$

$k=6 \quad e=0.023$

→ As k increases the value for error approach to 0.

Q1 S.P. ~~Sto~~

$C(s) = \frac{k}{s}$ $G(s) = \frac{1}{s+0.4}$ $G_{CH}(s) = \frac{0.7}{s^2+1.7s+0.75}$

$G(s) = C(s)G(s)G_{CH}(s) = \left(\frac{k}{s}\right)\left(\frac{1}{s+0.4}\right)\left(\frac{0.7}{s^2+1.7s+0.25}\right)$

$G(s) = \frac{0.7k}{s^4+2.1s^3+0.43s^2+0.1s}$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 0.93 | 0.7k |
| $s^3$ | 2.1 | 0.1 | 0 |
| $s^2$ | 0.892 | 0.7k | 0 |
| $s^1$ | $-1.67k+0.3$ | 0 | 0 |
| $s^0$ | $k(1.169k-0.4)$ | | |

$\rightarrow$ Stable when $a_3$ & $a_4 \geq 0$

$a_3 = -1.67k + 0.1 \geq 0$

$k \leq 0.0594$

$a_4 = k(1.169k - 0.07) \geq 0$

$k \geq 0.0594$ $k \geq 0$

$\therefore$ system is stable when $k \geq 0.0594$

Q2 $T_{ss} = \lim_{s \to 0} \cancel{s}\frac{1}{\cancel{s}}\left(\frac{0.7k}{s^4+2.1s^3+0.43s^2+0.1s+0.7k}\right)$

$e_{ss} = 1 - T_{ss} = 1 - 1 = 0$

For $k=0$ $e_{ss}=0$ $k=0.67$ $e_{ss}=0$ $k=6$ $e_{ss}=0$

$k=0.5$ $e_{ss}=0$ $k=2$ $e_{ss}=0$

There is no effect on $e_{ss}$ whatever the value of k is.