Mechanical Components Design for Mechatronic Systems

Design Project Report

Design Project Team 5

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Introduction

The primary goal for the design project is to create a transmission system for a single-speed electric drill. This tool is designed to be used in heavy-duty applications such as mining with long time continuous usage.

The goal for this specific report is designing suitable gears set that meets all the performance specifications and constraints of the described electric drill.

Performance Specifications

Objectives

Based on the project descriptions, the following objectives are obtained in order to design a suitable gearbox:

- *Size*: The size of the gearbox should not exceed the dimension of the existing motor, which has an overall length of 88.9mm and a diameter of 46.831mm.
- *Weight*: Due to the fact that the drill is supposed to be operate by hand, the weight of the gearbox should be minimized.
- *Cost*: Since the drill is a commercial product, the related manufacturing expenditure should be minimized in order to keep a competitive price.
- *Efficiency*: The efficiency of the gearbox should be maximized in order to minimize energy loss during the transmission process and reduce the energy consumption.

These objectives are only applicable for the gears of drill. Different objectives for shaft, bearing, components connections, fasteners and housing will be generated in the final report.

Constraints

- *Motor*: The motor has a nominal operating voltage of 40 VDC, a torque constant of 8.474 N.mm/A and a voltage constant of 1125 rpm/V, the input pinion will be mounted on the output shaft of the motor.
- *Gears*: For this design project, the spur gear and helical gear are the only two available choices, the pressure angles are limited to be 14.5 or 20 degrees, the gear system design is required to be a two-stage reduction, all gears have to be selected from Boston gear catalog.
- *Lifetime*: The drill is designed to have a duty requirement of 8 hours a day, 5 days a week, for 50 weeks in the year, which is 3200 hours per year, the drill has a minimum warranty of 2 years, which means the drill has to last 6400 hours in operation.
- *Output*: the output torque for the drill is required to be 55 N.m, and the output speed of 575 rpm, to meet this requirement, the gear ratio will be implemented.

Assumptions

The following assumption has been given in the design specification:

- A gearing efficiency of 90%, each reduction stage has a gearing efficiency of 94.87%.
- Neglect any limits on the current the battery is able to provide
- The associated heat generated is adequately dissipated by the cooling fan mounted on the rotor
- The input pinion of the gearbox will be integral to the rotor of the electric motor

Gear Specification

Motor Analysis

Given specifications:

- *Torque constant*: $8.474 \times 10^{-3}\ N \cdot m/A$
- *Voltage constant*: $1125rpm/V$
- *No-load current*: $2.6A$
- *Armature resistance*: $0.072ohm$
- *Voltage*: $40V$
- *Back EMF constant*: $K_e = \frac{1}{volatge\ constant} = 8.89e - 4\ V/rpm$

Assumptions:

- Speed and torque have linear relationship
- Current and torque have linear relationship
- The motor is modelled as general DC Brush/Brushless motor

Mathematic Model of the DC motor:

$$V_o = (I \times R) + V_e$$

$$V_o = (I \times R) + \omega \times K_e$$

$$V_o = \left(\frac{T}{K_t} \times R\right) + \omega \times K_e$$

- No-load speed:

$$40V = (2.6A \times 0.072\Omega) + \omega \times 8.89e - 4\ V/rpm$$

$$\omega = 44789.4\ rpm$$

- Stall torque:

$$40V = I \times 0.072ohm$$

$$I = 555.6A$$

$$T_{stall} = I \times K_t = 4.708\ N \cdot m$$

The peak power occurred at half way of the speed torque graph. The speed and torque relationship of this motor is $\omega = -9513.5T + 44789.4$, At peak power, the torque is $2.354\ N \cdot m$, speed is $22395\ rpm$. The peak power can be calculated as

$$P_{peak} = T \cdot \omega = 2 \cdot \pi \cdot \left(\frac{n_{rpm}}{60}\right) \cdot T = 5520W$$

Considering the 90% gear efficiency, the peak output power is then 4968W. The required output torque is $55\ N \cdot m$ and the required output speed is $575\ rpm$. Therefore, the required output power is 3.31KW. The motor is able to product the output power. In order to archive the required power, the motor will be running at less than peak power.

The required output of the motor: $\frac{H}{gear\ effciency} = \frac{3.31kw}{90\%} = 3678W$

$$H = T \times \omega \times 0.105$$

$$3678W = T \times (-9513.5T + 44789.4) \times 0.105$$

$$T = 3.717N \cdot m\ or\ 0.99N \cdot m$$

At $T = 3.717N \cdot m$, the speed is 9428 rpm, the required gear ratio is 16.4

At $T = 0.99N \cdot m$, the speed is 35371 rpm, the required gear ratio is 61.7

The smaller gear ratio is generally more preferred and more likely to have inertia matching; therefore, motor is set to be running at $torque = 3.717N \cdot m,\ speed = 9426\,rpm.$

Gear Analysis

Gear Ratio

As recommended by American Gear Manufactures Association (AGMA), gear ratio of each pair is advised to be less than 1:8. After going through the Spur and Helical sections of the Boston gear sheet, the closet combined gear ratio is 16. This gear ratio can be achieved by using a compound gear train, a 1:4 pair followed by another 1:4 pair.

The exact gear ratio of 16.4 cannot be achieved exactly, with the gear ratio of 16 implemented, the output torque is slightly lower than required (The output torque is $53.6\,N \cdot m$, 2.5% less).

The exact torque and speed output cannot be achieved using this specified gear ratio 16 as the torque and speed curve does not cross this output point. To achieve the exact output, PWM voltage control needs to be implemented to adjust the voltage for the torque and speed curve to cross the exact point.

In the following discussion, only gearbox with 16 gear ratios is implemented, PWM voltage control is not implemented. The primary reason for allowing this slight variance in torque and speed output is that this mathematic modelling is only an approximation, the 2.5% difference introduced by the gearbox is acceptable.

Gear Interference

As instructed on the textbook, gear inference needs to be avoided at all cost. To avoid gear interference, the smallest number of teeth on pinion and largest number of teeth on the larger gear is calculated. Standard full teeth are used.

Spur gear selection

K=1

Gear ratio m=4

Normal pressure angle $\emptyset = 20$ degree

Minimal number of required teeth on pinion:

$$N_p = \frac{2k}{(1+2m)\sin^2\emptyset}\left(m + \sqrt{m^2 + (1+2m)\sin^2\emptyset}\right)$$

$$N_p = 15.4\ teeth$$

Therefore, the smallest number of teeth on the pinion needs to be at least 16 teeth to avoid interference.

Maximum number of allowed teeth on larger gear:

$$N_a = \frac{N_p^2 sin^2\emptyset - 4k^2}{4k - 2N_p sin^2\emptyset}$$

$$N_a = 101.4\ teeth$$

Therefore, the largest number of teeth on the larger gear needs to be less than 101 teeth to avoid interference with 16 teeth pinion.

The following gear sets are selected for Spur gear speed reducer:

Pair	Teeth	Pitch Diameter	Diametral Pitch	Datasheet page
1	16	0.500	32	39
1	64	2.000	32	39
2	16	1.000	16	43
2	64	4.000	16	43

Helical gear selection

Normal pressure angle $\emptyset_{\mathrm{n}} = 14.5$ degree

Helix angle $\emptyset_{\mathrm{t}} = 45$ degree

Gear ratio m=4, K=1

Tangential pressure angle:

$$\emptyset_t = tan^{-1}\frac{(tan\emptyset_n)}{cos\emptyset}$$

$$\emptyset_t = 20.1°$$

Minimal number of required teeth on pinion:

$$N_p = \frac{2kcos\psi}{(1+2m)\sin^2\emptyset}(m+\sqrt{m^2+(1+2m)\sin^2\emptyset})$$

$$N_p = 10.8\ teeth$$

Therefore, the smallest number of teeth on the pinion needs to be at least 11 teeth to avoid interference.

Maximum number of allowed teeth on larger gear:

$$N_G = \frac{N_p^2 sin^2\emptyset_t - 4k^2 cos^2\psi}{4kcos\psi - 2N_p sin^2\emptyset_t}$$

$$N_G = \frac{14.99}{-0.00357}$$

The denominator being a very small negative number is due to numerical calculation rounding, N_G in this case is approaching positive infinity.

The following gear is selected for helical speed reducer:

Pair	Teeth	Pitch Diameter	Diametral Pitch	Datasheet page
1	12	0.750	16	64
1	48	3.000	16	64
2	12	0.750	16	64
2	48	3.000	16	64

Gear Box Design

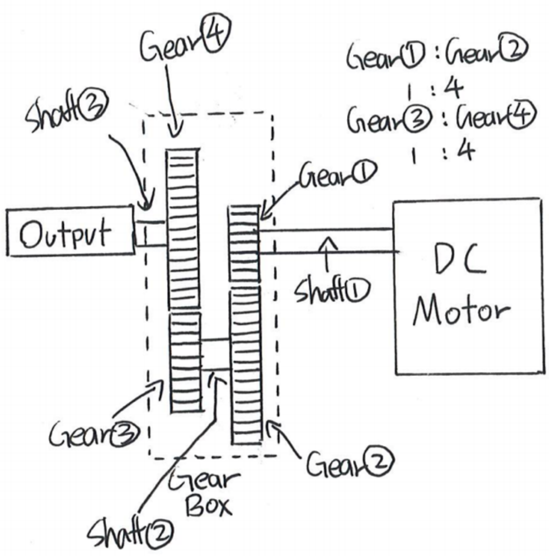
Two configurations of gear choice are listed above, the chosen configuration of gearbox design is Helical Gear. Helical gears are favored because helical gear offers smoother power transmission and more compact size than Spur gear.

Parallel configuration of helical gear is chosen against crossed configuration for its higher efficiency and reduced thrust load.

The final design of gear box is presented below:

Pair	Teeth	Pitch Diameter	Diametral Pitch	Datasheet page
1	12	0.750	16	64
1	48	3.000	16	64
2	12	0.750	16	64
2	48	3.000	16	64

Estimation of the torque within the shafts



Based on the graph above, shaft 1 will experience least torque and shaft 3 will experience most torque, the gear ratio between gear 1 and gear 2, gear 3 and gear 4 are both 1:4. Thus, the torque exerted by DC motor is the torque that shaft 1 will experience, which is 3.717N.m.

$$T_{shaft1} = T_{DCmotor} = 3.717N.m$$

As stated in assumption, with a gearing efficiency of 90%, each pair of gears will have a efficiency of 94.87%, which is the square root of 90%, the torque within shaft 2 can then be calculated as:

$$T_{shaft2} = \mu_{reduction1} * 4 * T_{shaft1} = 0.9487 * 4 * 3.717 N.m = 14.105 N.m$$

Similarly, the torque within shaft 3, which is the output torque can also calculate as:

$$T_{shaft3} = \mu_{reduction2} * 4 * T_{shaft2} = 0.9487 * 4 * 14.105 = 53.525 N.m$$

The forces on the pinions and gears and their dimensions

Based on calculations made in part 2, a gear ratio of 16.4:1 was determined, to evenly distribute this gear ratio to two reduction stages, both reduction stage implemented a 12 teeth pinion and a 48 teeth gear in order to achieve a gear ratio of 1:4. After searching the Boston Gears catalog, a pinion with 16 inch transverse diametral pitch, 12 teeth, 0.75 pitch diameter, 0.375 bore, 1/16 x 1/32 keyway, A style, right hand with a catalog number of H1612R was determined, the dimension for following 3 gears and pinion was determined in a similar way, the specific dimensions for each gears and pinions can be found in the table below:

	Transverse Diametral Pitch	Number of Teeth	Pitch Diameter	Bore	Keyway	Style	Direction	Catalog Number
Pinion 1	16	12	0.75	0.375	1/16x1/32	A	Right Hand	H1612R
Gear 2	16	48	3.0	0.5	1/8x1/16	A	Left Hand	H1648L
Pinion 3	16	12	0.75	0.375	1/16x1/32	A	Left Hand	H1612L
Gear 4	16	48	3.0	0.5	1/8x1/16	A	Right Hand	H1648R

A series of force analysis was implemented, the force on the pinions and gears was then obtained, using the formula provided on 13-40, page 705, Shigley's Mechanical Engineering *Design:*

$$W_t = \frac{T}{r}$$

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi$$

$$W = \frac{W_t}{\cos \psi \cos \phi_n}$$

Converting inches to meters:

$$0.75\ inch = 0.01905m$$

$$3.00\ inch = 0.0762m$$

The forces between pinion 1 and gear 2 can be calculated based on previous given formulas:

$$W_{t12} = \frac{T}{r} = \frac{14.105N.m}{0.009525m} = 1480.84N$$

$$W_{r12} = W_{t12} \tan \phi_t = 1480.84 * \tan 20.09 = 541.62N$$

$$W_{a12} = W_{t12} \tan \psi = 1480.84 * \tan 45 = 1480.84N$$

$$W = \frac{W_t}{\cos \psi \cos \phi_n} = \frac{1480.84}{\cos 14.5 \cos 45} = 2163.12N$$

The forces between pinion 3 and gear 4 can be calculated based on previous given formulas:

$$W_{t12} = \frac{T}{r} = \frac{53.525N.m}{0.0381m} = 1404.86N$$

$$W_{r12} = W_{t12} \tan \phi_t = 1404.86 * \tan 20.09 = 513.83N$$

$$W_{a12} = W_{t12} \tan \psi = 1404.86 * \tan 45 = 1404.86N$$

$$W = \frac{W_t}{\cos \psi \cos \phi_n} = \frac{1404.86}{\cos 14.5 \cos 45} = 2052.14N$$

Stress Analysis

As this stage of the report, the size, torque, and other necessary information on performing analysis of the gears are obtained. In order to analyze the gears, the bending stress, contact stress, Safety Factor and Wear Safety factor needs to be calculated. Using the provided equation in the Shigley's Mechanical Engineering design, the following equation is obtained in order to calculate the bending stress in the gear:

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

- W^t is the tangential transmitted load. The value of the load is provided as seen in the previous sections.
- K_O is the overload factor of the gears. It can be found using the following table:

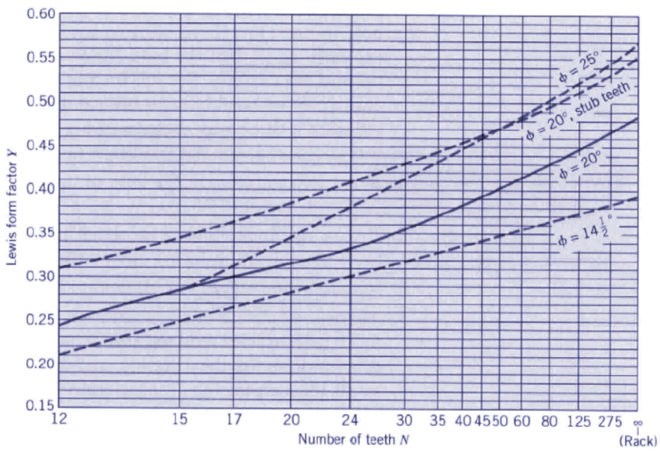
Impact from Prime Mover	Impact from Load Side of Machine		
	Uniform Load	Medium Impact Load	Heavy Impact Load
Uniform Load (Motor, Turbine, Hydraulic Motor)	1.0	1.25	1.75
Light Impact Load (Multicylinder Engine)	1.25	1.5	2.0
Medium Impact Load (Single Cylinder Engine)	1.5	1.75	2.25

In the Boston Gear Catalog, the following table is also provided for reference for the overload factor. (pg.66)

Service Factor	Operating Conditions
.8	Uniform – not more than 15 minutes in 2 hours.
1.0	Moderate Shock – not more than 15 minutes in 2 hours. Uniform – not more than 10 hours per day.
1.25	Moderate Shock – not more than 10 hours per day. Uniform – more than 10 hours per day.
1.50	Heavy Shock – not more than 15 minutes in 2 hours. Moderate Shock – more than 10 hours per day.
1.75	Heavy Shock – not more than 10 hours per day.
2.0	Heavy Shock – more than 10 hours per day.

With a moderate shock and a uniform load, we can assume that the overload factor should be 1.25.

- K_v , the Dynamic Factor of the gear is obtained as by assuming a quality number Q_V of 5 as the gears are commercial-quality. Using the provided equations, the dynamic factors are obtained,
- K_s The Size factor is obtained using the diametral pitch, face width and Lewis form factor. As the normal pressure angle in our used gear is 14.5 degres, the following table for the Lewis Factor is used:



https://www.engineersedge.com/gears/lewis-factor.htm

- K_m if the load distribution factor. Our gear is uncrowned, so $C_{mc} = 1$. With $S1/S<0.173$, $C_{pm}=1$. C_{ma} is calculated with the condition of commercial and enclosed units. And C_e is neither for gearing adjusted at assembly or compatibility improved by lapping, so $c_e=0.8$. Using this the provided value, K_m can be solved.
- The Rim-thickness factor is 1 as m_B is greater than 1.2 .
- Although it would be difficult to find a datasheet for our gear, using this table: https://eclass.upatras.gr/modules/document/file.php/MECH1178/18.%20Μετωπικοί%20οδοντωτοί%20τροχοί/AGMA-908-B89.pdf , the approximate value for the

bending-strength geometry factor is 0.364.

- Since we are using steel for the gear and pinion, the elastic coefficient is 2300 as in described in the book.
- The surface condition factor C_f is 1 for our gear as the gears are steel-heardened and polished.
- The geometry Factor I of the gear is obtained through the described equation in the Shigley's.

Using the numbers above, the bending stress and contact stress of the gears can be calculated.

Next, the factor of safety is calculated. The factor of safety of tooth bending is given in the book.

- The formula of allowable bending stress is given in figure 14-2 in Shigley's. Assuming a Brinell hardness of 321, the allowable bending stress is obtained as 49,142.
- Stress Cycle Factors Y_N and Z_N can be obtained through Figure 14-14 and Figure 14-15 by using the calculated value of cycles.
- The reliability factor is assumed to be 1 with reliability of 99%
- The temperature factor is unity as the operational temperature would be below 120 degree Celsius.

Hardness-Ratio Factor C_H is 1 as the gear use same material and would have a ratio for pinion and gear Brinell hardness below 1.2.

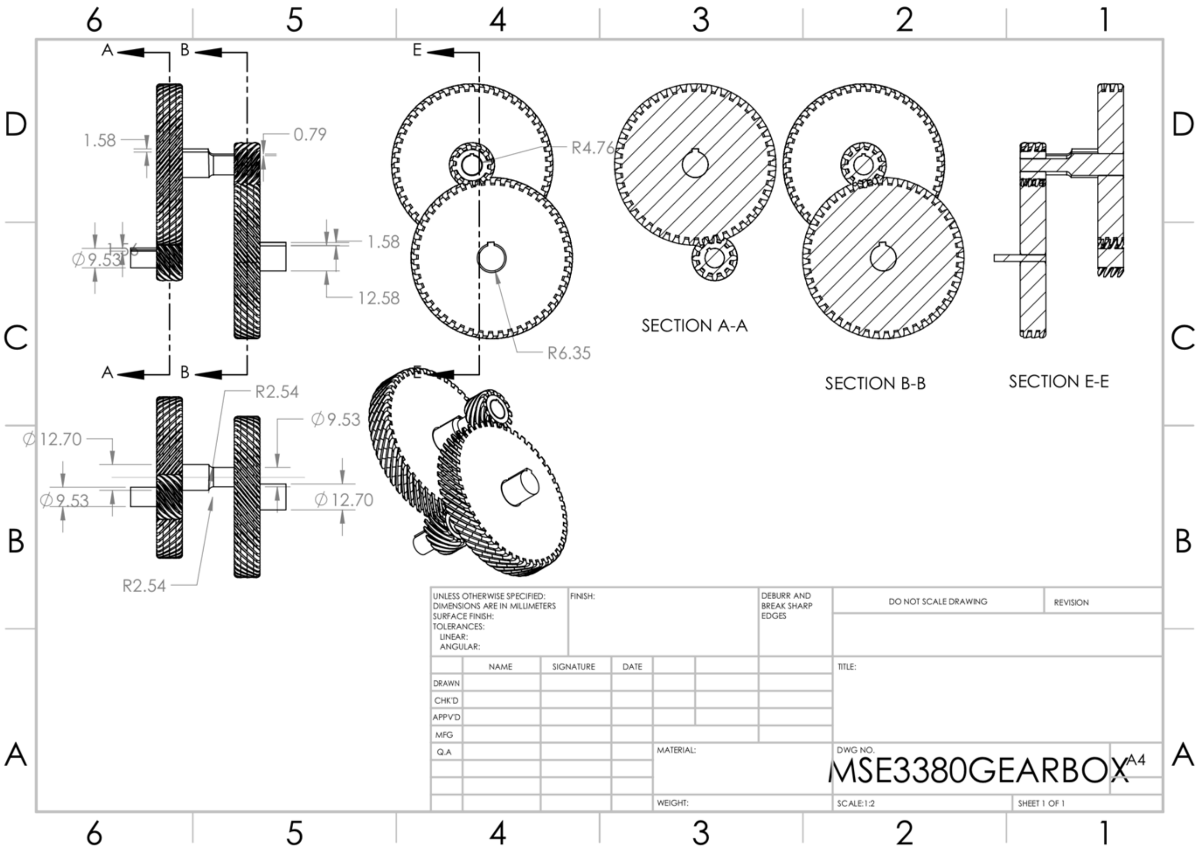
Bending Stress	42209	40015	34607	33254
Contact Stress	167743	94295	151888	85960
Bending FOS	1.0	1.10	1.27	1.40
Contact FOS	0.75	1.38	1.1	1.55

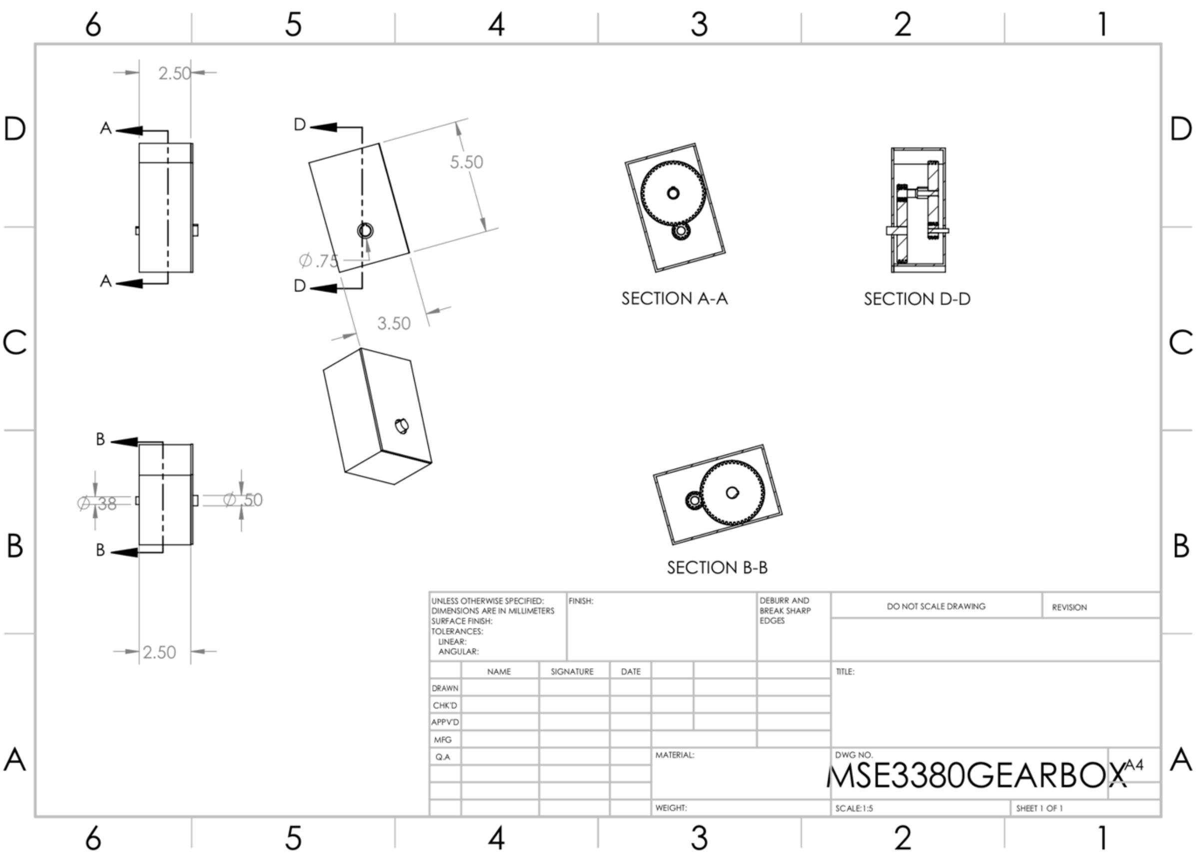
Stress calculation is presented in both Matlab and Excel files. Excel file include all 4 gears calculations, Matlab file includes gear 1 and 2 calculations.

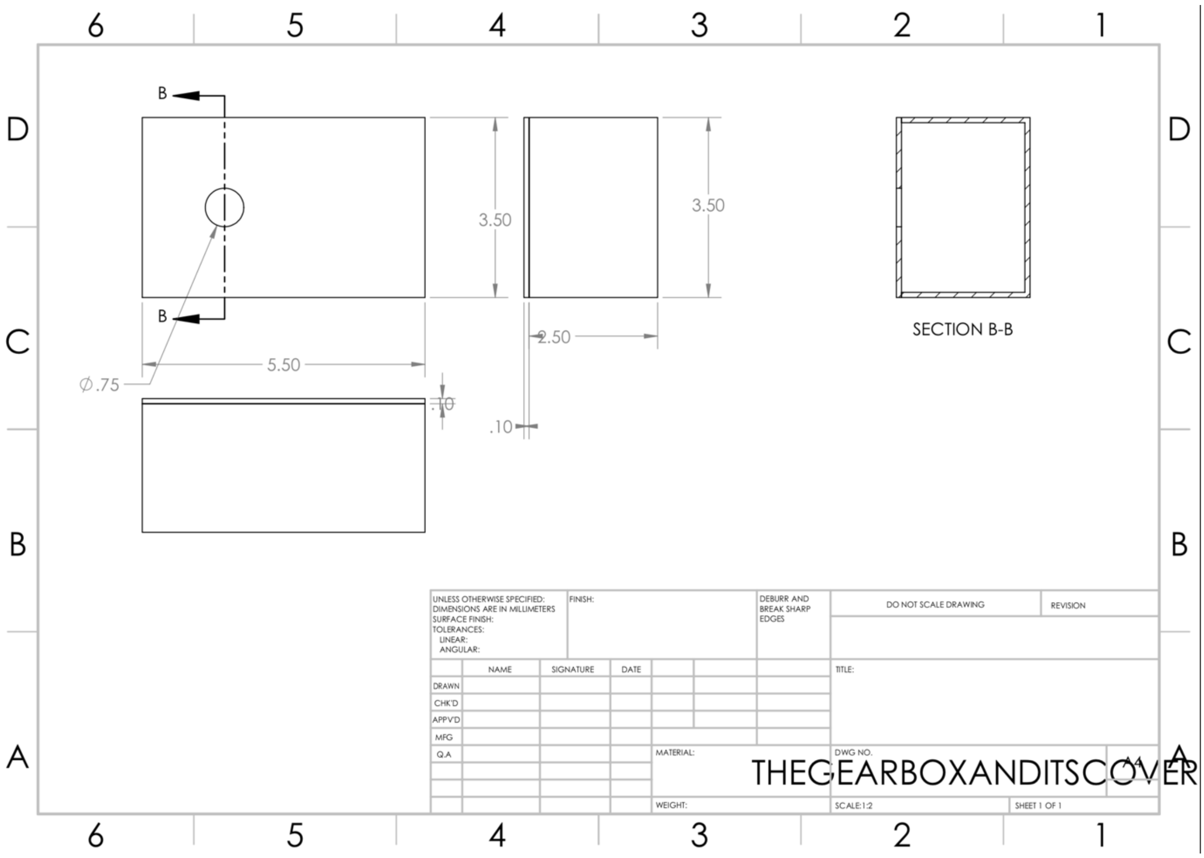
Discussion

Based on the stress calculation, the pinion gear attached on the motor shaft cannot withstand the contact stress with the required duty cycle. All other gears can fully meet the requirements. The failing pinion gear is iterated to meet the requirement. The failing pinion gear and its pair is changed to 8 transverse diametral pitch gear. Everything else remain. After the iteration, the pinion gear installed on the motor shaft can fully meet the requirement.

Appendix A- 2D Assembly Drawing For Gear, Box and Gearbox







Citation

ASM Aerospace Specification Metals Inc., "AISI 4340 Steel, normalized, 100 mm (4 in.) round", [Online]. Available: http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=m434ae. [Accessed March 21, 2019]

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