

# Problem Set 8

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## Problem 1

- (a) The piano and trumpet waveforms are shown in Figures 1 and 2. The results of FFT of these waveforms are shown in Figure 3 (piano) and Figure 4 (trumpet), where the first 10000 coefficients are plotted. From Figures 3 and 4, we can see that the note played by the piano is much “purer” in the sense that its waveform is dominated by a single sinusoidal component while the other components do not contribute much, whereas the note played by the trumpet has multiple harmonics dominating the waveform, each contributing a good amount to the sound. In short, the piano note has weaker overtones than the trumpet note.
- (b) In Figures 3 and 4, the first peak occurs approximately at the 1190-th coefficient, which should correspond to the fundamental frequency. With a sampling rate of 44100 samples per second, or 44100 Hz, and 100000 samples, the frequency that this coefficient corresponds to is

$$1190 \times \frac{44100 \text{ Hz}}{100000} = 525 \text{ Hz}.$$

Given that the frequency of middle C is 261 Hz, this (fundamental) frequency of 525 Hz very likely represents the high C note (an octave higher than middle C) since it is roughly double the former. Thus, the note played by the instruments is high C (or C5).

## Problem 2

- (a) I used `scipy.integrate.solve_ivp` to solve the given set of equations. The plot of  $y(t)$  is shown in Figure 5. The motion is indeed chaotic and unpredictable.
- (b) A plot of  $z(t)$  versus  $x(t)$  is shown in Figure 6. I indeed see a “strange attractor” of the Lorenz equations in the shape of a lop-sided butterfly.

My Github link: <https://github.com/ziqui-wang/phys-ua210>.

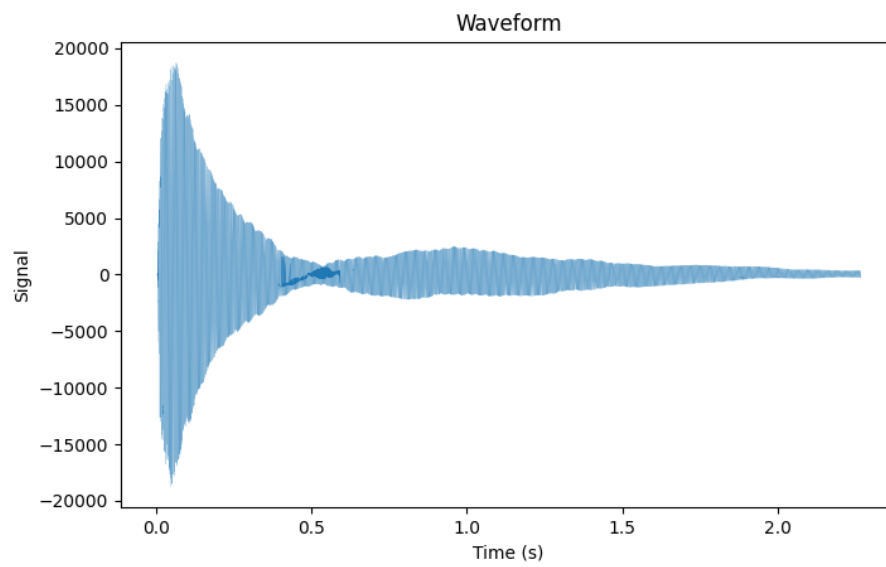


Figure 1: Waveform of the note played by the piano.

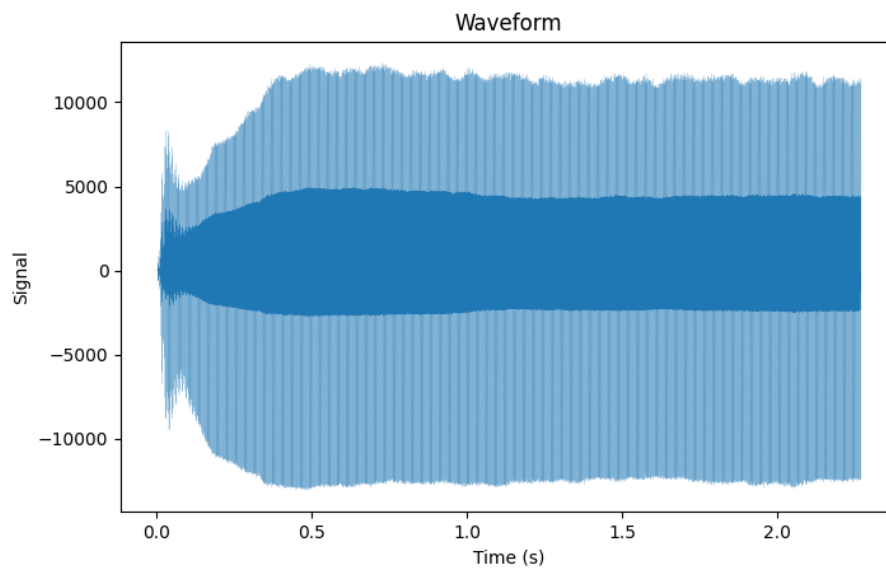


Figure 2: Waveform of the note played by the trumpet.

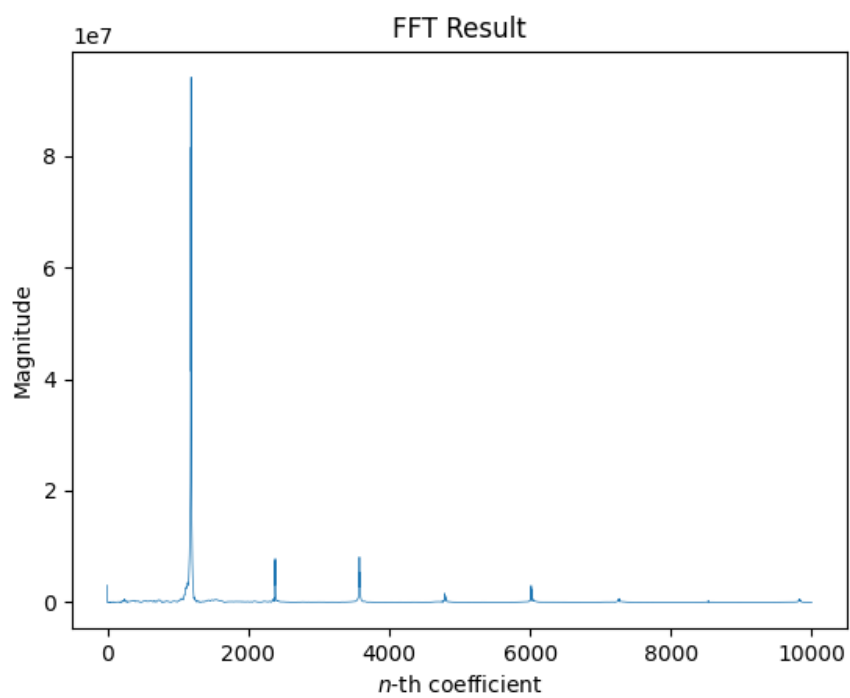


Figure 3: FFT of the piano waveform.

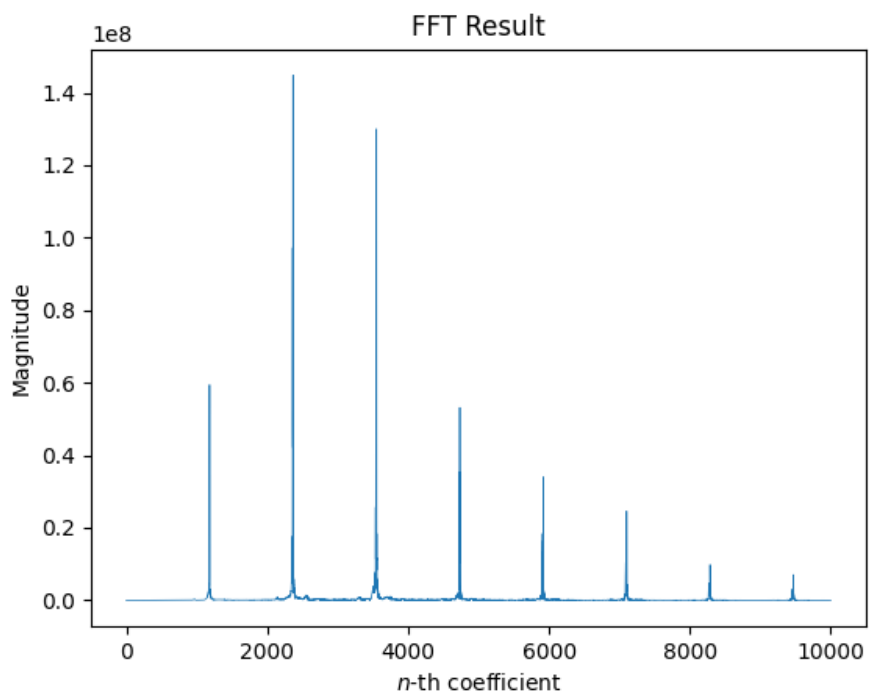


Figure 4: FFT of the trumpet waveform.

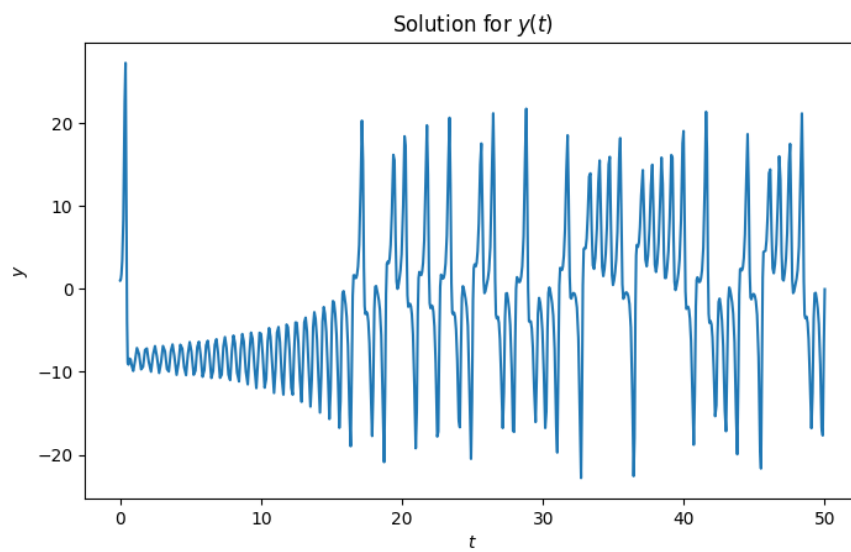


Figure 5: Chaotic motion:  $y(t)$ .

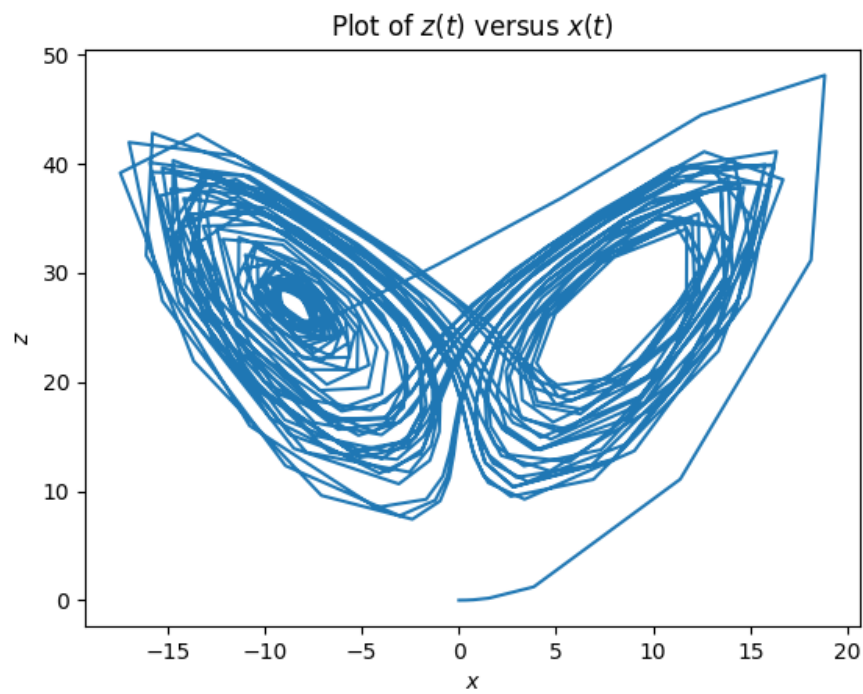


Figure 6: Plot of  $z(t)$  versus  $x(t)$ .