Problem Set 3

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Problem 1

(a) With $\delta = 10^{-2}$, my program gives 1.010000000000001. The accurate answer, calculated by evaluating the derivative at $x = 1$, is 1. Therefore, there is a 1% error in the answer given by my program. This is reasonable because to calculate the derivative accurately, I would in theory need to make δ infinitely small (taking the limit as δ goes to 0), but here the value of δ I used is much larger than 0.

(b) The absolute errors versus δ are plotted on a log-log plot in Figure 1. It can be seen that the accuracy first gets better as δ gets smaller; this is because a smaller δ is closer to 0, and thus our "linear approximation" to the function is more accurate. However, as δ gets smaller than 10^{-8}, the accuracy of my program begins to get worse. The reason for this drop in accuracy is that the level of precision in Python is only 16 significant digits (Newman 128). When my program computes $f(x+\delta) = (x+\delta) * (x+\delta-1)$ for $\delta < 10^{-8}$, the decimal part of the result has more than 16 significant figures, thus giving rise to rounding errors. The effect of these errors gets larger as δ gets even smaller (since the result requires more significant digits to be expressed accurately), so the accuracy of my program continues to drop.

Problem 2

For both the for-loop method and the "dot" method, the computation time versus N^3 graphs are shown in Figure 2 (N runs from 10 to 290 with a step size of 20). The curve for the for-loop method looks just like a straight line with a constant positive slope, thus suggesting that the computation time indeed increases as N^3. However, the computation time of the "dot" method remains very small and only increases by a little as N gets larger (see Figure 3). Therefore, the "dot" method is much faster than using for-loops.

Problem 3

The curves for this problem are shown in Figure 4. The shapes of the curves make sense: the number of ^{213}Bi atoms decreases exponentially while the number of ^{209}Bi atoms increases with time, rapidly at first but more and more slowly as time increases. The number of Pb atoms first reaches a small maximum but then slowly drops down, corresponding not only to the initial rapid and likely decay of ^{213}Bi atoms to Pb, but also to the decrease in the decay rate of ^{213}Bi atoms as time advances. Lastly, the number of Tl atoms stays quite small for the entire time, which is consistent with the fact that the probability that a ^{213}Bi atom decays to Tl is very small.

Problem 4

For this problem, I used Equation (10.10) with $\mu = \ln 2/\tau$ (the transformation method). The plot of number of atoms that have not decayed versus time is shown in Figure 5. The curve resembles an exponential decay as expected.

My Github link: https://github.com/ziqiu-wang/phys-ua210.

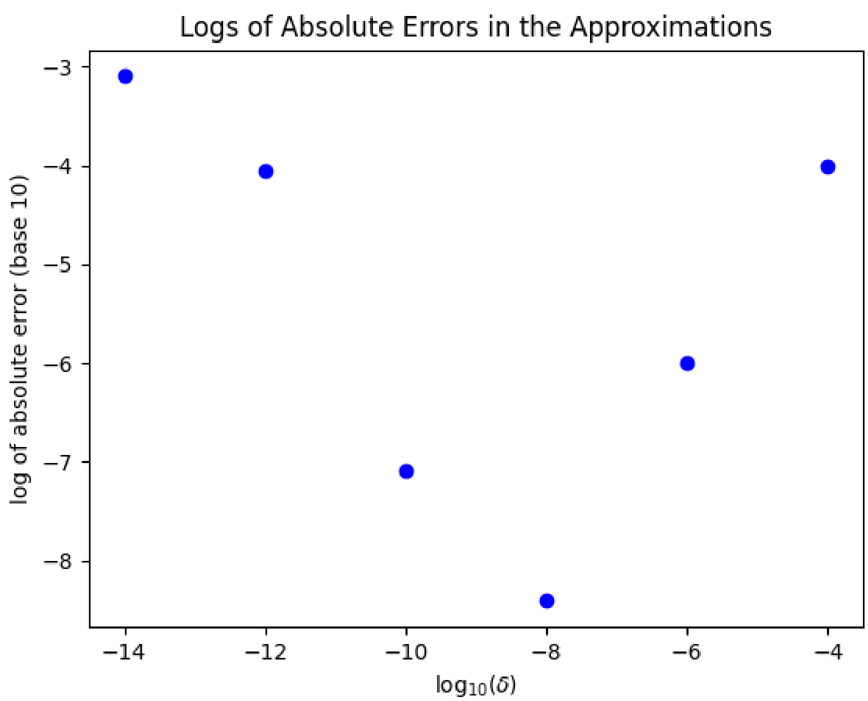

Figure 1: Plot for Problem 1.

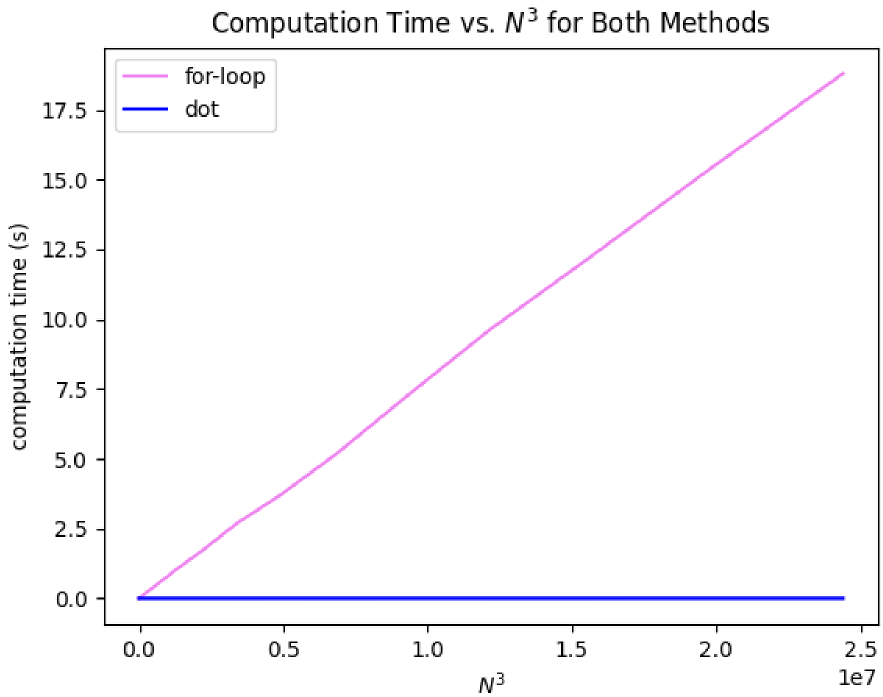

Figure 2: Plot for Problem 2.

```
10 : 0.0021739160000038282
30 : 1.2709000000349684e-05
50 : 2.8334000000668007e-05
70 : 0.0005644169999925452
90 : 0.00020070800000837608
110 : 0.00032633300000384224
130 : 0.0003543339999936279
150 : 0.0005464170000095692
170 : 0.0007965420000033419
190 : 0.0004958749999985912
210 : 0.0008187920000040094
230 : 0.000681999999997629
250 : 0.0007016670000012937
270 : 0.0013669580000055248
290 : 0.0013243750000100363
```

Figure 3: Computation time for the “dot” method. The number to the left of the colon is N, and the number to the right is the computation time in seconds. The computation time has a general tendency to increase as N increases.

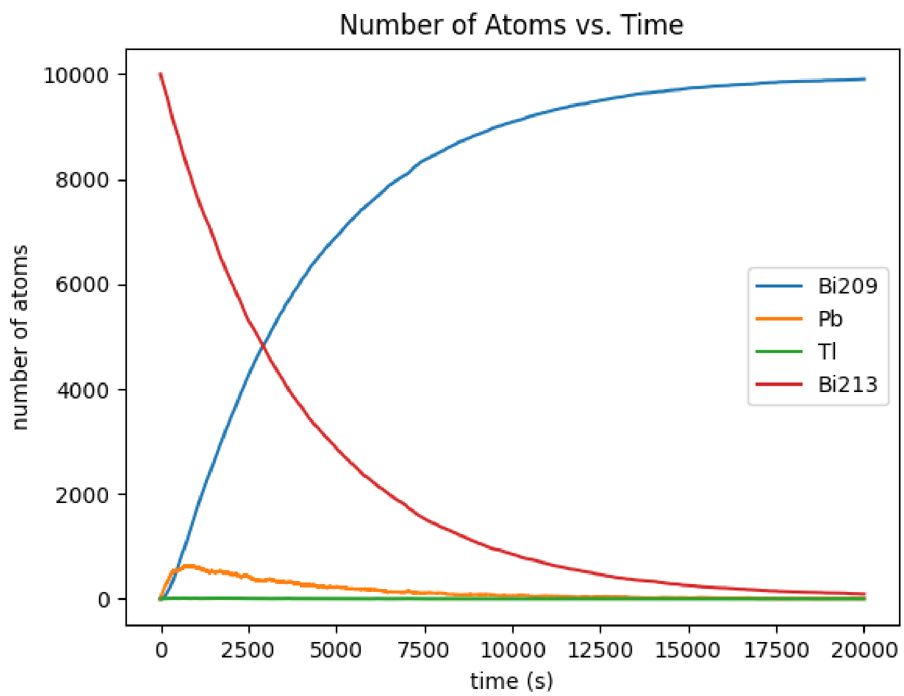

Figure 4: Plot for Problem 3.

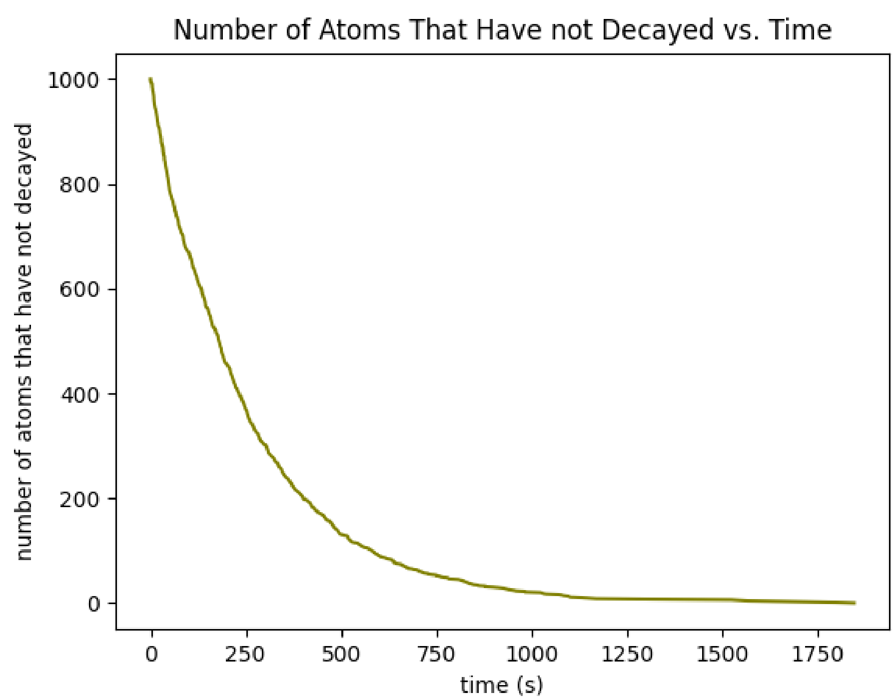

Figure 5: Plot for Problem 4.