Problem Set 7

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Problem 1

Brent's method for minimization is implemented by combining the quadratic interpolation method with the golden section search method. For the given function $f(x) = (x - 0.3)^2 e^x$, my implementation of Brent's method gives the minimizing result $x = 0.2999999999849663$, while `scipy.optimize.brent` gives $x = 0.29999999998599436$. The exact answer should be $x = 0.3$, so my implementation gives the correct result.

Problem 2

Using `scipy.optimize.minimize` and an initial guess of $\beta_0 = \beta_1 = 0.5$, I find the maximum likelihood values to be

$$\beta_0 = -5.62023964 \pm 0.60445528$$
$$\beta_1 = 0.10956353 \pm 0.01195889$$

where the uncertainties (errors) are calculated from the covariance matrix of β_0 and β_1:

$$\begin{bmatrix} 3.65366187e-01 & -6.89908441e-03 \\ -6.89908441e-03 & 1.43015143e-04 \end{bmatrix}.$$

The logistic model, together with the original data (blue dots), is plotted in Figure 1. Also shown in Figure 1 (orange dots) are the averages of the original data within each bin of 5 years (16 bins in total). It can be seen that the model matches the averages quite well. The model predicts that an older person is more likely to answer "yes" than a younger person, which is a trend that is also evident in the original data. Indeed, older people are more likely to know the meaning of the phrase "Be kind, rewind" than younger people.

My Github link: https://github.com/ziqiu-wang/phys-ua210.

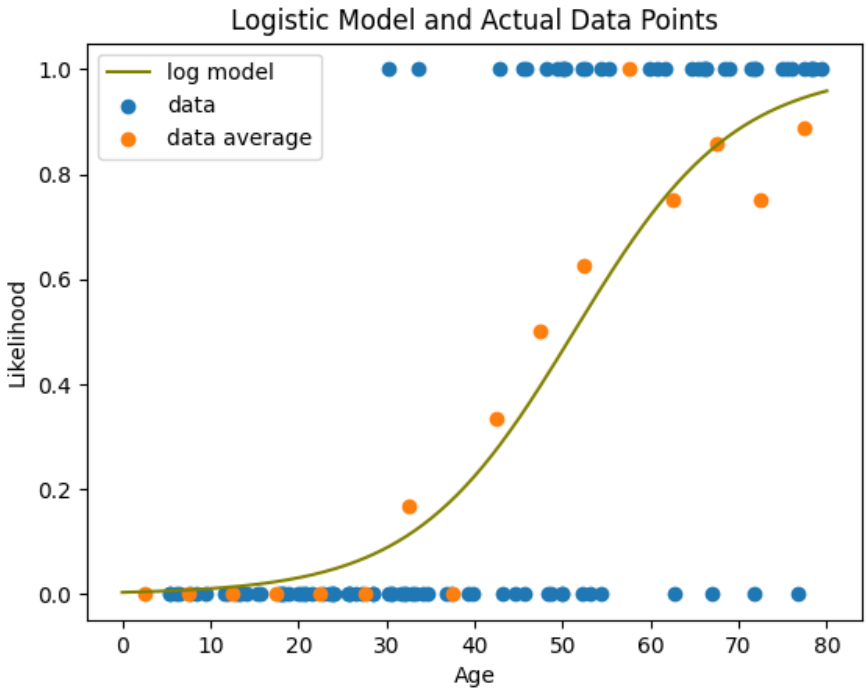

Figure 1: Logistic model, original data, and averaged data calculated using 16 bins, each with a size of 5 years.