Problem Set 2

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Problem 1

We divide the number each time by 2 and see whether the quotient is odd or even.

$$\begin{gathered}
121 \text{ is odd} \rightarrow 1 \\
60 \text{ is even} \rightarrow 0 \\
30 \text{ is even} \rightarrow 0 \\
15 \text{ is odd} \rightarrow 1 \\
7 \text{ is odd} \rightarrow 1 \\
3 \text{ is odd} \rightarrow 1 \\
1 \text{ is odd} \rightarrow 1
\end{gathered}$$

Going from bottom to top, the binary representation for 121 is 1111001.

Problem 2

For this problem, I set $L = 150$. The first method uses 3 for-loops, and the Madelung constant is calculated to be -1.74372838. The second method does not use any for-loop, and the Madelung constant is also calculated to be -1.74372838. The time used by the first method is about 54.7 seconds, while the second method only used 1.2 seconds. Therefore, the second method without the use of for-loops is a lot faster.

Problem 3

For this problem, I set $N = 3500$. The plot of the Mandelbrot set is shown in Figure 1.

Problem 4

(a) Using the standard formula, my program gives the two roots:

$$\begin{aligned}
x_1 &= -9.999894245993346 \times 10^{-07}, \\
x_2 &= -999999.999999.
\end{aligned}$$

(b) Derivation:

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{(-b \pm \sqrt{b^2 - 4ac})(-b \mp \sqrt{b^2 - 4ac})}{2a(-b \mp \sqrt{b^2 - 4ac})} \\
&= \frac{b^2 - b^2 + 4ac}{2a(-b \mp \sqrt{b^2 - 4ac})} \\
&= \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}
\end{aligned}
$$

Using the above formula, my program gives

$$
\begin{aligned}
x_1' &= -1.000000000001 \times 10^{-6}, \\
x_2' &= -1000010.5755125057.
\end{aligned}
$$

I notice that the first method used in part (a) gives a quite accurate result for the second root (x_2, which is almost exactly the expected answer, -1000000), but a much less accurate result for the first root (x_1, which differs from the expected answer, -1×10^{-6}, by more than 0.001%). However, the second method gives a quite accurate result for the first root (x_1') but not for the second root (x_2'). These significant errors are subtractive cancellation errors. For the first method, since $-b + \sqrt{b^2 - 4ac}$ is very close to 0, the rounding error of subtracting $4ac$ (which is a very small number) from b^2 (which is very big compared to $4ac$) gets magnified, so x_1 has a relatively large error; but the same is not true for the second root x_2 since $-b - \sqrt{b^2 - 4ac}$ is an addition between two negative numbers that are almost equal in size and is far greater in magnitude than the rounding error from $b^2 - 4ac$, and therefore x_2 is very accurate. The exact opposite is true for the second method, so x_1' is more accurate than x_2'.

(c) I need to avoid the subtractive cancellation errors. That is, I need to avoid letting $-b \pm \sqrt{b^2 - 4ac}$ be very close to 0. Analysis from part (b) suggests that this can be achieved by letting $-b$ and $\pm\sqrt{b^2 - 4ac}$ to have the same sign. Then, I can simply choose the appropriate expression for each of the two roots.

My Github link: https://github.com/ziqiu-wang/phys-ua210.

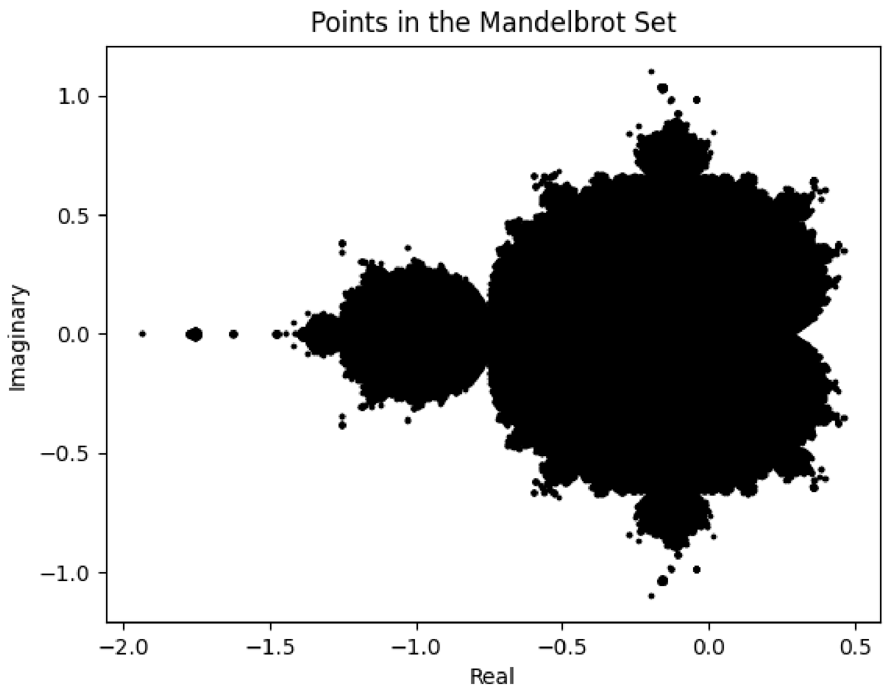

Figure 1: Points in the Mandelbrot set in the complex plane for Problem 3.