Solusi Lembar Tugas Mahasiswa ke-2
Fisika Dasar IA (FI-1101)

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1. Suppose that you lift a $4\,kg$ book from the floor to a shelf $2\,m$ high.
   (a) What force must you apply to move the book at constant velocity?
   (b) What work is done by this force?
   (c) What force must you apply to the book to carry it slowly at constant velocity from one shelf to an adjoining one $3\,m$ away but at the same level? (Assuming no friction between the book and the shelf)
   (d) How much work is done by this force?

Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

➢ Hukum Kekekalan Energi

$$\begin{aligned} E_m &= E_m' \\ E_p + E_k &= E_p' + E_k \end{aligned}$$

Diketahui:

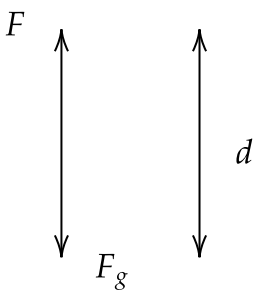
$$m = 4\,kg \qquad h = 2\,m$$

Ditanya:

$$F_1 = ...? \qquad W_{F_1} = ...? \qquad F_2 = ...? \qquad W_{F_2} = ...?$$

Jawab.

➢ Diagram Benda Bebas



➢ a) kecepatan konstan $\rightarrow a = 0$

$$\begin{aligned} \Sigma F &= ma \\ F - F_g &= 0 \\ F &= F_g \\ &= mg \\ &= 4 \cdot 9.8 \\ &= 39.2\,\hat{j}\,N \end{aligned}$$ □

➢ b)

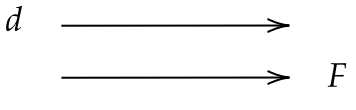
Cara I

$$\begin{aligned} W &= F \cdot d \\ &= 39.2 \cdot 2 \\ &= 78.4\,J \end{aligned}$$ □

Cara II *(Hukum Kekekalan Energi)*

$$\begin{aligned} E_m &= E_m' \\ E_p + E_k &= E_p' + E_k' \\ 0 + E_k &= mgh + 0 \\ E_k &= mgh \\ W &= 4 \cdot 9.8 \cdot 2 \\ &= 78.4\,J \end{aligned}$$

➢ Diagram Benda Bebas



➢ c) kecepatan konstan $\rightarrow a = 0$

$$\begin{aligned} \Sigma F &= ma \\ F &= 0\,N \end{aligned}$$ □

➢ d)

$$\begin{aligned} W &= F \cdot d \\ &= 0 \cdot 3 \\ &= 0\,J \end{aligned}$$ □

2. Sixteen kg sled is pulled by a rope over a wet snow for a horizontal distance of 3.2 m. The tension in the rope remains constant at 5.8 N and the rope is at 37° from horizontal. Suppose that the sled is moving at a constant velocity. Determine
   (a) the work done by the rope on the sled,
   (b) the work done by the frictional force on the sled, and
   (c) the coefficient of kinetic friction at the snow-sled interface

Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

Diketahui:

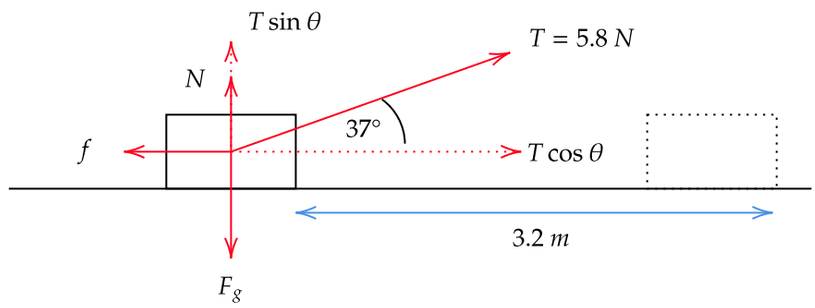
$$m = 16\ kg \qquad d = 3.2\ m \qquad T = 5.8\ N \qquad \theta = 37°$$

Ditanya:

$$W_T = ...? \qquad W_f = ...? \qquad \mu_k = ...?$$

Jawab.

➢ *Diagram Benda Bebas*



$$\begin{aligned} sumbu - x \\ \Sigma F_x &= ma \\ T\cos\theta - f &= 0 \\ f &= T\cos\theta \\ f &= 5.8\cos 37° \\ &= 5.8 \cdot 0.8 \\ &= 4.64\ N \end{aligned}$$

$$\begin{aligned} sumbu - y \\ \Sigma F_y &= 0 \\ F_g - (N + T\sin\theta) &= 0 \\ N &= F_g - T\sin\theta \\ &= mg - T\sin 37° \\ &= 16 \cdot 9.8 - 5.8 \cdot 0.6 \\ &= 153.32\ N \end{aligned}$$

➢ a)

$$\begin{aligned} W_T &= F_T \cdot d \\ &= T\cos\theta \cdot d \\ &= 4.64 \cdot 3.2 \\ &= 14.848\ J \end{aligned}$$

□

➢ b)

$$\begin{aligned} W_f &= F_f \cdot d \\ &= 4.64 \cdot 3.2 \\ &= -14.848\ J \end{aligned}$$

□

➢ c)

$$\begin{aligned} f &= \mu_k \cdot N \\ \mu_k &= \frac{f}{N} \\ &= \frac{4.64}{153.32} \\ &= 0.03 \end{aligned}$$

□

3. A $1.5\,kg$ ball attached to a light string is whirled in a circular path of radius $0.75\,m$.
   If the path is a horizontal circle,
   (a) how much work is done by the earth's gravitational force on the ball as it moves halfway around the circle, and
   (b) how much work is done by the tension force in the string?
   If the path is a vertical circle,
   (c) evaluate the work done by the earth's gravitational force as the ball moves from the highest point to the lowest point in the circle, and
   (d) how much work is done by the tension force in the string?

Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

Diketahui:

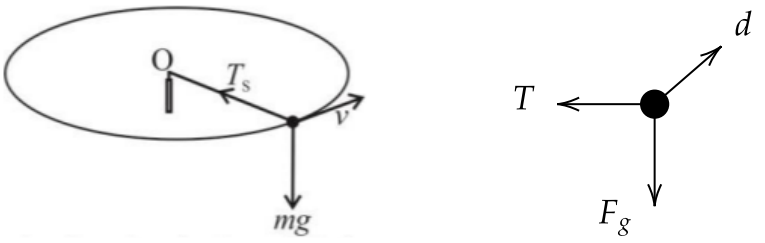
$$m = 1.5\,kg \qquad r = 0.75\,m$$

Ditanya:

$$W_{Fg} = ...? \qquad W_T = ...? \qquad W_{Fg}' = ...? \qquad W_T' = ...?$$

Jawab.

➢ *Diagram Benda Bebas (Horizontal)*



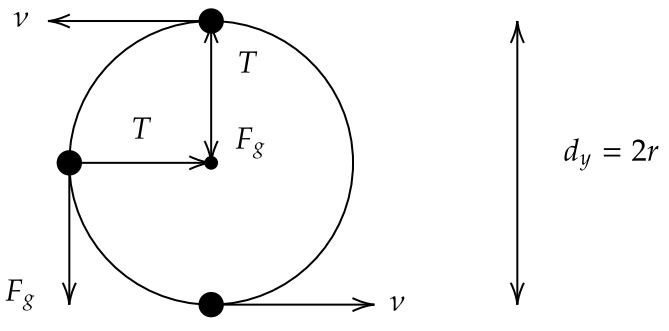
➢ a) *Gaya grativasi selalu tegak lurus dengan posisi*

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= \overrightarrow{F_g} \cdot \vec{d} \\ &= -\hat{i} \cdot \hat{j} \\ &= 0 \end{aligned}$$ □

➢ b) T*egangan tali selalu tegak lurus dengan posisi*

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= \vec{T} \cdot \vec{d} \\ &= \hat{i} \cdot \hat{j} \\ &= 0 \end{aligned}$$ □

➢ *Diagram Benda Bebas (Vertikal)*



➢ c) *Gaya gravitasi sejajar dengan perubahan posisi vertikal bola*

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= F_g \cdot d_y \\ &= mg \cdot 2r \\ &= (1.5)(9.8) \cdot 2(0.75) \\ &= 22.05\,J \end{aligned}$$ □

➢ d) *Tegangan tali selalu tegak lurus dengan posisi*

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= \vec{T} \cdot \vec{d} \\ &= \hat{i} \cdot \hat{j} \\ &= 0 \end{aligned}$$ □

4. One end of a light string is slipped around a peg fixed in a horizontal tabletop, while the other end is tied to a $0.50\,kg$ puck. The puck is given an initial velocity of magnitude $3.4\,m/s$ so that it moves in a horizontal circle of radius $0.75\,m$. The object comes to rest after completing 2.5 revolutions.
   (a) For the entire motion, what work is done by frictional force?
   (b) Assume that the magnitude of the frictional force is constant, determine the coefficient of kinetic friction at the interface.
   (c) Determine the tension in the string at the instant that the puck complete the first revolution.
   (d) How much work is done by the tension force in the string?

Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

Diketahui:

$$m = 0.5\,kg \qquad v_0 = 3.4\,m/s \qquad r = 0.75\,m \qquad n = 2.5\,rev$$

Ditanya:

$$W_f = ...? \qquad \mu_k = ...? \qquad T = ...? \qquad W_T = ...?$$

Jawab.

➢ *Keliling Lingkaran*

$$\begin{aligned} K &= 2\pi r \\ &= 2\pi(0.75) \\ &= 1.5\pi\ m \end{aligned}$$

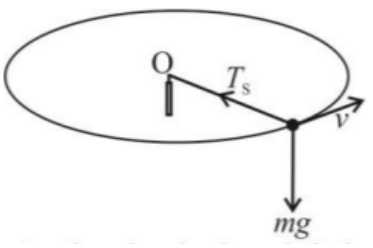
➢ *Jarak yang ditempuh*

$$\begin{aligned} s &= K \cdot n \\ &= 1.5\pi \cdot 2.5 \\ &= 3.75\pi\ m \end{aligned}$$

➢ *Percepatan*

$$\begin{aligned} {v_t}^2 &= {v_0}^2 + 2as \\ 0 &= (3.4)^2 + 2 \cdot a \cdot (3.75\pi) \\ a &= -\frac{11.56}{7.5\pi} \\ &= \frac{-1.54}{\pi}\ m/s \end{aligned}$$

➢ *Diagram Benda Bebas*



$$\begin{aligned} &sumbu - x \\ \Sigma F_x &= ma \\ f_k &= ma \\ &= 0.5 \cdot \left(\frac{-1.54}{\pi}\right) \\ &= \frac{-0.77}{\pi}\ N \end{aligned}$$

$$\begin{aligned} &sumbu - y \\ \Sigma F_y &= 0 \\ N - F_g &= 0 \\ N &= F_g \\ &= mg \\ &= (0.5)(9.8) \\ &= 4.9\ N \end{aligned}$$

➢ a)

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= f_k \cdot 3.75\pi \\ &= \frac{-0.77}{\pi} \cdot 3.75\pi \\ &\approx -2.89\ J \end{aligned}$$ □

➢ b)

$$\begin{aligned} f_k &= \mu_k \cdot N \\ \mu_k &= \frac{f_k}{N} \\ &= \frac{0.77}{4.9\pi} \\ &\approx 0.05 \end{aligned}$$ □

➢ c)

jarak yang ditempuh dalam satu revolusi

$$\begin{aligned} s &= K \cdot n \\ &= 1.5\pi \cdot 1 \\ &= 1.5\pi\ m \end{aligned}$$

kecepatannya

$$\begin{aligned} {v_t}^2 &= {v_0}^2 + 2as \\ &= (3.4)^2 + 2 \cdot \left(-\frac{1.54}{\pi}\right) \cdot (1.5\pi) \\ &\approx 2.63\ m/s \end{aligned}$$

proyeksi yang sejajar dengan tegangan tali

$$\begin{aligned} \Sigma F &= ma \\ T &= ma_s \\ &= m \cdot \frac{{v_t}^2}{r} \\ &= 0.5 \cdot \frac{(2.63)^2}{0.75} \\ &\approx 4.624\ N \end{aligned}$$ □

➢ d)

Tegangan tali selalu tegak lurus dengan posisi

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= \vec{T} \cdot \vec{d} \\ &= \hat{i} \cdot \hat{j} \\ &= 0 \end{aligned}$$ □

5. Assume that the drag force exerted by the water on a barge is proportional to the speed of the barge to water. A tug delivers $230\ hp$ to the barge when they travel at constant speed of $0.25\ m/s$.
   (a) What power is required to move the barge at $0.75\ m/s$?
   (b) What force does the tug exert on the barge at the lower speed?
   (c) at the higher speed? $(1\ hp = 746\ W)$

Materi: Usaha dan Energi

➢ *Usaha*

$$W = \vec{F} \cdot \vec{d}$$

➢ *Gaya Hambat*

$$F_d = k \cdot v$$

Keterangan:
k = konstanta hambat

Diketahui:

$$P_{low} = 230\ hp \approx 171580\ watt \qquad v_{low} = 0.25\ m/s \qquad v_{high} = 0.75\ m/s$$

Ditanya:

$$P_{high} = ...? \qquad F_{low} = ...? \qquad F_{high} = ...?$$

Jawab.

➢ a)

mencari nilai konstanta gaya

$$\begin{aligned} P_{low} &= F \cdot v_{low} \\ P_{low} &= k \cdot {v_{low}}^2 \\ k &= \frac{P_{low}}{{v_{low}}^2} \\ &= \frac{230}{(0.25)^2} \end{aligned}$$

substitusi

$$\begin{aligned} P_{high} &= k \cdot {v_{high}}^2 \\ &= \frac{230}{(0.25)^2} \cdot (0.75)^2 \\ &= 2070\ hp \\ &= 1544220\ W \\ &= 15442.2\ kW \end{aligned}$$ □

➢ b)

$$\begin{aligned} F_{low} &= k \cdot v_{low} \\ &= \frac{230}{(0.25)^2} \cdot (0.25) \\ &= 920\ hp\ s/m \\ &= 686320\ N \end{aligned}$$ □

➢ c)

$$\begin{aligned} F_{high} &= k \cdot v_{high} \\ &= \frac{230}{(0.25)^2} \cdot (0.75) \\ &= 2760\ hp\ s/m \\ &= 2058960\ N \end{aligned}$$ □

6. The potential energy of interaction between neutral atoms is sometimes approximated by "12-6" potential energy function.

$$U(x) = V_0 \left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6} \right]$$

where x is the separation between centers of the atoms and V_0 and a are constants.

(a) Construct a graph of the function with $a = 0.3\ nm$ and $V_0 = 3.2 \times 10^{-21}$ J for $0.28\ nm \leq x \leq 0.47\ nm$.

(b) determine from the graph the equilibrium separation where $F_x = 0$.

(c) Check the value by using the connection $F_x = -dU/dx$.

(d) Suppose one turning point is at $x = 0.40\ nm$, locate the other turning point.

(e) Determine the mechanical energy for the motion between these turning points.

Materi: Usaha dan Energi

➢ Energi Mekanik

$$E_m = E_k + E_p$$

Diketahui:

$$a = 0.3\ nm \qquad V_0 = 3.2 \cdot 10^{-21}\ J \qquad x_{example} = 0.40\ nm$$

Ditanya:

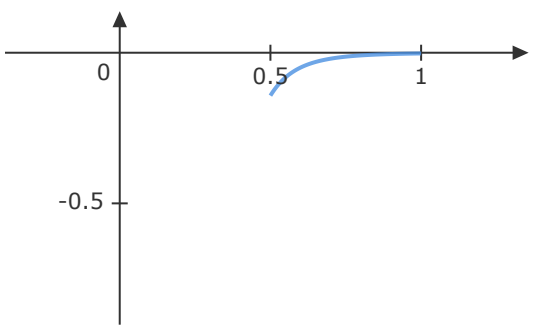
$$r_{eq} = ...? \qquad x_{example}' = ...? \qquad E_m = ...?$$

Jawab.

➢ a)

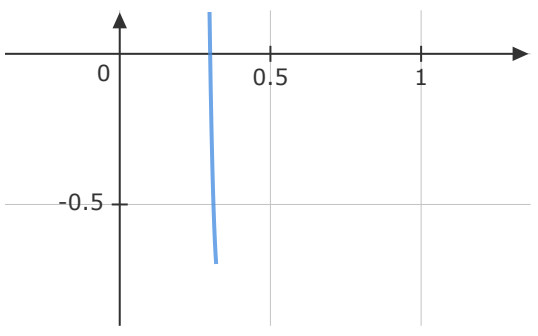
Tinjau untuk nilai $x \to \infty$

$$\lim_{x \to \infty} \frac{a}{x} = 0 \quad \textit{sehingga} \quad \lim_{x \to \infty} U(x) = 0$$



Tinjau untuk nilai $x \to 0$

$$\lim_{x \to 0} \frac{a}{x} = \infty \quad \textit{sehingga} \quad \lim_{x \to 0} U(x) = \infty$$



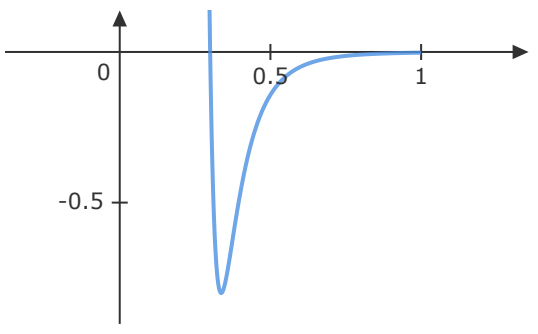
memotong sumbu-x di titik

$$U(x) = V_0 \left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6} \right]$$

$$0 = 3.2 \times 10^{-21} \left[\left(\frac{0.3}{x}\right)^{12} - \left(\frac{0.3}{x}\right)^{6} \right] \quad \rightarrow \quad \textit{titik } (0.3,\ 0)$$

$$x = 0.3$$

fungsi $U(x)$ *merupakan bentuk polinomial sehingga fungsi* $U(x)$ *kontinu dan akan ada suatu titik balik sebagai penghubung*



mencari nilai y *untuk interval* $0.28 \leqslant x \leqslant 0.47$

$$U(0.28) = V_0 \left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6} \right]$$

$$= 3.2 \times 10^{-21} \left[\left(\frac{0.3}{0.28}\right)^{12} - \left(\frac{0.3}{0.28}\right)^{6} \right]$$

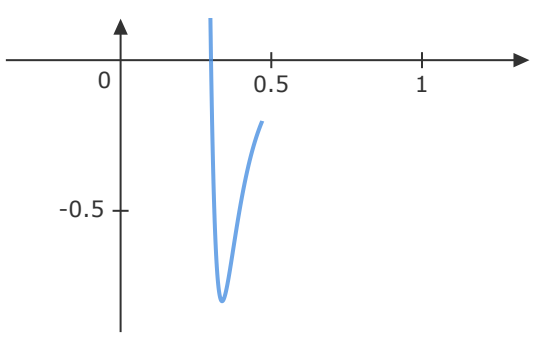
$$\approx 2.48 \times 10^{-21}\ J$$

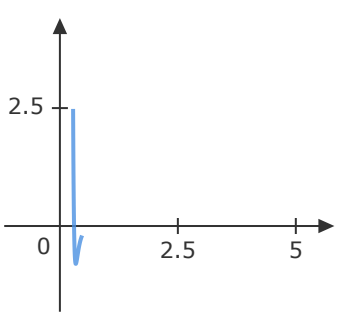
$$U(0.47) = V_0 \left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6} \right]$$

$$= 3.2 \times 10^{-21} \left[\left(\frac{0.3}{0.47}\right)^{12} - \left(\frac{0.3}{0.47}\right)^{6} \right]$$

$$\approx -0.2 \times 10^{-21}\ J$$

grafik $U(x)$ *untuk interval* $0.28 \leqslant x \leqslant 0.47$

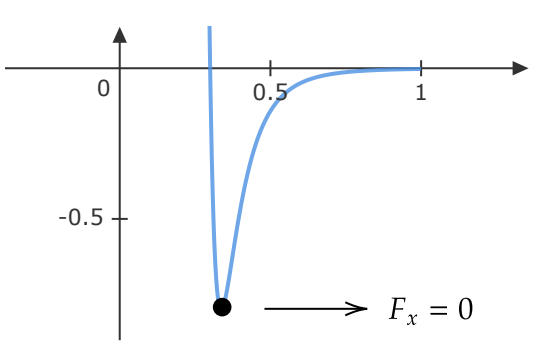




➢ b)

F_x *merupakan turunan dari* $U(x)$

maka $F_x = 0$ *terjadi di titik balik kurva* $U(x)$ *yang mana gradien garisnya* $= 0$



➢ c)

$$
\begin{aligned}
F_x &= -\frac{dU}{dx} \\
0 &= \frac{d}{dx}\left(-V_0 \left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6} \right] \right) \\
0 &= -V_0 \left[a^{12}\, \frac{d}{dx}\left(\frac{1}{x}\right)^{12} - a^6\, \frac{d}{dx}\left(\frac{1}{x}\right)^{6} \right] \\
0 &= -V_0 \left[a^{12}\, \frac{d}{dx}\left(x^{-12}\right) - a^6\, \frac{d}{dx}\left(x^{-6}\right) \right] \\
0 &= -V_0 \cdot a^{12} \cdot (-12) \cdot \frac{1}{x^{13}} + V_0 \cdot a^6 \cdot (-6) \cdot \frac{1}{x^7} \\
0 &= 12 \cdot V_0 \cdot a^{12} \cdot \frac{1}{x^{13}} - 6 \cdot V_0 \cdot a^6 \cdot \frac{1}{x^7} \\
\frac{6 \cdot V_0 \cdot a^6}{x^7} &= \frac{12 \cdot V_0 \cdot a^{12}}{x^{13}} \\
x^6 &= 2\, a^6 \\
x &= 2^{\frac{1}{6}} \cdot a \\
&= \sqrt[6]{2} \cdot 0.3 \\
&\approx 0.337\ nm
\end{aligned}
$$

➢ d)

energi potensial di titik balik $x = 0.4\ nm$

$$
\begin{aligned}
U(0.4) &= 3.2 \times 10^{-21} \left[\left(\frac{0.3}{0.4}\right)^{12} - \left(\frac{0.3}{0.4}\right)^{6} \right] \\
&\approx -4.68 \times 10^{-22}\ J
\end{aligned}
$$

misalkan $\beta = \left(\frac{a}{x}\right)^6$ *maka salah satu akar dari* $\beta = \left(\frac{0.3}{0.4}\right)^6 = 0.1779$

$$
\begin{aligned}
U(0.4) &= V_0 \left[\left(\frac{a}{x}\right)^{12} - \left(\frac{a}{x}\right)^{6} \right] \\
U(0.4) &= V_0 \left[\beta^2 - \beta \right] \\
\beta^2 - \beta - \frac{U(0.4)}{V_0} &= 0 \\
\beta^2 - \beta + \frac{4.68 \times 10^{-22}}{3.2 \times 10^{-21}} &= 0 \\
\beta^2 - \beta + 0.14625 &= 0 \\
(\beta - 0.1779)(\beta + k) &= 0 \qquad (-0.1779)(k) \approx 0.14625 \\
(\beta - 0.1779)(\beta - 0.8.221) &= 0 \qquad -0.1779 + k = -1 \\
\beta &= 0.8221 \\
\left(\frac{a}{x}\right)^6 &= 0.8221 \\
x^6 &= \frac{a^6}{0.8221} \\
x &= \sqrt[6]{\frac{1}{0.8221}} \cdot a \\
&= \sqrt[6]{\frac{1}{0.8221}} \cdot 0.3 \\
&\approx 0.31\ nm
\end{aligned}
$$

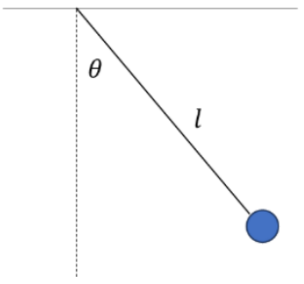
□

➢ e)

saat turning point, E_p *maks dan* $E_k = 0$

$$
\begin{aligned}
E_m &= E_k + E_p \\
&= 0 + U_{maks} \\
&= U(0.4) \\
&= 3.2 \times 10^{-21} \left[\left(\frac{0.3}{0.4}\right)^{12} - \left(\frac{0.3}{0.4}\right)^{6} \right] \\
&\approx -4.68 \times 10^{-22}\ J \\
&\approx 4.68 \times 10^{-22}\ J \qquad (\textit{berlawanan arah perpindahan partikel})
\end{aligned}
$$

7. A simple pendulum is formed by attaching a $40\ g$ ball to a one end of a light string; the other end of the string is held fixed, and the ball can swing in a vertical plane. Suppose the ball is released from rest from the position shown in the figure with $l = 500\ mm$ and $\theta = 60.0°$. Determine
   (a) the speed of the ball and
   (b) the tension in the string when the ball passes through its lowest position.
   (c) Construct a graph of the kinetic energy of the ball as a function of θ.



Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

Diketahui:

$$m = 40\ g \qquad l = 500\ mm \qquad \theta = 60°$$

Ditanya:

$$v = ...? \qquad T_{lowest} = ...?$$

Jawab.

➢ a)

$$\begin{aligned} E_m &= E_m' \\ E_p + E_k &= E_p' + E_k' \\ mgh + 0 &= 0 + \frac{1}{2}mv^2 \\ v &= \sqrt{2gh} \\ &= \sqrt{2g(L - L\cos\theta)} \\ &= \sqrt{2gL\,(1 - \cos\theta)} \\ &= \sqrt{2 \cdot 9.8 \cdot 0.5\,(1 - \cos 60°)} \\ &= \frac{7}{\sqrt{10}} \\ &\approx 2.21\ m/s \end{aligned}$$

□

➢ b)

$$posisi\ terendah\ maka\ \theta = 60°$$

$$\begin{aligned} T - F_g &= ma_s \\ T - mg &= m \cdot \frac{v^2}{r} \\ T &= m\left(\frac{\left(\sqrt{2gL(1 - \cos\theta)}\right)^2}{L} + g\right) \\ &= mg((2 - 2\cos\theta) + 1) \\ &= mg(3 - 2\cos\theta) \\ &= 0.04 \cdot 9.8 \cdot (3 - 2\cos 60°) \\ &= 0.784\ N \end{aligned}$$

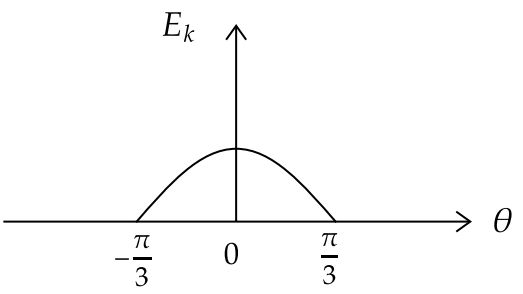
□

➢ c)

$$\begin{aligned} E_k &= E_m - E_p \\ &= E_{k\,maks} - E_p \\ &= \frac{1}{2}m\left(\sqrt{2gL(1 - \cos 60°)}\right)^2 - mgL(1 - \cos\theta) \\ &= mgL\left(\frac{1}{2}\right) - mgL(1 - \cos\theta) \\ &= mgL\left(\frac{1}{2} - 1 + \cos\theta\right) \\ &= mgL\left(\cos\theta - \frac{1}{2}\right) \end{aligned}$$

$$dengan\ interval\ \theta = \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$$

E_k *maksimum terjadi saat* $\theta = 0$ *dan minimum terjadi saat* $\theta = \frac{\pi}{3}$



8. The gravitational-potential-energy- function $U(y)$ is graphed in the figure for a $10.2\,kg$ object close to the earth's surface; $y = 0$ corresponds to ground level. Suppose the mechanical energy of the system is $0.20\,kJ$. From the graph determine
   (a) the maximum height of the object,
   (b) the maximum kinetic energy and the point where the object has that maximum kinetic energy,
   (c) the location of the object when its kinetic energy equals the potential energy,
   (d) the force on the object at that instant.

Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

➢ Hukum Kekekalan Energi

$$E_m = E_m'$$
$$E_p + E_k = E_p' + E_k$$

Diketahui:

$$E_m = 0.2\,kJ = 200\,J \qquad m = 10.2\,kg$$

Ditanya:

$h_{maks} = ...?$ $E_{k\ maks} = ...?$ $h_{E_{k\ maks}} = ...?$ $h_{E_k = E_p} = ...?$ $F = ...?$

Jawab.

➢ a)

$$\begin{aligned} E_m &= E_m' \\ &= E_p' + E_k' \\ &= mgh + 0 \\ 200 &= mgy + 0 \\ y &= \frac{200}{10.2 \cdot 9.8} \\ &\approx 2\,m \end{aligned}$$

□

➢ b)

$$E_{k\ maks\ ketika\ y = 0}$$

$$\begin{aligned} E_m &= E_p + E_k \\ &= mgh + E_k \\ &= mgy + E_k \\ 0.2 &= 0 + E_k \\ E_k &= 0.2\ kJ \end{aligned}$$

➢ c)

$$saat\ E_p = E_k$$

$$\begin{aligned} E_m &= E_p + E_k \\ &= E_p + E_p \\ &= 2E_p \\ 200 &= 2mgh \\ h &= \frac{200}{2 \cdot 10.2 \cdot 9.8} \\ &\approx 1\,m \end{aligned}$$

□

➢ d

$$\begin{aligned} F &= -\frac{dU}{dx} \\ &= -\frac{(200 - 0)}{(2 - 0)} \\ &= -100\,N \\ &= 100\,N \quad (arahnya\ ke\ bawah) \end{aligned}$$

□

9. Calculate the energy required to raise $1\,kg$ to a height of $100\,km$. Do in two ways:
   (a) assume g stays constant;
   (b) use the exact expression of the potential energy in the earth's field.
   (c) What is the percentage difference in the two calculations?

Materi: Usaha dan Energi

➢ Usaha

$$W = \vec{F} \cdot \vec{d}$$

➢ Energi Potensial Gravitasi Bumi

$$U = \frac{GMm}{r}$$

dimana:

G = *konstanta gravitasi* $\left(6.67 \times 10^{-11}\ m^3/kg\,s^2\right)$

M = *massa bumi* $\left(5.97 \times 10^{24}\ kg\right)$

r = *radius bumi* $\left(6.37 \times 10^{6}\ m\right)$

m = *massa objek yang ditinjau*

Diketahui:

$P = 230\,hp \approx 171580\ watt$ $\quad v = 0.25\,m/s \quad v' = 0.75\,m/s$

Ditanya:

$W_P = ...?$ $\quad F_{low} = ...?$ $\quad F_{high} = ...?$

Jawab.

➢ a)

$$\begin{aligned} W &= F \cdot d \\ &= mg \cdot 10^5 \\ &= 1 \cdot 9.8 \cdot 10^5 \\ &= 980000\,J \end{aligned}$$

□

➢ b)

$$\begin{aligned} W &= \Delta U \\ &= U_{akhir} - U_{permukaan} \\ &= \frac{GMm}{R_b + h} - \frac{GMm}{R_b} \\ &= \frac{GMm \cdot h}{R_b(R_b + h)} \\ &= \frac{6.67 \times 10^{-11} \cdot 5.97 \times 10^{24} \cdot 1 \cdot 10^5}{6.37 \times 10^6\left(6.37 \times 10^6 + 10^5\right)} \\ &\approx 966176\,J \end{aligned}$$

➢ c)

$$\begin{aligned} \%\ perbedaan &= \frac{W' - W}{W} \times 100\% \\ &= \frac{980000 - 966176}{966176} \times 100\% \\ &= 1.43\% \end{aligned}$$

10. Compute the work done by the force $\vec{F} = xy\hat{i} + (x - y)\hat{j}$ acting on an object when the object moves from point $(1, 2)$ to point $(2, 8)$ along
    (a) a linear path and
    (b) a parabolic path passing through the origin.
    (c) Is the force conservative? $\vec{F}$ in newton and x and y in meters

Materi: Usaha dan Energi

➢ Usaha

$$W = \int F\, dt$$

➢ Gradien Garis

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

➢ Persamaan Garis Lurus

$$y - y_1 = m(x - x_1)$$

Diketahui:

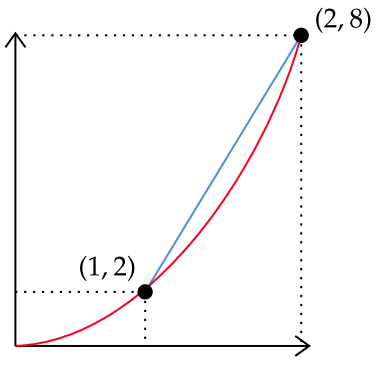
$$x_1 = 1 \qquad y_1 = 2 \qquad x_2 = 2 \qquad x_3 = 8$$

Ditanya:

$$W_{linier} = ...? \qquad W_{parabola} = ...?$$

Jawab.

➢ Grafik



➢ a) garis linier

dengan mencari m *dan* c

$$\begin{aligned} y &= mx + c \\ &= 6x - 4 \\ dy &= 6\, dx \end{aligned}$$

substitusi

$$\begin{aligned} W_{AB} &= \int_A^B (xy\, dx + (x - y)\, dy \\ &= \int_A^B (x(6x - 4)\, dx + (x - (6x - 4))\, 6\, dx \\ &= \int_1^2 \left(6x^2 - 4x\right)\, dx + (24 - 30x)\, dx \\ &= \int_1^2 \left(6x^2 - 34x + 24\right)\, dx \\ &= \left[2x^3 - 17x^2 + 24x\right]_1^2 \\ &= -13\, J \end{aligned}$$

➢ b) garis parabola

dengan diketahui 3 *titik yang berada di parabola*

$$\begin{aligned} y &= 2x^2 \\ dy &= 4x\, dx \end{aligned}$$

substitusi

$$\begin{aligned} W_{AB} &= \int_A^B xy\, dx + (x - y)\, dy \\ &= \int_A^B x\left(2x^2\right)\, dx + \left(x - 2x^2\right)\, 4x\, dx \\ &= \int_1^2 \left(2x^3\right)\, dx + \left(4x^2 - 8x^3\right)\, dx \\ &= \int_1^2 \left(-6x^3 + 4x^2\right)\, dx \\ &= \left[-\frac{3}{2}x^4 + \frac{4}{3}x^3\right]_1^2 \\ &= -\frac{79}{6} \\ &= -13.17\, J \end{aligned}$$

➢ c)

The force is non-conservative because it depends on the path

11. On an air hockey table, a floating disk with a speed of $4\ m/s$ strikes another identical disk which is at rest. The first disk glances off at an angle of $30°$ from its original direction with a speed of v_1. The second disk is knocked at an angle of $60°$ from that original direction with a speed of v_2. What are the values of v_1 and v_2?

Materi: Momentum Linier

➢ *Hukum Kekekalan Momentum*

$$p = p'$$
$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Diketahui:

$$v_1 = 4\ m/s \qquad \theta = 30° \qquad m_1 = m_2$$
$$v_2 = 0\ m/s \qquad \phi = 60°$$

Ditanya:

$$v_1' = ...? \qquad v_2 = ...?$$

Jawab.

➢ a)

komponen-y

$$\begin{aligned} mv_{y1} + mv_{y2} &= mv_{y1}' - mv_{y2}' \\ m(v_{y1} + v_{y2}) &= m(v_1' \sin\theta - v_2' \sin\phi) \\ m(0+0) &= mv_1' \sin\theta - mv_2' \sin\phi \\ 0 &= mv_1' \sin\theta - mv_2' \sin\phi \\ mv_2' \sin\phi &= mv_1' \sin\theta \\ v_2' &= v_1' \cdot \frac{\sin\theta}{\sin\phi} \qquad \text{... (1)} \end{aligned}$$

komponen-x

$$\begin{aligned} mv_{x1} + mv_{x2} &= mv_{x1}' + mv_{x2}' \\ mv_{x1} + mv_{x2} &= mv_1' \cos\theta + mv_2' \cos\phi \\ m(4) + m(0) &= mv_1' \cos\theta + mv_2' \cos\phi \\ 4m &= m(v_1' \cos\theta + v_2' \cos\phi) \\ 4 &= v_1' \cos\theta + v_1' \cdot \frac{\sin\theta}{\sin\phi} \cos\phi \\ 4 &= v_1' \left(\cos\theta + \frac{\sin\theta}{\sin\phi}\cos\phi\right) \\ v_1' &= \frac{4}{\left(\cos\theta + \frac{\sin\theta}{\sin\phi}\cos\phi\right)} \\ &= \frac{4}{\left(\cos 30° + \frac{\sin 30°}{\sin 60°}\cos 60°\right)} \\ &= 2\sqrt{3}\ m/s \end{aligned}$$ □

dan

$$\begin{aligned} v_2' &= v_1' \cdot \frac{\sin\theta}{\sin\phi} \\ &= v_1' \cdot \frac{\sin 30°}{\sin 60°} \\ &= 2\ m/s \end{aligned}$$ □

12. A $50\ g$ tennis ball strikes a racquet perpendicular to its surface. Its speed is $10\ m/s$ and it rebounds with the same speed. The force applied to the ball increases linearly to a maximum in $0.01\ s$ and then decreases linearly to zero in the next $0.01\ s$.
    (a) what is the change of momentum of the ball?
    (b) What is the average force exerted by the racquet on the ball?
    (c) What is the maximum force exerted by the racquet on the ball?

Materi: Momentum Linier

➢ *Perubahan momentum*

$$\Delta p = p - p_0$$

➢ *Impuls*

$$\vec{I} = \int F\,dt$$

$$\Delta p = F \cdot \Delta t$$

Diketahui:

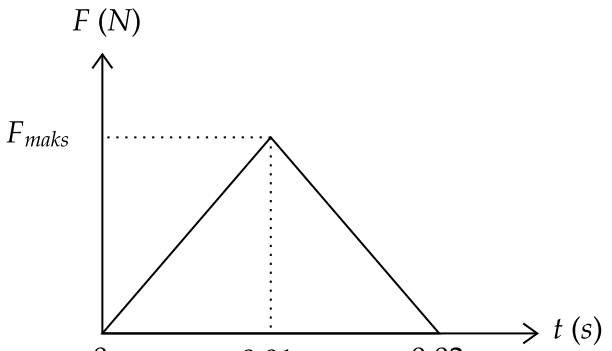
$$m = 50\ g = 0.05\ kg \qquad v_i = -10\ m/s \qquad v_f = 10\ m/s \qquad \Delta t = 0.02\ s$$

Ditanya:

$$\Delta p = ...? \qquad F_{avg} = ...? \qquad F_{maks} = ...?$$

Jawab.

➢ *Grafik F terhadap t*



➢ a)

$$\begin{aligned}\Delta p &= p - p_0 \\ &= m(v_f - v_i) \\ &= 0.05(10 - (-10) \\ &= 1\ kg\ m/s\end{aligned}$$ □

➢ b)

$$\vec{I} = \int F\,dt$$

$$1 = F \cdot \Delta t$$

$$F = \frac{1}{0.01 - (-0.01)}$$

$$= 50\ N$$ □

➢ c)

$$L_{\triangle} = \frac{1}{2} \cdot a \cdot t$$

$$I = \frac{1}{2} \cdot \Delta t \cdot F_{maks}$$

$$F_{maks} = \frac{2I}{\Delta t}$$

$$= \frac{2}{0.02}$$

$$= 100\ N$$ □

13. A rocket ship emits hot gases at a speed of $2000\ m/s$ with respect to the ship. If it burns and shoots out $100\ kg$ of mass every second, what is the resultant thrust on the rocket?

Materi: Momentum Linier

➢ *Rocket Equation*

$$F = \frac{dp}{dt}$$

Diketahui:

$$v = 2000\ m/s \qquad \frac{dm}{dt} = 100\ kg/s$$

Ditanya:

$$F = \ldots\ ?$$

Jawab.

➢ a)

$$\begin{aligned} F &= \frac{dp}{dt} \\ &= \frac{d}{dt}(mv) \\ &= m\frac{dv}{dt} + v\frac{dm}{dt} \end{aligned}$$

saat v *konstan dan* m *sebagai variabel*

$$\begin{aligned} F &= m\frac{dv}{dt} \\ &= 100 \cdot 2000 \\ &= 200000 \\ &= 200\ kN \end{aligned}$$

□

14. Consider a system of three particles whose masses and positions t are given in the table. Determine
    (a) the position,
    (b) velocity, and
    (c) acceleration of the center of mass of the system

Particles	Masses (kg)	x (m)	y (m)	z (m)
Particles 1	1	$2t$	$3t^2-2$	$4t^2$
Particles 2	3	2	t^2	0
Particles 3	2	-3	$-4t$	$-t$

Materi: Momentum Linier

➢ *Pusat massa*

$$x_{pm} = \frac{1}{M}\sum_{i=1}^{n} m_i r_{xi}$$

Diketahui

pada tabel

Ditanya:

$$\vec{r} = ...? \qquad \vec{v} = ...? \qquad \vec{a} = ...?$$

Jawab.

➢ a)

$$\begin{aligned} x_{pm} &= \frac{1}{M}\sum_{i=1}^{n} m_i r_{xi} \\ &= \frac{m_1 r_{x1} + m_2 r_{x2} + m_3 r_{x3}}{m_1 + m_2 + m_3} \\ &= \frac{1\cdot 2t + 3\cdot 2 + 2\cdot(-3)}{1+3+2} \\ &= \frac{2t}{6} \\ &= \frac{t}{3} \end{aligned}$$

$$\begin{aligned} y_{pm} &= \frac{1}{M}\sum_{i=1}^{n} m_i r_{yi} \\ &= \frac{m_1 r_{y1} + m_2 r_{y2} + m_3 r_{y3}}{m_1 + m_2 + m_3} \\ &= \frac{1\cdot\left(3t^2-2\right) + 3\cdot\left(t^2\right) + 2\cdot(-4t)}{1+3+2} \\ &= \frac{6t^2-8t-2}{6} \\ &= \frac{3t^2-4t-1}{3} \end{aligned}$$

$$\begin{aligned} z_{pm} &= \frac{1}{M}\sum_{i=1}^{n} m_i r_{zi} \\ &= \frac{m_1 r_{z1} + m_2 r_{z2} + m_3 r_{z3}}{m_1 + m_2 + m_3} \\ &= \frac{1\cdot\left(4t^2\right) + 3\cdot(0) + 2\cdot(-t)}{1+3+2} \\ &= \frac{4t^2-2t}{6} \\ &= \frac{2t^2-t}{3} \end{aligned}$$

posisi pusat massa

$$\vec{r}_{pm} = \left(\frac{t}{3}\,\hat{i} + \frac{3t^2-4t-1}{3}\,\hat{j} + \frac{2t^2-t}{3}\,\hat{k}\right) m$$

➢ b)

kecepatan pusat massa

$$\frac{d}{dt}\vec{r}_{pm} = \frac{d}{dt}\left(\frac{t}{3}\,\hat{i} + \frac{3t^2-4t-1}{3}\,\hat{j} + \frac{2t^2-t}{3}\,\hat{k}\right)$$

$$\vec{v}_{pm} = \left(\frac{1}{3}\hat{i} + \frac{6t-4}{3}\,\hat{j} + \frac{4t-1}{3}\,\hat{k}\right) m/s$$

➢ c)

percepatan pusat massa

$$\frac{d}{dt}\vec{v}_{pm} = \frac{d}{dt}\left(\frac{1}{3}\hat{i} + \frac{6t-4}{3}\,\hat{j} + \frac{4t-1}{3}\,\hat{k}\right)$$

$$\vec{a}_{pm} = \left(2\,\hat{j} + \frac{4}{3}\,\hat{k}\right) m/s^2$$

15. Position particle 1 of mass m at time t is $\vec{r_1} = 5\,(\hat{i}\cos(at+b) + \hat{j}\sin(at+b))$ while position of particle 2 of mass $5m$ is $\vec{r_2} = \hat{i}\cos(ct+d) + \hat{j}\sin(ct+d)$. $a,\ b,\ c,\ d$ are constants. Determine
(a) the position and
(b) the velocity of the center of mass of the system.
(c) Specify the requirement that the position of the center of mass does not move.
(d) Specify the requirements that the position and velocity of the center of mass are perpendicular to each other.

Materi: Momentum Linier

➢ *Pusat massa*

$$x_{pm} = \frac{1}{M}\sum_{i=1}^{n} m_i r_{xi}$$

Diketahui:

$$m_1 = m \qquad m_2 = 5m$$

Ditanya:

$$\vec{r} = ...? \qquad \vec{v} = ...?$$

Jawab.

➢ a)

$$\begin{aligned} x_{pm} &= \frac{1}{M}\sum_{i=1}^{n} m_i r_{xi} \\ &= \frac{m_1 r_{x1} + m_2 r_{x2}}{m_1 + m_2} \\ &= \frac{m\cdot 5\cos(at+b) + 5m\cos(ct+d)}{m+5m} \\ &= \frac{5m\,[\cos(at+b) + \cos(ct+d)]}{6m} \\ &= \frac{5}{6}\cdot[\cos(at+b) + \cos(ct+d)] \end{aligned}$$

$$\begin{aligned} y_{pm} &= \frac{1}{M}\sum_{i=1}^{n} m_i r_{yi} \\ &= \frac{m_1 r_{y1} + m_2 r_{y2}}{m_1 + m_2} \\ &= \frac{m\cdot 5\sin(at+b) + 5m\cdot\sin(ct+d)}{m+5m} \\ &= \frac{5m\,[\sin(at+b) + \sin(ct+d)]}{6m} \\ &= \frac{5}{6}\cdot[\sin(at+b) + \sin(ct+d)] \end{aligned}$$

posisi pusat massa

$$\vec{x}_{pm} = \left(\frac{5}{6}\cdot[\cos(at+b) + \cos(ct+d)]\,\hat{i} + \frac{5}{6}\cdot[\sin(at+b) + \sin(ct+d)]\,\hat{j}\right) m$$

➢ b)

kecepatan pusat massa

$$\frac{d}{dt}\vec{x}_{pm} = \frac{d}{dt}\left(\frac{5}{6}\cdot[\cos(at+b) + \cos(ct+d)]\,\hat{i} + \frac{5}{6}\cdot[\sin(at+b) + \sin(ct+d)]\,\hat{j}\right)$$

$$\vec{v}_{pm} = \left(-\frac{5}{6}\cdot[a\sin(at+b) + c\sin(ct+d)]\,\hat{i} - \frac{5}{6}\cdot[a\cos(at+b) + c\cos(ct+d)]\,\hat{j}\right) m/s$$

➢ c)

pusat massa tidak bergerak apabila $\vec{v}_{pm} = 0$ *sehingga* $a = c = 0$

➢ d)

vektor tegak lurus apabila

$$\vec{r}\cdot\vec{v} = 0$$