

EDUNEX ITB



## Modul 4: Adversarial Search

### Adversarial Search

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Inteligensi Buatan  
(Artificial Intelligence)



## Adversarial Search

Approximate solution: strategy

Multiagent: competitive environment (games)

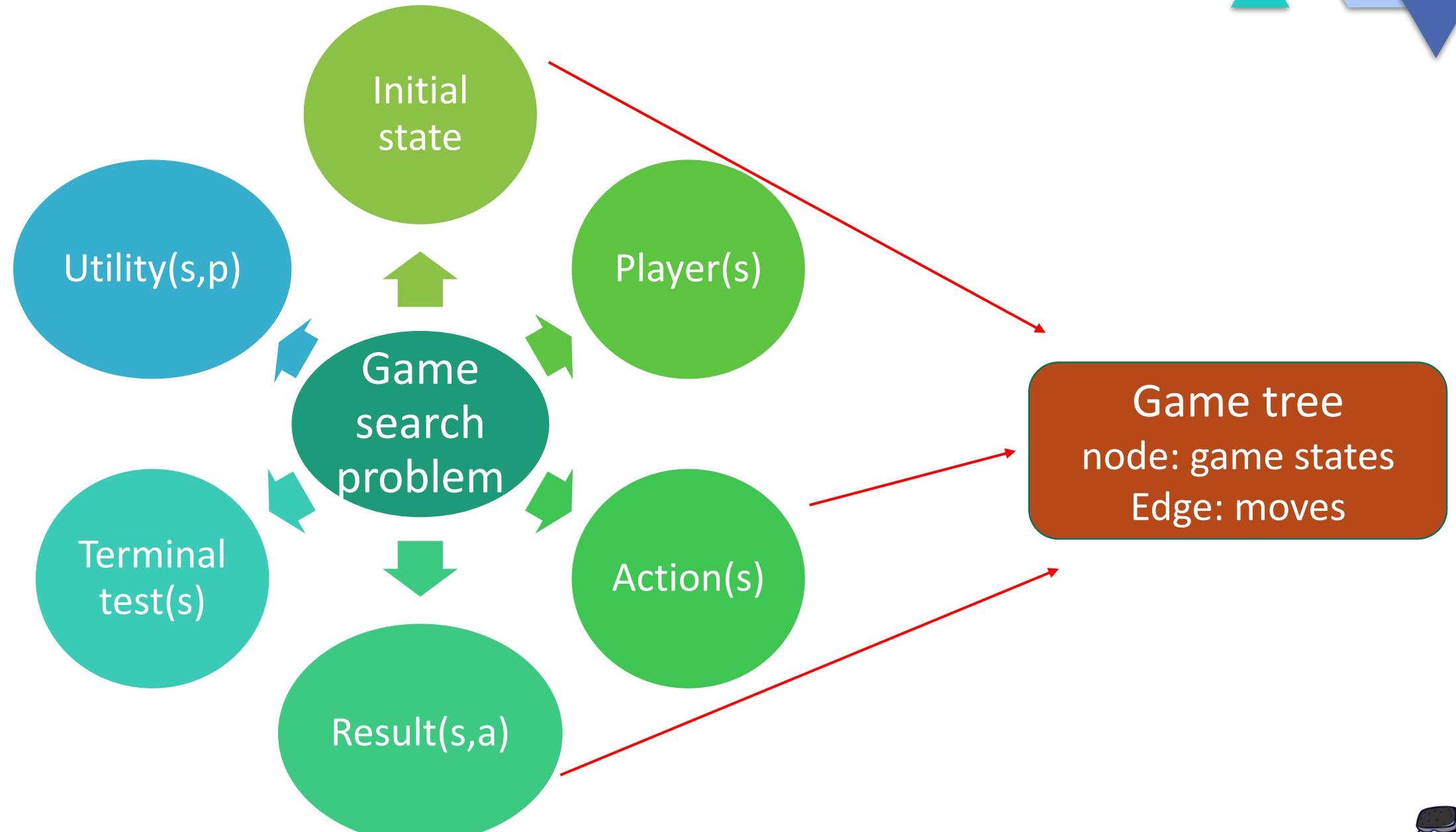
## Non-Adversarial Search

Optimize solution: path to goal or solution state

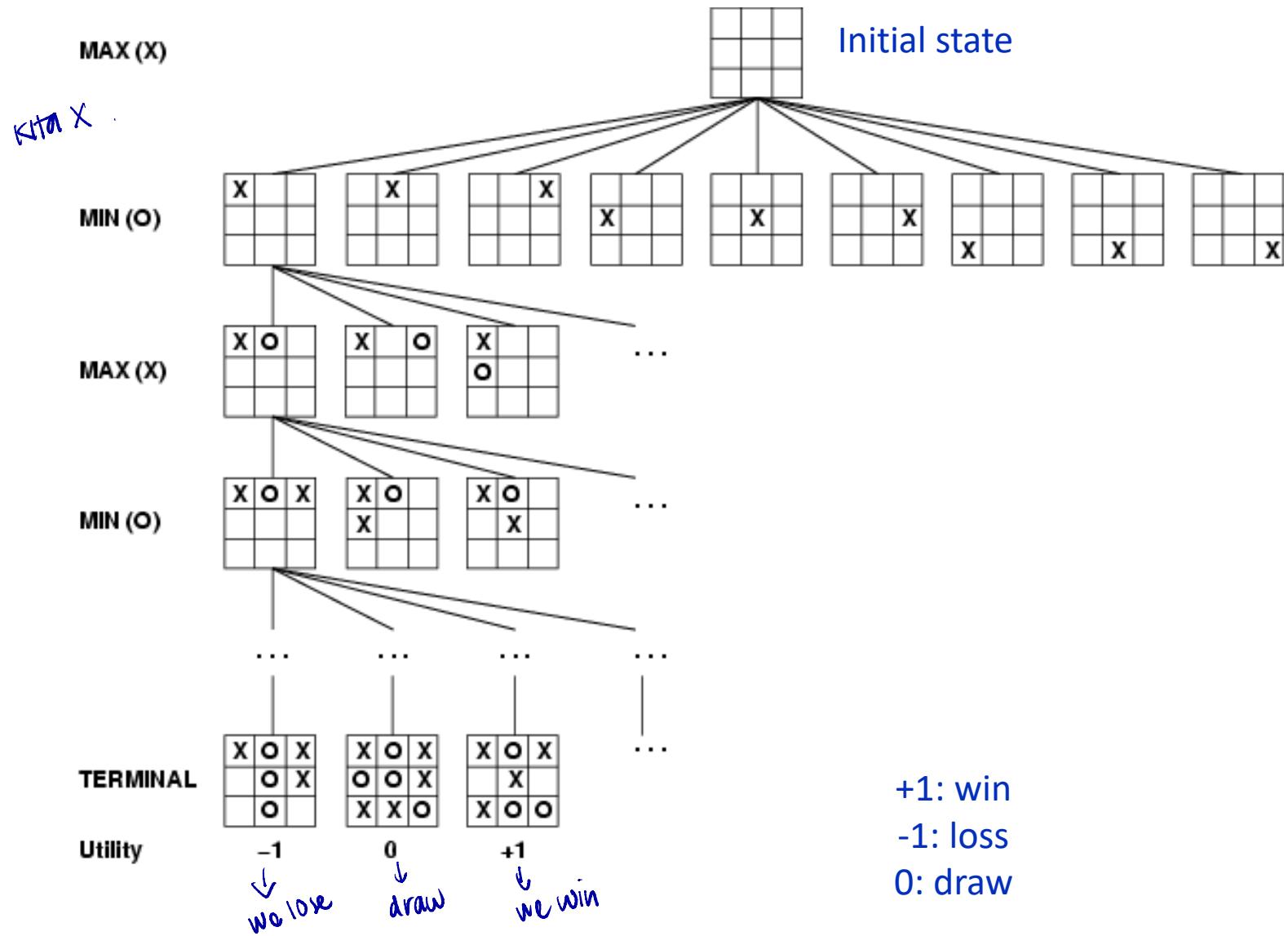
Deterministic and fully observable

Turn-taking





# Game tree (2-players, tic-tac-toe)



node: game states  
Edge: moves



# Optimal Decision with Minimax Algorithm

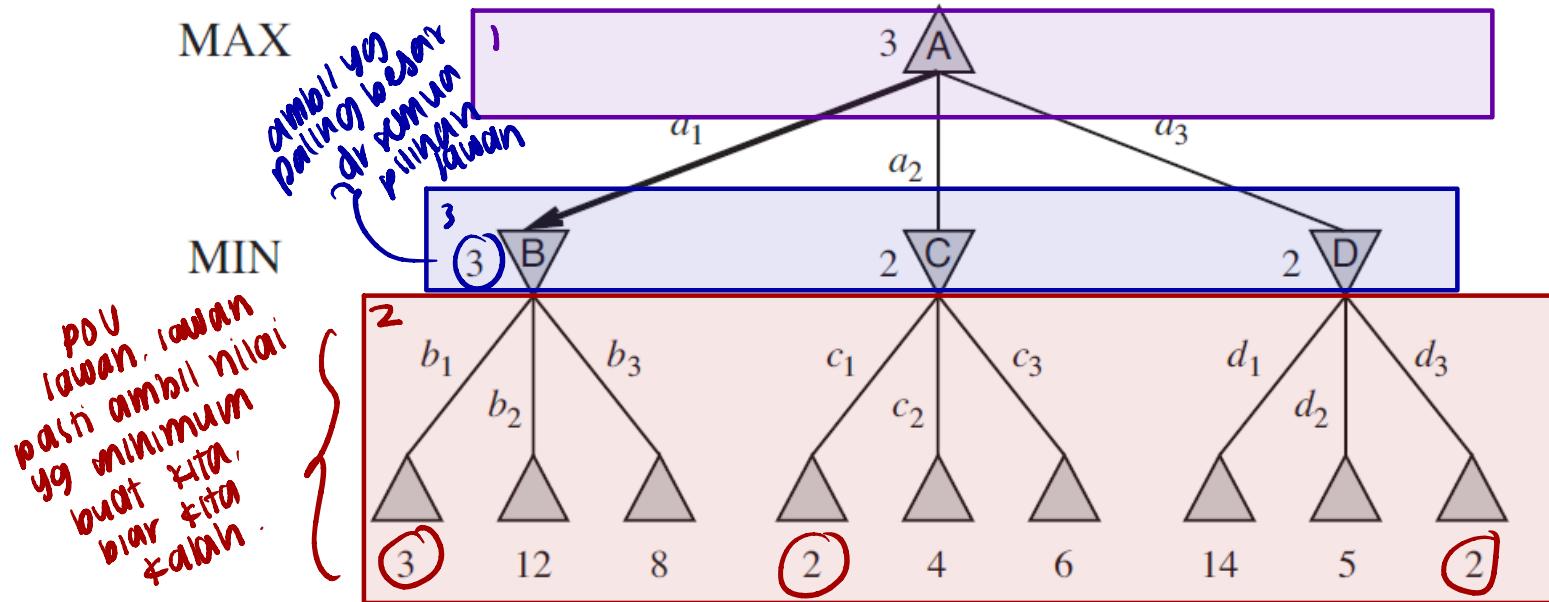
```
function MINIMAX-DECISION(state) returns an action
    return  $\arg \max_{a \in \text{ACTIONS}(s)} \text{MIN-VALUE}(\text{RESULT}(s, a))$ 
```

**function** MIN-VALUE(*state*) **returns** *a utility value*  
**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)  
 $v \leftarrow \infty$  → nilai maks.  
**for each** *a* in ACTIONS(*state*) **do**  
 $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a)))$   
**return** *v*

**function** MAX-VALUE(*state*) **returns** *a utility value*  
**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)  
 $v \leftarrow -\infty$   
**for each** *a* in ACTIONS(*state*) **do**  
 $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a)))$   
**return** *v*



# Minimax: 2-ply Game Tree



$\text{Minimax}(B) = \min(\text{Minimax}(\text{result}(B, b_1)), \text{Minimax}(\text{result}(B, b_2)), \text{Minimax}(\text{result}(B, b_3)))$   
 $= \min(3, 12, 8) = 3$

$\text{Minimax}(A) = \max(\text{Minimax}(B), \text{Minimax}(C), \text{Minimax}(D))$   
 $= \max(3, 2, 2) = 3$

$\text{MINIMAX}(s) =$

$$\begin{cases} \text{UTILITY}(s) & \text{if TERMINAL-TEST}(s) \\ \max_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if } \text{PLAYER}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if } \text{PLAYER}(s) = \text{MIN} \end{cases}$$



# Minimax 3-ply in Multiplayer Games

to move

A

B

C

A

(1, 2, 6)

(1, 2, 6)

(1, 5, 2)

(1, 2, 6)

(6, 1, 2)

(1, 5, 2)

(5, 4, 5)

(1, 2, 6)

(4, 2, 3)

(6, 1, 2)

(7, 4, 1)

(5, 1, 1)

(1, 5, 2)

(7, 7, 1)

(5, 4, 5)



# Minimax Properties

Complete?

Yes (if tree is finite)

Optimal?

Yes (against an optimal opponent)

Time complexity?

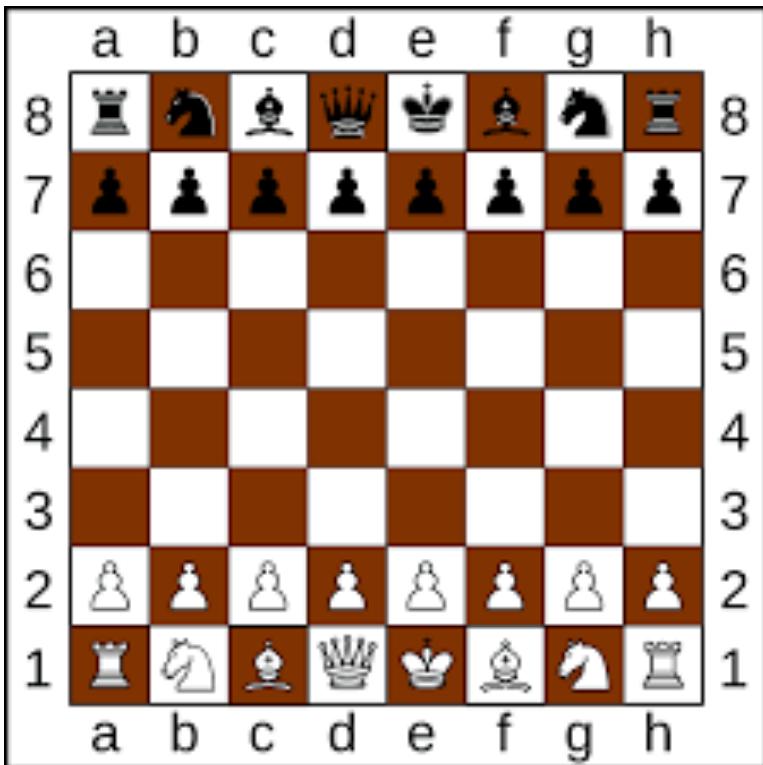
$O(b^m)$

Space complexity?

$O(bm)$  (depth-first exploration)



# Minimax for Chess



[https://commons.wikimedia.org/wiki/File:AAA\\_SVG\\_Chessboard\\_and\\_chess\\_pieces\\_02.svg](https://commons.wikimedia.org/wiki/File:AAA_SVG_Chessboard_and_chess_pieces_02.svg)

- Branching factor: 35 (avg)
- Games often 50 moves for each player →  $m=100$
- Game states is exponential in the depth of game tree.
- Exact solution is completely infeasible

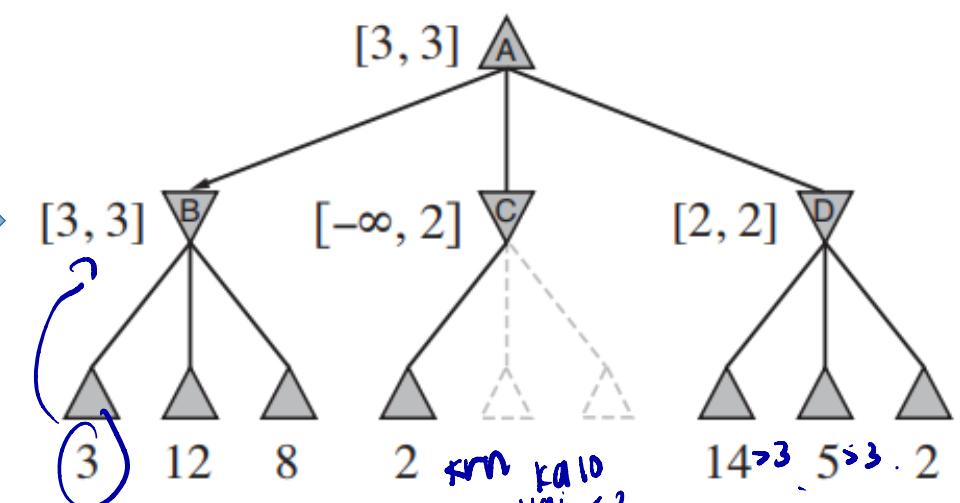
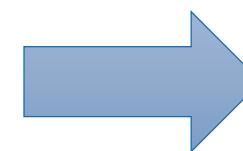
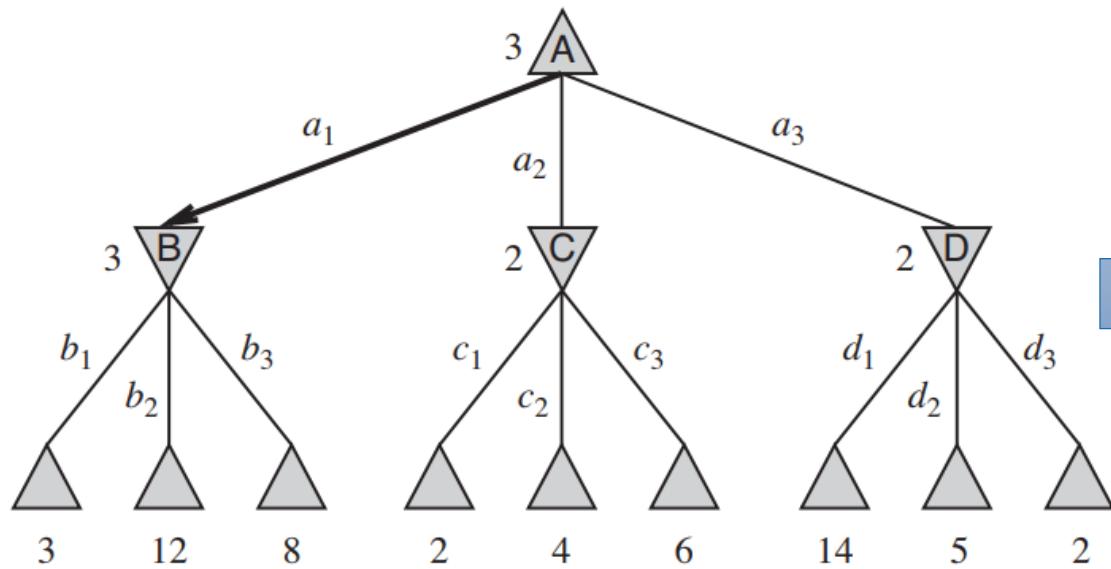


# $\alpha\beta$ Search: Minimax with Pruning

tidak mengubah hasil akhir,  
namanya memang kasih  
pohon pencarian.

MAX

MIN



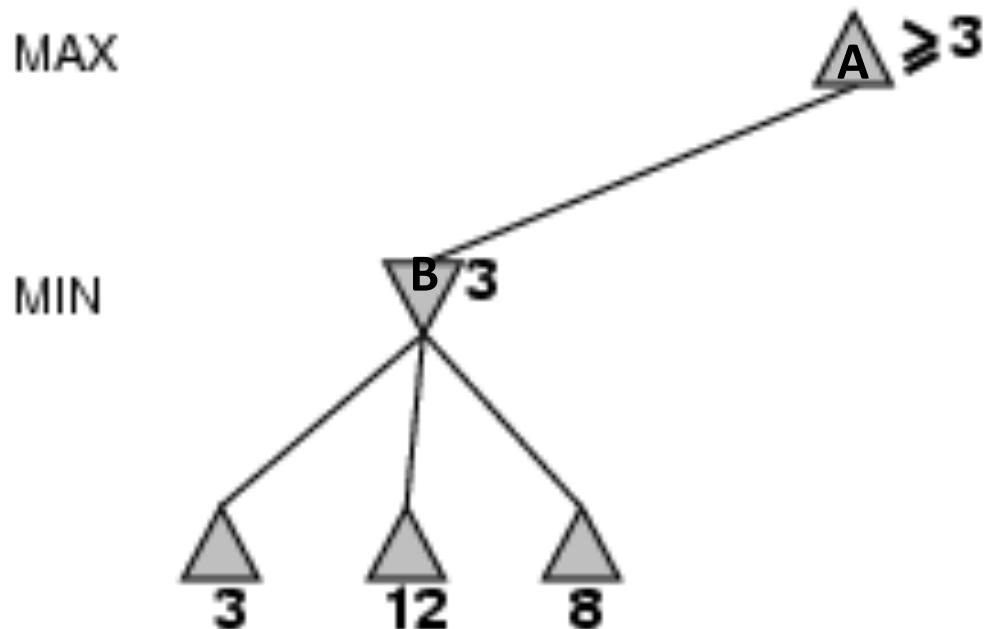
身 kalo  
ada nilai < 2,  
akan dipilih sama MIN,  
tapi maks gara-gara min itu  
karena sebagian nilai kita  
dh ada 3.

Pruning **does not** affect final result. It returns the same move as minimax would, but prunes away branches that cannot possibly influence the final decision.



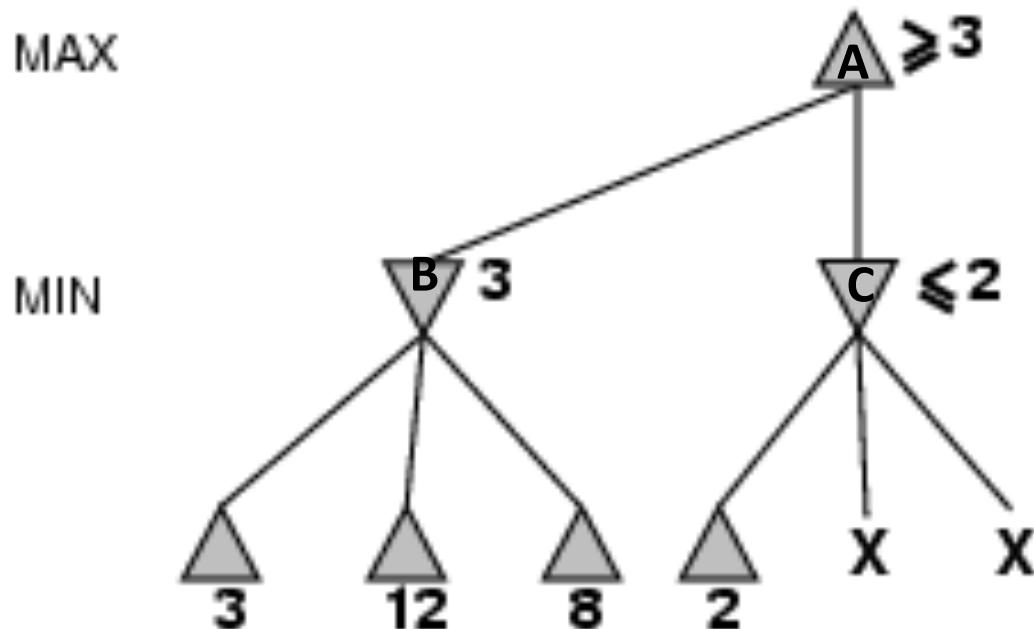


# $\alpha$ - $\beta$ pruning example

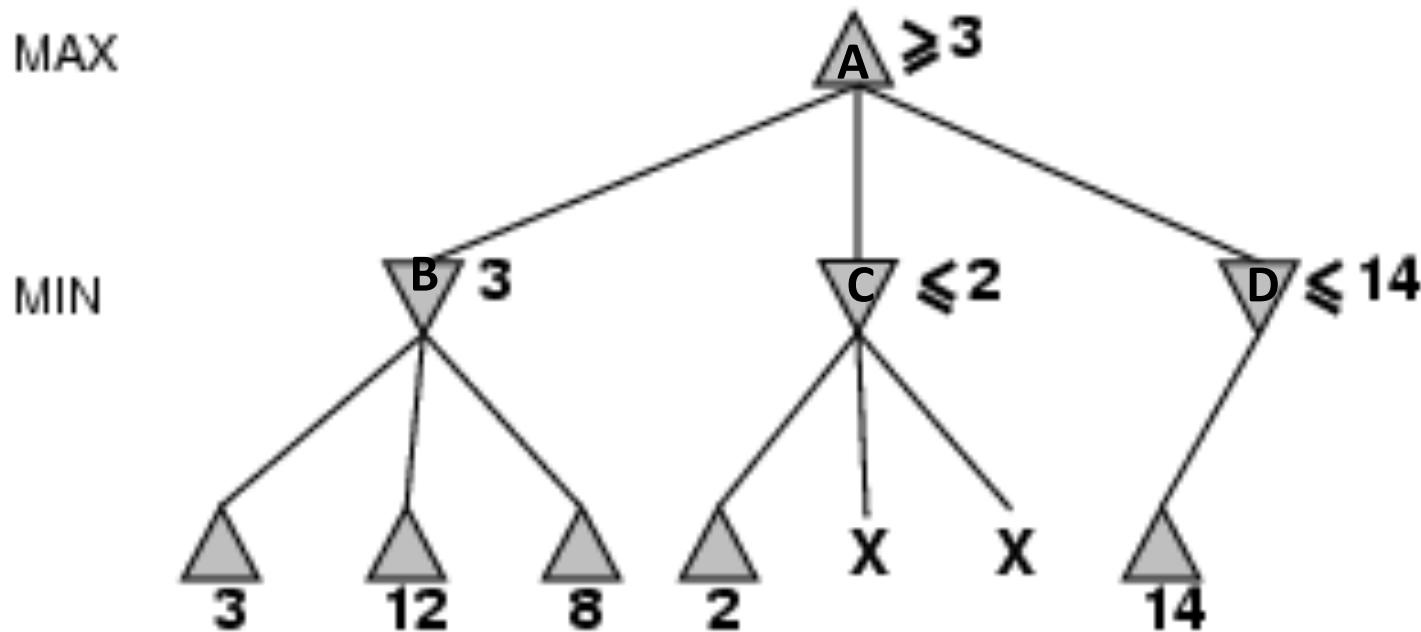




# $\alpha$ - $\beta$ pruning example

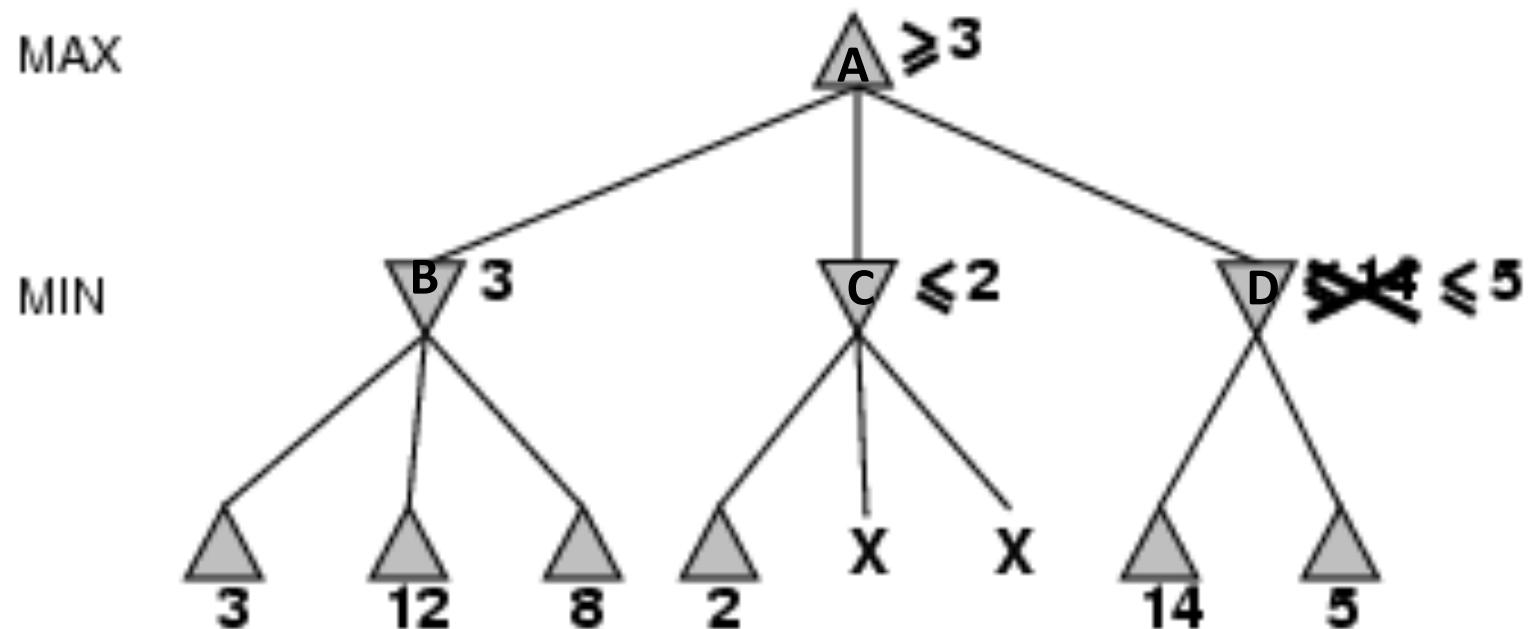


# $\alpha$ - $\beta$ pruning example



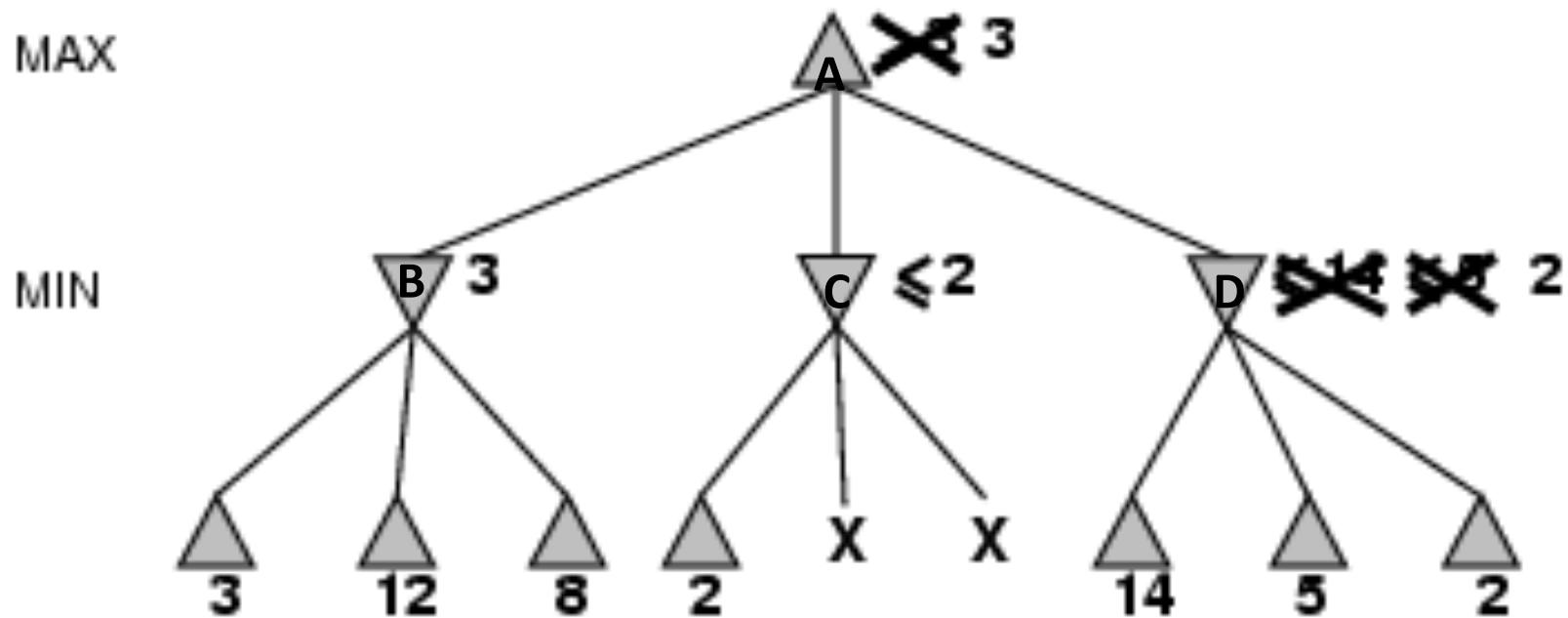


# $\alpha$ - $\beta$ pruning example

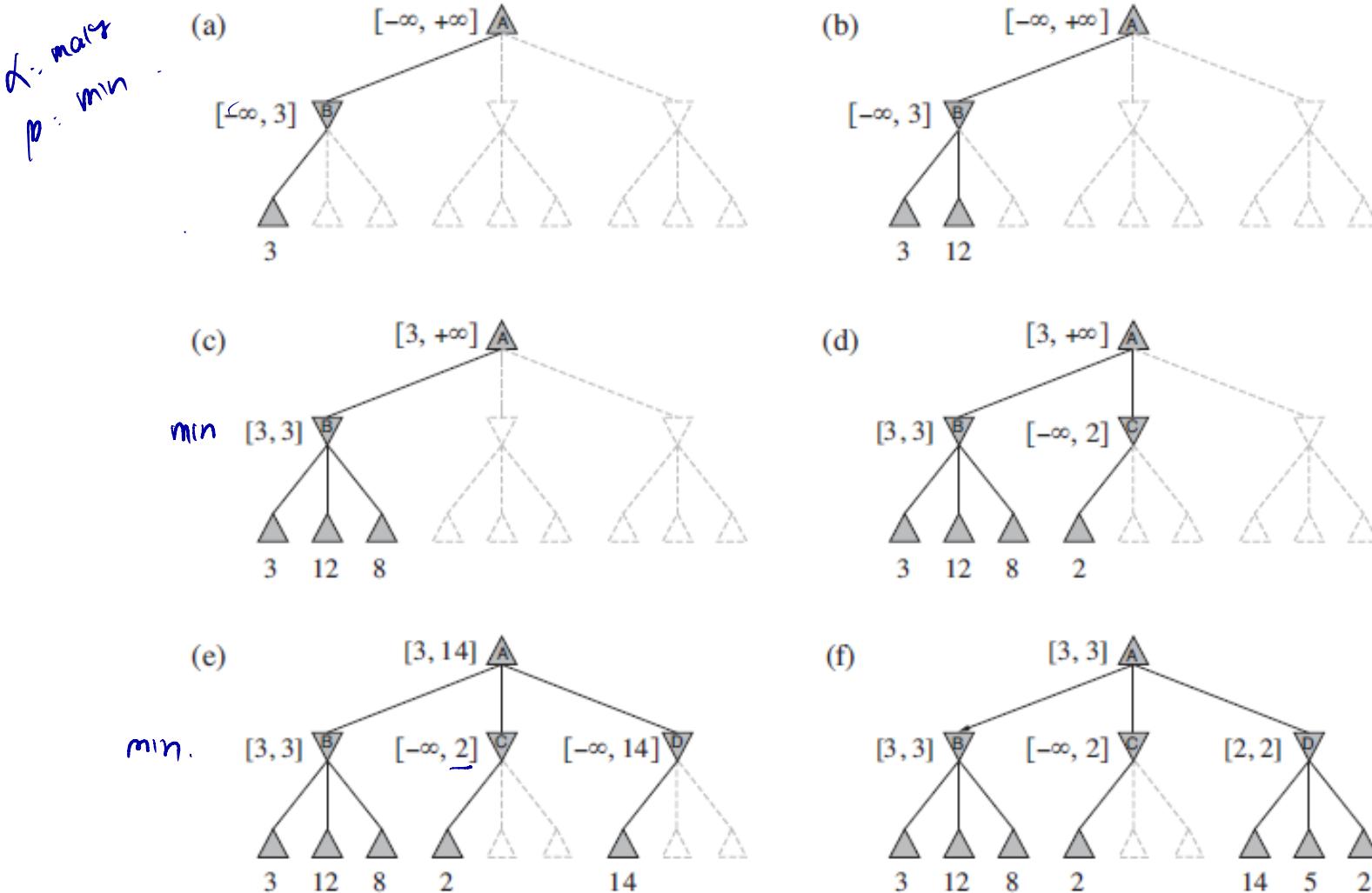




# $\alpha$ - $\beta$ pruning example



# Algorithm illustration with $\alpha$ - $\beta$ value



# Summary

Adversarial  
search

Minimax  
search

$\alpha\beta$ Search





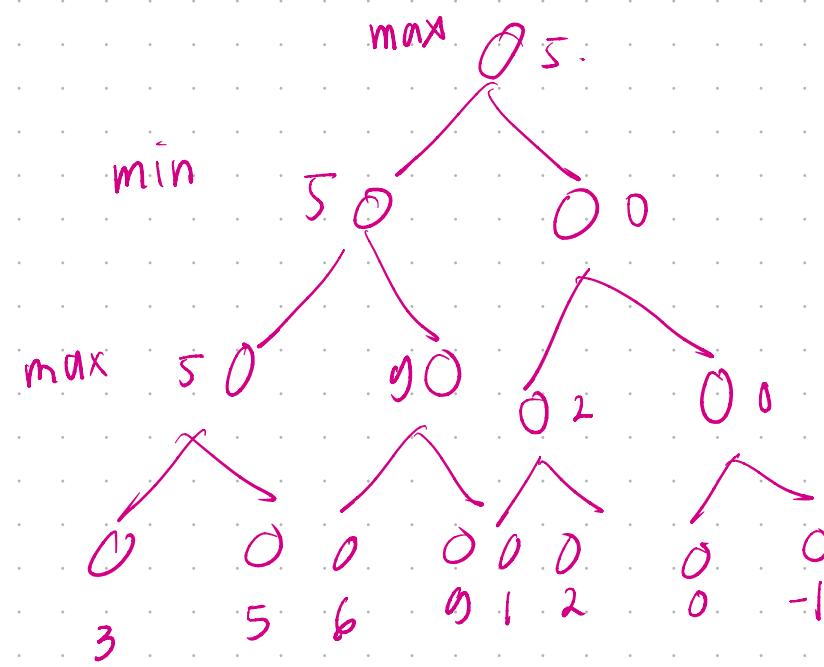
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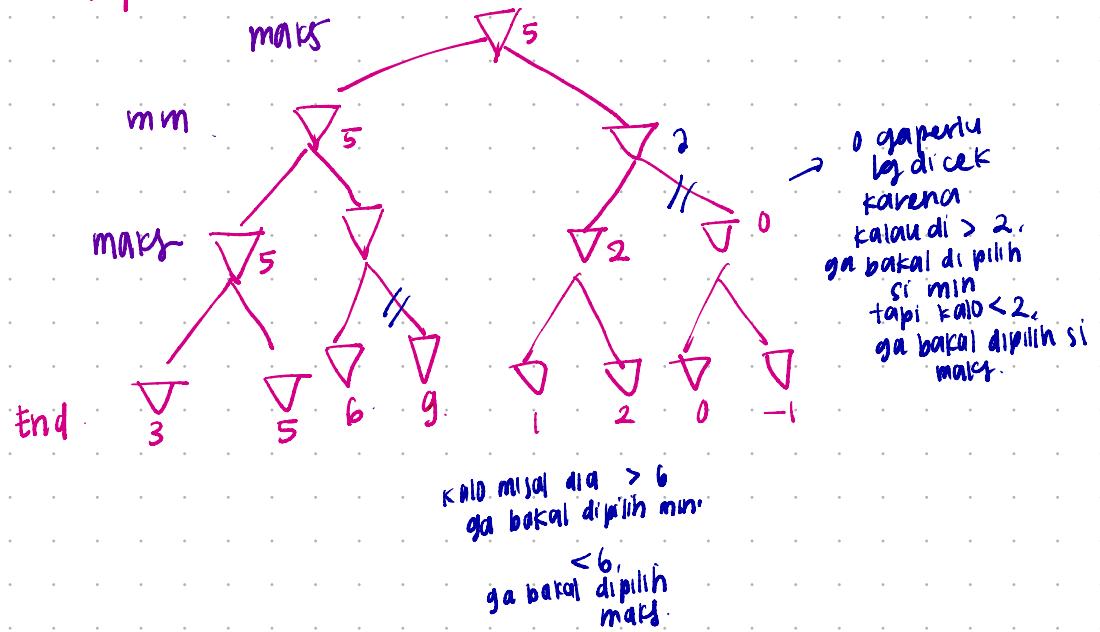
## Soal Adversarial Search

Terdapat suatu permainan berupa *3-ply game* sebagai berikut. Langkah pertama MAX yang bergerak dan memiliki dua kemungkinan langkah yaitu L atau R. Langkah berikutnya dilakukan MIN dengan dua kemungkinan langkah juga yaitu L atau R. Langkah terakhir dilakukan oleh MAX dengan dua kemungkinan langkah yaitu L atau R. Banyaknya kemungkinan urutan langkah adalah 8. Nilai pay-offs untuk MAX pada setiap kemungkinan urutan langkah adalah sebagai berikut:  $LLL = 3$ ;  $LLR = 5$ ;  $LRL = 6$ ;  $LRR = 9$ ;  $RLL = 1$ ;  $RLR = 2$ ;  $RRL = 0$ ; dan  $RRR = -1$ . MAX berusaha memaksimalkan nilai payoffs nya, sedangkan MIN berusaha meminimalkan nilai payoffs MAX.

- Gambarkan pohon *3-ply game* tersebut lengkap dengan nilai payoffs dari MAX pada setiap simpul pohon, dengan asumsi semua pemain memilih aksi secara rasional.
- Terapkan *alpha-beta pruning* saat melakukan pencarian, dan ilustrasikan dengan gambar pohon untuk setiap *pruning* yang mungkin dilakukan pada cabang pohon dan jelaskan alasannya. Aksi L dieksplorasi terlebih dahulu sebelum aksi R dalam tahapan pencarian.



## alpha beta-pruning



$\alpha$  mengubah  
nilai maks.