PROBLEM 1

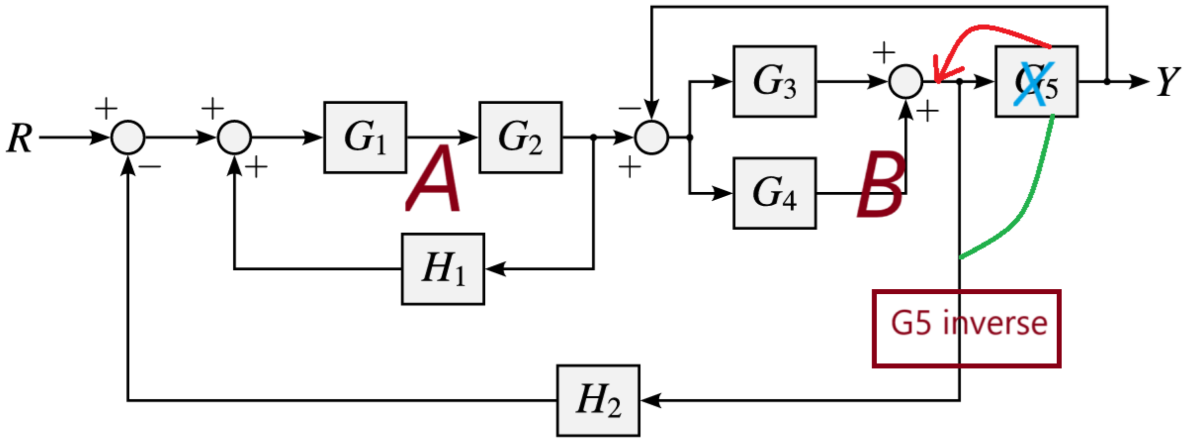

Figure 1: Complicated block diagram

1. A part:
$$A = \frac{G_1G_2}{1 - G_1G_2H_1}$$
2. B part:
$$B = \frac{(G_3 + G_4)G_5}{1 + (G_3 + G_4)G_5}$$
3. Final formula:
$$C_{\text{final}} = \frac{AB}{1 + ABH_2G_5^{-1}}$$

Substituting A and B into the final formula:

$$C_{\text{final}} = \frac{\left(\frac{G_1G_2}{1-G_1G_2H_1}\right)\left(\frac{(G_3+G_4)G_5}{1+(G_3+G_4)G_5}\right)}{1 + \left(\frac{G_1G_2}{1-G_1G_2H_1}\right)\left(\frac{(G_3+G_4)G_5}{1+(G_3+G_4)G_5}\right)H_2G_5^{-1}}$$

Multiplying the fraction by the denominator in the numerator

$$AB = \frac{G_1G_2(G_3 + G_4)G_5}{(1 - G_1G_2H_1)(1 + (G_3 + G_4)G_5)}$$

Multiply by the denominator of the numerator The denominator of AB is:

$$(1 - G_1G_2H_1)(1 + (G_3 + G_4)G_5)$$

Multiplying both numerator and denominator of C_{final} by this:

$$C_{\text{final}} = \frac{G_1G_2(G_3 + G_4)G_5}{(1 - G_1G_2H_1)(1 + (G_3 + G_4)G_5) + G_1G_2(G_3 + G_4)H_2}$$

PROBLEM 2

2a

Controllability matrix for this case has the rank of 2, which is equal to the number of states, so the system is **controllable**. The controllability matrix C_o:

$$C_o = \begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$$

The rank of the controllability matrix:

$$\text{Rank}(C_o) = 2$$

The eigenvalues of matrix A are both zero:

$$\lambda_1 = 0, \quad \lambda_2 = 0$$

2b

Full-State Feedback Gain K: -5.4140 6.8280

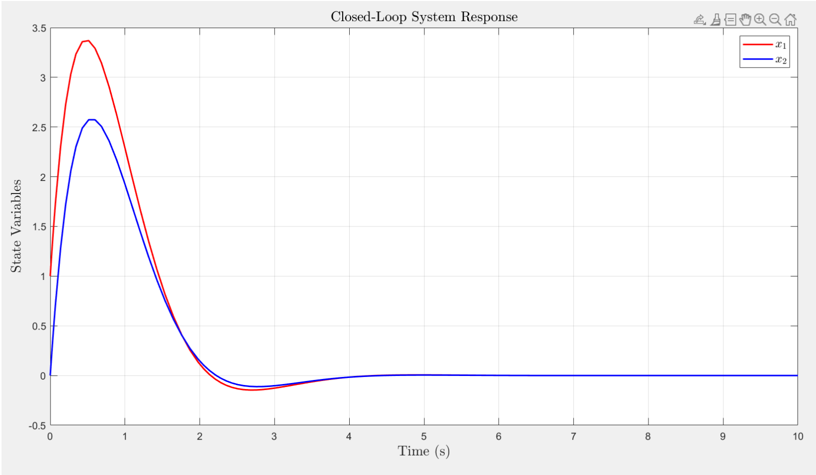

Figure 2: Closed Loop System Response

2c

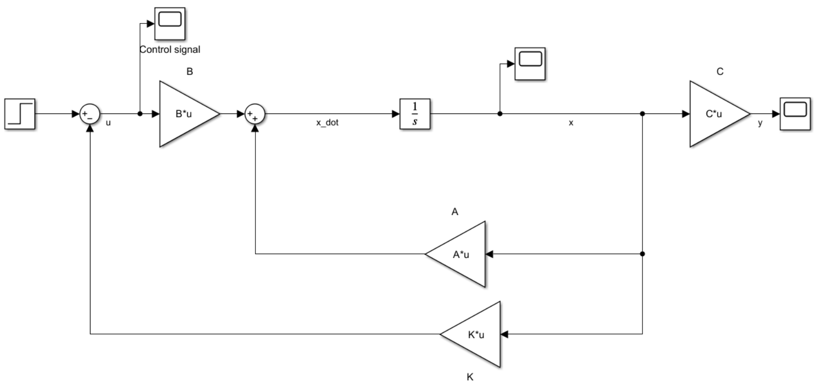

Figure 3: Simulink

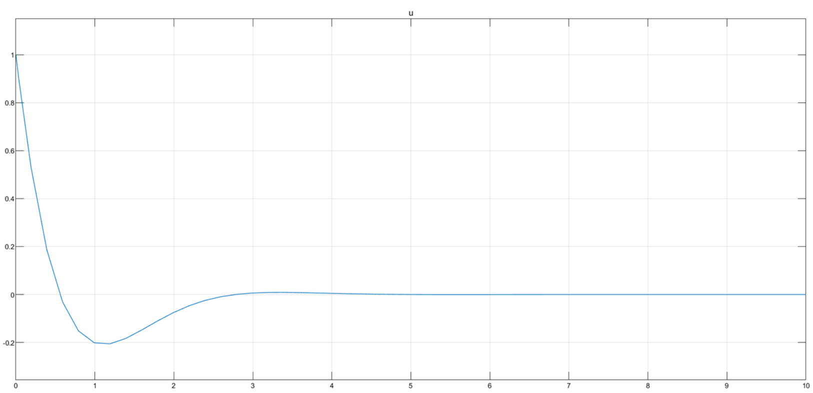

Figure 4: Control Signal

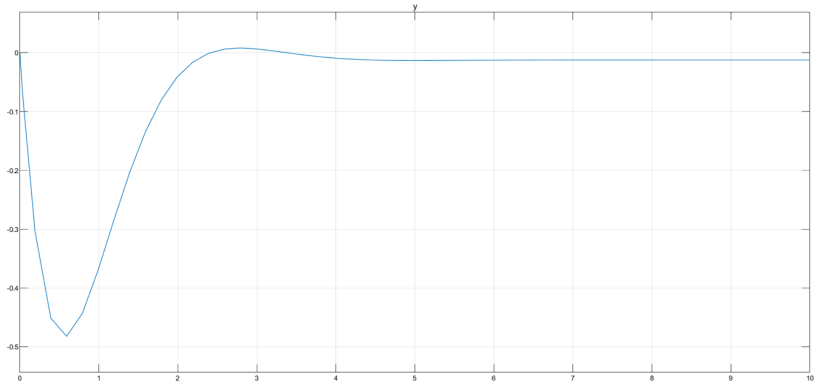

Figure 5: y(t)

2d

Plots

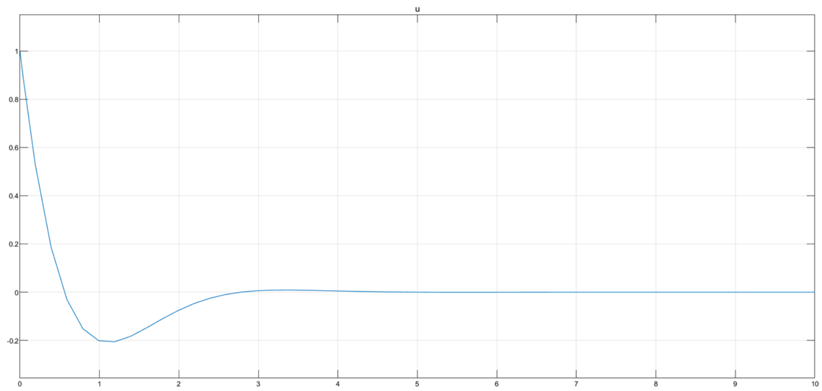

Figure 6: Control Signal

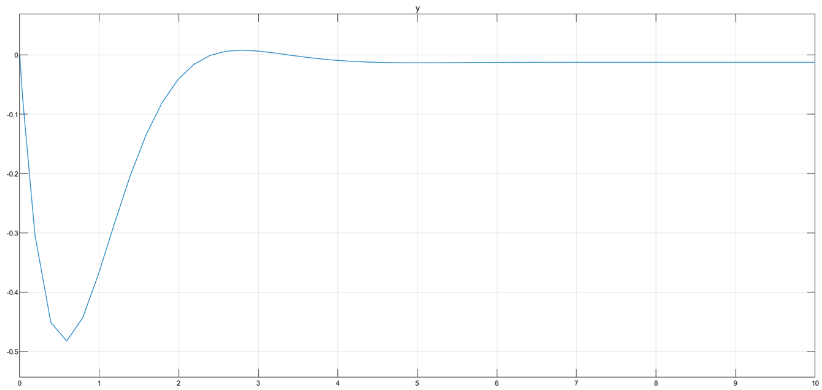

Figure 7: y(t)

2e

The step responses in both cases are nearly identical. I would say: there is no significant difference in steady-state value, they even look similar - settling time, or overshoot. The control signals also follow a similar trend. Any variations are minimal. The system remains stable and performs the same. The controller is robust to changes in BB.

PROBLEM 3

3a

The eigenvalues of matrix A are:

$$\lambda = \begin{bmatrix} 0 \\ -22.6 \\ -4644.1 \end{bmatrix}$$

3b

Design of a full-state feedback controller with desired closed-loop eigenvalues

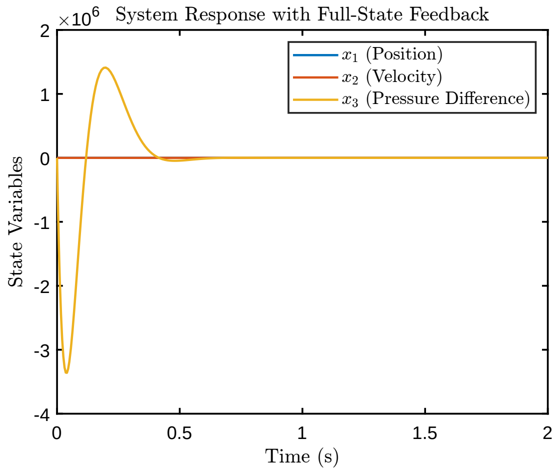

Figure 8: Result Problem 3b

3c

The steady-state error exists because the system lacks integral action to reject constant disturbances. From the simulation, the steady-state error for an external force of $f = 500N$ is :

$$e_{ss} = -0.0076$$

This confirms that the system does not fully compensate for the external force. The following simulation figure shows the steady-state error response due to the applied external force:

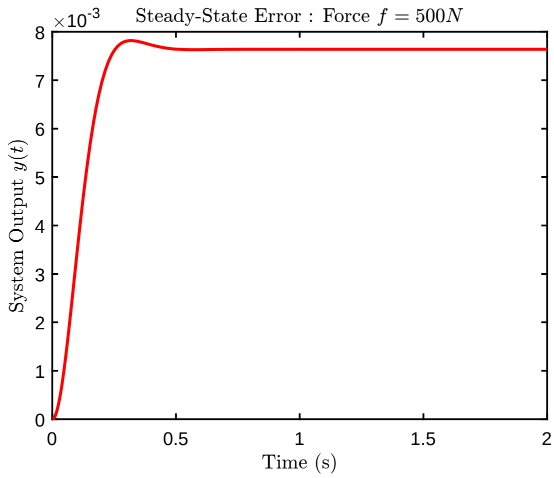

Figure 9: Result Problem 3c