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# PROBLEM 1

## 1a

Lyapunov function:

$$V(x_1, x_2) = \ln(1 + x_1^2) + x_2^2$$

Time derivative of $V$:

$$\dot{V}(x_1, x_2) = -\frac{2x_1^2}{1 + x_1^2}$$

System (a) $\dot{V}(x) \leq 0$, negative semi-definite. $\rightarrow$ Origin is Lyapunov stable, not asymptotically stable.

## 1b

Lyapunov function:

$$V(x_1, x_2) = a - a\cos(x_1) + \frac{1}{2}x_2^2$$

Time derivative of $V$:

$$\dot{V}(x_1, x_2) = 0$$

System (b): $\dot{V}(x) = 0$ (conservative). $\rightarrow$ Origin is Lyapunov stable, not asymptotically stable.

## 1c

Lyapunov function:

$$V(x_1, x_2) = 2x_1^2 - 4x_1x_2 + x_2^2$$

Time derivative of $V$:

$$\dot{V}(x_1, x_2) = -(4x_1 - 2x_2)(x_1 + x_2(x_1^2 - 1)) - x_2(4x_1 - 4x_2)$$

System (c): $V(x)$ is not positive definite. $\rightarrow$ Stability cannot be concluded from this function, since function can have zero, positive and negative values.

## 1d

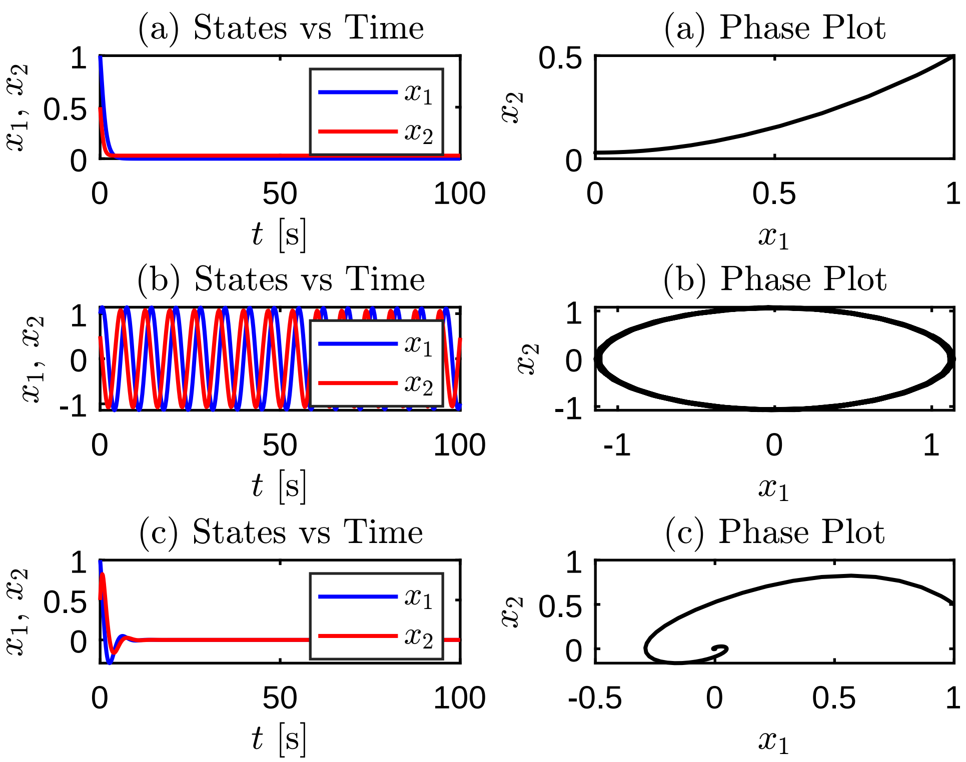



Figure 1: Simulation of all systems from an initial condition of $x_0 = [1; 0.5]$

**System (a)**: From the plots, it can be seen that $x_1(t)$ approaches zero, while $x_2(t)$ settles to a nonzero constant (seems like around 0.029 in the figure).That matches our conclusion that the entire line $\{x_1 = 0\}$ is an equilibrium, and hence the origin is only Lyapunov stable but not asymptotically stable.
**System (b)**: In that case, continual oscillations in both states are shown where we neither have decay nor blow-up. That is what we expect from a conservative system ($\dot{V} \equiv 0$). In that type of system the energy is constant and solution orbits remain on closed loops (Phase Plot b). So the equilibrium at the origin is stable but not asymptotically stable.
**System (c)**: We were not be able to asses the stability of this system earlier. The time-domain plot shows both $x_1$ and $x_2$ converging to exactly zero; the phase plane, on the other hand plots a spiral path that goes inward to the origin. I believe that confirms asymptotic convergence to $(0, 0)$, even though, given Lyapunov function was indefinite i.e, $\dot{V}$ had + - and zero values. So I would say this one is asymptotically stable using the simulaiton.

# PROBLEM 2

## 2a

The equilibrium points of the system are:

$$(0, 0), \quad (-1, 1), \quad (1, 1)$$

## 2b

- At $(0, 0)$: Eigenvalues $= -1,\ 1 \rightarrow$ **(Saddle Point) Unstable**
- At $(-1, 1)$: Eigenvalues $= -0.5 \pm 1.3229i \rightarrow$ **Stable**
- At $(1, 1)$: Eigenvalues $= 1,\ 2 \rightarrow$ **Unstable**

## 2c

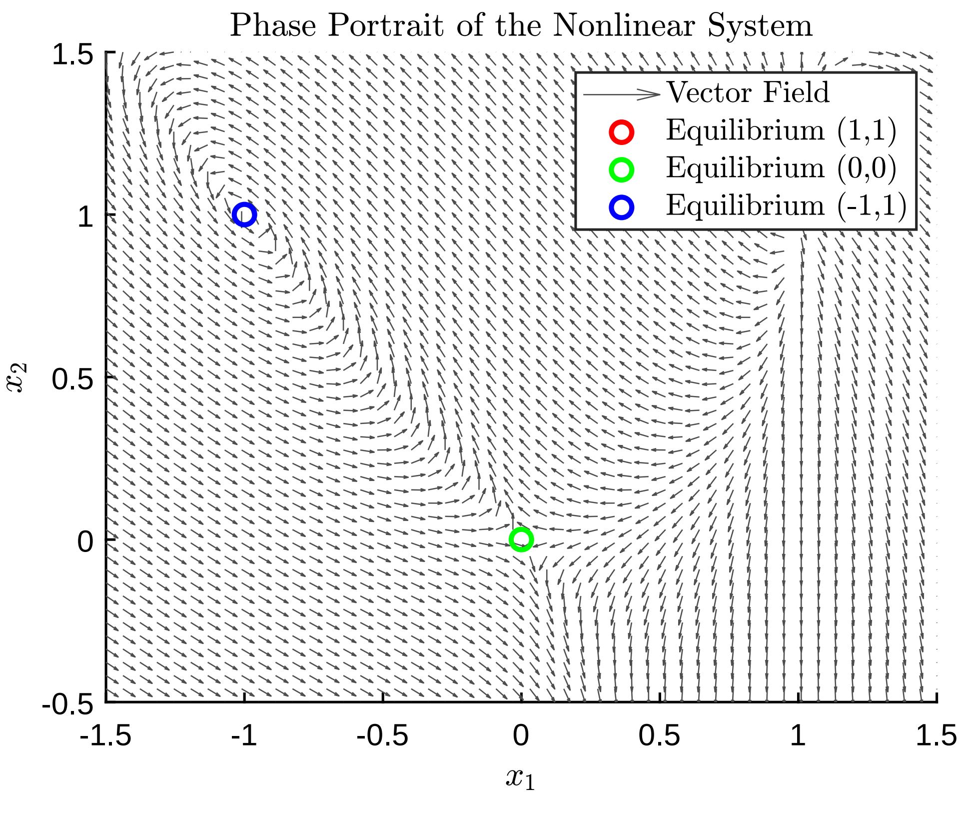



Figure 2: Equilibrium points of the system

The phase image verifies the equilibrium points at $(1, 1)$, $(0, 0)$, and $(-1, 1)$. The vector field diverges from $(1, 1)$ and $(0, 0)$, which denotes instability; however, trajectories converge toward $(-1, 1)$, proving its stability. This is consistent with the findings from parts (a) and (b).

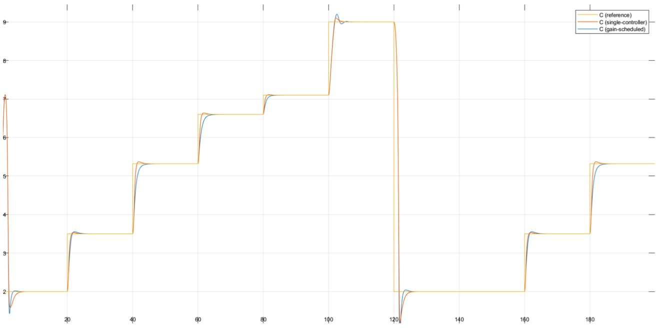
# PROBLEM 3

## 3a

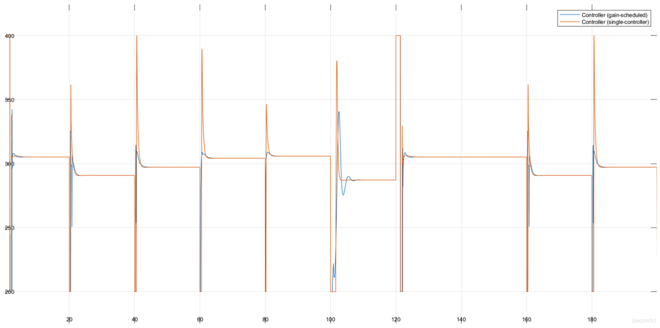
The Simulink model has two varieties of controllers: a fixed gain linear controller and 2) a gain-scheduled controller. Both of them are PIDF controllers. The fixed controller employs a singular set of PID gains tailored for a particular operating situation. These gains remain constant throughout the experiment. The gain-scheduled controller employs several gain settings, each tailored for distinct concentration values ( which can be changed in the code, I mention that in the next part). It modifies the profits during the simulation by integration according to the current concentration value. Both controllers include actuator constraints by application of upper and lower bounds on the control signal. The gain-scheduled controller manages varying concentration levels by seamlessly adjusting its gains in real time.

## 3b

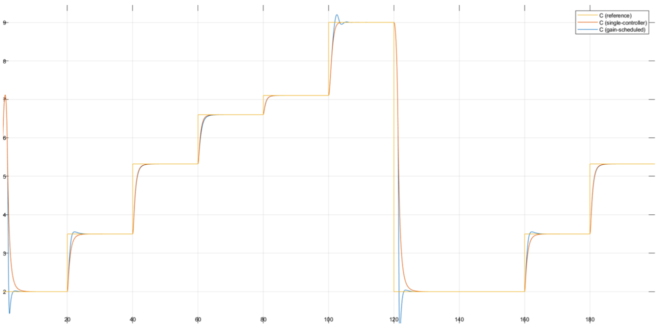
The concentration vector was defined as $C = [\mathbf{2}\ 3\ 4\ 5\ \mathbf{5.5}\ 6\ 7\ 8\ 9]^T$.
For the case $C = 2$, $I = 1$.
For the case $C = 5.5$, $I = 5$.



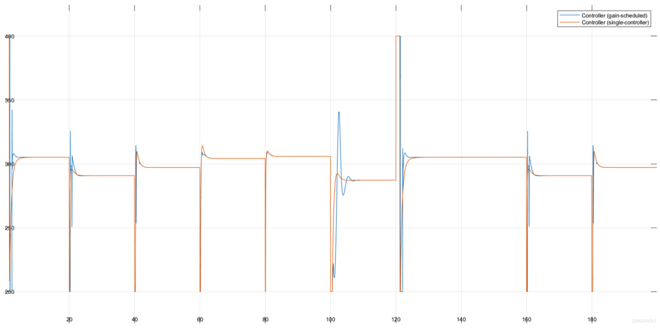

Output tracking (Ref 1, single PID at $C = 2$)



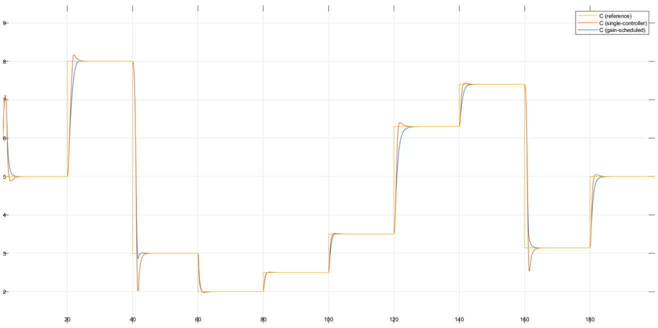

Controller signals (Ref 1, single PID at $C = 2$)



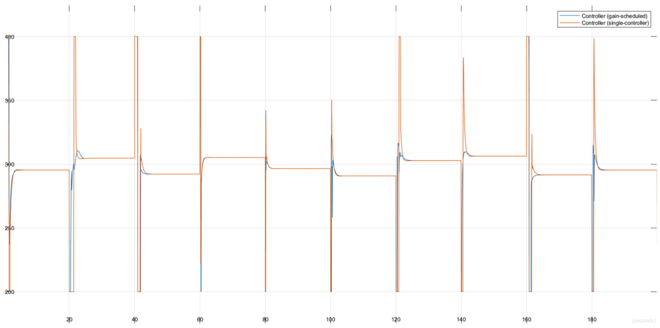

Output tracking (Ref 1, single PID at $C = 5.5$)



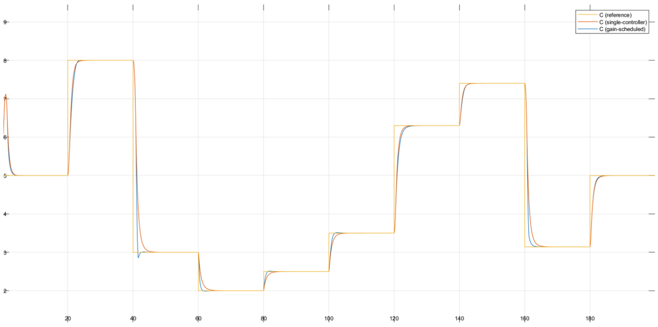

Controller signals (Ref 1, single PID at $C = 5.5$)



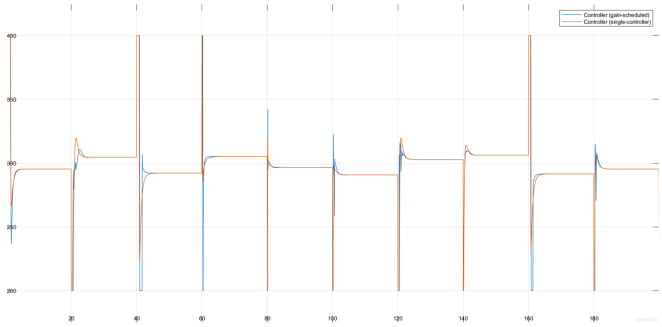

Output tracking (Ref 2, single PID at $C = 2$)




Controller signals (Ref 2, single PID at $C = 2$)




Output tracking (Ref 2, single PID at $C = 5.5$)




Controller signals (Ref 2, single PID at $C = 5.5$)

### 3c

We evaluated the performance of single-gain linear PID controllers (tuned to C = 2 and C = 5.5) and a gain-scheduled controller in regulating the system using two distinct reference signals. According to the initial reference, the controller calibrated at C = 2 had a slower convergence and larger overshoot while functioning beyond its design point, whereas the controller calibrated at C = 5.5 achieved faster settling and smoother tracking. In contrast, according to the second reference, the C = 5.5 controller exhibited somewhat worse accuracy, while the C = 2 controller had comparatively superior performance. These findings highlight the susceptibility of single-gain controllers to the particular operating point for which they were developed. Conversely, the gain-scheduled controller, evaluated independently, is anticipated to provide more seamless and resilient performance across various situations by interpolating between distinct gain settings based on real-time values of C. Simultaneously, the control signals exhibited varying degrees of aggressiveness, with off-design controllers requiring bigger or more oscillatory inputs to get the appropriate output tracking.

## PROBLEM 4

### 4a

The ideal controller parameters in Direct MRAC are:

$$\theta_1 = \frac{b_m}{b_p}, \quad \theta_2 = \frac{a_m - a_p}{b_p}$$

Given:

$$a_m = -1, \quad b_m = 1$$

Before fault:

$$a_p = 1, \quad b_p = 2$$

$$\theta_1 = \frac{1}{2} = 0.5, \quad \theta_2 = \frac{-1-1}{2} = \frac{-2}{2} = -1$$

After fault:

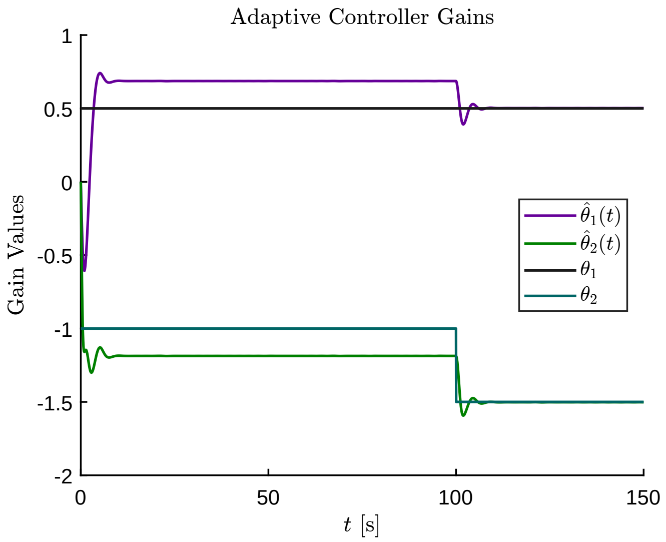
$$a_p = 2, \quad b_p = 2$$

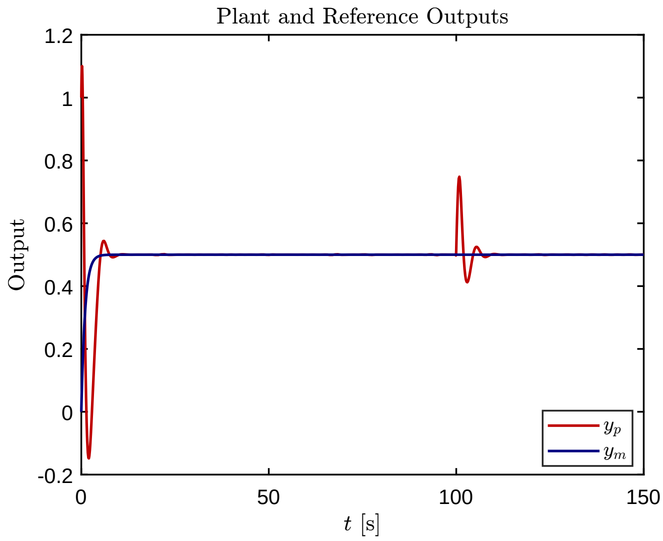
$$\theta_1 = \frac{1}{2} = 0.5, \quad \theta_2 = \frac{-1-2}{2} = \frac{-3}{2} = -1.5$$

Final results:

$$\theta_1 = 0.5 \quad \text{(unchanged)}, \quad \theta_2 = -1 \text{ (before)}, \ -1.5 \text{ (after)}$$

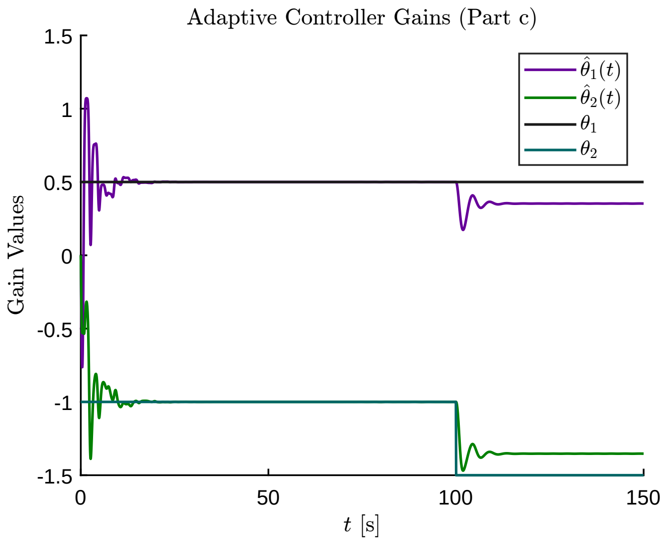
## 4b

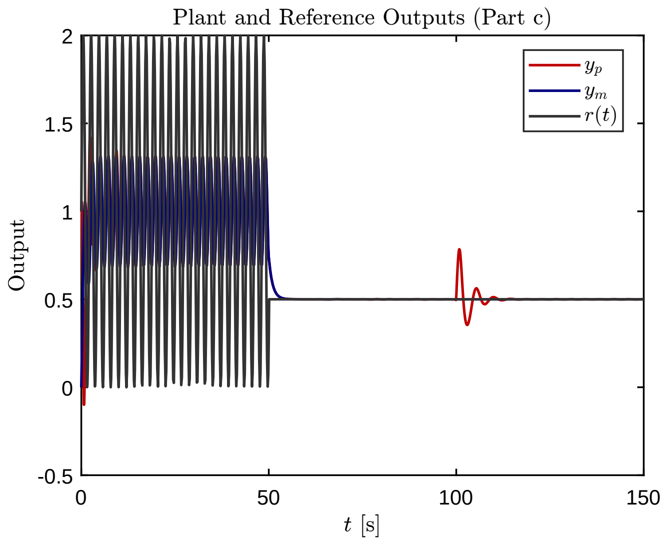



I run MRAC simulation via a constant reference input $r = 0.5$. As we can see from the plot, the system started with known parameters, and then a fault was introduced at $t = 100$ seconds: at that time point, the plant dynamics changed. The output plot tells us that the plant output $y_p$ quickly adjusts to follow the reference model $y_m$, and even after the fault, the system recovers and continues tracking well. The gain plot in its turn, shows how the controller parameters adapt: $\hat{\theta}_1$ smoothly converges to its ideal value (0.5), and $\hat{\theta}_2$ first settles around -1 and then shifts toward -1.5 after the fault. This indicates that the adaptive controller handles the fault effectively by adjusting the gains in real-time.

## 4c




Here, I change the reference signal as he question states -to be time-varying at first, with $r(t) = \sin(3t) + 1$ until $t = 50$, and then constant afterward. Firstly, the output plot shows that the plant output $y_p$ tracks the model output $y_m$ well, even when the change in reference happens quickly. There's a little bit of lag early on due to the rapid changes, but in overall I would say the tracking is solid. We can see that, when the fault at $t = 100$ occurs, the system still manages to adapt and settle nicely. The gain plot, moreover, shows both $\hat{\theta}_1$ and $\hat{\theta}_2$ responding more actively during the fast-changing input, and then gradually stabilizing around their correct values.