Assignment 2 Basic R: Matrices

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Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = \mathbf{0}$
(b) Replace the third column of A by the sum of the second and third columns

---

First, produce A

```
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A %*% A %*% A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

Then, add the columns 2 and 3 and assign the sum to the third column

```
A[,3] <- A[,2] + A[,3]

A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

---

2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \cdots & \cdots & \cdots \\ 10 & -10 & 10 \end{bmatrix}$$

```
B <- matrix(c(10,-10,10), nrow = 15, ncol = 3, byrow = TRUE)
B
```

```
##       [,1] [,2] [,3]
##  [1,]   10  -10   10
##  [2,]   10  -10   10
##  [3,]   10  -10   10
##  [4,]   10  -10   10
##  [5,]   10  -10   10
##  [6,]   10  -10   10
##  [7,]   10  -10   10
##  [8,]   10  -10   10
##  [9,]   10  -10   10
## [10,]   10  -10   10
## [11,]   10  -10   10
## [12,]   10  -10   10
## [13,]   10  -10   10
## [14,]   10  -10   10
## [15,]   10  -10   10
```

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function crossprod(). See the documentaion.

```
crossprod(B)
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

---

3. Create a 6 x 6 matrix matE with every element equal to 0. check what the functions row() and col() return when applied to matE.

Now, create the 6 x 6 matix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here is matE, a 6x6 matrix of 0's followed by row(matE) and col(matE)

```
matE <- matrix(rep(0,36), nrow = 6, byrow = TRUE)

# Note what the functions row() and col() do
row(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    1    1    1    1
## [2,]    2    2    2    2    2    2
## [3,]    3    3    3    3    3    3
## [4,]    4    4    4    4    4    4
## [5,]    5    5    5    5    5    5
## [6,]    6    6    6    6    6    6
```

```
col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    2    3    4    5    6
## [2,]    1    2    3    4    5    6
## [3,]    1    2    3    4    5    6
## [4,]    1    2    3    4    5    6
## [5,]    1    2    3    4    5    6
## [6,]    1    2    3    4    5    6
```

```
# With a little experimentation you would see
# that the specified pattern is in the |1|'s
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0   -1   -2   -3   -4   -5
## [2,]    1    0   -1   -2   -3   -4
## [3,]    2    1    0   -1   -2   -3
## [4,]    3    2    1    0   -1   -2
## [5,]    4    3    2    1    0   -1
## [6,]    5    4    3    2    1    0
```

```
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

---

4. Look at the help for the function outer(). Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

Use outer() a little more to make sure you get it.

```
B <- outer(a,a, "*")
B
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    0    0    0    0
## [2,]    0    1    2    3    4
## [3,]    0    2    4    6    8
## [4,]    0    3    6    9   12
## [5,]    0    4    8   12   16
```

```
# and
b <- 5:10
C <- outer(a,b,"+")
C
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    5    6    7    8    9   10
## [2,]    6    7    8    9   10   11
## [3,]    7    8    9   10   11   12
## [4,]    8    9   10   11   12   13
## [5,]    9   10   11   12   13   14
```

```r
# and finally  --  make sure you check the values.
D <- outer(b,a, "%%")
D
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   NA    0    1    2    1
## [2,]   NA    0    0    0    2
## [3,]   NA    0    1    1    3
## [4,]   NA    0    0    2    0
## [5,]   NA    0    1    0    1
## [6,]   NA    0    0    1    2
```

---

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```r
a <- 0:4
A <- outer(a,a,"+") %% 5
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
b <- 0:9
B <- outer(b,b,"+") %% 10
B
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
##  [5,]    4    5    6    7    8    9    0    1    2     3
```

```
##  [6,]    5    6    7    8    9    0    1    2    3    4
##  [7,]    6    7    8    9    0    1    2    3    4    5
##  [8,]    7    8    9    0    1    2    3    4    5    6
##  [9,]    8    9    0    1    2    3    4    5    6    7
## [10,]    9    0    1    2    3    4    5    6    7    8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```
c <- 0:8
C <- (outer(c,c,"-")  + 9) %% 9
C
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

---

6. Solve the following system of linear equations by setting up and solving the matrix equation $\mathbf{Ax} = \mathbf{y}$.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 7 \\ 2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 &= -1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 &= -3 \\ 4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 &= 5 \\ 5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 &= 17 \end{aligned}$$

```
A <- matrix(0, nrow = 5, ncol = 5)
A <- abs(col(A)-row(A)) + 1
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    3    4    5
## [2,]    2    1    2    3    4
## [3,]    3    2    1    2    3
## [4,]    4    3    2    1    2
## [5,]    5    4    3    2    1
```

```
y <- c(7, -1, -3, 5, 17)
solve(A,t(t(y)))
```

```
##      [,1]
## [1,]   -2
## [2,]    3
## [3,]    5
## [4,]    2
## [5,]   -4
```

---

7. Create a 6 x 10 matrix of random integers chosen from 1,2,. . . ,10 by executing the following two lines of code:

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
```

Use the matrix you have created to answer these questions:

(a) Find the number of entries in each row which are greater than 4.

```
aMat
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    3    6    7    7    2    4    3    7    1     4
## [2,]    1    9    8    7    2    6   10    9    5     2
## [3,]    7   10    8    4   10    5    4    8    4     4
## [4,]    4    3    1    1    3    3    9    7    4     2
## [5,]    1    8    1    9    9    8    1    3    7     7
## [6,]    2    6    7    5    6   10    4    6   10     1
```

```
apply(aMat, 1, function(x){sum(x>4)})
```

```
## [1] 4 7 6 2 6 7
```

(b) Which rows contain exactly two occurrences of the number seven?

```
A <- apply(aMat, 1, function(x){sum(x==7)==2})
A
```

```
## [1] FALSE FALSE FALSE FALSE  TRUE FALSE
```

```
which(A == TRUE)
```

```
## [1] 5
```

(c) Find those pairs of columns whose total (over both columns) is greater than 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for exalmple, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

```
colSums(aMat)
```

```
##  [1] 18 42 32 33 32 36 31 40 31 20
```

```
B <- outer(colSums(aMat), colSums(aMat), "+")
which(B > 75, arr.ind=TRUE)
```

```
##      row col
## [1,]   2   2
## [2,]   6   2
## [3,]   8   2
## [4,]   2   6
## [5,]   8   6
## [6,]   2   8
## [7,]   6   8
## [8,]   8   8
```

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

```
colSums(aMat)
```

```
##  [1] 18 42 32 33 32 36 31 40 31 20
```

```
B <- outer(colSums(aMat), colSums(aMat), "+")
B[(row(B) < col(B)) == FALSE] <- 0
B
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0   60   50   51   50   54   49   58   49    38
##  [2,]    0    0   74   75   74   78   73   82   73    62
##  [3,]    0    0    0   65   64   68   63   72   63    52
##  [4,]    0    0    0    0   65   69   64   73   64    53
##  [5,]    0    0    0    0    0   68   63   72   63    52
##  [6,]    0    0    0    0    0    0   67   76   67    56
##  [7,]    0    0    0    0    0    0    0   71   62    51
##  [8,]    0    0    0    0    0    0    0    0   71    60
##  [9,]    0    0    0    0    0    0    0    0    0    51
## [10,]    0    0    0    0    0    0    0    0    0     0
```

```
which(B > 75, arr.ind=TRUE)
```

```
##      row col
## [1,]   2   6
## [2,]   2   8
## [3,]   6   8
```

---

8. Calculate

(a) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+j)}$

```
sum((1:20)^4) * sum(1/(3+(1:5)))
```

```
## [1] 639215.3
```

```
# or

sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+ij)}$

```r
sum((1:20) ^ 4 / (3 + outer(1:20, 1:5, "*")))
```

```
## [1] 89912.02
```

(c) $\sum_{i=1}^{10} \sum_{j=1}^{i} \frac{i^4}{(3+ij)}$

```r
sum(outer(1:10, 1:10, function(i,j){(i>=j)*i^4/(3+i*j)}))
```

```
## [1] 6944.743
```
