

```
###Install and load the required packages and libraries
library(pROC)
```

```
## Type 'citation("pROC")' for a citation.
```

```
##
```

```
## Attaching package: 'pROC'
```

```
## The following objects are masked from 'package:stats':
```

```
##
```

```
##      cov, smooth, var
```

```
library(ROCR)
library(readxl)
library(tree)
library(magrittr)
library(tidyr)
```

```
##
```

```
## Attaching package: 'tidyr'
```

```
## The following object is masked from 'package:magrittr':
```

```
##
```

```
##      extract
```

```
library(partykit)
```

```
## Loading required package: grid
```

```
## Loading required package: libcoin
```

```
## Loading required package: mvtnorm
```

```
library(MASS)
library(dplyr)
```

```
##
```

```
## Attaching package: 'dplyr'
```

```
## The following object is masked from 'package:MASS':
```

```
##
```

```
##      select
```

```
## The following objects are masked from 'package:stats':
```

```
##
```

```
##      filter, lag
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##      intersect, setdiff, setequal, union
```

```
###Data preparation
#Prepare training data
bag_data <- read_excel('auction_clean version.xlsx') #Choose auction_clean version.xlsx
head(bag_data)
```

```
## # A tibble: 6 x 14
##   brand bag      'Production Ye~' status Location 'Auction year' color
##   <chr> <chr>      <chr>      <chr> <chr>      <dbl> <chr>
## 1 Hermès Nata Clemence Mi~ 2021      Bid di~ Hong Ko~      2022 non--
## 2 Hermès Nata Swift In & ~ 2021      Bid di~ Hong Ko~      2022 non--
## 3 Hermès White Matte Nilo~ 2014      Bid di~ Hong Ko~      2022 non--
## 4 Hermès Nata Chèvre Myso~ 2021      Bid di~ Hong Ko~      2022 non--
## 5 Hermès Metallic Silver ~ 2005      Bid di~ Hong Ko~      2022 non--
## 6 Hermès Noir Swift and C~ 2021      Bid di~ Hong Ko~      2022 non--
## # ... with 7 more variables: special_leather <dbl>, birkin <dbl>,
## #   over_sold <dbl>, lowerestimate_USD <dbl>, upperestimate_USD <dbl>,
## #   soldprice_USD <dbl>, amount_oversold <dbl>
```

```
colnames(bag_data)
```

```
## [1] "brand"      "bag"      "Production Year"
## [4] "status"     "Location" "Auction year"
## [7] "color"      "special_leather" "birkin"
## [10] "over_sold"  "lowerestimate_USD" "upperestimate_USD"
## [13] "soldprice_USD" "amount_oversold"
```

```
class(bag_data$soldprice_USD)
```

```
## [1] "numeric"
```

```
class(bag_data$brand)
```

```
## [1] "character"
```

```
bag_data$special_leather[is.na(bag_data$special_leather)] <- 0
Hermes <- ifelse(bag_data$brand == "Hermès", 1, 0)
Chanel <- ifelse(bag_data$brand == "Chanel", 1, 0)
Black <- ifelse(bag_data$color == "black", 1, 0)
bag_data <- cbind(bag_data, Hermes, Chanel, Black)
bag_data <- as.data.frame(bag_data)
head(bag_data)
```

```
##   brand
## 1 Hermès
## 2 Hermès
## 3 Hermès
## 4 Hermès
## 5 Hermès
## 6 Hermès
##
```

bag

```

## 1          Nata Clemence Mini Amazon Evelyne TPM Gold Hardware, 2021
## 2          Nata Swift In & Out Kelly 25 Retourne Palladium Hardware, 2021
## 3 White Matte Niloticus Crocodile Himalaya Birkin 30 Palladium Hardware, 2014
## 4          Nata Chèvre Mysore Geta Palladium Hardware, 2021
## 5          Metallic Silver and Bronze Chevre Birkin 25 Palladium Hardware, 2005
## 6          Noir Swift and Canvas Birkin Fray 35, 2021
##   Production Year      status Location Auction year    color
## 1          2021 Bidding is closed Hong Kong      2022 non-black
## 2          2021 Bidding is closed Hong Kong      2022 non-black
## 3          2014 Bidding is closed Hong Kong      2022 non-black
## 4          2021 Bidding is closed Hong Kong      2022 non-black
## 5          2005 Bidding is closed Hong Kong      2022 non-black
## 6          2021 Bidding is closed Hong Kong      2022 non-black
##   special_leather birkin over_sold lowerestimate_USD upperestimate_USD
## 1              0      0          1          1911.45          2803.46
## 2              0      0          1          20388.80          25486.00
## 3              1      1          1          114687.00          152916.00
## 4              0      0          0           7008.65          10194.40
## 5              0      1          1          50972.00          76458.00
## 6              0      1          0          19114.50          31857.50
##   soldprice_USD amount_oversold Hermes Chanel Black
## 1         4495.730         1692.270      1      0      0
## 2        35323.596         9837.596      1      0      0
## 3        208730.340        55814.340      1      0      0
## 4          7706.966         -2487.434      1      0      0
## 5       136477.530        60019.530      1      0      0
## 6       28901.124        -2956.376      1      0      0

```

```

bag_data <- bag_data[bag_data$brand == "Hermès" | bag_data$brand == "Chanel", ]
bag_data$production_year <- as.numeric(bag_data$`Production Year`)
bag_data$auction_year <- as.numeric(bag_data$`Auction year`)
summary(bag_data)

```

```

##      brand           bag      Production Year      status
## Length:915      Length:915      Length:915      Length:915
## Class :character Class :character Class :character Class :character
## Mode :character Mode :character Mode :character Mode :character
##
##
##
##      Location      Auction year      color      special_leather
## Length:915      Min. :2020      Length:915      Min. :0.0000
## Class :character 1st Qu.:2022      Class :character 1st Qu.:0.0000
## Mode :character  Median :2022      Mode :character  Median :0.0000
##                  Mean :2022                  Mean :0.1322
##                  3rd Qu.:2022                  3rd Qu.:0.0000
##                  Max. :2022                  Max. :1.0000
##
##      birkin      over_sold      lowerestimate_USD upperestimate_USD
## Min. :0.0000      Min. :0.0000      Min. : 100      Min. : 200
## 1st Qu.:0.0000      1st Qu.:1.0000      1st Qu.: 2242      1st Qu.: 3020
## Median :0.0000      Median :1.0000      Median : 6000      Median : 8000
## Mean :0.1989      Mean :0.7648      Mean : 8979      Mean : 11781

```

```
## 3rd Qu.:0.0000 3rd Qu.:1.0000 3rd Qu.: 12000 3rd Qu.: 15292
## Max. :1.0000 Max. :1.0000 Max. :120000 Max. :152916
## NA's :35
## soldprice_USD amount_oversold Hermes Chanel
## Min. : 126 Min. :-7697.7 Min. :0.0000 Min. :0.0000
## 1st Qu.: 3780 1st Qu.: 37.8 1st Qu.:0.0000 1st Qu.:0.0000
## Median : 8831 Median : 868.7 Median :1.0000 Median :0.0000
## Mean : 14156 Mean : 2325.9 Mean :0.7082 Mean :0.2918
## 3rd Qu.: 18900 3rd Qu.: 2851.1 3rd Qu.:1.0000 3rd Qu.:1.0000
## Max. :208730 Max. :60019.5 Max. :1.0000 Max. :1.0000
## NA's :35 NA's :35
## Black production_year auction_year
## Min. :0.0000 Min. :1956 Min. :2020
## 1st Qu.:0.0000 1st Qu.:2009 1st Qu.:2022
## Median :0.0000 Median :2016 Median :2022
## Mean :0.1137 Mean :2013 Mean :2022
## 3rd Qu.:0.0000 3rd Qu.:2021 3rd Qu.:2022
## Max. :1.0000 Max. :2022 Max. :2022
## NA's :121
```

```
#Prepare testing data
bag_prediction <- read_excel('auction_prediction.xlsx') #Choose auction_prediction.xlsx
head(bag_prediction)
```

```
## # A tibble: 6 x 15
## brand bag production_year status location auction_year color special_leather
## <chr> <chr> <dbl> <chr> <chr> <dbl> <chr> <dbl>
## 1 Herm~ Whit~ 2020 Bid di~ Hong Ko~ 2022 non~~ 1
## 2 Herm~ Whit~ 2021 Bid di~ Hong Ko~ 2022 non~~ 1
## 3 Herm~ Gris~ 2021 Bid di~ Hong Ko~ 2022 non~~ 1
## 4 Herm~ Beto~ 2020 Bid di~ Hong Ko~ 2022 non~~ NA
## 5 Herm~ Nata~ 2022 Bid di~ Hong Ko~ 2022 non~~ NA
## 6 Herm~ Limi~ 2021 Bid di~ Hong Ko~ 2022 non~~ NA
## # ... with 7 more variables: birkin <dbl>, oversold <dbl>,
## # lowerestimate_USD <dbl>, upperestimate_USD <dbl>, soldprice_USD <dbl>,
## # amount_oversold <dbl>, exchange_rate <dbl>
```

```
colnames(bag_prediction)
```

```
## [1] "brand" "bag" "production_year"
## [4] "status" "location" "auction_year"
## [7] "color" "special_leather" "birkin"
## [10] "oversold" "lowerestimate_USD" "upperestimate_USD"
## [13] "soldprice_USD" "amount_oversold" "exchange_rate"
```

```
class(bag_prediction$soldprice_USD)
```

```
## [1] "numeric"
```

```
class(bag_prediction$brand)
```

```
## [1] "character"
```

```

bag_prediction$special_leather[is.na(bag_prediction$special_leather)] <- 0
Hermes <- ifelse(bag_prediction$brand == "Hermès", 1, 0)
Chanel <- ifelse(bag_prediction$brand == "Chanel", 1, 0)
Black <- ifelse(bag_prediction$color == "black", 1, 0)
bag_prediction <- cbind(bag_prediction, Hermes, Chanel, Black)
bag_prediction <- as.data.frame(bag_prediction)
head(bag_prediction)

```

```

##      brand
## 1 Hermès
## 2 Hermès
## 3 Hermès
## 4 Hermès
## 5 Hermès
## 6 Hermès
##
##                                     bag
## 1      White Matte Niloticus Crocodile Himalaya Birkin 30 Palladium Hardware, 2020
## 2 White Matte Niloticus Crocodile Himalaya Kelly 25 Retourné Palladium Hardware, 2021
## 3      Gris Perle and Kraft Matte Alligator Mini Kelly II 20 HSS Gold Hardware, 2021
## 4      Beton and Abricot Clemence Birkin 25 HSS Brushed Gold Hardware, 2020
## 5      Nata Epsom Kelly 28 Sellier Gold Hardware, 2022
## 6      Limited Edition Nata Swift In & Out Kelly 25 Retourne Palladium Hardware, 2021
##  production_year      status  location auction_year      color
## 1      2020 Bidding is closed Hong Kong      2022 non-black
## 2      2021 Bidding is closed Hong Kong      2022 non-black
## 3      2021 Bidding is closed Hong Kong      2022 non-black
## 4      2020 Bidding is closed Hong Kong      2022 non-black
## 5      2022 Bidding is closed Hong Kong      2022 non-black
## 6      2021 Bidding is closed Hong Kong      2022 non-black
##  special_leather birkin oversold lowerestimate_USD upperestimate_USD
## 1      1      1      0      114655.52      152874.03
## 2      1      0      1      127395.03      191092.54
## 3      1      0      1      50958.01      63697.51
## 4      0      1      0      19109.25      31848.76
## 5      0      0      1      11465.55      19109.25
## 6      0      0      0      22931.10      33122.71
##  soldprice_USD amount_oversold exchange_rate Hermes Chanel Black
## 1      152491.85      -382.1851      7.8496      1      0      0
## 2      240776.60      49684.0603      NA      1      0      0
## 3      192621.28      128923.7668      NA      1      0      0
## 4      24077.66      -7771.0966      NA      1      0      0
## 5      22472.48      3363.2287      NA      1      0      0
## 6      28893.19      -4229.5149      NA      1      0      0

```

```

bag_prediction <- bag_prediction[bag_prediction$brand == "Hermès" | bag_prediction$brand == "Chanel", ]
bag_prediction$production_year <- as.numeric(bag_prediction$production_year)
bag_prediction$auction_year <- as.numeric(bag_prediction$auction_year)
summary(bag_prediction)

```

```

##      brand      bag      production_year      status
## Length:94      Length:94      Min. :2001      Length:94
## Class :character      Class :character      1st Qu.:2017      Class :character

```

```

## Mode :character Mode :character Median :2020 Mode :character
## Mean :2018
## 3rd Qu.:2022
## Max. :2022
## NA's :2
## location auction_year color special_leather
## Length:94 Min. :2022 Length:94 Min. :0.0000
## Class :character 1st Qu.:2022 Class :character 1st Qu.:0.0000
## Mode :character Median :2022 Mode :character Median :0.0000
## Mean :2022 Mean :0.1915
## 3rd Qu.:2022 3rd Qu.:0.0000
## Max. :2022 Max. :1.0000
##
## birkin oversold lowerestimate_USD upperestimate_USD
## Min. :0.0000 Min. :0.0000 Min. : 573.3 Min. : 828.1
## 1st Qu.:0.0000 1st Qu.:0.0000 1st Qu.: 4458.8 1st Qu.: 7006.7
## Median :0.0000 Median :0.0000 Median : 7643.7 Median : 10191.6
## Mean :0.2447 Mean :0.4681 Mean : 13304.0 Mean : 19032.7
## 3rd Qu.:0.0000 3rd Qu.:1.0000 3rd Qu.: 14013.5 3rd Qu.: 20383.2
## Max. :1.0000 Max. :1.0000 Max. :127395.0 Max. :191092.5
##
## soldprice_USD amount_oversold exchange_rate Hermes
## Min. : 963.1 Min. : -7771.1 Min. :7.85 Min. :0.0000
## 1st Qu.: 6179.9 1st Qu.: -1220.4 1st Qu.:7.85 1st Qu.:1.0000
## Median : 11236.2 Median : -108.9 Median :7.85 Median :1.0000
## Mean : 20877.5 Mean : 1844.9 Mean :7.85 Mean :0.9574
## 3rd Qu.: 20466.0 3rd Qu.: 1031.9 3rd Qu.:7.85 3rd Qu.:1.0000
## Max. :240776.6 Max. :128923.8 Max. :7.85 Max. :1.0000
## NA's :93
## Chanel Black
## Min. :0.00000 Min. :0.00000
## 1st Qu.:0.00000 1st Qu.:0.00000
## Median :0.00000 Median :0.00000
## Mean :0.04255 Mean :0.01064
## 3rd Qu.:0.00000 3rd Qu.:0.00000
## Max. :1.00000 Max. :1.00000
##

```

### ###Linear regression

#### #lm for sold\_price

```

bag_data$production_year <- as.numeric(bag_data$production_year)
bag_data$auction_year <- as.numeric(bag_data$auction_year)
price_predict_model <- lm(soldprice_USD ~ auction_year + special_leather +
                          birkin + upperestimate_USD + Hermes + Black,
                          data = bag_data)
summary(price_predict_model)

```

```

##
## Call:
## lm(formula = soldprice_USD ~ auction_year + special_leather +
##     birkin + upperestimate_USD + Hermes + Black, data = bag_data)
##
## Residuals:

```

```
##      Min      1Q Median      3Q      Max
## -14002 -1608   -406    989  51568
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.613e+06  5.057e+05  -3.190  0.00147 **
## auction_year   7.983e+02  2.502e+02   3.191  0.00147 **
## special_leather -1.548e+03  5.214e+02  -2.970  0.00306 **
## birkin         1.073e+03  4.451e+02   2.410  0.01616 *
## upperestimate_USD 1.173e+00  1.342e-02  87.381 < 2e-16 ***
## Hermes        -5.353e+02  4.389e+02  -1.220  0.22288
## Black         -2.355e+02  5.646e+02  -0.417  0.67668
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4769 on 873 degrees of freedom
## (35 observations deleted due to missingness)
## Multiple R-squared:  0.9267, Adjusted R-squared:  0.9262
## F-statistic: 1839 on 6 and 873 DF, p-value: < 2.2e-16
```

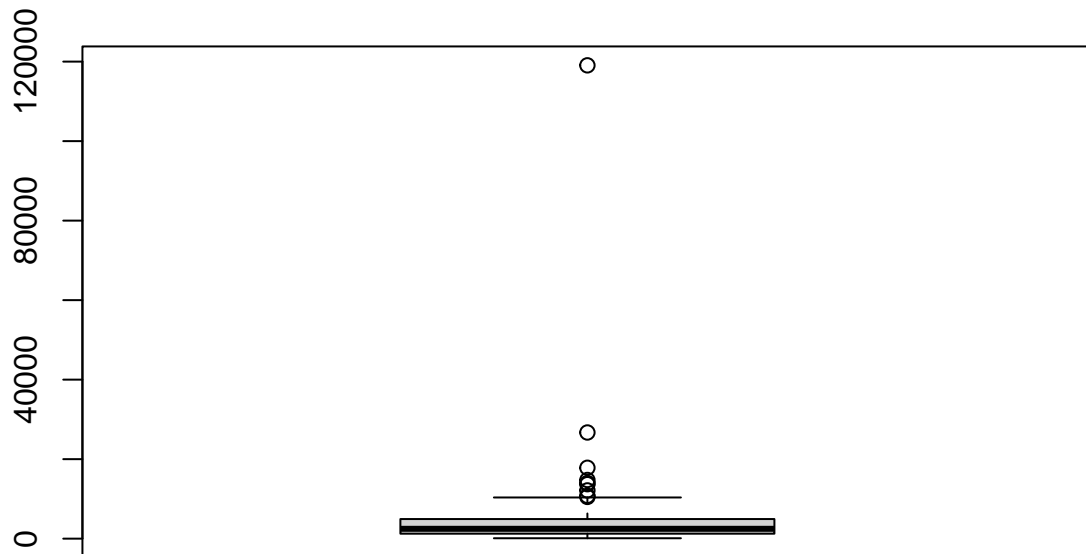
```
predicted_sold_price <- predict(price_predict_model, newdata = bag_prediction)
bag_prediction <- cbind(bag_prediction, predicted_sold_price)
diff <- abs(bag_prediction$soldprice_USD - bag_prediction$predicted_sold_price)
bag_prediction <- cbind(bag_prediction, diff)
min(diff)
```

```
## [1] 106.0959
```

```
max(diff)
```

```
## [1] 119060.8
```

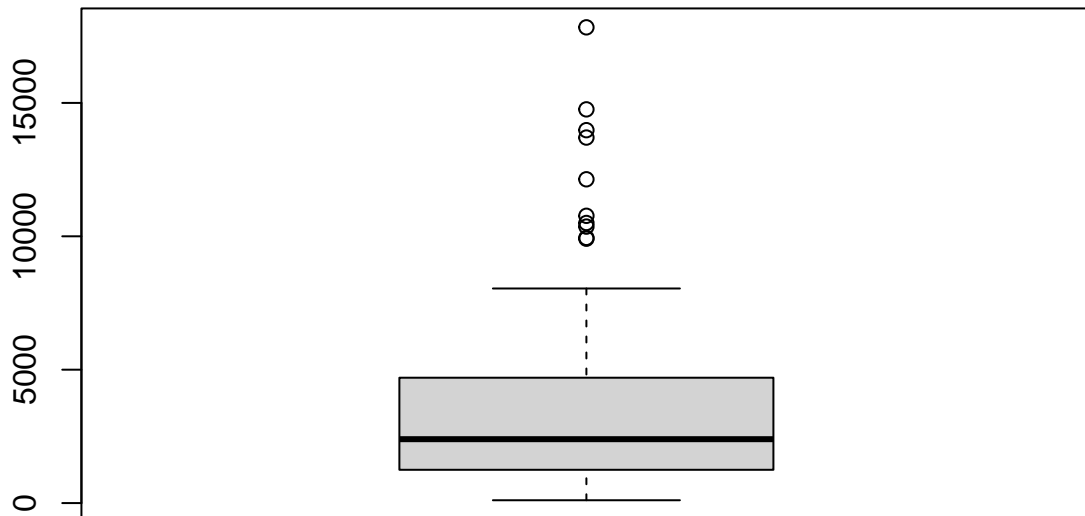
```
boxplot(diff)
```



```
#Drop outlier  
bag_prediction <- bag_prediction[-c(1,3),]  
  
#Plot the boxplot of differences as shown in figure 4  
boxplot(bag_prediction$diff, main = "Distribution of Difference ($) Between Actual and Predicted")
```

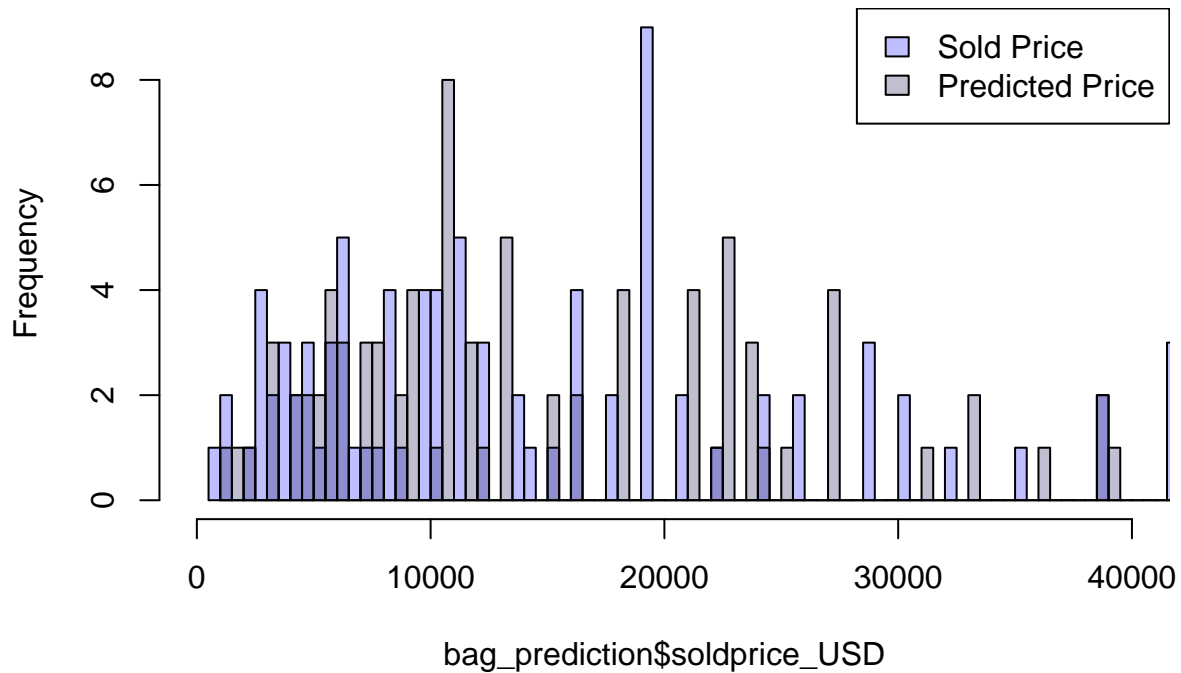


## Distribution of Difference (\$) Between Actual and Predicted



```
#Plot the histogram of bag prices as shown in figure 4
hgA <- hist(bag_prediction$soldprice_USD, breaks = 500, plot = FALSE)
hgB <- hist(bag_prediction$predicted_sold_price, breaks = 500, plot = FALSE)
plot(hgA, xlim = c(0, 40000), col=rgb(0,0,1,1/4), main = "Histogram of Bag Price")
plot(hgB, xlim = c(0, 40000), col=rgb(0,0,1/4,1/4), add = TRUE)
legend('topright', c('Sold Price', 'Predicted Price'), fill=c(rgb(0,0,1,1/4), rgb(0,0,1/4,1/4)))
```

## Histogram of Bag Price



```
###5-fold Cross validation of linear regression models
nfold <- 5
n <- nrow(bag_data)
foldid <- rep(1:nfold,each=ceiling(n/nfold))[sample(1:n)]
### create an empty dataframe of results
OOS <- data.frame(linear1=rep(NA,nfold), linear2=rep(NA,nfold),linear3=rep(NA,nfold))
bag_data <- bag_data %>% drop_na()
deviance <- function(y, pred, family=c("gaussian","binomial")){
  family <- match.arg(family)
  if(family=="gaussian"){
    return( sum( (y-pred)^2 ) )
  }else{
    if(is.factor(y)) y <- as.numeric(y)>1
    return( -2*sum( y*log(pred) + (1-y)*log(1-pred) ) )
  }
}

R2 <- function(y, pred, family=c("gaussian","binomial")){
  fam <- match.arg(family)
  if(fam=="binomial"){
    if(is.factor(y)){ y <- as.numeric(y)>1 }
  }
  dev <- deviance(y, pred, family=fam)
  dev0 <- deviance(y, mean(y), family=fam)
  return(1-dev/dev0)
}
```

```

for(k in 1:nfold){
  train <- which(foldid!=k) # train on all but fold `k`
  colnames(bag_data)
  model.linear1 <-lm(soldprice_USD ~ auction_year + special_leather +
                    birkin + upperestimate_USD + Hermes + Black,
                    data = bag_data, subset=train)
  model.linear2 <-lm(soldprice_USD ~ special_leather +
                    birkin + Hermes + Black,
                    data = bag_data, subset=train)
  model.linear3 <-lm(soldprice_USD ~ birkin + Hermes + Black,
                    data = bag_data, subset=train)

  pred.linear1 <- predict(model.linear1, newdata=bag_data[-train,])
  pred.linear2 <- predict(model.linear2, newdata=bag_data[-train,])
  pred.linear3 <- predict(model.linear3, newdata=bag_data[-train,])

  ## calculate and log R2
  OOS$linear1[k] <- R2(y=bag_data$soldprice_USD[-train], pred=pred.linear1, family = "gaussian")
  OOS$linear1[k]
  OOS$linear2[k] <- R2(y=bag_data$soldprice_USD[-train], pred=pred.linear2, family = "gaussian")
  OOS$linear2[k]
  OOS$linear3[k] <- R2(y=bag_data$soldprice_USD[-train], pred=pred.linear3, family = "gaussian")
  OOS$linear3[k]
}
OOS

```

```

##      linear1  linear2  linear3
## 1 0.9031282 0.1767346 0.1583123
## 2 0.9420573 0.2913947 0.2012516
## 3 0.8392498 0.2249945 0.2269542
## 4 0.9373173 0.2216785 0.1440821
## 5 0.9389078 0.2814375 0.1637521

```

```

#We have nfold values in OOS for each model, this computes the mean of them
colMeans(OOS)

```

```

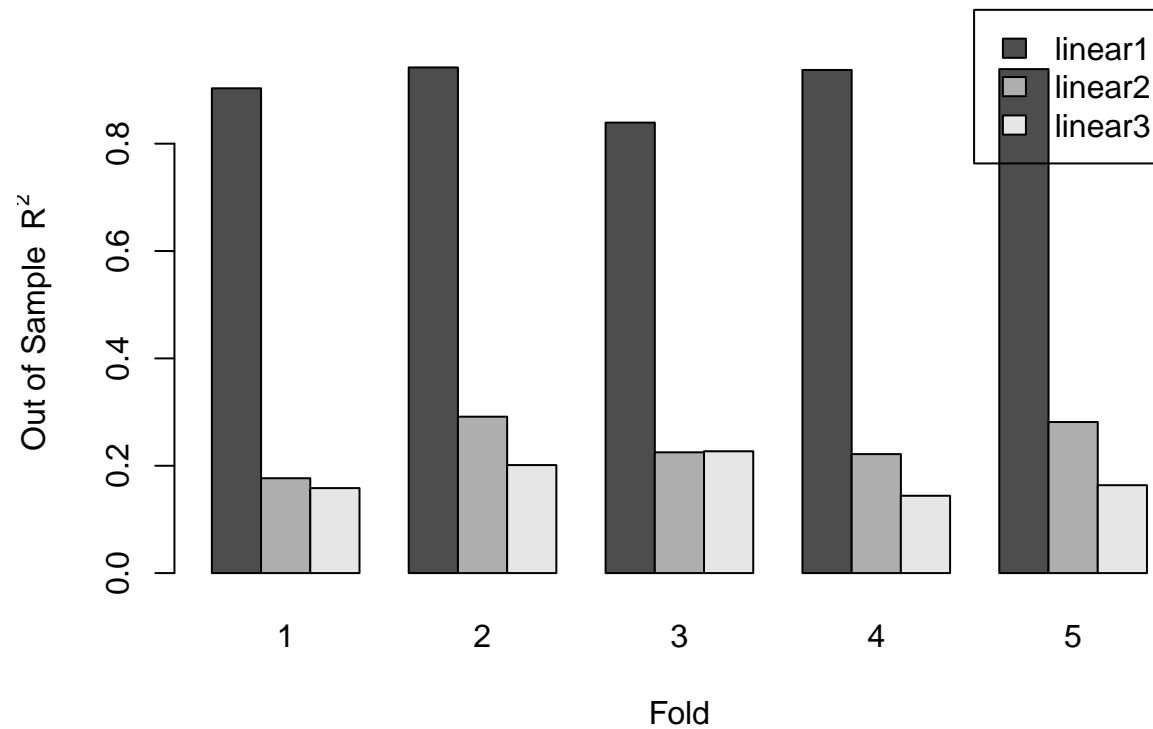
##      linear1  linear2  linear3
## 0.9121321 0.2392480 0.1788705

```

```

m.OOS <- as.matrix(OOS)
rownames(m.OOS) <- c(1:nfold)
barplot(t(as.matrix(OOS)), beside=TRUE, legend=TRUE, args.legend=c(xjust=1, yjust=0.5),
        ylab= bquote( "Out of Sample " ~ R^2), xlab="Fold", names.arg = c(1:5))

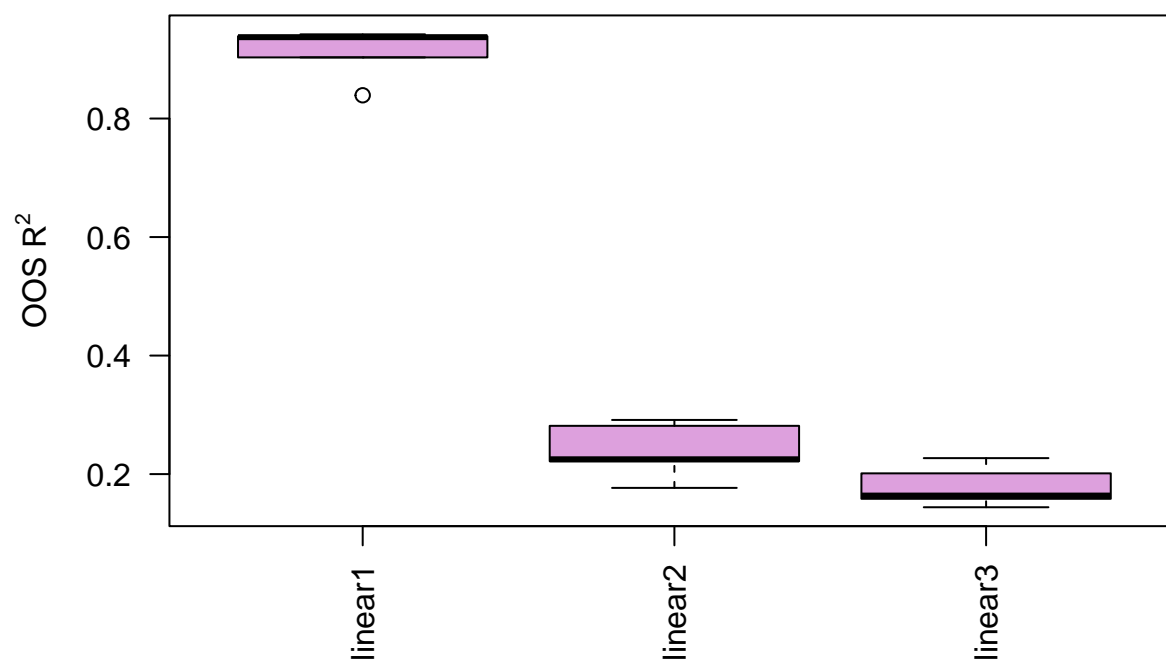
```



*#We then plotted the boxplot and concluded that model.linear1 performs the best, as shown in Figure 9*

```
if (nfold >= 5){
  boxplot(OOS, col="plum", las = 2, ylab=expression(paste("OOS ", R^2)), xlab=c(""), main="5-fold Cross V
}
```

## 5-fold Cross Validation



###Logistic regression models:

```
bag_data <- rename(bag_data, oversold = over_sold)
```

```
bag_data <- rename(bag_data, location = Location)
```

```
model.logistic <- glm(oversold~Hermes + birkin + Black +special_leather + auction_year + location, data = bag_data)
summary(model.logistic)
```

##

## Call:

```
## glm(formula = oversold ~ Hermes + birkin + Black + special_leather +
##      auction_year + location, family = "binomial", data = bag_data)
```

##

## Deviance Residuals:

```
##      Min       1Q   Median       3Q      Max
## -2.1356  0.4644  0.5893  0.8208  1.2905
```

##

## Coefficients:

```
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -564.38897   510.50486  -1.106   0.2689
## Hermes         0.04730    0.27463   0.172   0.8633
## birkin         0.43961    0.23238   1.892   0.0585 .
## Black        -0.07019    0.32626  -0.215   0.8297
## special_leather -0.58435    0.24703  -2.365   0.0180 *
## auction_year   0.27970    0.25258   1.107   0.2681
## locationHong Kong -0.29443    0.32734  -0.899   0.3684
## locationLondon -0.60716    0.35497  -1.710   0.0872 .
```

```
## locationNew York      0.52371    0.35030    1.495    0.1349
## locationParis        -0.09624    0.52881   -0.182    0.8556
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 838.47  on 758  degrees of freedom
## Residual deviance: 799.96  on 749  degrees of freedom
## AIC: 819.96
##
## Number of Fisher Scoring iterations: 4
```

*#We can compute the R squared*

```
Rsq <- 1 - model.logistic$deviance/model.logistic$null.deviance
Rsq
```

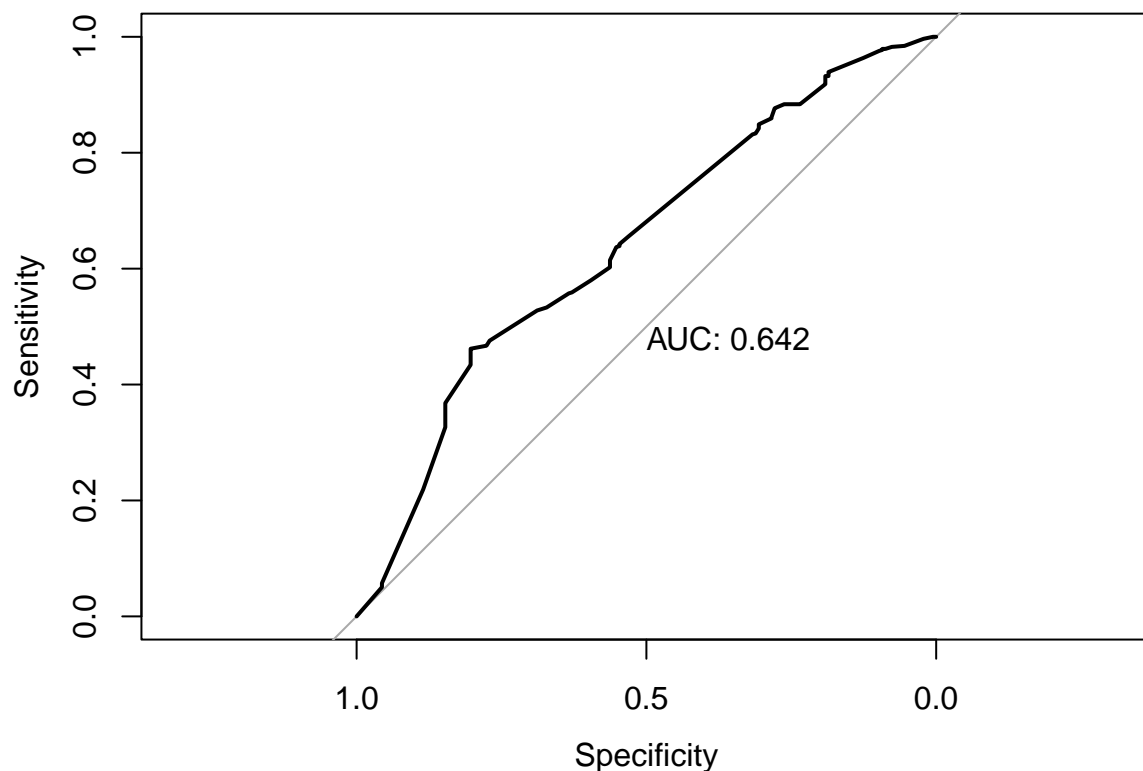
```
## [1] 0.04592939
```

*#Overall ROC, as shown in Figure 5*

```
test_prob.all = predict(model.logistic, newdata = bag_data, type = "response")
test_roc.all = roc(bag_data$oversold ~ test_prob.all, plot = TRUE, print.auc = TRUE)
```

```
## Setting levels: control = 0, case = 1
```

```
## Setting direction: controls < cases
```



```
#We also created two separate datasets for Hermes and Chanel (in Appendix)
```

```
hermes <- bag_data[bag_data$brand == "Hermès", ]
```

```
chanel <- bag_data[bag_data$brand == "Chanel", ]
```

```
#Hermes logistics regression:
```

```
model.logistic.hermes <- glm(oversold~ birkin + Black + special_leather + auction_year + location, data = hermes)
summary(model.logistic.hermes)
```

```
##
```

```
## Call:
```

```
## glm(formula = oversold ~ birkin + Black + special_leather + auction_year +
```

```
## location, family = "binomial", data = hermes)
```

```
##
```

```
## Deviance Residuals:
```

```
##      Min       1Q   Median       3Q      Max
```

```
## -2.12300  0.00026  0.61606  0.84708  1.23283
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std. Error z value Pr(>|z|)
```

```
## (Intercept)   -363.7427    785.5103  -0.463   0.6433
```

```
## birkin         0.4570     0.2330   1.961   0.0499 *
```

```
## Black         15.0945    522.0514   0.029   0.9769
```

```
## special_leather -0.5770     0.2527  -2.283   0.0224 *
```

```
## auction_year   0.1804     0.3886   0.464   0.6424
```

```
## locationHong Kong -0.2436     0.3575  -0.681   0.4956
```

```
## locationLondon  -0.4559     0.3812  -1.196   0.2317
```

```
## locationNew York  0.6016     0.3877   1.552   0.1207
```

```
## locationParis    0.0132     0.5714   0.023   0.9816
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
```

```
## (Dispersion parameter for binomial family taken to be 1)
```

```
##
```

```
## Null deviance: 643.38 on 575 degrees of freedom
```

```
## Residual deviance: 606.81 on 567 degrees of freedom
```

```
## AIC: 624.81
```

```
##
```

```
## Number of Fisher Scoring iterations: 15
```

```
#We can compute the R squared
```

```
Rsq2 <- 1 - model.logistic.hermes$deviance/model.logistic.hermes$null.deviance
```

```
Rsq2
```

```
## [1] 0.05683745
```

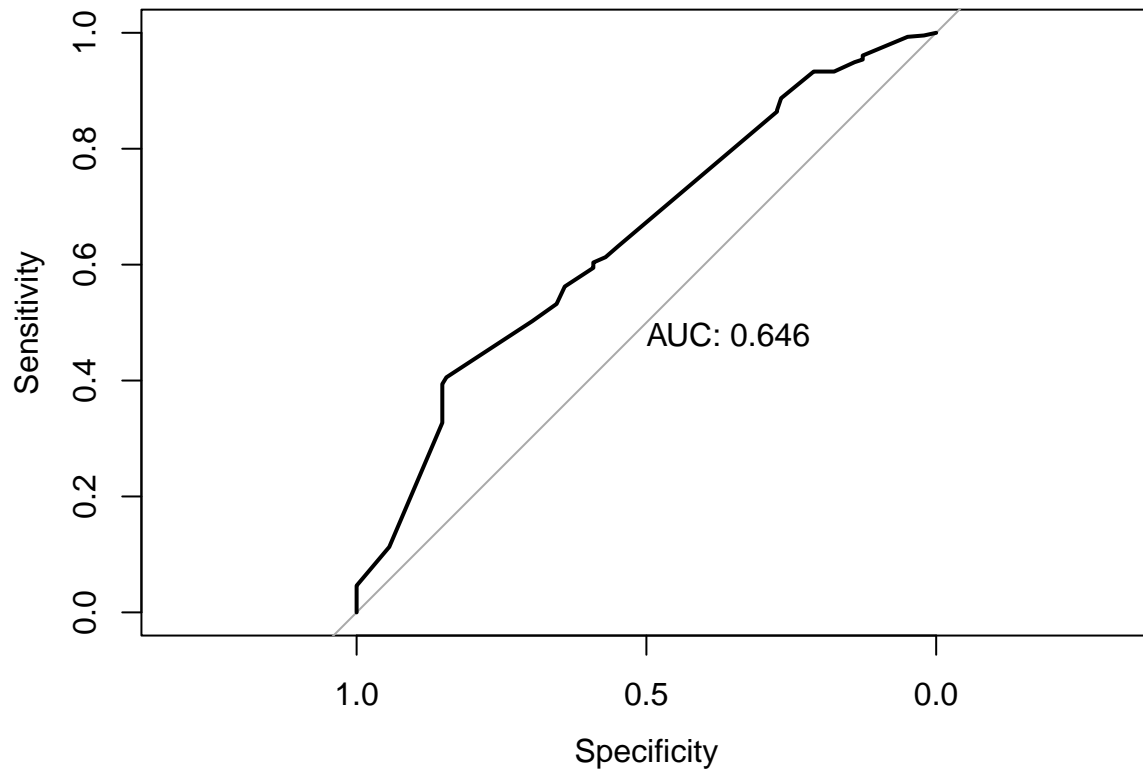
```
#Hermes ROC
```

```
test_prob = predict(model.logistic.hermes, newdata = hermes, type = "response")
```

```
test_roc = roc(hermes$oversold ~ test_prob, plot = TRUE, print.auc = TRUE)
```

```
## Setting levels: control = 0, case = 1
```

```
## Setting direction: controls < cases
```



```
###Logistic regression predictions:
#For all bags prediction:
prediction.all <- predict(model.logistic, newdata=bag_prediction, type="response")
prediction.all
```

```
##      2      4      5      6      7      8      9     10
## 0.5818854 0.7948612 0.7139958 0.7139958 0.7139958 0.7139958 0.7139958 0.7948612
##      11      12      13      14      15      16      17      18
## 0.7139958 0.7948612 0.7139958 0.7139958 0.7139958 0.7948612 0.7139958 0.7139958
##      19      20      21      22      23      24      25      26
## 0.7139958 0.7139958 0.7139958 0.7139958 0.7139958 0.7139958 0.7139958 0.7139958
##      27      28      29      30      31      32      33      34
## 0.7139958 0.7948612 0.7139958 0.7139958 0.7139958 0.7948612 0.7948612 0.7948612
##      35      36      37      38      39      40      41      42
## 0.7948612 0.7139958 0.5818854 0.7042402 0.7139958 0.7139958 0.7948612 0.5818854
##      43      44      45      46      47      48      49      50
## 0.7139958 0.7139958 0.7948612 0.5818854 0.7139958 0.6835487 0.7948612 0.7139958
##      51      52      53      54      55      56      57      58
## 0.7948612 0.7139958 0.6835487 0.5818854 0.6835487 0.5818854 0.5818854 0.7831799
##      59      60      61      62      63      64      65      66
## 0.5818854 0.7139958 0.7948612 0.7139958 0.7139958 0.7139958 0.5818854 0.7139958
##      67      68      69      70      71      72      73      74
## 0.5818854 0.7139958 0.7139958 0.5818854 0.7948612 0.7139958 0.7139958 0.7139958
##      75      76      77      78      79      80      81      82
## 0.7139958 0.7139958 0.7139958 0.7139958 0.7042402 0.7948612 0.7948612 0.7139958
##      83      84      85      86      87      88      89      90
```



```
## 0.7042402 0.7139958 0.7139958 0.7139958 0.7139958 0.7042402 0.7139958 0.7139958
##      91      92      93      94
## 0.7948612 0.7139958 0.5818854 0.5818854
```

*#Combining the predictions with the actual oversold status of the 94 bags:*

```
pred <- prediction(prediction.all,bag_prediction$oversold)
pred
```

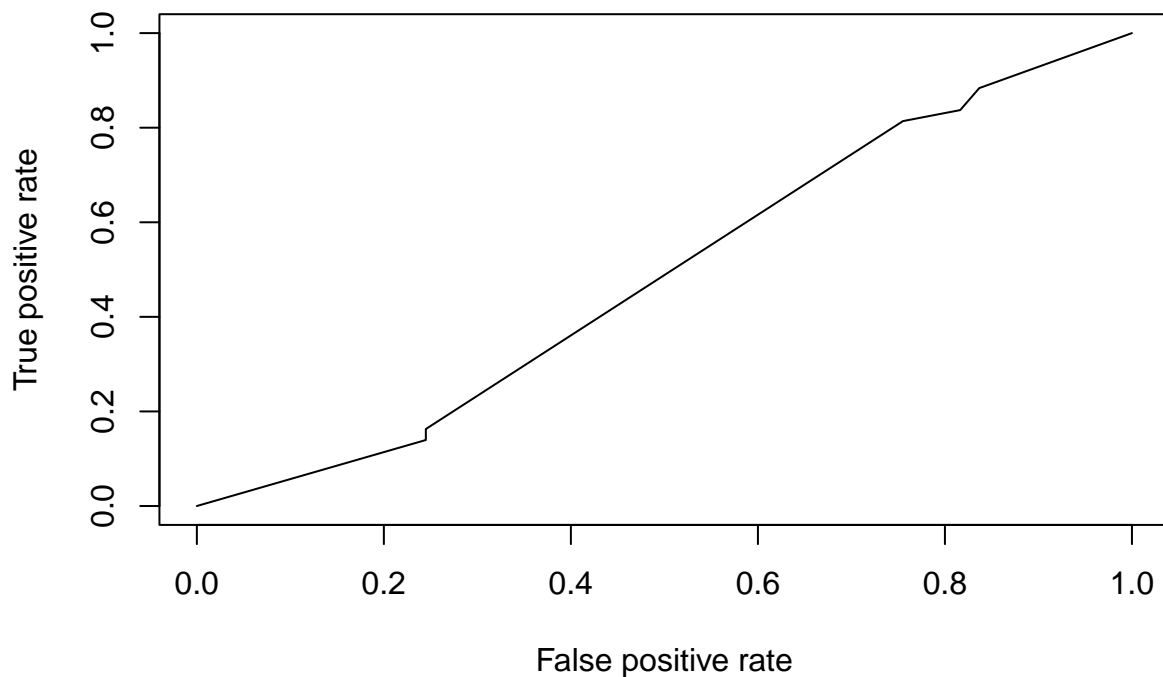
```
## A prediction instance
##   with 92 data points
```

*#We then try to find the optimal cutoff that maximized TPR while minimizing FPR*

```
perf <- performance(pred,"tpr","fpr")
perf
```

```
## A performance instance
##   'False positive rate' vs. 'True positive rate' (alpha: 'Cutoff')
##   with 7 data points
```

```
plot(perf)
```



```
pred@cutoffs[[1]][which.min(perf@y.values[[1]])]
```

```
##
## Inf
```

```

opt.cut = function(perf, pred){
  cut.ind = mapply(FUN=function(x, y, p){
    d = (x - 0)^2 + (y-1)^2
    ind = which(d == min(d))
    c(sensitivity = y[[ind]], specificity = 1-x[[ind]],
      cutoff = p[[ind]])
  }, perf@x.values, perf@y.values, pred@cutoffs)
}
optimal.cutoff <- opt.cut(perf, pred)[3]
optimal.cutoff

```

```
## [1] 0.7139958
```

```

#Using the optimal cutoff, we then proceeded to assign the oversold status to the 94 new bags
predicted_oversold <- ifelse(prediction.all >= optimal.cutoff,1,0)
predicted_oversold

```

```

##  2  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
##  0  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
## 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54
##  1  1  1  1  1  1  1  1  1  0  0  1  1  1  0  1  1  1  0  1  0  1  1  1  0  0
## 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
##  0  0  0  1  0  1  1  1  1  1  0  1  0  1  1  0  1  1  1  1  1  1  1  1  0  1
## 81 82 83 84 85 86 87 88 89 90 91 92 93 94
##  1  1  0  1  1  1  1  0  1  1  1  1  0  0

```

```

comparison <- cbind(predicted_oversold,bag_prediction$oversold)
colnames(comparison) <- c('predicted','actual')
comparison <- data.frame(comparison)

```

```

#Calculating accuracy of our prediction:
TP <- sum(comparison$predicted == 1 & comparison$actual == 1)
TP

```

```
## [1] 35
```

```

TN <- sum(comparison$predicted == 0 & comparison$actual == 0)
TN

```

```
## [1] 12
```

```

FP <- sum(comparison$predicted == 1 & comparison$actual == 0)
FP

```

```
## [1] 37
```

```

FN <- sum(comparison$predicted == 0 & comparison$actual == 1)
FN

```

```
## [1] 8
```

```
accuracy <- (TP + TN) / nrow(bag_prediction)
accuracy
```

```
## [1] 0.5108696
```

```
#For Hermes bags prediction:
```

```
prediction.hermes <- predict(model.logistic.hermes, newdata=bag_prediction[bag_prediction$brand == "Hermès",])
prediction.hermes
```

```
##          2          4          5          6          7          8          9         10
## 0.5654508 0.7853839 0.6985336 0.6985336 0.6985336 0.6985336 0.6985336 0.7853839
##          11          12          13          14          15          16          17          18
## 0.6985336 0.7853839 0.6985336 0.6985336 0.6985336 0.7853839 0.6985336 0.6985336
##          19          20          21          22          23          24          25          26
## 0.6985336 0.6985336 0.6985336 0.6985336 0.6985336 0.6985336 0.6985336 0.6985336
##          27          28          29          30          31          32          33          34
## 0.6985336 0.7853839 0.6985336 0.6985336 0.6985336 0.7853839 0.7853839 0.7853839
##          35          36          37          39          40          41          42          43
## 0.7853839 0.6985336 0.5654508 0.6985336 0.6985336 0.7853839 0.5654508 0.6985336
##          44          45          46          47          48          49          50          51
## 0.6985336 0.7853839 0.5654508 0.6985336 0.6726755 0.7853839 0.6985336 0.7853839
##          52          53          54          55          56          57          58          59
## 0.6985336 0.6726755 0.5654508 0.6726755 0.5654508 0.5654508 0.9999999 0.5654508
##          60          61          62          63          64          65          66          67
## 0.6985336 0.7853839 0.6985336 0.6985336 0.6985336 0.5654508 0.6985336 0.5654508
##          68          69          70          71          72          73          74          75
## 0.6985336 0.6985336 0.5654508 0.7853839 0.6985336 0.6985336 0.6985336 0.6985336
##          76          77          78          80          81          82          84          85
## 0.6985336 0.6985336 0.6985336 0.7853839 0.7853839 0.6985336 0.6985336 0.6985336
##          86          87          89          90          91          92          93          94
## 0.6985336 0.6985336 0.6985336 0.6985336 0.7853839 0.6985336 0.5654508 0.5654508
```

```
#Combing the predictions with the actual oversold status of the new Hermes bags:
```

```
pred.hermes <- prediction(prediction.hermes,bag_prediction$oversold[bag_prediction$brand == "Hermès"])
pred.hermes
```

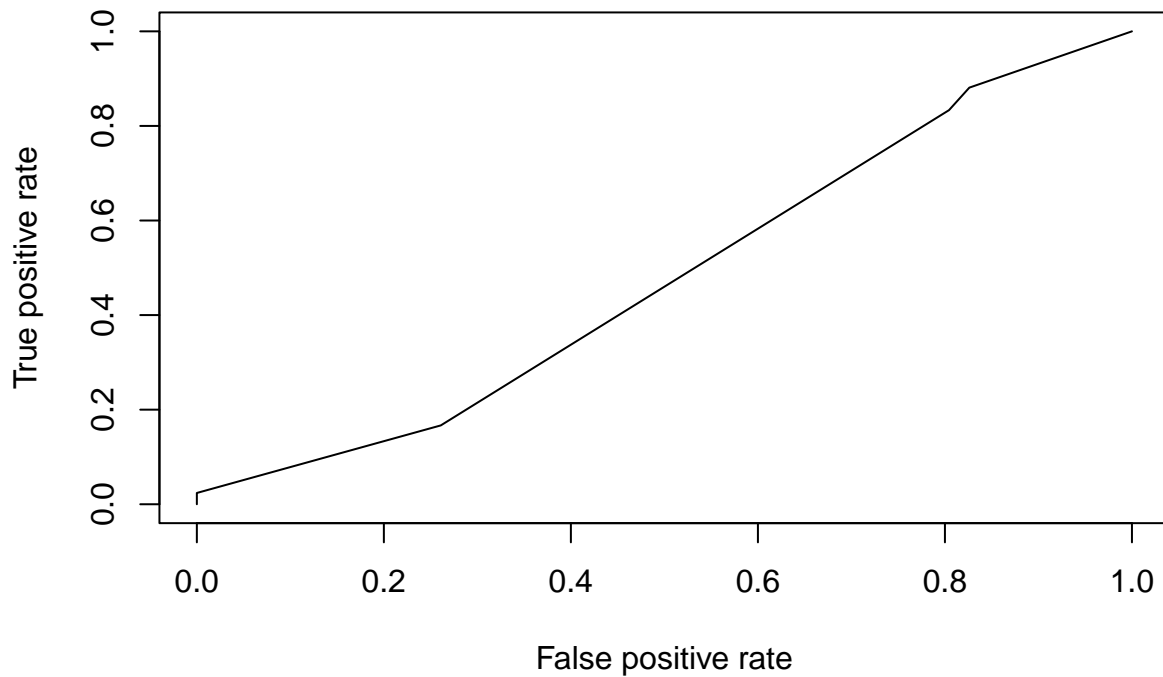
```
## A prediction instance
## with 88 data points
```

```
#We then try to find the optimal cutoff that maxmized TPR while minimizing FPR
```

```
perf.hermes <- performance(pred.hermes,"tpr","fpr")
perf.hermes
```

```
## A performance instance
## 'False positive rate' vs. 'True positive rate' (alpha: 'Cutoff')
## with 6 data points
```

```
plot(perf.hermes)
```



```
pred.hermes@cutoffs[[1]][which.min(perf.hermes@y.values[[1]])]
```

```
##  
## Inf
```

```
opt.cut.hermes = function(perf.hermes, pred.hermes){  
  cut.ind = mapply(FUN=function(x, y, p){  
    d = (x - 0)^2 + (y-1)^2  
    ind = which(d == min(d))  
    c(sensitivity = y[[ind]], specificity = 1-x[[ind]],  
      cutoff = p[[ind]])  
  }, perf.hermes@x.values, perf.hermes@y.values, pred.hermes@cutoffs)  
}  
optimal.cutoff.hermes <- opt.cut.hermes(perf.hermes, pred.hermes)[3]  
optimal.cutoff.hermes
```

```
## [1] 0.6985336
```

```
#Using the optimal cutoff, we then proceeded to assign the oversold status to the 94 new bags  
predicted_oversold.hermes <- ifelse(prediction.hermes >= optimal.cutoff.hermes,1,0)  
predicted_oversold.hermes
```

```
## 2 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28  
## 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
## 29 30 31 32 33 34 35 36 37 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55
## 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 0 1 0 1 1 1 1 0 0 0
## 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 80 81 82
## 0 0 1 0 1 1 1 1 1 0 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1
## 84 85 86 87 89 90 91 92 93 94
## 1 1 1 1 1 1 1 1 0 0
```

```
comparison.hermes <- cbind(predicted_oversold.hermes,bag_prediction$oversold[bag_prediction$brand == "Hermes"])
colnames(comparison.hermes) <- c('predicted','actual')
comparison.hermes <- data.frame(comparison.hermes)
```

```
#Calculating accuracy of our prediction:
```

```
TP.hermes <- sum(comparison.hermes$predicted == 1 & comparison.hermes$actual == 1)
TP.hermes
```

```
## [1] 35
```

```
TN.hermes <- sum(comparison.hermes$predicted == 0 & comparison.hermes$actual == 0)
TN.hermes
```

```
## [1] 9
```

```
FP.hermes <- sum(comparison.hermes$predicted == 1 & comparison.hermes$actual == 0)
FP.hermes
```

```
## [1] 37
```

```
FN.hermes <- sum(comparison.hermes$predicted == 0 & comparison.hermes$actual == 1)
FN.hermes
```

```
## [1] 7
```

```
accuracy.hermes <- (TP.hermes + TN.hermes) / length(prediction.hermes)
accuracy.hermes
```

```
## [1] 0.5
```

```
###Classification Tree
```

```
bagtree <- tree(oversold ~ Hermes + Chanel + Black + birkin + special_leather, data=bag_data, mindev = 0.0001)
summary(bagtree)
```

```
##
```

```
## Regression tree:
```

```
## tree(formula = oversold ~ Hermes + Chanel + Black + birkin +
##       special_leather, data = bag_data, mindev = 0, minsize = 2)
```

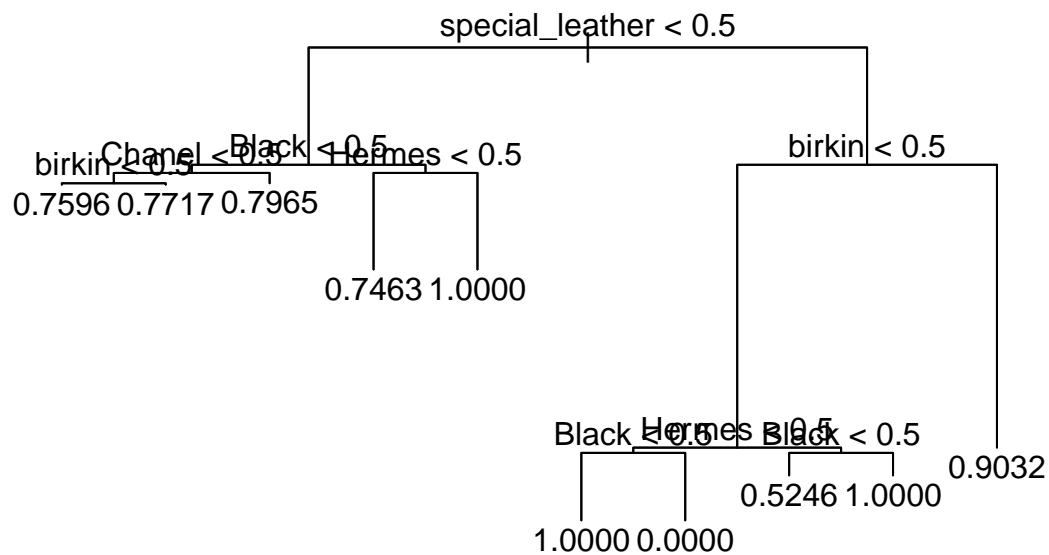
```
## Number of terminal nodes: 10
```

```
## Residual mean deviance: 0.1774 = 132.8 / 749
```

```
## Distribution of residuals:
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -0.9032  0.0000  0.2283  0.0000  0.2404  0.4754
```

```
#We plotted the tree
plot(bagtree)
#We also added the labels, as shown in Figure 6
text(bagtree, label="yval")
```

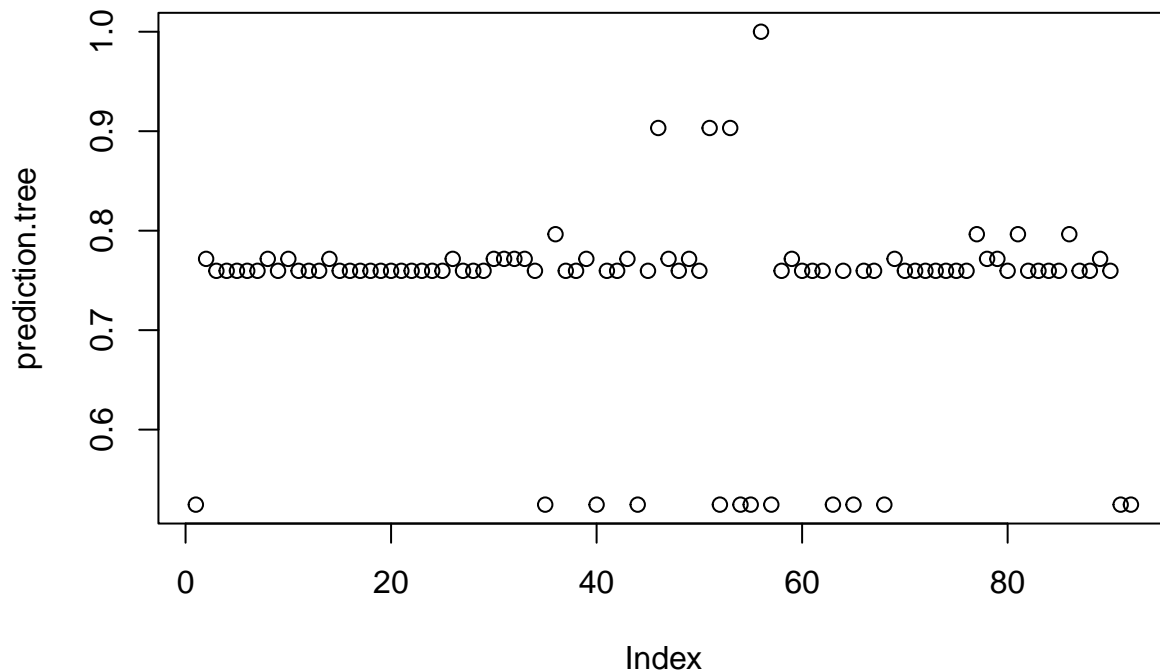


```
###Classification Tree Prediction
prediction.tree <- predict(bagtree, newdata = bag_prediction, type = "vector")
prediction.tree
```

```
##      2      4      5      6      7      8      9      10
## 0.5245902 0.7716535 0.7596439 0.7596439 0.7596439 0.7596439 0.7596439 0.7716535
##      11      12      13      14      15      16      17      18
## 0.7596439 0.7716535 0.7596439 0.7596439 0.7596439 0.7716535 0.7596439 0.7596439
##      19      20      21      22      23      24      25      26
## 0.7596439 0.7596439 0.7596439 0.7596439 0.7596439 0.7596439 0.7596439 0.7596439
##      27      28      29      30      31      32      33      34
## 0.7596439 0.7716535 0.7596439 0.7596439 0.7596439 0.7716535 0.7716535 0.7716535
##      35      36      37      38      39      40      41      42
## 0.7716535 0.7596439 0.5245902 0.7964602 0.7596439 0.7596439 0.7716535 0.5245902
##      43      44      45      46      47      48      49      50
## 0.7596439 0.7596439 0.7716535 0.5245902 0.7596439 0.9032258 0.7716535 0.7596439
##      51      52      53      54      55      56      57      58
## 0.7716535 0.7596439 0.9032258 0.5245902 0.9032258 0.5245902 0.5245902 1.0000000
##      59      60      61      62      63      64      65      66
## 0.5245902 0.7596439 0.7716535 0.7596439 0.7596439 0.7596439 0.5245902 0.7596439
```

```
##      67      68      69      70      71      72      73      74
## 0.5245902 0.7596439 0.7596439 0.5245902 0.7716535 0.7596439 0.7596439 0.7596439
##      75      76      77      78      79      80      81      82
## 0.7596439 0.7596439 0.7596439 0.7596439 0.7964602 0.7716535 0.7716535 0.7596439
##      83      84      85      86      87      88      89      90
## 0.7964602 0.7596439 0.7596439 0.7596439 0.7596439 0.7964602 0.7596439 0.7596439
##      91      92      93      94
## 0.7716535 0.7596439 0.5245902 0.5245902
```

```
plot(prediction.tree)
```



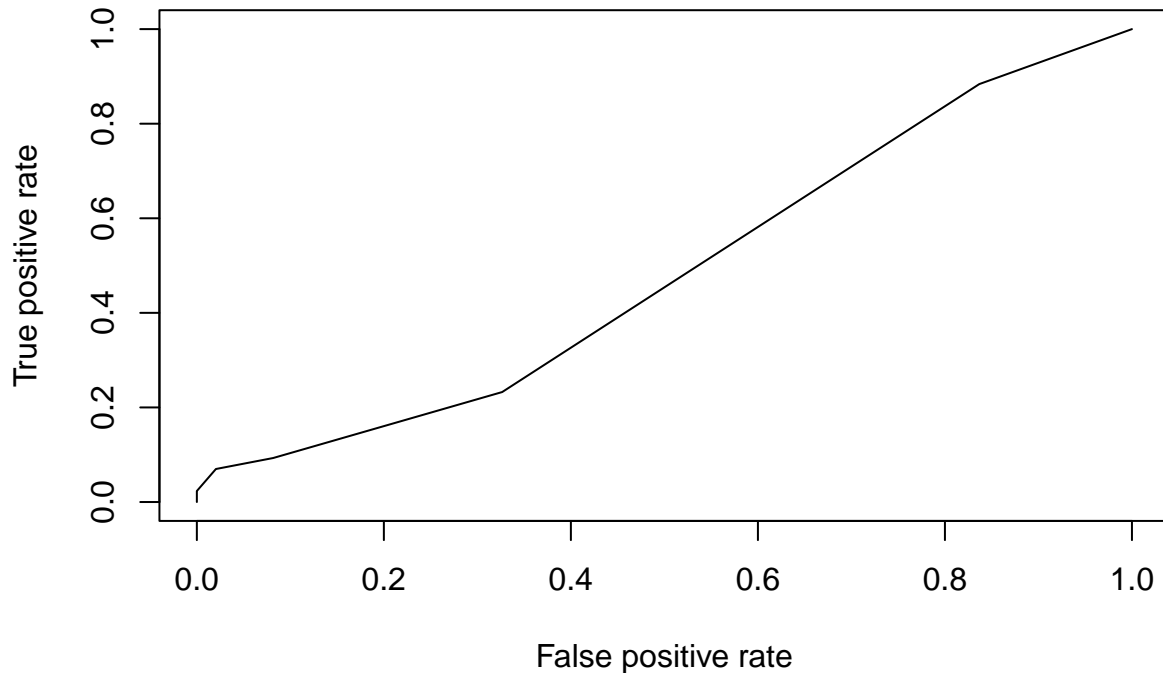
```
#Combing the predictions with the actual oversold status of the 94 bags:
pred.tree <- prediction(prediction.tree,bag_prediction$oversold)
pred.tree
```

```
## A prediction instance
## with 92 data points
```

```
perf.tree <- performance(pred.tree,"tpr","fpr")
perf.tree
```

```
## A performance instance
## 'False positive rate' vs. 'True positive rate' (alpha: 'Cutoff')
## with 7 data points
```

```
plot(perf.tree)
```



```
pred.tree@cutoffs[[1]][which.min(perf.tree@y.values[[1]])]
```

```
##  
## Inf
```

```
opt.cut.tree = function(perf.tree, pred.tree){  
  cut.ind = mapply(FUN=function(x, y, p){  
    d = (x - 0)^2 + (y-1)^2  
    ind = which(d == min(d))  
    c(sensitivity = y[[ind]], specificity = 1-x[[ind]],  
      cutoff = p[[ind]])  
  }, perf.tree@x.values, perf.tree@y.values, pred.tree@cutoffs)  
}  
optimal.cutoff.tree <- opt.cut.tree(perf.tree, pred.tree)[3]  
optimal.cutoff.tree
```

```
## [1] 0.7716535
```

```
#Using the optimal cutoff, we then proceeded to assign the oversold status to the 94 new bags  
predicted_oversold.tree <- ifelse(prediction.tree >= optimal.cutoff.tree,1,0)  
predicted_oversold.tree
```



```
## 2 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
## 0 1 0 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
## 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54
## 0 0 0 1 1 1 1 0 0 1 0 0 1 0 0 0 1 0 0 1 1 0 1 0 1 0
## 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
## 1 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 1
## 81 82 83 84 85 86 87 88 89 90 91 92 93 94
## 1 0 1 0 0 0 0 1 0 0 1 0 0 0
```

```
comparison.tree <- cbind(predicted_oversold.tree, bag_prediction$oversold)
colnames(comparison.tree) <- c('predicted', 'actual')
comparison.tree <- data.frame(comparison.tree)
```

*#Calculating accuracy of our prediction:*

```
TP.tree <- sum(comparison.tree$predicted == 1 & comparison.tree$actual == 1)
TP.tree
```

```
## [1] 10
```

```
TN.tree <- sum(comparison.tree$predicted == 0 & comparison.tree$actual == 0)
TN.tree
```

```
## [1] 33
```

```
FP.tree <- sum(comparison.tree$predicted == 1 & comparison.tree$actual == 0)
FP.tree
```

```
## [1] 16
```

```
FN.tree <- sum(comparison.tree$predicted == 0 & comparison.tree$actual == 1)
FN.tree
```

```
## [1] 33
```

```
accuracy.tree <- (TP.tree + TN.tree) / nrow(bag_prediction)
accuracy.tree
```

```
## [1] 0.4673913
```

*###Linear Discriminant Analysis*

```
set.seed(555)
```

*#Train & test*

```
temp <- bag_data[,c(8:12,15,16,17)]
temp <- temp %>% drop_na()
temp <- scale(temp)
train_size <- 0.8*nrow(temp)
train_index <- sample(x = 1:nrow(temp), size = train_size, replace = F)
train_set <- as.data.frame(temp[train_index, ])
test_set <- as.data.frame(temp[-train_index, ])
```

```
LDA <- lda(oversold ~ lowerestimate_USD + upperestimate_USD + special_leather + birkin + Hermes + Black
LDA
```

```
## Call:
## lda(oversold ~ lowerestimate_USD + upperestimate_USD + special_leather +
##     birkin + Hermes + Black, data = train_set)
##
## Prior probabilities of groups:
## -1.77296193579809  0.56328478168585
##      0.2421746      0.7578254
##
## Group means:
##               lowerestimate_USD upperestimate_USD special_leather
## -1.77296193579809      -0.13319494      -0.08228658      0.11058885
##  0.56328478168585      0.04309862      0.02521216      -0.04040373
##               birkin      Hermes      Black
## -1.77296193579809 -0.12237272  0.007035563 -0.06462577
##  0.56328478168585  0.03084894 -0.010619303  0.01809522
##
## Coefficients of linear discriminants:
##               LD1
## lowerestimate_USD  8.8178495
## upperestimate_USD -8.5052362
## special_leather   -0.4154099
## birkin            0.1321538
## Hermes            0.1108569
## Black             0.1399917
```

#### *#In-sample test*

```
LDA_training_pred = predict(LDA, train_set)
LDA_training_pred
```

```
## $class
## [1] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [5] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
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## [37] 0.56328478168585 -1.77296193579809 0.56328478168585 0.56328478168585
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## [45] -1.77296193579809 0.56328478168585 0.56328478168585 0.56328478168585
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## [69] 0.56328478168585 0.56328478168585 0.56328478168585 -1.77296193579809
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## [89] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [93] 0.56328478168585 0.56328478168585 0.56328478168585 -1.77296193579809
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[illegible]

[illegible]

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## [537] 0.56328478168585 0.56328478168585 0.56328478168585 -1.77296193579809
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## [549] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
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## [569] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
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## [589] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [593] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [597] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [601] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [605] 0.56328478168585 0.56328478168585 0.56328478168585
## Levels: -1.77296193579809 0.56328478168585
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## $posterior
##      -1.77296193579809 0.56328478168585
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## 2      0.237762232      0.7622378
## 3      0.209821685      0.7901783
## 4      0.193029720      0.8069703
## 5      0.175989282      0.8240107
## 6      0.246485804      0.7535142
## 7      0.446861055      0.5531389
## 8      0.223662324      0.7763377
## 9      0.210221272      0.7897787
## 10     0.403776339      0.5962237
## 11     0.217476070      0.7825239
## 12     0.196226666      0.8037733
## 13     0.238636298      0.7613637
## 14     0.297555493      0.7024445
## 15     0.214051931      0.7859481
## 16     0.154172825      0.8458272
## 17     0.178498354      0.8215016
## 18     0.156244495      0.8437555
## 19     0.197467558      0.8025324
## 20     0.193029720      0.8069703
## 21     0.200603220      0.7993968
## 22     0.242476504      0.7575235
## 23     0.795766273      0.2042337
## 24     0.155709004      0.8442910
## 25     0.169139888      0.8308601
## 26     0.032086782      0.9679132
## 27     0.240790140      0.7592099
## 28     0.213536066      0.7864639
## 29     0.212949611      0.7870504
## 30     0.203278092      0.7967219

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## 31	0.227145622	0.7728544
## 32	0.176035444	0.8239646
## 33	0.209821685	0.7901783
## 34	0.175342567	0.8246574
## 35	0.439073972	0.5609260
## 36	0.198914589	0.8010854
## 37	0.302950357	0.6970496
## 38	0.595599527	0.4044005
## 39	0.183749553	0.8162504
## 40	0.212507055	0.7874929
## 41	0.464229135	0.5357709
## 42	0.099393858	0.9006061
## 43	0.138483768	0.8615162
## 44	0.114332712	0.8856673
## 45	0.797210192	0.2027898
## 46	0.031395239	0.9686048
## 47	0.354159092	0.6458409
## 48	0.216700234	0.7832998
## 49	0.136682949	0.8633171
## 50	0.065482532	0.9345175
## 51	0.010447160	0.9895528
## 52	0.754699377	0.2453006
## 53	0.243002289	0.7569977
## 54	0.066673313	0.9333267
## 55	0.221857234	0.7781428
## 56	0.620882901	0.3791171
## 57	0.209491148	0.7905089
## 58	0.282381404	0.7176186
## 59	0.481665117	0.5183349
## 60	0.169139888	0.8308601
## 61	0.195557888	0.8044421
## 62	0.274805305	0.7251947
## 63	0.342849353	0.6571506
## 64	0.229151770	0.7708482
## 65	0.212507055	0.7874929
## 66	0.307637284	0.6923627
## 67	0.196420361	0.8035796
## 68	0.210123067	0.7898769
## 69	0.186615422	0.8133846
## 70	0.067275306	0.9327247
## 71	0.156880562	0.8431194
## 72	0.869631812	0.1303682
## 73	0.183749553	0.8162504
## 74	0.213536066	0.7864639
## 75	0.175989282	0.8240107
## 76	0.258680733	0.7413193
## 77	0.216700234	0.7832998
## 78	0.170807201	0.8291928
## 79	0.214051931	0.7859481
## 80	0.185072201	0.8149278
## 81	0.299280278	0.7007197
## 82	0.115581708	0.8844183
## 83	0.193179827	0.8068202
## 84	0.175989282	0.8240107

## 85	0.209413998	0.7905860
## 86	0.116778183	0.8832218
## 87	0.211481670	0.7885183
## 88	0.367876008	0.6321240
## 89	0.121852109	0.8781479
## 90	0.197734660	0.8022653
## 91	0.129420610	0.8705794
## 92	0.174554236	0.8254458
## 93	0.216893922	0.7831061
## 94	0.355011517	0.6449885
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## 96	0.554160047	0.4458400
## 97	0.103006777	0.8969932
## 98	0.175849621	0.8241504
## 99	0.792203297	0.2077967
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## 101	0.156880562	0.8431194
## 102	0.237762232	0.7622378
## 103	0.094532682	0.9054673
## 104	0.161225808	0.8387742
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## 108	0.211481670	0.7885183
## 109	0.563921288	0.4360787
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## 111	0.100465829	0.8995342
## 112	0.212600415	0.7873996
## 113	0.149465232	0.8505348
## 114	0.285213898	0.7147861
## 115	0.216700234	0.7832998
## 116	0.210221272	0.7897787
## 117	0.169907693	0.8300923
## 118	0.193029720	0.8069703
## 119	0.151592306	0.8484077
## 120	0.200603220	0.7993968
## 121	0.175733265	0.8242667
## 122	0.113143529	0.8868565
## 123	0.317328921	0.6826711
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## 125	0.200226499	0.7997735
## 126	0.242179589	0.7578204
## 127	0.171490503	0.8285095
## 128	0.265283464	0.7347165
## 129	0.355011517	0.6449885
## 130	0.152523904	0.8474761
## 131	0.353532694	0.6464673
## 132	0.212507055	0.7874929
## 133	0.057298934	0.9427011
## 134	0.240790140	0.7592099
## 135	0.363596766	0.6364032
## 136	0.257744268	0.7422557
## 137	0.276681114	0.7233189
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## 139	0.202937315	0.7970627
## 140	0.154316112	0.8456839
## 141	0.246121896	0.7538781
## 142	0.276226503	0.7237735
## 143	0.129135824	0.8708642
## 144	0.135491903	0.8645081
## 145	0.197508113	0.8024919
## 146	0.056088347	0.9439117
## 147	0.309447794	0.6905522
## 148	0.150926477	0.8490735
## 149	0.175989282	0.8240107
## 150	0.180673825	0.8193262
## 151	0.152056874	0.8479431
## 152	0.312120309	0.6878797
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## 158	0.469913720	0.5300863
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## 160	0.102443350	0.8975566
## 161	0.192654839	0.8073452
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## 168	0.230607817	0.7693922
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## 170	0.238636298	0.7613637
## 171	0.238636298	0.7613637
## 172	0.094532682	0.9054673
## 173	0.304041043	0.6959590
## 174	0.193029720	0.8069703
## 175	0.237048477	0.7629515
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## 177	0.212143415	0.7878566
## 178	0.398253873	0.6017461
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## 181	0.766928511	0.2330715
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## 189	0.326561379	0.6734386
## 190	0.341153629	0.6588464
## 191	0.203478452	0.7965215
## 192	0.316000709	0.6839993



## 193	0.129135824	0.8708642
## 194	0.329736673	0.6702633
## 195	0.212507055	0.7874929
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## 206	0.217476070	0.7825239
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## 210	0.085819361	0.9141806
## 211	0.446861055	0.5531389
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## 213	0.179205813	0.8207942
## 214	0.171413344	0.8285867
## 215	0.298742551	0.7012574
## 216	0.063725180	0.9362748
## 217	0.341153629	0.6588464
## 218	0.396049741	0.6039503
## 219	0.121852109	0.8781479
## 220	0.300136885	0.6998631
## 221	0.247830213	0.7521698
## 222	0.195557888	0.8044421
## 223	0.247381531	0.7526185
## 224	0.280657100	0.7193429
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## 273	0.149465232	0.8505348
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## 279	0.350420090	0.6495799
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## 289	0.331420147	0.6685799
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## 300	0.192654839	0.8073452

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## 303	0.300136885	0.6998631
## 304	0.212143415	0.7878566
## 305	0.176035444	0.8239646
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## 308	0.354770263	0.6452297
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## 310	0.186615422	0.8133846
## 311	0.170543302	0.8294567
## 312	0.214738024	0.7852620
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## 314	0.293076506	0.7069235
## 315	0.398254135	0.6017459
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## 320	0.214163092	0.7858369
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## 322	0.175989282	0.8240107
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## 329	0.341153629	0.6588464
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## 349	0.210537719	0.7894623
## 350	0.293076506	0.7069235
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## 353	0.195709485	0.8042905
## 354	0.149465232	0.8505348

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## 371	0.246121896	0.7538781
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## 389 0.207211289
## 390 -0.460033444
## 391 1.842275293
## 392 -1.411434909
```

## 393 0.029036509  
## 394 -0.055597129  
## 395 0.360941509  
## 396 1.604220836  
## 397 -0.133608045  
## 398 0.144134766  
## 399 -0.173744895  
## 400 0.127235032  
## 401 -0.881855293  
## 402 0.116590249  
## 403 -0.431204276  
## 404 0.599468898  
## 405 0.540690673  
## 406 0.823592750  
## 407 0.127235032  
## 408 -1.023212396  
## 409 -0.576599194  
## 410 0.627091561  
## 411 0.094160397  
## 412 1.842275293  
## 413 0.561191423  
## 414 0.079535588  
## 415 0.388520451  
## 416 0.280770753  
## 417 0.057903774  
## 418 -1.086518876  
## 419 0.949413936  
## 420 0.362256108  
## 421 -0.232635571  
## 422 -3.196049039  
## 423 0.263228582  
## 424 -2.389889186  
## 425 0.360507174  
## 426 0.037602760  
## 427 2.402459483  
## 428 -0.677642120  
## 429 0.447117927  
## 430 -0.781025775  
## 431 1.908175431  
## 432 0.067147562  
## 433 -0.218129178  
## 434 -0.481911502  
## 435 0.741385000  
## 436 -0.049976491  
## 437 -0.230663673  
## 438 0.028325311  
## 439 0.015485042  
## 440 0.061310493  
## 441 0.160845336  
## 442 -0.039510128  
## 443 -0.068730317  
## 444 0.152904784  
## 445 0.426407312  
## 446 0.206309803



## 447 -0.035956704  
## 448 0.548045439  
## 449 0.254614469  
## 450 1.020390682  
## 451 0.797480651  
## 452 -0.329243220  
## 453 -0.076031967  
## 454 -1.139555074  
## 455 -0.076031967  
## 456 0.160845336  
## 457 -1.139555074  
## 458 -2.418796699  
## 459 -1.204177669  
## 460 -0.033265921  
## 461 -0.033265921  
## 462 -0.068730317  
## 463 -1.710232630  
## 464 1.307819696  
## 465 0.827590443  
## 466 -1.444022802  
## 467 -1.169861240  
## 468 0.555640842  
## 469 -0.225512420  
## 470 0.060503212  
## 471 0.166395917  
## 472 0.040328201  
## 473 0.145449364  
## 474 0.627091561  
## 475 1.215998323  
## 476 -1.169861240  
## 477 5.118953188  
## 478 -0.205565233  
## 479 0.007841585  
## 480 -0.105470941  
## 481 0.007841585  
## 482 -0.193813319  
## 483 -0.172955290  
## 484 0.476395312  
## 485 -0.230663673  
## 486 0.050166705  
## 487 0.263228582  
## 488 0.426407312  
## 489 -2.657776711  
## 490 0.404448740  
## 491 -1.169861240  
## 492 0.028325311  
## 493 0.007841585  
## 494 -0.032244727  
## 495 0.007841585  
## 496 0.364113117  
## 497 -3.370937121  
## 498 0.549067850  
## 499 0.372489081  
## 500 1.277451833

## 501 -0.214872647  
## 502 -0.959788743  
## 503 0.360941509  
## 504 1.976721721  
## 505 1.209621172  
## 506 -0.033265921  
## 507 0.116757143  
## 508 0.949413936  
## 509 0.522029441  
## 510 -0.158189937  
## 511 -0.132733377  
## 512 1.339801245  
## 513 1.615971258  
## 514 -0.544148972  
## 515 0.167418328  
## 516 0.045978724  
## 517 0.254614469  
## 518 0.426407312  
## 519 -0.903438142  
## 520 -0.439199663  
## 521 0.364113117  
## 522 -0.162499450  
## 523 0.026353413  
## 524 1.353537015  
## 525 -0.683525255  
## 526 0.206309803  
## 527 -4.380892569  
## 528 0.628221019  
## 529 0.662696540  
## 530 -0.832508046  
## 531 -1.701856666  
## 532 1.353394909  
## 533 -1.895177237  
## 534 0.076968357  
## 535 0.038184799  
## 536 0.166172953  
## 537 0.050166705  
## 538 0.007841585  
## 539 0.144134766  
## 540 -1.786706982  
## 541 -1.444022802  
## 542 0.516230515  
## 543 -1.676728776  
## 544 0.046043468  
## 545 -0.787993216  
## 546 0.103997916  
## 547 -0.231978272  
## 548 0.160845336  
## 549 -0.847538865  
## 550 -0.930881229  
## 551 0.083163851  
## 552 -0.188368002  
## 553 -0.700277182  
## 554 0.075294596

## 555 -0.205565233  
## 556 0.041790742  
## 557 -1.006946750  
## 558 0.184926780  
## 559 0.748729688  
## 560 -0.167820387  
## 561 -0.327719302  
## 562 -2.146343900  
## 563 0.627091561  
## 564 0.057903774  
## 565 -0.072228382  
## 566 -0.033244972  
## 567 -0.031879964  
## 568 0.694874441  
## 569 -0.218129178  
## 570 -0.195087343  
## 571 0.321461959  
## 572 -0.456700830  
## 573 -0.230663673  
## 574 -0.033265921  
## 575 0.109254175  
## 576 0.204995205  
## 577 0.561191423  
## 578 -0.427793317  
## 579 -2.179816688  
## 580 0.063789708  
## 581 -0.452921207  
## 582 0.007841585  
## 583 -0.232549859  
## 584 -0.070553189  
## 585 1.738438844  
## 586 -0.146933489  
## 587 -0.230663673  
## 588 -0.195087343  
## 589 0.160845336  
## 590 -0.602811981  
## 591 0.536502691  
## 592 0.225723064  
## 593 -1.204177669  
## 594 0.549067850  
## 595 0.153184242  
## 596 -0.847538865  
## 597 -1.324967426  
## 598 0.377085421  
## 599 -0.431204276  
## 600 -0.197189269  
## 601 0.038184799  
## 602 0.360507174  
## 603 -0.117511595  
## 604 0.108798449  
## 605 1.700181953  
## 606 -0.209753214  
## 607 -0.847538865

```
names(LDA_training_pred)
```

```
## [1] "class"      "posterior" "x"
```

```
head(LDA_training_pred$class)
```

```
## [1] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [5] 0.56328478168585 0.56328478168585
## Levels: -1.77296193579809 0.56328478168585
```

```
head(LDA_training_pred$posterior)
```

```
##      -1.77296193579809 0.56328478168585
## 1      0.2126004      0.7873996
## 2      0.2377622      0.7622378
## 3      0.2098217      0.7901783
## 4      0.1930297      0.8069703
## 5      0.1759893      0.8240107
## 6      0.2464858      0.7535142
```

```
head(LDA_training_pred$x)
```

```
##      LD1
## 1 0.04102959
## 2 -0.15592646
## 3 0.06378971
## 4 0.20630980
## 5 0.36094151
## 6 -0.22080419
```

```
#Find accuracy of model, 78% correctly predicted
mean(LDA_training_pred$class == train_set$oversold)
```

```
## [1] 0.7545305
```

```
table(predicted_LDA = LDA_training_pred$class, actual = train_set$oversold)
```

```
##      actual
## predicted_LDA -1.77296193579809 0.56328478168585
## -1.77296193579809      13      15
## 0.56328478168585      134      445
```

```
#out-sample test
```

```
LDA_testing_pred = predict(LDA, test_set)
head(LDA_testing_pred$class)
```

```
## [1] -1.77296193579809 0.56328478168585 0.56328478168585 0.56328478168585
## [5] 0.56328478168585 0.56328478168585
## Levels: -1.77296193579809 0.56328478168585
```

```
head(LDA_testing_pred$posterior)
```

```
##      -1.77296193579809 0.56328478168585
## 1      0.8774817      0.1225183
## 2      0.4386059      0.5613941
## 3      0.2461219      0.7538781
## 4      0.1756525      0.8243475
## 5      0.2034586      0.7965414
## 6      0.3564183      0.6435817
```

```
head(LDA_testing_pred$x)
```

```
##      LD1
## 1 -4.4322390
## 2 -1.4088413
## 3 -0.2181292
## 4  0.3641131
## 5  0.1167571
## 6 -0.9392572
```

```
#find accuracy of model, 77% correctly predicted
mean(LDA_testing_pred$class == test_set$oversold)
```

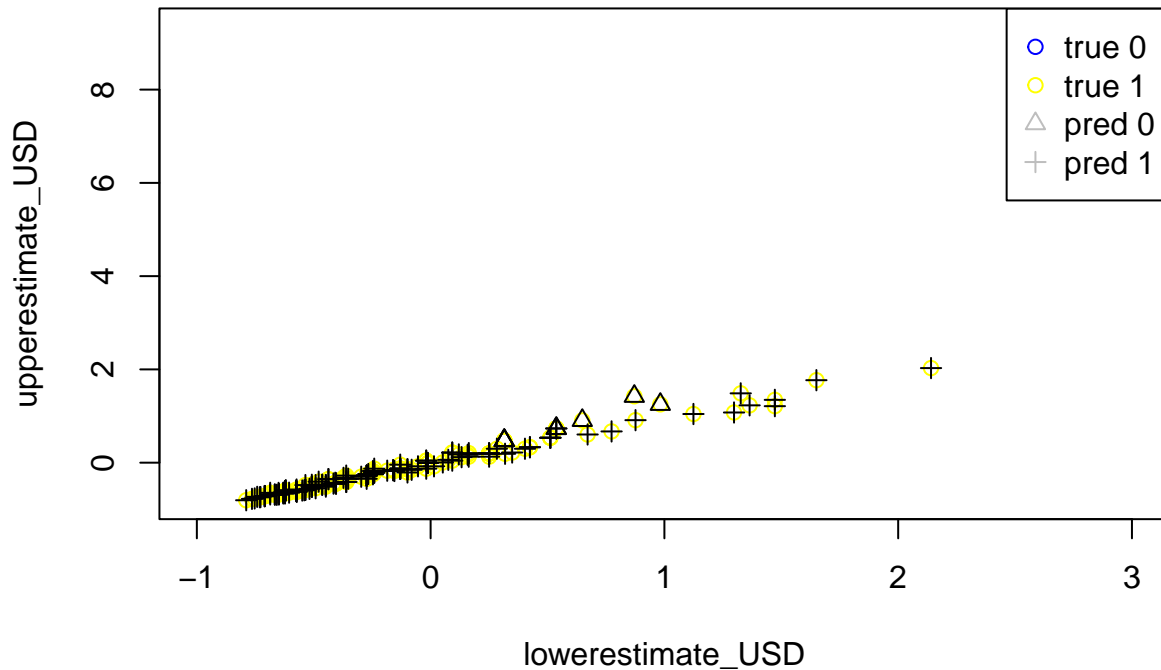
```
## [1] 0.75
```

```
table(predicted_LDA = LDA_testing_pred$class, actual = test_set$oversold)
```

```
##              actual
## predicted_LDA -1.77296193579809 0.56328478168585
##   -1.77296193579809             4             6
##   0.56328478168585          32          110
```

```
plot(test_set$lowerestimate_USD, test_set$upperestimate_USD,
     col = c("blue", "yellow")[test_set$oversold], xlim = c(-1,3),
     xlab = "lowerestimate_USD", ylab = "upperestimate_USD",
     main = "Out-Sample True class vs Predicted class by LDA")
points(test_set$lowerestimate_USD, test_set$upperestimate_USD, pch = c(2,3)[LDA_testing_pred$class])
legend("topright", c("true 0", "true 1",
                    "pred 0", "pred 1"),
     col = c("blue", "yellow", "grey", "grey"), pch = c(1, 1, 2, 3))
```

## Out-Sample True class vs Predicted class by LDA



```
#Predict new dataset
#Clean data
bag_prediction_test <- bag_prediction[,c(8:12,16,17,18)]
bag_prediction_test <- bag_prediction_test %>% drop_na()
bag_prediction_scale <- as.data.frame(scale(bag_prediction_test))

LDA_new_testing_pred = predict(LDA, bag_prediction_scale)
names(LDA_new_testing_pred)
```

```
## [1] "class"      "posterior" "x"
```

```
head(LDA_new_testing_pred$class)
```

```
## [1] 0.56328478168585 0.56328478168585 0.56328478168585 0.56328478168585
## [5] 0.56328478168585 0.56328478168585
## Levels: -1.77296193579809 0.56328478168585
```

```
head(LDA_new_testing_pred$posterior)
```

```
## -1.77296193579809 0.56328478168585
## 1      0.1911567      0.8088433
## 2      0.3575822      0.6424178
## 3      0.3427573      0.6572427
## 4      0.1603872      0.8396128
```

```
## 5      0.2731471      0.7268529
## 6      0.2268944      0.7731056
```

```
head(LDA_new_testing_pred$x)
```

```
##      LD1
## 1  0.22277902
## 2 -0.94617628
## 3 -0.85726419
## 4  0.51321503
## 5 -0.41011272
## 6 -0.07276407
```

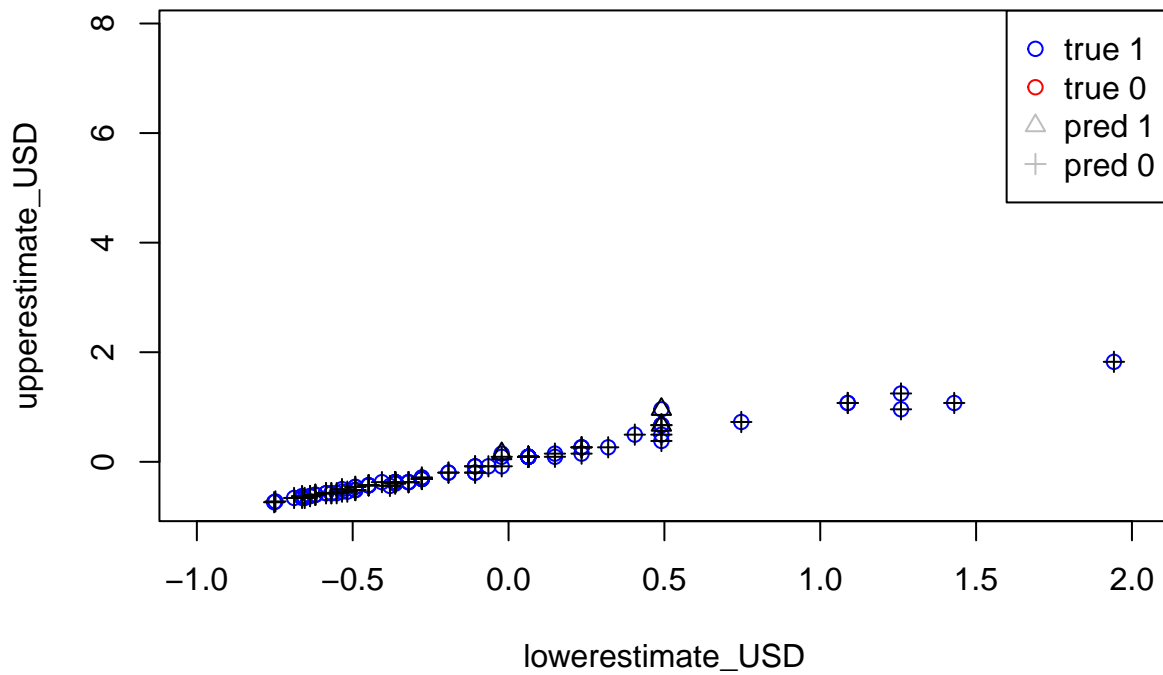
```
#find accuracy of model
```

```
table(predicted_LDA = LDA_new_testing_pred$class, actual = bag_prediction_test$oversold)
```

```
##      actual
## predicted_LDA      0  1
## -1.77296193579809  2  1
##  0.56328478168585 47 42
```

```
plot(bag_prediction_scale$lowerestimate_USD, bag_prediction_scale$upperestimate_USD,
     col = c("blue", "red")[bag_prediction_test$oversold],
     xlab = "lowerestimate_USD", ylab = "upperestimate_USD", xlim = c(-1,2),
     main = "New Dataset True class vs Predicted class by LDA")
points(bag_prediction_scale$lowerestimate_USD, bag_prediction_scale$upperestimate_USD, pch = c(2,3)[LDA,],
       legend("topright", c("true 1", "true 0",
                           "pred 1", "pred 0"),
              col = c("blue", "red", "grey", "grey"), pch = c(1, 1, 2, 3))
```

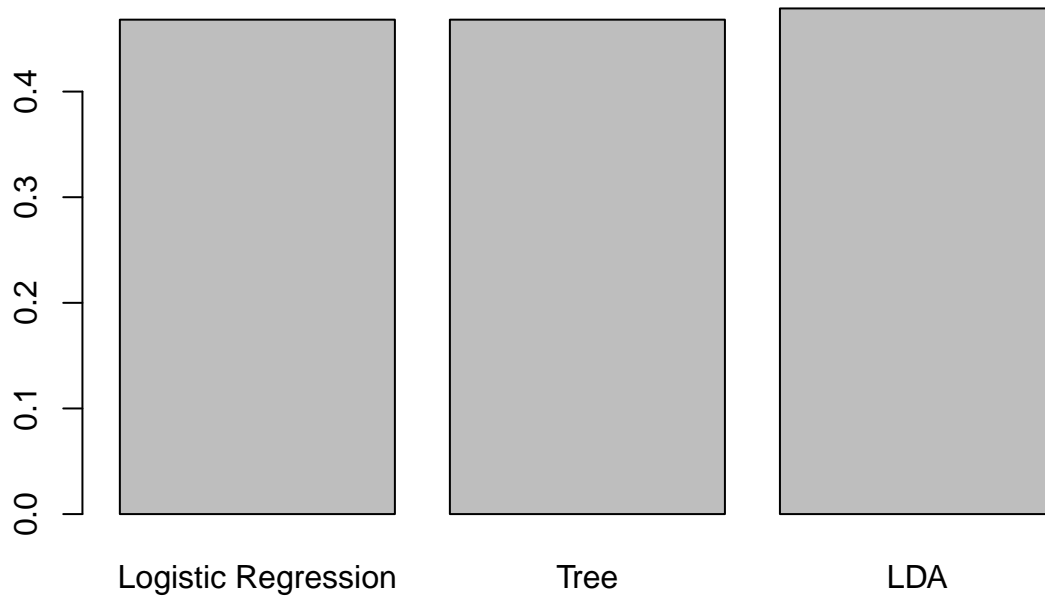
## New Dataset True class vs Predicted class by LDA



```
#accuracy table
log_acc <- 44/94
class_acc <- 44/94
lda_acc <- 45/94
barplot(c(log_acc, class_acc, lda_acc), main="Accuracy for three models Use 0.5 as cut-off",
        names.arg=c("Logistic Regression", "Tree", "LDA"))
```



### Accuracy for three models Use 0.5 as cut-off



```
log_acc2 <- 47/94
class_acc2 <- 43/94
lda_acc2 <- 45/94
barplot(c(log_acc2, class_acc2, lda_acc2), main="Accuracy for three models Use Optimal as cut-off",
        names.arg=c("Logistic Regression", "Tree", "LDA"))
```

**Accuracy for three models Use Optimal as cut-off**

