

Lookup Table-Based Consensus Algorithm for Real-Time Longitudinal Motion Control of Connected and Automated Vehicles

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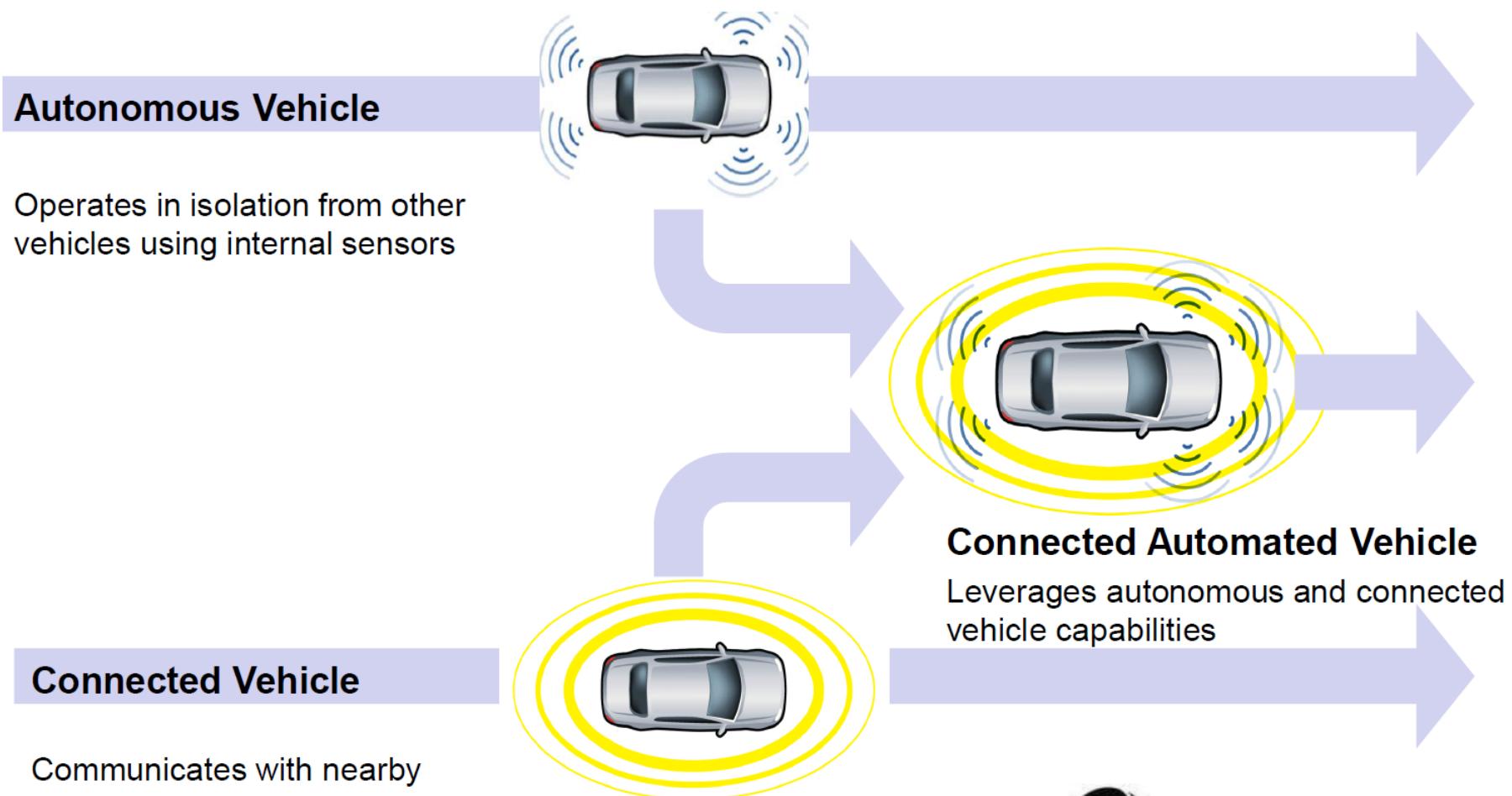
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INTRODUCTION AND BACKGROUND

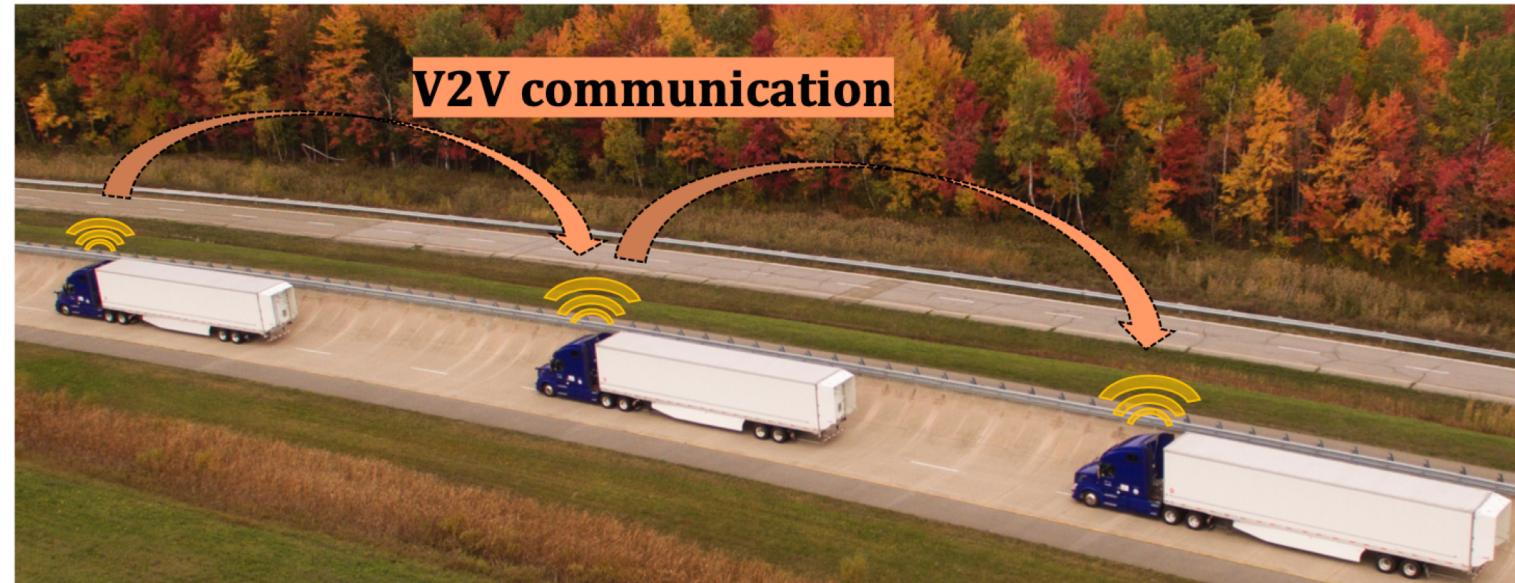
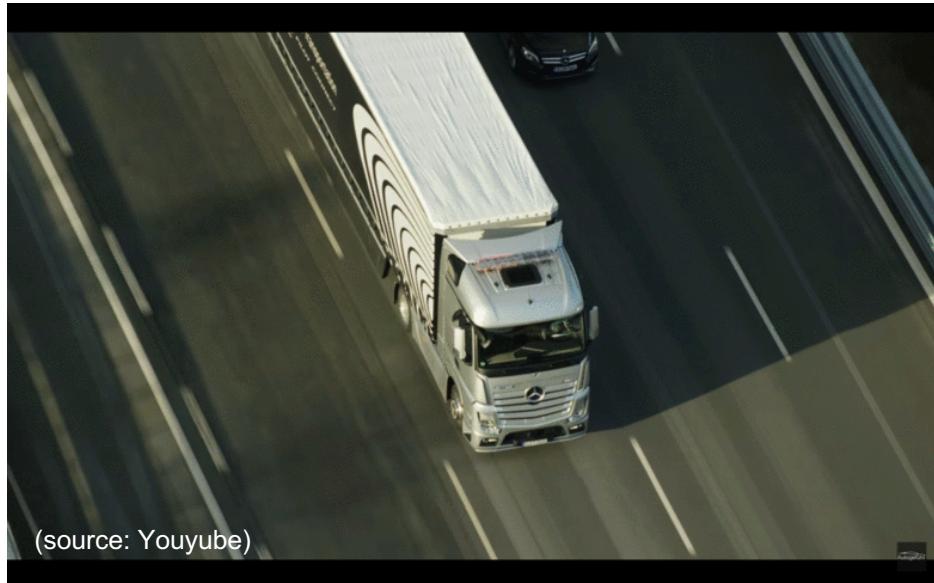
Connected and Automated Vehicles



U.S. Department of Transportation
ITS Joint Program Office

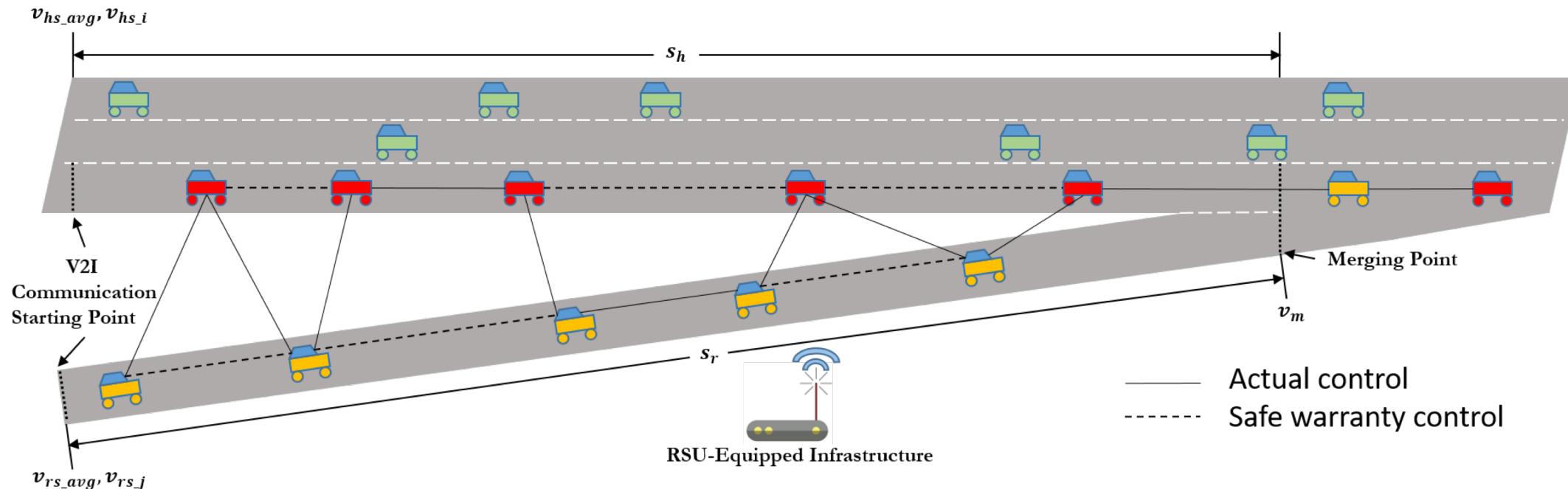
Cooperative Adaptive Cruise Control

- **Safer** than human driving by taking a lot of danger out of the equation
- Roadway **capacity** is increased due to the reduction of inter-vehicle time gap
- **Fuel** consumption and pollutant emissions are reduced due to the mitigation of unnecessary stop and go, and aerodynamic drag of following vehicles



Cooperative Merging at Highway On-Ramps

- Cooperative merging at highway on-ramps
 - Take advantage of V2V and I2V communication
 - Adopt “ghost vehicle” concept
 - Complete longitudinal formation before merging



LOOKUP TABLE-BASED CONSENSUS ALGORITHM

Distributed Consensus Algorithms for Car Following

Dynamics of a connected and automated vehicle

$$\begin{aligned}\dot{r}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= a_i(t)\end{aligned}$$

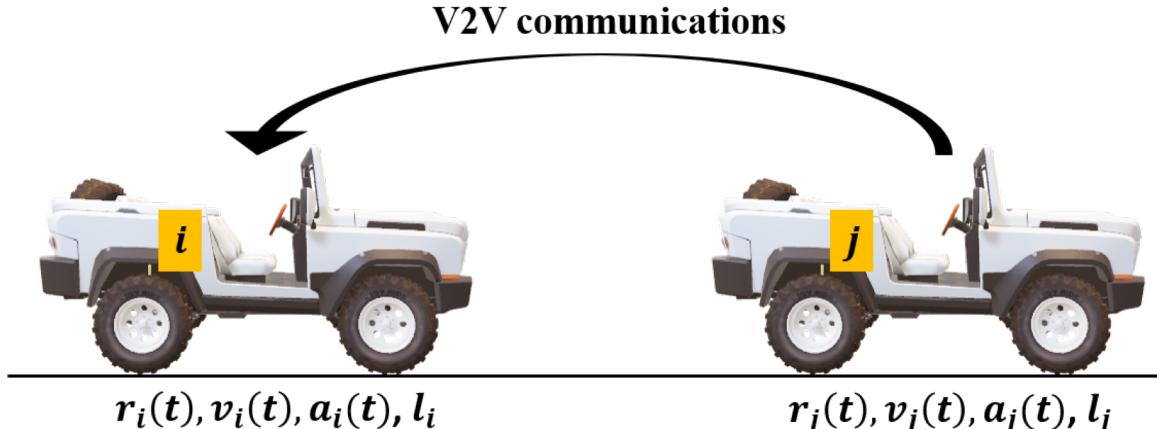
- **First-order consensus algorithm**

$$v_i(t) = - \sum_{j=1}^{n-1} a_{ij} k_{ij} (r_i(t) - r_j(t) - gap), \quad i = 2, \dots, n, j = i - 1$$

- **Second-order consensus algorithm**

$$a_i(t) = - \sum_{j=1}^{n-1} a_{ij} k_{ij} \left[(r_i(t) - r_j(t) - gap) + \gamma (v_i(t) - v_j(t)) \right], \quad i = 2, \dots, n, j = i - 1$$

where a_{ij} is the adjacency matrix of the associated communication graph, k_{ij} and γ are control gains

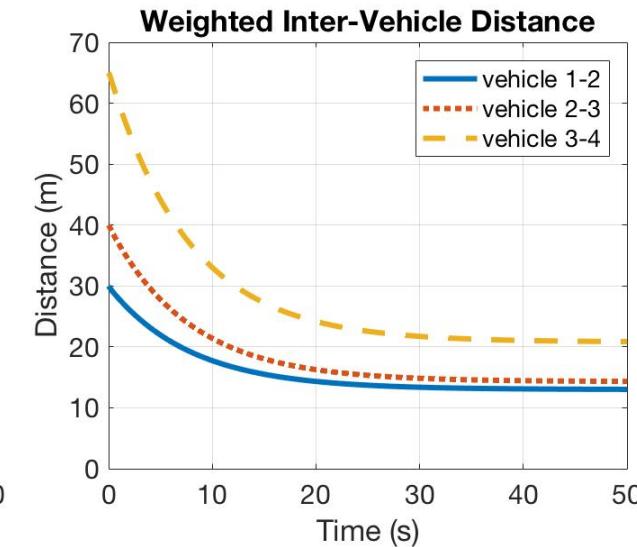
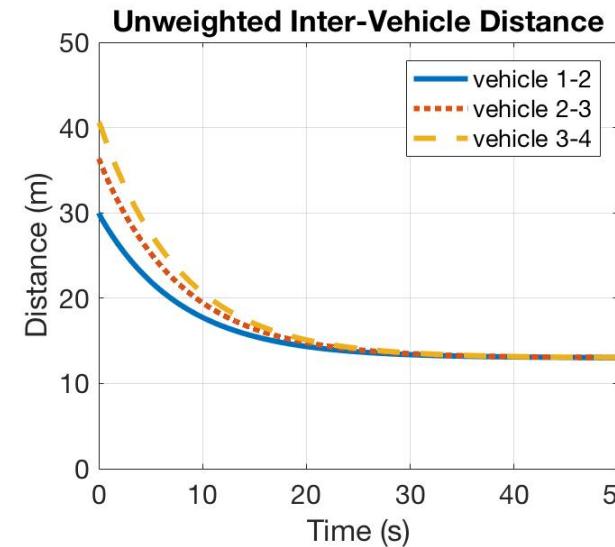
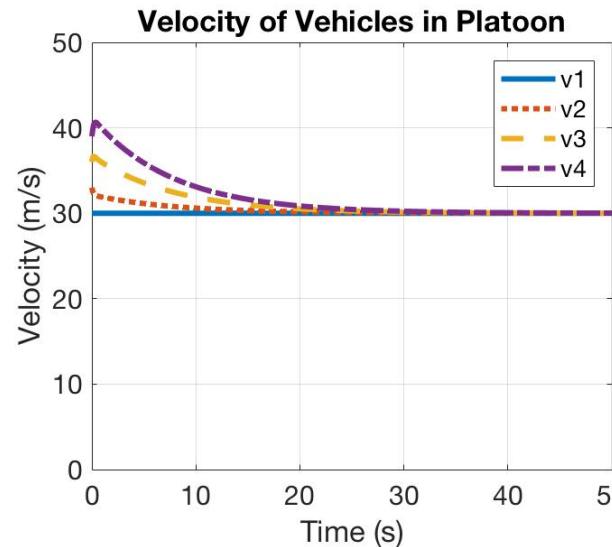
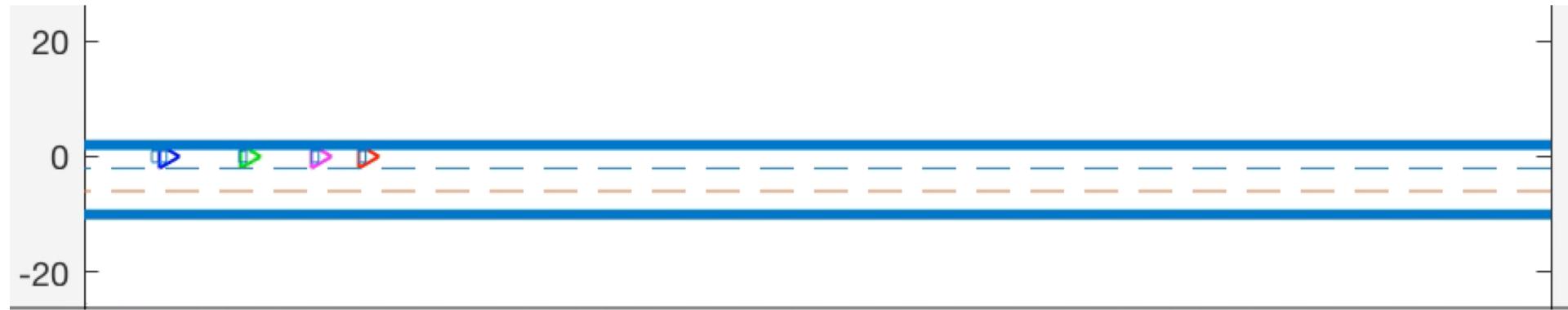


Distributed Consensus Algorithms for Car Following

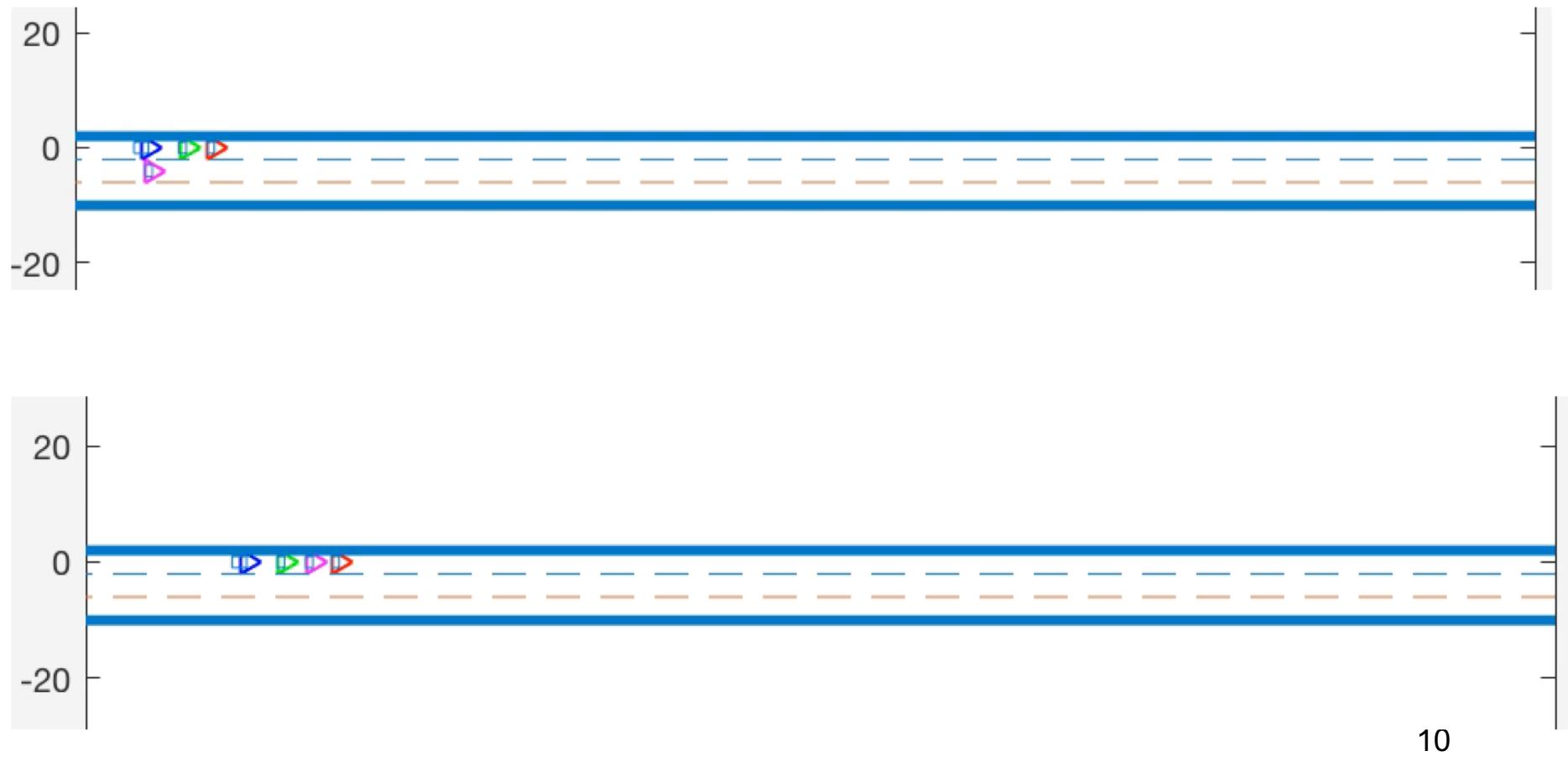
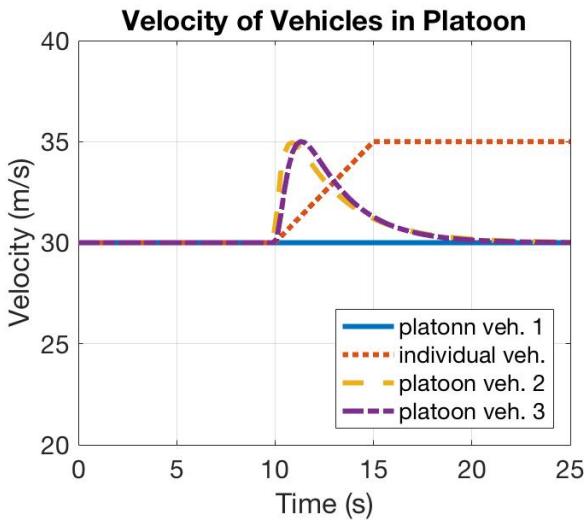
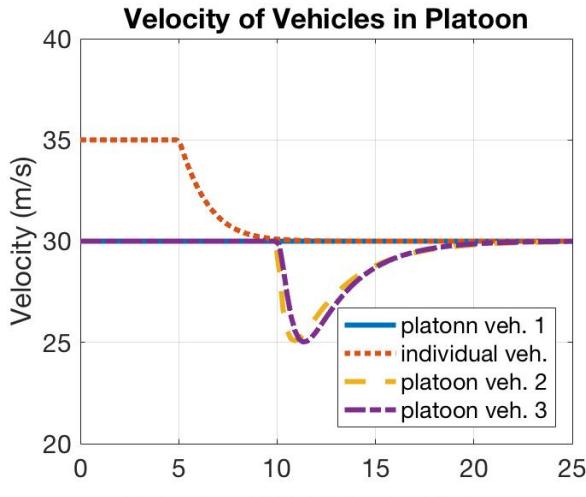
$$\begin{cases} \dot{r}_i(t) = v_i(t) \\ \dot{v}_i(t) = -a_{ij}k_{ij}[r_i(t) - r_j(t - \tau_{ij}(t)) + l_{if} + l_{jr} + v_i(t - \tau_{ij}(t))(t_{ij}^g + \tau_{ij}(t))b_i] \\ \quad - \cancel{\gamma a_{ij}k_{ij}} [v_i(t) - v_j(t - \tau_{ij}(t))] \end{cases} \quad i = 2, \dots, n, j = i - 1$$

$r_i(t)$	Longitudinal position of vehicle i at time t	t_{ij}^g	Inter-vehicle time gap
$v_i(t)$	Longitudinal speed of vehicle i at time t	l_{if}	Length between GPS antenna to front bumper
$\dot{v}_i(t)$	Longitudinal acceleration of vehicle i at time t	l_{jr}	Length between GPS antenna to rear bumper
a_{ij}	(i, j) th entry of the adjacency matrix	b_i	Braking factor of vehicle i
$\tau_{ij}(t)$	Communication delay at time t	γ, k_{ij}	Tuning parameter

Cooperative Adaptive Cruise Control



Cooperative Merging and Splitting



Necessity to Tune Control Gain

- Build lookup table for the control gain

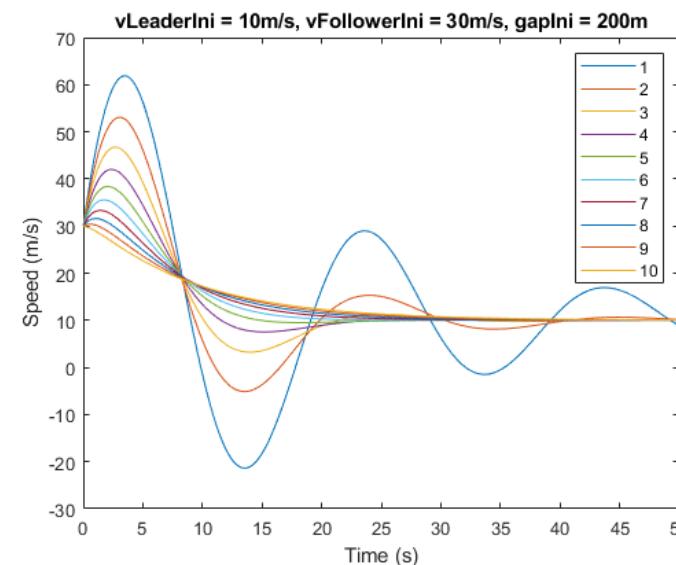
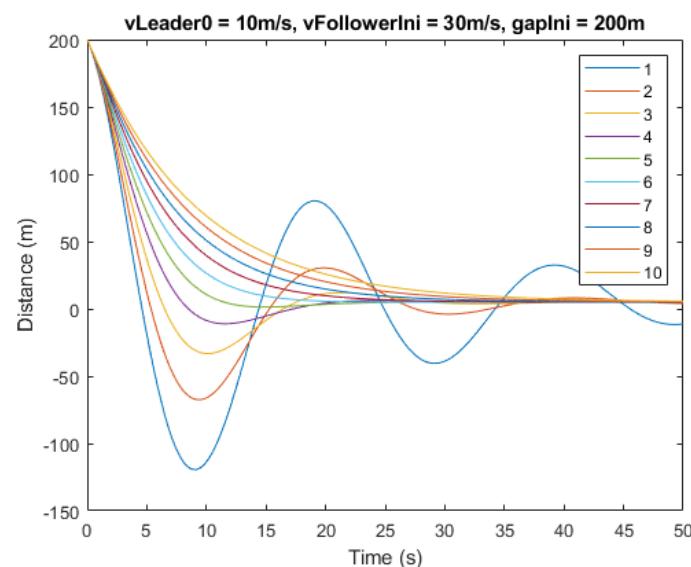
Initial states $(\Delta r_{ij}(t_0), v_i(t_0), v_j(t_0 - \tau_{ij}(t_0))$) varies every time the algorithm is switched on by vehicles



Initial states of vehicles highly affect the convergence of the consensus algorithm



Build up a lookup table to find the suitable value of control gains with respect to different initial conditions



Constraints to Build Lookup Table

Safety Constraint (1st priority)

Evaluated by headway overshoot

$$r_j(t - \tau_{ij}(t)) - r_i(t) > l_j, t \in [t_0, t_{consensus}]$$

Efficiency Constraint (2nd priority)

Evaluated by convergence time

$$\begin{aligned} \left| r_j(t_{consensus} - \tau_{ij}(t_{consensus})) - r_i(t_{consensus}) \right| &\leq \eta_r \cdot \left[l_j + v_i(t_{consensus}) \cdot \left(t_{ij}^g(t_{consensus}) + \tau_{ij}(t_{consensus}) \right) \right] \\ \left| v_j(t_{consensus} - \tau_{ij}(t_{consensus})) - v_i(t_{consensus}) \right| &\leq \eta_v \cdot v_j(t_{consensus} - \tau_{ij}(t_{consensus})) \\ |a_i(t_{consensus})| &\leq \delta_a \\ |jerk_i(t_{consensus})| &\leq \delta_{jerk} \end{aligned}$$

Comfort Constraint (3rd priority)

Evaluated by maximum acceleration/deceleration and maximum jerk

$$\Omega_i = \omega_1 \cdot \max_{t \in [t_0, t_{consensus}]} (|a_i^{\max}(t)|, |d_i^{\max}(t)|) + \omega_2 \cdot \max_{t \in [t_0, t_{consensus}]} (|jerk_i^{\max}(t)|, |jerk_i^{\min}(t)|), t \in [t_0, t_{consensus}]$$

Algorithms to Build Lookup Table

Algorithm 1: Build lookup table offline

Input: $\Pi_{\Delta r_{ij}} = \{\Delta r_{ij_1}, \Delta r_{ij_2}, \dots, \Delta r_{ij_{\zeta_1}}\}$, $\Pi_{v_i} = \{v_{i_1}, v_{i_2}, \dots, v_{i_{\zeta_2}}\}$,
 $\Pi_{v_j} = \{v_{j_1}, v_{j_2}, \dots, v_{j_{\zeta_3}}\}$, $\Pi_\gamma = \{v_{\gamma_1}, v_{\gamma_2}, \dots, v_{\gamma_{\zeta_4}}\}$, $\Pi_k = \{v_{k_1}, v_{k_2}, \dots, v_{k_{\zeta_5}}\}$, $\zeta_1 = |\Pi_{\Delta r_{ij}}|$, $\zeta_2 = |\Pi_{v_i}|$, $\zeta_3 = |\Pi_{v_j}|$
Output: 3-dimension table with size $\zeta_1 \times \zeta_2 \times \zeta_3$

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01: for  $\xi_1 \in [1, \zeta_1], \xi_2 \in [1, \zeta_2], \xi_3 \in [1, \zeta_3], \Delta r_{ij_{\xi_1}} \in \Pi_{\Delta r_{ij}}$ ,
     $v_{i_{\xi_2}} \in \Pi_{v_i}, v_{j_{\xi_3}} \in \Pi_{v_j}$  do
02:   run algorithm (9) with  $\Delta r_{ij_{\xi_1}}, v_{i_{\xi_2}}$  and  $v_{j_{\xi_3}}$ 
03:   find  $\Lambda_\gamma \subseteq \Pi_\gamma, \Lambda_k \subseteq \Pi_k$  satisfy Constraint 1
04:   if  $\Lambda_\gamma = \emptyset \text{ || } \Lambda_k = \emptyset$  then
05:      $\gamma_{(\xi_1, \xi_2, \xi_3)} = NaN, k_{(\xi_1, \xi_2, \xi_3)} = NaN$ 
06:   else
07:     find  $\Psi_\gamma \subseteq \Lambda_\gamma, \Psi_k \subseteq \Lambda_k$  satisfy Constraint 2
08:     if  $|\Psi_\gamma| == |\Psi_k| == 1$  then
09:        $\gamma_{(\xi_1, \xi_2, \xi_3)} \in \Psi_\gamma, k_{(\xi_1, \xi_2, \xi_3)} \in \Psi_k$ 
10:     else
11:       find  $\Phi_\gamma \subseteq \Psi_\gamma, \Phi_k \subseteq \Psi_k$  satisfy Constraint 3
12:       if  $|\Phi_\gamma| == |\Phi_k| == 1$  then
13:          $\gamma_{(\xi_1, \xi_2, \xi_3)} \in \Phi_\gamma, k_{(\xi_1, \xi_2, \xi_3)} \in \Phi_k$ 
14:       else
15:          $\gamma_{(\xi_1, \xi_2, \xi_3)} = \min_{\gamma \in \Phi_\gamma} \Phi_\gamma, k_{(\xi_1, \xi_2, \xi_3)} = \min_{k \in \Phi_k} \Phi_k$ 
16:       end if
17:     end if
18:   end if
19: end for

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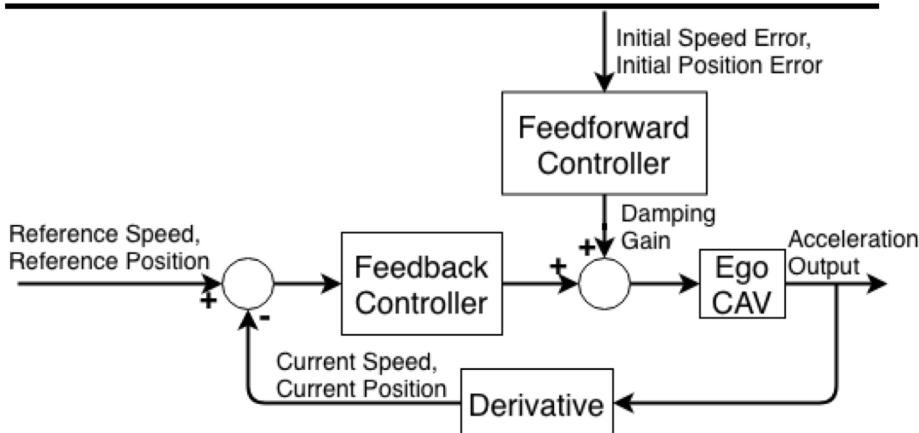
Algorithm 2: Search lookup table in real time

Input: $(\Delta r_{ij}(t_0), v_i(t_0), v_j(t_0 - \tau_{ij}(t_0)))$, ($\zeta_1 \times \zeta_2 \times \zeta_3$) size of lookup table
Output: values of control gains k and γ

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01: for  $\Delta r_{ij}(t_0), v_i(t_0), v_j(t_0 - \tau_{ij}(t_0))$  do
02:   if  $\Delta r_{ij}(t_0) < \Delta r_{ij_1} \text{ || } \Delta r_{ij}(t_0) > \Delta r_{ij_{\zeta_1}} \text{ || } v_i(t_0) < v_{i_1} \text{ || } v_i(t_0)$ 
       $> v_{i_{\zeta_2}} \text{ || } v_j(t_0 - \tau_{ij}(t_0)) < v_{j_1} \text{ || } v_j(t_0 - \tau_{ij}(t_0)) > v_{j_{\zeta_3}}$  then
03:     return  $\gamma = NaN, k = NaN$ 
04:   else
05:     find  $(\Delta r_{ij_{\xi_1}} \mid \min |\Delta r_{ij}(t_0) - \Delta r_{ij_{\xi_1}}|),$ 
         $(v_{i_{\xi_2}} \mid \min |v_i(t_0) - v_{i_{\xi_2}}|),$ 
         $(v_{j_{\xi_3}} \mid \min |v_j(t_0 - \tau_{ij}(t_0)) - v_{j_{\xi_3}}|)$ 
06:     return  $\gamma = \gamma(\Delta r_{ij_{\xi_1}}, v_{i_{\xi_2}}, v_{j_{\xi_3}}), k = k(\Delta r_{ij_{\xi_1}}, v_{i_{\xi_2}}, v_{j_{\xi_3}})$ 
07:   end if
08: end for

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Numerical Simulation Results

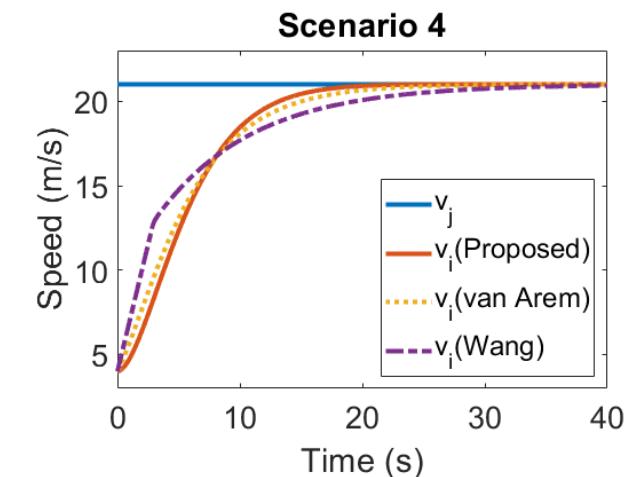
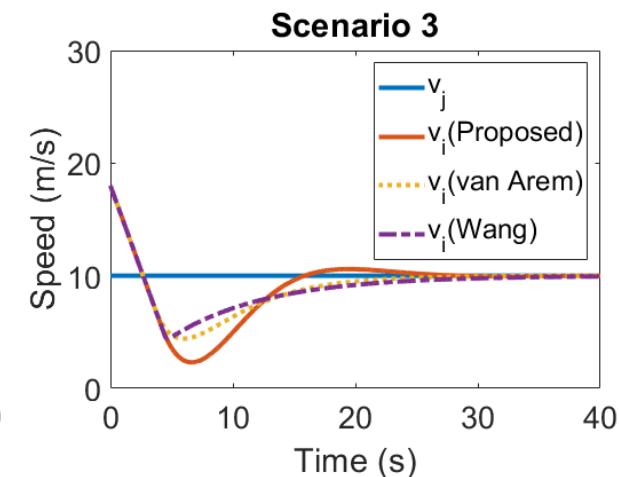
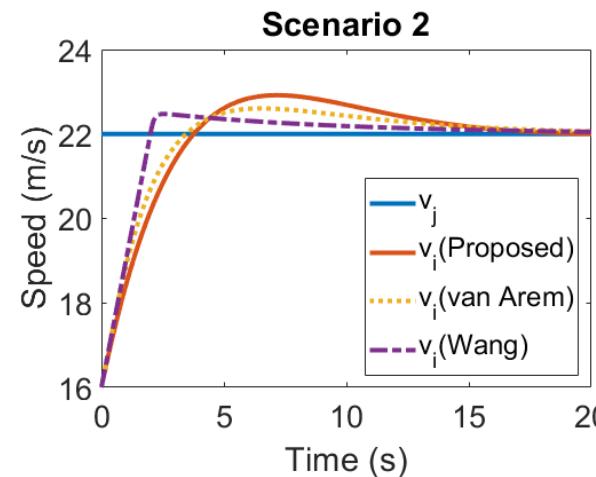
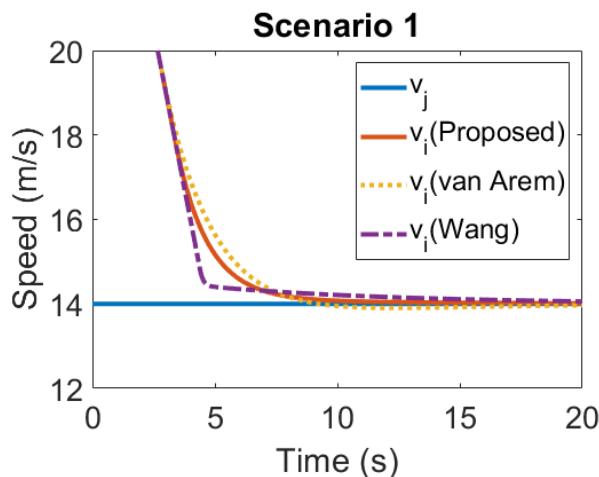
- Compare to our existing algorithm, as well as van Arem's CACC algorithm

TABLE I. Settings of Simulation Scenarios

	$\Delta r_{ij}(t_0)$ (m)	$v_i(t_0)$ (m/s)	$v_j(t_0 - \tau_{ij}(t_0))$ (m/s)
Scenario 1	50	28	14
Scenario 2	20	16	22
Scenario 3	-30	18	10
Scenario 4	-80	4	21

TABLE II. SIMULATION RESULTS

Scenario	Convergence time (s)				Maximum jerk (m/s ³)			
	1	2	3	4	1	2	3	4
Wang	35.9	35.0	56.5	57.6	21.2	20.7	25.7	13.4
van Arem	29.3	32.1	41.8	40.1	1.5	1.6	2.3	0.7
Proposed	24.9	22.9	32.1	28.3	2.3	0.8	1.6	1.6

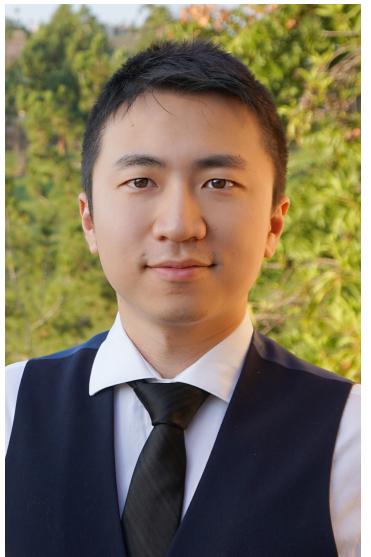


FUTURE WORK AND ACKNOWLEDGEMENT

Future Work

- Build a more **comprehensive** lookup table especially when leading vehicle's speed is dynamically changing
- Consider higher-order vehicle **dynamics** to consider acceleration and jerk of the leading vehicle
- Test the effectiveness of the proposed methodology in various real-world traffic scenarios

Acknowledgement



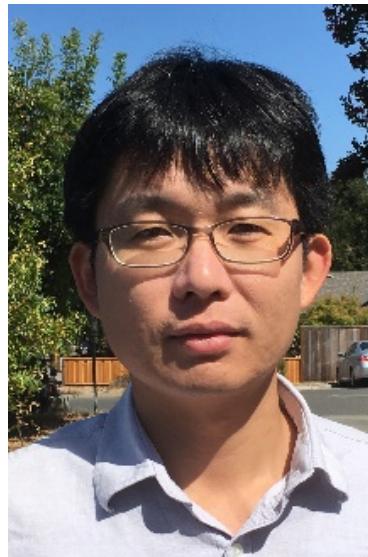
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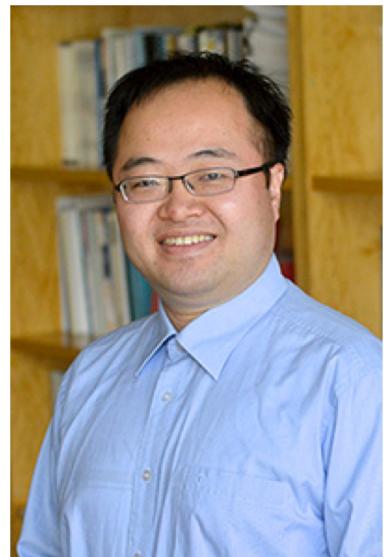
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Thank you all! QUESTIONS?