Developing a Distributed Consensus-Based Cooperative Adaptive Cruise Control (CACC) System

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ABSTRACT

Autopilot driving and autonomous driving have become increasingly popular in recent years. As an example, a Cooperative Adaptive Cruise Control (CACC) system allows autopilot or autonomous vehicles to communicate with each other, and coordinate their maneuvers in a form of platoon, where one vehicle follows another with a constant velocity and/or time headway. A CACC system brings about several benefits, including the improvement of driving safety on a highway, the increase of roadway capacity, and the decrease of fuel consumption and pollutant emissions. In this paper, we propose a novel CACC system based on the *distributed consensus theory*, where distributed consensus algorithm and protocol are designed for platoon formation, merging maneuvers, and splitting maneuvers. Unlike many existing CACC systems, our system only uses local cooperation to achieve a global goal, hence the communication complexity is reduced. Moreover, different from most studies assuming the type and dynamics of all the vehicles in a platoon to be homogenous, our distributed consensus algorithm takes into account the length and braking ability of different vehicles. Communication delay is also included in the algorithm, making the system more realistic and applicable. A simulation study is conducted under different scenarios, including normal platoon formation, platoon restoration from disturbances, and merging and splitting maneuvers. We also carry out sensitivity analysis on the distributed consensus algorithm, investigating the effect of the damping term on driving safety and driving comfort of our CACC system.

INTRODUCTION

Recently, the rapid development of transportation systems has led to a world-wide economic prosperity, where transportation for both passengers and goods is much more convenient than before domestically and internationally. The number of motor vehicles worldwide is estimated to be more than 1 billion nowadays, and will be doubled within one or two decades (*1*). Such a huge quantity of motor vehicles and intensive transportation activities have brought about various social-economic issues. For example, more than 30,000 people die from roadway crashes on US highways every year (*2*). In terms of traffic congestion, more than $100 billion are cost annually in the U.S. alone due to fuel waste and time loss (*3*).

Significant efforts have been made around the world to address the above issues. Many people have been considering to expand existing infrastructure to solve these traffic-related problems, or at least relieve the induced pressures. However, this is not only costly, but also overwhelmed by the chain effects from the neighborhood environment. Some indirect solutions to the increases of roadway capacity include the development of intelligent transportation systems which can help better regulate the traffic, thus improving traffic safety, mobility, and reliability without building more infrastructures. One of the promising ITS applications is the Cooperative Adaptive Cruise Control (CACC), which is a combination of Adaptive Cruise Control (ACC) and Connected Vehicle (CV) technology (e.g., mainly via vehicle-to-vehicle communication) (*4*). With the information shared among vehicles, the CACC system allows vehicles to form platoons and be driven at the same velocity and with constant time headway. The main advantages of the CACC system are: a) autonomous or autopilot driving is safer than human driving by taking a lot of danger (e.g., being distracted) out of the equation; b) roadway capacity is increased due to the reduction of inter-vehicle time gap without compromising the safety; c) fuel consumption and pollutant emissions are reduced due to the mitigation of aerodynamic drag of following vehicles in the platoon and potentially less incidents.

In view of the merits mentioned above and the advances in wireless communication, many studies have been conducted to develop and improve the CACC system (*5-9*). Weighted and constrained protocols are proposed in (*10*) to coordinate vehicles in a CACC system. The goal of it is to understand the influence of communication network topologies on the platooning dynamics by applying a discrete-time Markov chain based algorithm. However, the protocols only focus on the positions of vehicles (as their states), without considering the constraints on the values of velocity. In addition, the inter-vehicle distances are described as constant values without taking into account the variations of vehicles' characteristics (e.g., vehicle type, braking capability) within the platoon. This discounts the protocols' practicability in the real-world implementation. In this study, we propose a distributed consensus methodology for the CACC system based on the second-order consensus algorithm (*11*). In our methodology, we use vehicle velocity, time headway together with vehicle length to adjust inter-vehicle distance. Besides analyzing the normal platoon formation scenario, platoon restoration from disturbances, merging and splitting maneuvers are also addressed by the methodology we proposed.

The remainder of this paper is organized as follows: The next section reviews the related work in the research area of CACC. Section 3 describes the methodology used for distributed consensus-based CACC system. Section 4 carries out detailed simulation study and sensitivity analysis where the results are further discussed. The last section provides a summary of the paper and outlines some future work.

RELATED WORK

This paper is focused more on the algorithm and protocol of CACC that enable vehicles to form and stabilize the platoon. We assume a standard Dedicated Short Range Communications (DSRC) and Wireless Access in the Vehicular Environment (WAVE) (*12*, *13*) access network with beaconing messages.

The core of a CACC system is the vehicle-following model, which efficiently describes the vehicle dynamics and cooperative maneuvers residing in the system. In (*14*), a nonlinear model is used to describe the vehicle longitudinal dynamics, where the engine power, gravity, road and tire resistance, and aerodynamics drag are all considered. However, since the complexity of such nonlinear models introduces inconvenience to system analysis, a linearized model is usually more practical for field deployment (*15*). In terms of inter-vehicle gap in motion (at relatively high speed), the existing vehicle-following models can be divided into two categories: one regulates the spatial gap, where one vehicle follows its preceding vehicle with a fixed inter-vehicle distance (*16*). The other is based on time gap or velocity dependent distance, where the inter-vehicle distance may vary with vehicle velocity and vehicle length by keeping a constant time headway. Our study falls into the second category.

The performance and robustness of a CACC consensus algorithm are discussed in (*17*), where packet loss, network failures and beaconing frequencies are all taken into consideration when the simulation framework is built with the CACC controller developed by (*18*). A consensus-based platooning approach is proposed by (*19*), where the importance of wireless Inter-Vehicle Communications (IVC) is discussed. The approach is analyzed in three wireless network settings, including uncorrelated Bernoullian losses, correlated losses using a Gilbert-Elliott channel, and a realistic traffic scenario with interferences caused by other vehicles.

Stability is a basic requirement to ensure the safety of a CACC system. Particularly, propagation of motion signals is attenuated by adjusting the controller parameters in the system, which guarantees the so called string stability of the platoon (*20*). It is shown in (*21*) that a predecessor-following platoon is fragile to external disturbances. However, the standard constant time-gap spacing policy can guarantee string stability as long as a sufficient large time gap is maintained (*22*). Since the distributed consensus algorithm adopted in our paper relies on the time-gap spacing policy, string stability can be guaranteed when the requirement of time gap suffices.

METHODOLOGY

Mathematical Preliminaries and Nomenclature

We represent the interaction topology of a distributed network of vehicles by using a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \dots, n\}$ is a finite nonempty node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set of ordered pairs of nodes, called edges. The edge $(i, j) \in \mathcal{E}$ denotes that vehicle j can obtain information from vehicle i, however, this is not necessarily true vice versa. The neighbors of vehicle i are denoted by $\mathcal{N}_i = \{j \in \mathcal{V}: (i, j) \in \mathcal{E}\}$. The topology of the graph is associated to an adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}$, which is defined such that $a_{ii} = 0$ and a_{ij} is a positive weight if edge $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ if edge $(j, i) \notin \mathcal{E}$. All graphs are weighted, and $a_{ij} = 1$ for all edge $(j, i) \in \mathcal{E}$ if the weights are not relevant. $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}$ (i.e., $\ell_{ij} = -a_{ij}, i \neq j, \ell_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$) is the nonsymmetrical Laplacian matrix associated with $\mathcal{G}$. A directed spanning tree is a directed tree formed by graph edges that connect all the nodes of the graph.

Before proceeding to design our distributed consensus algorithm for the CACC system, we recall here some basic consensus algorithms which can be used to apply similar dynamics on the information states of vehicles. If the communication between vehicles in the distributed networks is continuous, then a differential equation can be used to model the information state update of each vehicle.

The single-integrator consensus algorithm (*23*) is given by

$$\dot{x}_i(t) = -\sum_{j=1}^{n} g_{ij} k_{ij} \left(x_i(t) - x_j(t)\right), i \in V \quad (2)$$

$x_i \in \mathbb{R}$, $k_{ij} > 0$, $g_{ij} = 1$ if information flows from vehicle j to i and 0 otherwise, $\forall i \neq j$. The adjacency matrix A of the interaction topology is defined accordingly as $a_{ii} = 0$ and $a_{ij} = g_{ij} k_{ij}$, $\forall i \neq j$. This consensus algorithm guarantees convergence of multiple agents to a collective decision via local interactions.

The consensus algorithm (2) can be extended to double-integrator dynamics to better model the movement of a physical entity, such as an autonomous vehicle. For a double-integrator modeled vehicle, the consensus algorithm (*11*) is given by

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = -\sum_{j=1}^{n} g_{ij} \left[k_{ij}\left(x_i(t) - x_j(t)\right) + \gamma k_{ij}\left(v_i(t) - v_j(t)\right)\right], i \in V \end{cases} \quad (3)$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$, $k_{ij} > 0$, $\gamma > 0$, and $g_{ij} = 1$ if information flows from vehicle j to i and 0 otherwise, $\forall i \neq j$.

Distributed Consensus Algorithm for the CACC System

The objective of the distributed consensus-based CACC system is to use algorithm and protocol to ensure consensus of a platoon of vehicles. Toward this end, the meaning of consensus is two-fold: one is the position consensus, where one vehicle maintains a certain distance with its preceding one; the other is the velocity consensus, where one vehicle maintains the same velocity with its preceding one. Taking into account second-order vehicle dynamics, we propose the following distributed consensus algorithm for the CACC system

$$\begin{aligned} \ddot{x}_i(t) = & -\left[x_i(t) - x_j\left(t - \tau_{ij}(t)\right) + c_j + \dot{x}_j\left(t - \tau_{ij}(t)\right) t_{ij}^g b_i \right] \\ & -\gamma\left[\dot{x}_i(t) - \dot{x}_j\left(t - \tau_{ij}(t)\right)\right],\ i = 2, \dots, n, j = i - 1 \end{aligned} \quad (4)$$

where vehicle i's preceding vehicle is vehicle j; $x_i(t)$ is the position of vehicle i at time t; $\tau_{ij}(t)$ is the unavoidable time-varying communication delay when information is transmitted from vehicle j to vehicle i; c_j is the length of vehicle j; t_{ij}^g is the desired inter-vehicle time gap between vehicle i and vehicle j; b_i is the braking factor of vehicle i; γ is a damping term. The first part $\left[x_i(t) - x_j\left(t - \tau_{ij}(t)\right) + c_j + \dot{x}_j\left(t - \tau_{ij}(t)\right) t_{ij}^g b_i \right]$ is the absolute position consensus term, and the second part $\gamma[\dot{x}_i(t) - \dot{x}_j\left(t - \tau_{ij}(t)\right)$ is the velocity consensus term.

The communication delay $\tau_{ij}(t)$ is bounded as $\tau_{ij}(t) \leq \tau$, and can be simply set to 100 ns since each message is associated with GPS-base time and has a precision better than 100 ns (*19)*. Since different types of vehicles may have different braking performance, we introduce a braking factor b_i into our distributed consensus algorithm to better simulate the real-world situations. We assume the braking factor is a known constant, and it is varied by different vehicle species. According to the braking distance of different kinds of vehicles (*24*,*25*), we can set a 4-door sedan with a braking factor of 1, a SUV with a braking factor of 1.1, and a truck with a braking factor of 1.6. We use time gap t_{ij}^g to adjust the inter-vehicle distances, which are subject to the change of vehicles' velocities. Furthermore, the time headway can be denoted as $t_h = t_{ij}^{g} b_i + \frac{c_j}{\dot{x}_j(t-\tau_{ij}(t))}$. The damping term γ needs to meet a specific requirement to ensure the convergence property of the distributed consensus algorithm, which will be analyzed in the fourth section of the paper.

Algorithm (4) is designed for all but the leading vehicle in our CACC system. The dynamics of the leading vehicle can be characterized as follows:

$$\begin{cases} \dot{x}_1\,(t) = v_1\,(t) \\ \dot{v}_1\,(t) = a_1\,(t) \end{cases} \tag{5}$$

where $x_1(t)$, $v_1(t)$ and $a_1(t)$ represent the absolute position, velocity, and acceleration of the leading vehicle, respectively. The leading vehicle of a platoon can be manually driven, where the velocity and acceleration may be time-varying. It can also be set to cruise at a certain velocity, with zero acceleration. Algorithm (4) will allow all the following vehicles in the platoon to track the dynamics of the leading vehicle on the above two occasions.

Distributed Consensus Protocol for the CACC System

In our distributed consensus-based CACC system, we assume every vehicle is equipped with on-board sensors and inertial measurement unit (IMU) to measure its absolute position and velocity. The IEEE 802.11p standard (*13*) is used to support the wireless information transmission among vehicles. Considering different scenarios in our system, two protocols are designed in the following.

Protocol 1: Normal Platoon Formation

This protocol is designed for vehicles to form a platoon. For vehicle i in our CACC system, it needs to check whether there is a preceding vehicle in a certain distance l (e.g., via RADAR, LIDAR, or computer vision) when the platoon formation mode is on.

a) If yes, then vehicle i will communicate with its preceding vehicle and algorithm (4) will be applied, which enables vehicle i to be a following vehicle;

b) If no, then vehicle i may become a leading vehicle of a platoon (where i = 1) and cruise at a constant velocity. The driver can also take over the control to drive however he/she wants, but the vehicle may still potentially act as a leading vehicle of the platoon.

After the above procedure, vehicle i is in the distributed consensus-based CACC system, whether it plays the role of a following vehicle or a leading vehicle. However, the "following" and "leading" role for vehicle i may switch under the following conditions:

a) For a following vehicle i, if all of the preceding vehicles of vehicle i move out of the distance l ahead of vehicle i, then vehicle i changes from a following vehicle to a leading vehicle, where i = 1;

b) For a leading vehicle i (i.e., i = 1), if one or more vehicles move into the distance l ahead of vehicle i, then vehicle i changes from a leading vehicle to a following vehicle, where i = 2, …, n.

If the length of a platoon is too long, it may bring about safety and road utility issues. Therefore, the aforementioned distance l should be constrained to a threshold value l_{max}. Intuitively, if vehicle i is too far away from a platoon in the front, itself may create another platoon. FIGURE 1 shows the flowchart of protocol 1 for the distributed consensus-based CACC system.

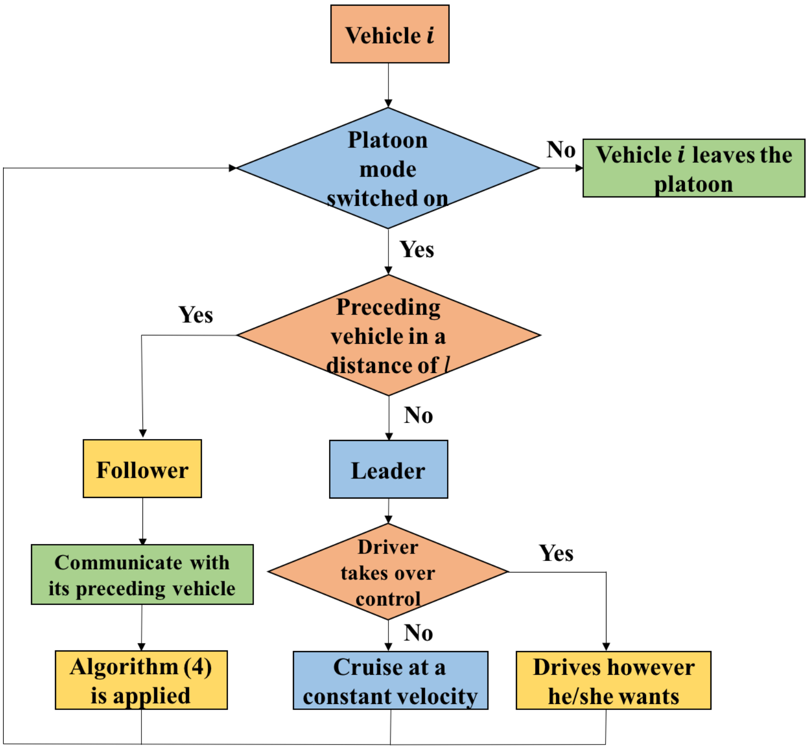


FIGURE 1 Protocol 1 for the distributed consensus-based CACC system.

Protocol 2: Merging and splitting maneuvers

Protocol 1 addresses the longitudinal maneuvers while Protocol 2 is aimed at handling the lateral maneuvers (i.e., lane change). It is introduced in (*26*) that there are four different cases for the lane change within the platoon maneuvers: 1) free-agent-to-free-agent lane change, 2) free-agent-to-platoon lane change, 3) platoon-to-free-agent lane change, and 4) platoon-to-platoon lane change. In our paper, we focus on the second and third cases.

For the case where vehicle i (as a free agent) tries to merge into a platoon on the adjacent lane, after the merging mode is activated, vehicle i will communicate with the platoon and decide which position it will be in the platoon. If it decides to be the jth vehicle of the platoon after merging maneuvers, then a "ghost" vehicle with respect to vehicle $j-1$ in the platoon will be created on the lane vehicle i is in. This "ghost" vehicle has all the same parameters but the lateral position as vehicle $j-1$. Then, vehicle i will autonomously adjust its absolute position and velocity with the "ghost" vehicle by algorithm (4). After that, vehicle i sends a merging signal to vehicle $j+1$ in the platoon. Upon receiving the merging signal, a "ghost" vehicle with respect to vehicle i is created in front of vehicle $j+1$, and vehicle $j+1$ starts to adjust its absolute position and velocity to create a gap for vehicle i by algorithm (4). The interaction topology of this platoon changes from a three-vehicle-platoon topology to a four-vehicle-platoon topology, where the "ghost" vehicle in front of vehicle $j+1$ also receives information from its preceding vehicle, which is vehicle $j-1$. After the gap is fully created, vehicle j+1 sends a confirmation signal to vehicle i, and vehicle i merges into the platoon.

The case where vehicle j (in the platoon) tries to split from the platoon is easier. It is studied in (*6*) that there are two strategies for splitting maneuvers, where the most efficient action is for the departing driver to do a simple lane change in the direction of the off-ramp. After the splitting mode is activated, the driver can take over the lateral control of the vehicle and perform the lane change without adjusting the velocity longitudinally. After vehicle j completes the lane change, vehicle $j+1$ will sense that its preceding vehicle changes from vehicle j to vehicle $j-1$, and therefore adjust its velocity to close the gap. A new platoon is formed, where vehicle $j+1$ becomes vehicle j, and vehicle $j+2$ becomes vehicle $j+1$, and so on.

SIMULATION STUDY

Simulation Scenarios and Results

We use MATLAB Simulink (*27*) to simulate three different scenarios of our distributed consensus-based CACC system. Results of vehicle velocity, weighted and unweighted inter-vehicle distance are shown in different scenarios.

Scenario 1: Normal Platoon Formation

In the first scenario, assume there are four connected vehicles driving at randomly varied velocities on the same lane of a highway. At a certain moment ($t = 0$), they all switch to the platoon mode. From then on, they adjust their absolute positions and velocities based on algorithm (4), algorithm (5) and protocol 1 to reach consensus and form a platoon. The vehicle parameters of this distributed consensus-based CACC system are listed in TABLE 1.

TABLE 1 Values of Vehicle Parameters in Scenario 1

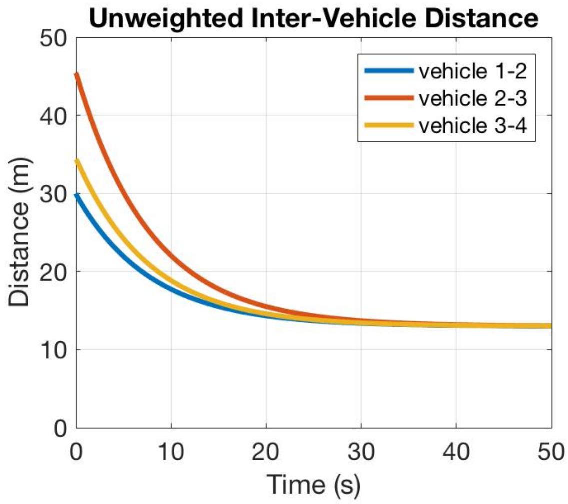
Vehicle Parameters	Vehicle 1	Vehicle 2	Vehicle 3	Vehicle 4
Length c_j	5 m	5 m	5 m	10 m
Braking factor b_i	1	1	1.1	1.6
Initial velocity $\dot{x}_{i0}$	30 m/s	33 m/s	36 m/s	39 m/s
Desired velocity $\dot{x}_i$	30 m/s	30 m/s	30 m/s	30 m/s
Initial time gap t_{ij0}^g	1.06 s	1.25 s	1.79 s	
Initial weighted inter-vehicle distance d_{ij0}	35 m	45 m	70 m	
Desired time gap t_{ij}^g	0.43 s	0.48 s	0.69 s	
Desired time headway t_{ij}^h	0.6 s	0.64 s	0.86 s	
Desired weighted inter-vehicle distance d_{ij}	13 m	14.3 m	20.8 m	
Desired unweighted inter-vehicle distance d_{ij}/b_i	13 m	13 m	13 m	

As we can see from TABLE 1, weighted inter-vehicle distance is used instead of time gap to measure the consensus of vehicles' absolute positions more intuitively. They can be performed as

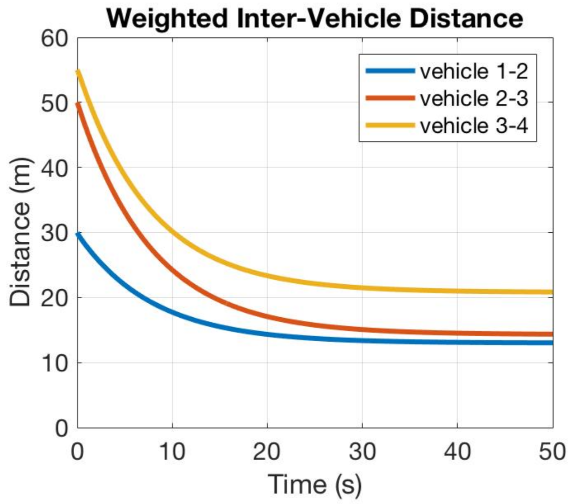
$$\begin{cases} d_{ij0} = \dot{x}_{i0}(t)\, t_{ij0}^g\, b_i \\ d_{ij} = \dot{x}_i(t)\, t_{ij}^g\, b_i \end{cases} \tag{6}$$

The damping term γ in algorithm (4) will effect the convergence speed of absolute position and velocity of all the vehicles in the platoon. In this study, γ =7.5 is preliminarily set to all three simulation scenarios. More detailed analysis on how the value of γ may affect the system performance (e.g., driving safety, driving comfort) will be conducted in the second half of this section.

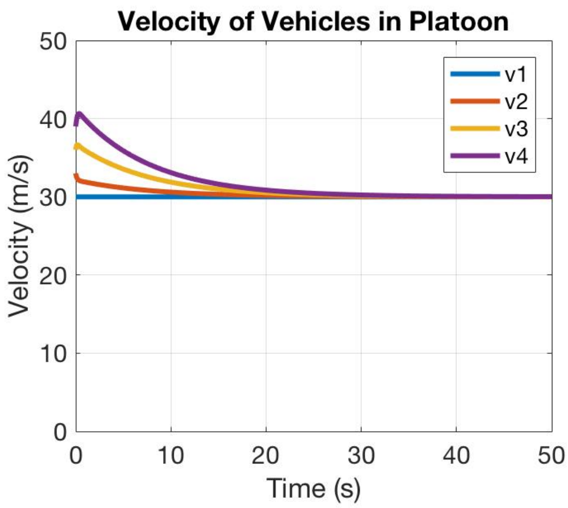
The simulation of our CACC system are shown in FIGURE 2 (*a*)-(*c*). Plot (*a*) shows that after the platoon mode is activated at $t = 0$, all of the three unweighted inter-vehicle distance converge to 13 m at around 40 seconds. This unweighted inter-vehicle distance is more like a target value we set for the system to process, not the "real" inter-vehicle distance. Plot (*b*) shows the results for weighted inter-vehicle distance. By introducing the braking factor, the steady state of weighted inter-vehicle distance varies with different vehicle pairs. The weighted inter-vehicle distance indicates the "real" value for inter-vehicle distance in our CACC system. In this case, at the steady state of the system, vehicle 1 and vehicle 2 have a 13 m (0.43 s) gap, vehicle 2 and vehicle 3 have a 14.3 m (0.48 s) gap, and vehicle 3 and vehicle 4 have a 20.8 m (0.69 s) gap. It is shown in plot (*c*) that velocities of the four vehicles converge within around 40 seconds after the platoon mode is activated. After running the distributed consensus algorithms (4) and (5), they all converge to 30 m/s, which is the constant velocity of the leading vehicle, and also the desired velocity of this platoon.



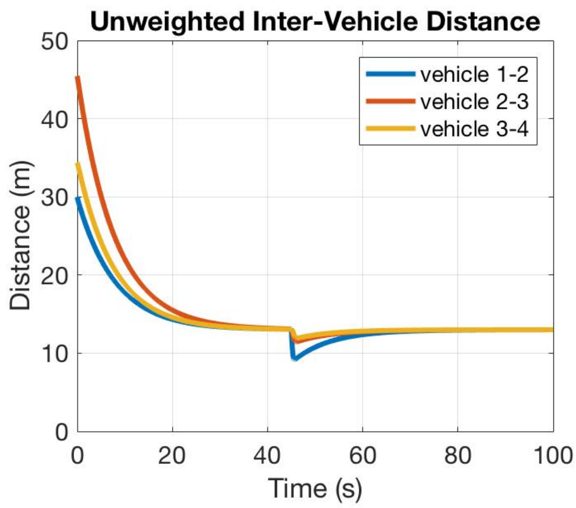
(*a*)



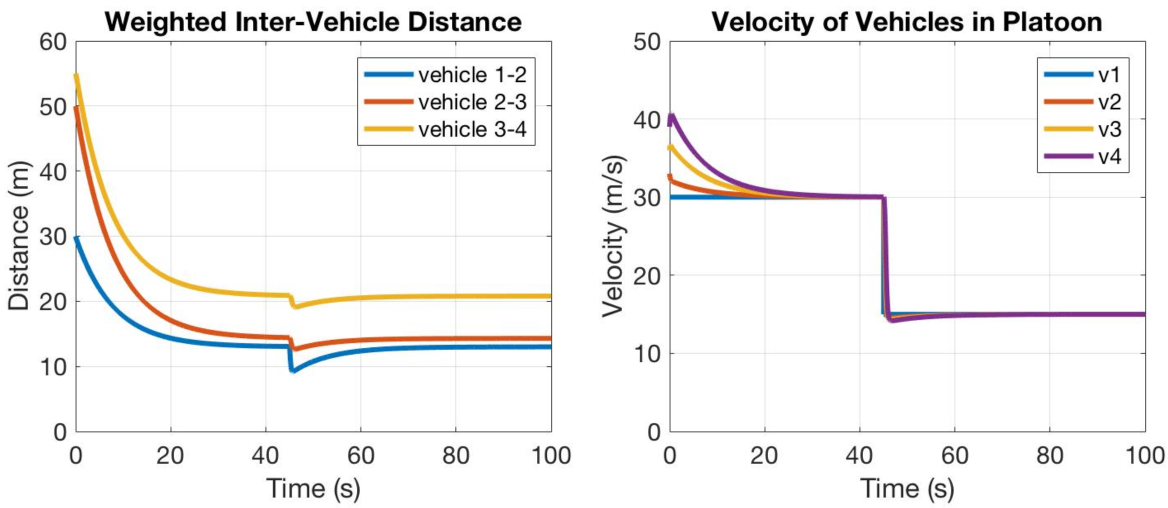
(*b*)



(*c*)



(*d*)



(e) (f)

FIGURE 2 Simulation results of scenario 1 and 2.

Scenario 2: Platoon Restoration from Disturbances

The distributed consensus algorithm (4) also has the capability to attenuate the impact of sudden disturbances. In the platoon mode of our distributed consensus-based CACC system, if one vehicle (e.g., leading vehicle) suddenly brakes due to emergency, then the following vehicles will decelerate accordingly to maintain certain weighted inter-vehicle distances.

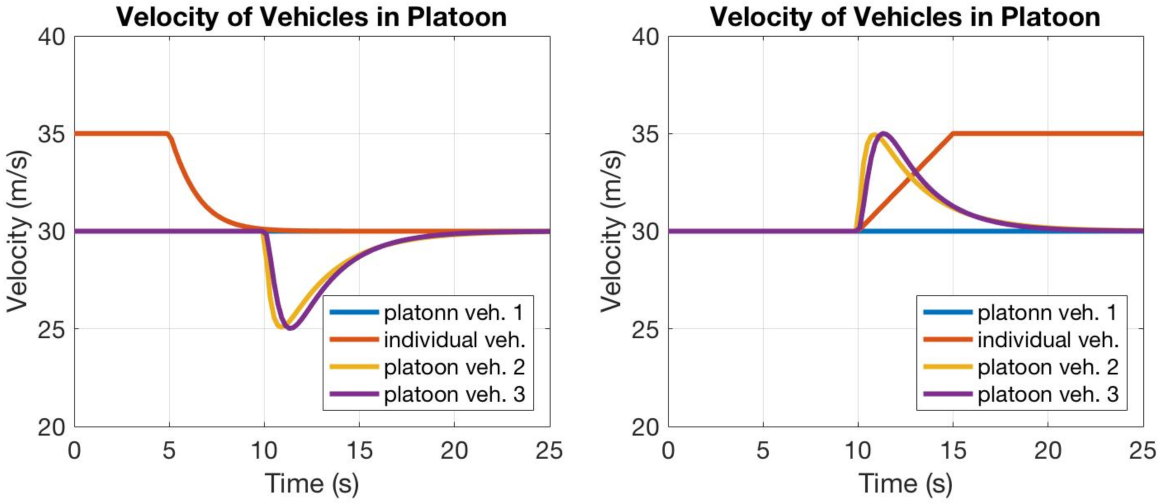
For example, we assume all the parameters remain the same as scenario 1. At time $t =$ 45 s, suppose the leading vehicle suddenly brakes due to a flat tire, and its velocity decreases from 30 m/s to 15 m/s. To simplify the scenario, we assume the brake happens only in a sudden ($\Delta t \approx 0$), i.e., a step change in leading vehicle's velocity.

The simulation results of sudden brake are shown in FIGURE 2 (*d*)-(*f*). Plot (*d*) shows that the unweighted inter-vehicle distance between vehicle 1 and vehicle 2 suffers an approximately 4 m decrease at time $t = 45$ s. However, the unweighted inter-vehicle distance between vehicle 2 and vehicle 3 only suffers an approximately 0.7 m decrease, and the one between vehicle 3 and vehicle 4 is even smaller. These maneuvers can be well explained by plot (*f*). The sudden brake originates from vehicle 1, and vehicle 2 tends to avoid the collision with vehicle 1 with a hard brake. The brake of vehicle 3 is not as hard as vehicle 2 (the slope is smaller), and the brake of vehicle 4 is even smoother than vehicle 3. The smoother their brakes are, the smaller the unweighted inter-vehicle distance will decrease. After the brake, the velocities of the three following vehicles slowly restore to the desired velocity, and the unweigted inter-vehicle distances also restore to consensus. Therefore, the simulation results of scenario 2 indicate that our distributed consensus-based CACC system is capable of attenuating sudden disturbances and restoring to the normal condition.

Scenario 3: Merging and Splitting Maneuvers

In this scenario, distributed consensus algorithm (4) and protocol 2 are used to perform the merging and splitting maneuvers.

Case 1: Merging Maneuvers Assume at time $t = 0$, a three-vehicle platoon (same parameters as vehicle 1, 3 and 4 in scenario 1) is operating at the steady state (i.e., cruising at the velocity of 30 m/s). Another individual vehicle (same parameters as vehicle 2 in scenario 1) traveling at the velocity of 35 m/s on adjacent lane plans to merge into the platoon, and the simulation result is shown in FIGURE 3 (*a*).



(*a*) (*b*)

FIGURE 3 Simulation results of scenario 3.

It can be observed that the individual vehicle switches on the merging mode at time $t = 5$ s. From then on, it adjusts its velocity from 35 m/s to 30 m/s, and then sends a merging signal to the vehicle next to it, which is the second vehicle of the platoon in this study case. Then a “ghost” vehicle (with respect to the merging vehicle) is created in front of the second vehicle of the platoon. Based on algorithm (4), both the second and third vehicle of the platoon decelerate to create a gap. Finally, the individual vehicle merges into the platoon, and the velocities of the other two following vehicles restore to consensus in 5 s.

Case 2: Splitting Maneuvers Assume at time $t = 0$, a four-vehicle platoon (same parameters as vehicle 1, 2, 3 and 4 in scenario 1) is cruising at the velocity of 30 m/s. The second vehicle will split from the platoon, and the simulation result is shown in FIGURE 3 (*b*).

The second vehicle of the platoon switches off the platoon mode and drives away (constantly accelerates from 30 m/s to 35 m/s) from platoon at time $t = 10$ s. After the second vehicle completes its lane change, the third vehicle confirms that its preceding vehicle has changes to the first vehicle of the platoon. Then it adjusts its velocity based on algorithm (4) to close the gap. The fourth vehicle accordingly adjusts its velocity to follow the movement of its preceding one.

Therefore, the simulation results of scenario 3 show that our distributed consensus-based CACC system is capable of carrying out merging and splitting maneuvers.

Sensitivity Analysis on the Distributed Consensus Algorithm

In this part, sensitivity analysis is conducted based on the normal platoon formation scenario, where the communication graph $\mathcal{G}$ is a directed spanning tree. The adjacency matrix can be written as $\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, and therefore the nonsymmetrical Laplacian matrix is $\mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$.

Recall our distributed consensus algorithm (4), there is a damping term γ inside of the velocity consensus term. We can get the conclusion based on (*28*) that algorithm (4) achieves consensus asymptotically if and only if directed graph $\mathcal{G}$ has a directed spanning tree and

$$\gamma > \max_{\forall \mu_i \neq 0} \frac{|\mathrm{Im}(\mu_i)|}{\sqrt{\mathrm{Re}(-\mu_i)}|\mu_i|}, \tag{7}$$

where $\mu_i, i = 1, \ldots, n$ denotes the ith eigenvalue of $-\mathcal{L}$. After calculation, $\gamma > 0$ is the requirement for algorithm (4) to reach consensus.

As mentioned in the simulation part, $\gamma = 7.5$ is preliminarily set for different simulation scenarios. However, the value of γ may have a significant effect on the convergence speed of algorithm (4), including the convergence speed of vehicle absolute position and the convergence speed of vehicle velocity. The change of vehicle absolute position is related to the safety of our CACC system, since if the weighted inter-vehicle distance drops to negative, collision may occur. The change of vehicle velocity is related to vehicle acceleration and jerk (time rate of change of acceleration), and it is studied in (*29*, *30*) that a limitation of ± 2.5 m/s^2 and ± 10 m/s^3 for acceleration and jerk separately will be comfortable for human passengers. Therefore, sensitivity analysis is conducted to study the effect of γ on driving safety and driving comfort.

TABLE 2 Parameters of Sensitivity Analysis

(*a*) Driving safety	Leading vehicle	Following vehicle
Vehicle length c_i	5 m	5 m
Vehicle braking factor b_i	1	1
Initial velocity $\dot{x}_{i0}$	30 m/s	40 m/s
Desired velocity $\dot{x}_i$	30 m/s	30 m/s
Desired weighted inter-vehicle distance d_{ij}	13 m	
(*b*) Driving comfort	Leading vehicle	Following vehicle
Vehicle length c_i	5 m	5 m
Vehicle braking factor b_i	1	1
Initial velocity $\dot{x}_{i0}$	30 m/s	33 m/s
Desired velocity $\dot{x}_i$	30 m/s	30 m/s
Initial weighted inter-vehicle distance d_{ij0}	35 m	
Desired weighted inter-vehicle distance d_{ij}	13 m	

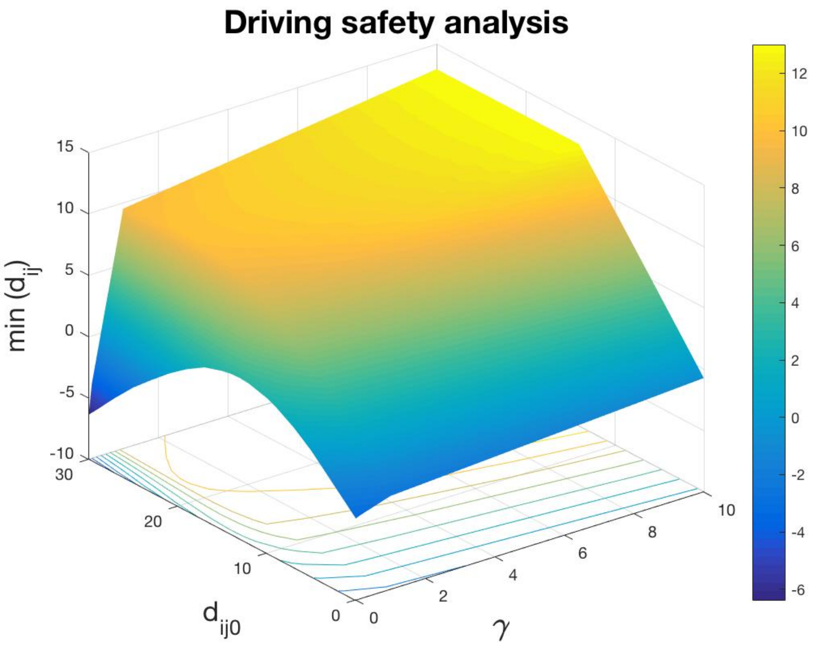
Driving Safety Analysis

In this part, we analyze the effect of γ on driving safety. We measure the value of minimum weighted inter-vehicle distance through normal platoon formation process, and check whether it goes to negative. If it does, then a collision between the leading vehicle and the following vehicle occurs.

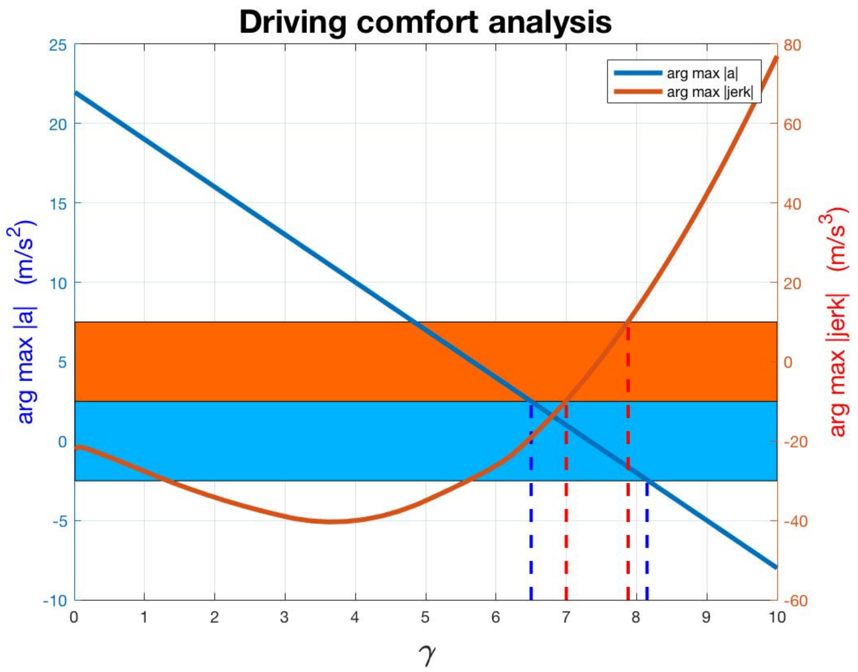
Parameters of this sensitivity analysis are set in TABLE 2 (*a*). We construct a model to study how the changes of γ and the initial weighted inter-vehicle distance d_{ij0} will affect the minimum weighted inter-vehicle distance min (d_{ij}).

The result of sensitivity analysis on driving safety is shown in FIGURE 4 (*a*). The dark blue areas indicate min $(d_{ij}) < 0$, which appear mostly when $d_{ij0} > 25$ m and meanwhile $\gamma < 1$. Although there is a linear blue area where $d_{ij0} \approx 0$, it is because at time t = 0, $\dot{x}_{20} > \dot{x}_{10}$ is given, and there is no way to choose a γ for the following vehicle to avoid the collision with such a short weighted inter-vehicle distance to the leading vehicle . If we fix the value of γ, it is found that the closer d_{ij0} approaches to d_{ij} (13 m), the larger min (d_{ij}) is. And it can also be noted that when $8 < d_{ij0} < 18$, $\min(d_{ij}) > 0$ is guaranteed whatever the value of γ is.

Under current parameter setting, the value of γ can better be chosen in the yellow area of FIGURE 4 (*a*) based on different initial weighted inter-vehicle distance to ensure $\min(d_{ij}) > 0$, where the normal platoon formation of our CACC system is safe with no collision. By conducting this sensitivity analysis under the same parameter settings as aforementioned simulation scenario 1, 2 and 3, we find the preliminary value we choose where $\gamma = 7.5$ is safe under all occasions.



(*a*)



(b)

FIGURE 4 Sensitivity analysis results.

Driving Comfort Analysis

In this part, we analyze the effect of γ on driving comfort. We measure the values of $\arg\max|a|$ and $\arg\max|jerk|$ through normal platoon formation process, and check under which value of γ will $-2.5\ \mathrm{m/s^2} < a < 2.5\ \mathrm{m/s^2}$ and $-10\ \mathrm{m/s^3} < jerk < 10\ \mathrm{m/s^3}$ be satisfied. If a and $jerk$ are in the range, then driving is comfortable for human passengers.

Parameters of this analysis are set in TABLE 2 (b), which are exactly the same as the first two vehicles in aforementioned simulation scenario 1, 2 and 3. As we can see from FIGURE 4 (b), $\arg\max|a|$ changes in a linear function along with the change of γ , while $\arg\max|jerk|$ changes in a quadratic function along with the change of γ. When $7 < \gamma < 7.8$, both the acceleration and the jerk are in the comfort areas. Therefore, a value of 7.5 can be chosen for γ, which satisfies both the requirements of driving safety and driving comfort. And when the parameter setting changes, the same method of sensitivity analysis can be applied to choose the value of γ, and ensure autopilot or autonomous driving in our CACC system to be safe and comfortable.

CONCLUSIONS AND FUTURE WORK

In this paper we have proposed a novel CACC system based on distributed consensus methodology. Distributed consensus algorithm with communication has been designed, where the length and braking ability of different vehicles have been taken into account. We have also designed distributed consensus protocol to allow our CACC system to process the algorithm under different scenarios. The algorithm and protocol have been implemented in MATLAB Simulink on top of a standard DSRC/WAVE communication infrastructure and the results have been evaluated in several realistic scenarios. Finally, sensitivity analysis on the algorithm regarding different respects of driving has been conducted.

However, this paper has only considered linearly distributed consensus control, while realistic systems are always with the feature of nonlinearity. Also, although the distributed consensus algorithm proposed has covered some environmental uncertainties like communication delay, many others still have not been considered in the paper, such as packet loss, signal fading, signal interference, etc. Furthermore, only the system-level (cyber-space) of vehicles has been taken into account in this paper, where actual vehicle dynamics model (physical-space) has been neglected. Combination of the cyber-space and the physical-space is another future goal of this study.

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