Product Relevance, Consumer Search, and Competition

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Abstract

When consumers incur search costs to evaluate match, the product sampled first more likely makes the sale. Product relevance (i.e., the probability of match) then becomes a source of competitive advantage, because all else equal consumers will first sample the product with a greater relevance. We examine relevance and price competition in a duopoly. Interestingly, even when consumers have homogeneous search cost, the ex ante symmetric firms choose different product relevance. The rationale is as follows. If the firms chose an identical relevance, the firm with even a slightly lower price would attract all consumers to first sample its product, intensifying price competition. Differentiation in relevance relaxes price rivalry, as a firm must undercut the competitor's price by a sufficiently large amount to alter consumers' search sequence. Moreover, relevance differentiation expands at low or high search costs but dwindles at intermediate search costs. When search costs are uniformly distributed, one firm chooses a greater relevance and a higher price than the other. Each product is prominent to a different segment of consumers: The consumers with relatively high (low) search costs first sample the product with greater (smaller) relevance.

1 Introduction

Product relevance is the likelihood that a firm's product or service can meet the needs of target consumers. The specific meaning of product relevance varies from one market to another. For example, product relevance may refer to the probability that an online dating site can find a suitable match for a consumer, or the chance that a consumer can find a right gardening book at Amazon.com. For a durable goods manufacturer like Sony, product relevance depends on whether the company has developed proper features to address latent customer needs. For a business school offering executive education programs, product relevance means the likelihood its course content addresses an applicant's career needs and its structure accommodates her schedule. In the rest of this paper, we will abstract away from these specific contexts and use product relevance to refer to the likelihood that a firm's offering matches with customer needs.

Product relevance is important when consumers evaluate alternatives to make a choice. In many markets, individual consumers have idiosyncratic tastes and are ex ante uncertain whether a product or service is right for them. Such uncertainty is resolved through costly search or evaluation. Common search and evaluation activities may include using internal deliberation to simulate usage occasions, traveling to retail locations or going online to inspect products, and sampling products through trials and test driving. For example, one has to read the introduction, table of content and other readers' comments to determine whether a gardening book fits her needs. A student candidate needs to talk to the program director to learn about the content and structure of a part-time executive education program. When consumers incur search costs to evaluate each alternative, all else equal they will first sample the product with a greater relevance, which allows them to find a match more efficiently. The firm offering a greater relevance then benefits from the resulting greater demand. It is widely believed that Amazon's success was in part driven by its highly relevant recommendations, which leads to a conversion rate estimated to be as high as 60%.

Firms invest in marketing research, information technology and operational efforts to improve their product relevance. For example, online retailers such as Amazon.com try to increase product assortment and develop new predictive models with state-of-the-art data science and machine learning techniques (Managalindan 2012, Robert 2008). Similarly, online dating sites need to recruit more subscribers and to improve the accuracy of their recommendation systems. A British marketing firm developed a new "relevance marketing" approach by using sophisticated data collection and predictive modeling techniques to advise supermarket clients like Tesco and Kroger on product assortment and inventory decisions (Jordan 2007). In a B2B market, CDW, a business product and service provider, invested in customer-community programs to hear customer voice and gauge what technology issues matter most to the customers (Lauren 2010).

Investments in product relevance can be costly, and firms need to understand how product relevance affects consumer choices and price competition with rivals. In many industries, it is not uncommon to see one firm offering a significantly higher product relevance than another. For example, online dating sites are reported to have different rates of matching success. Manufacturers within an industry often have different levels of success in new product launch. A few business schools' executive programs consistently rank higher than the others in student satisfaction surveys. To better understand these observations, we develop a duopoly model to study competition in *product relevance* with consumer search. The firms are ex ante symmetric in that they are endowed with the same technology. Each firm first determines and commits to its product relevance, and then sets a price. Observing both products' relevance and prices, each consumer chooses an optimal search strategy, i.e., which product to sample first, when to stop search, and whether and which product to purchase. Each firm maximizes its expected profits, and each consumer her expected surplus. When the search objects are ex ante heterogeneous, the optimal search sequence is usually not random, as shown by Weitzman (1979). As we shall see, in our model the firms use both product relevance and price to influence consumer search sequence. We are interested in the following questions. Would they choose the same or differentiated product relevance?

How would their product relevance drive their pricing strategies? How would change in consumer search cost affect the equilibrium product relevance? Answers to these questions have important and timely managerial implications, as practitioners are contemplating over how to respond to the significant changes in search cost due to rapidly evolving Internet and mobile technologies.

Our main model considers a setting with homogeneous consumer search costs. The first result is about the relation between equilibrium pricing and consumer search cost. Given the product relevance, the unique price equilibrium is in mixed strategies. Interestingly, the firm with a higher product relevance maintains an atom when search cost is low, and the firm with a lower product relevance employs an atom when search cost is sufficiently high. When search cost is low, the firm with a greater relevance has a limited competitive advantage and is content with the profits of selling only to its captive customers.[1] When search cost is high, the firm with a greater relevance has a sufficiently large competitive advantage. It can afford to pursue a more aggressive pricing strategy, forcing its rival to employ an atom.

Second, the ex ante symmetric firms choose different product relevance to relax price rivalry. Price competition is most fierce when the firms have an identical product relevance, as even with a slightly lower price than the rival either firm would attract all consumers to first sample its product. In contrast, when the firms differ in product relevance, a firm must price undercut its rival by a sufficiently large amount to alter consumers' search sequence. Moreover, the degree of differentiation has a U-shaped relation with search cost: Differentiation in product relevance expands when search cost is either high or low, but narrows when search cost is intermediate. The intuition is the following. When search cost is low, for given product relevance the difference between the products' absolute appeal is low, which intensifies price rivalry because even a relatively small price difference shifts the consumers' search sequence.[2] Consequently, the firms *have to* differentiate more in product relevance to relax

[1]When all consumers first sample product i, those realizing a poor match with i will continue to sample j ($i, j = 1, 2$, $i \neq j$). The consumer realizing a good match with j is called product j's captive customers.

[2]As we introduce in Subsection 2.1, each product's absolute appeal is represented by its *reservation price* (in the sence of Weitzman (1979)).

price competition. When search cost is high, given the firms' product relevance the difference between their absolute appeal is large. This weakens price rivalry, as their price difference must be sufficiently large to alter consumers' search sequence. In this case, the return from differentiation is high, which prompts the firms to differentiate more in product relevance. In contrast, their incentives to differentiate in relevance are weakest for intermediate levels of search costs.

We then analyze a duopoly with uniformly distributed consumer search costs. For given product relevance, such heterogeneity in search costs smooths each firm's demand function and may lead to a pure-strategy price equilibrium. The equilibrium product relevance may still be characterized with differentiation, with the firm choosing a greater relevance charging a higher price. Interestingly, heterogeneous search costs lead to a natural segmentation in consumer search strategy, and each product becomes prominent to a different segment of consumers. The consumers with relatively high search costs first sample the product with a higher relevance and higher price, while the remaining consumers first sample the product with a lower relevance and lower price. The intuition is that the former care more about minimizing total expected search costs and the latter care more about paying a lower price.

Our model is related to the literature on sequential search in product markets. In a vast majority of these models, the products are assumed to be ex ante homogeneous, and consumers search randomly (e.g., Wolinsky 1986, Anderson and Renault 1999, Kuksov and Villas-Boas 2010, Dukes and Liu 2015). That is, search sequence does not matter. When the products are ex ante heterogeneous, however, search sequence clearly matters. When the objects have different search costs and different reward distributions, Weitzman (1979) shows that the optimal search sequence is in descending order of reservation prices, and that search stops once the realized reward exceeds or equals the highest reservation price of the unsampled objects.[3] Our model is in the spirit of Weitzman (1979): The consumer freely observes each product's relevance and price and decides which product to sample first. Our

[3] An object's reservation price is a known reward so that when presented with this reward, the consumer is indifferent between accepting it and sampling the object. See Section 2 for a formal definition.

model is also related to the growing branch of literature on search prominence (Armstrong et al. 2009, Chen and He 2011, Amaldoss et al. 2015, Katona and Sarvary 2010, Jerath et al. 2011, Lu et al. 2015, Shin 2015), where all consumers first sample the prominent product (e.g., the top search result at a search engine or the ads in the front page of newspaper). Prominence is exogenous in Armstrong et al. (2009), and the firm has to pay for prominence in the other models. In our model, prominence arises *endogenously* through the deliberate choice of relevance and price. Each consumer will first sample the product with a higher reservation price (which is a function of the product's relevance and price and the consumer's search cost; see below).

vertical attribute can be materilize(quality) or informational(quality of information)

Our model adds to the literature on how search costs affects endogenous product design. Kuksov (2004) considers a horizontal duopoly, where consumers costlessly observe the product attributes but incur a search cost to discover price. He shows that lower search costs lead to greater product differentiation, which relaxes price competition. Bar-Isaac et al. (2011) and Larson (2013) are oligopoly models of product design. Bar-Isaac et al. (2011) show that lower search costs allow consumers to find a better suited product and induce firms to choose extremal (most generic or most niche) product designs. Larson (2013) finds that firms choose niche designs when search costs are low and generic designs when search costs are high. When search costs are lower, consumers become more "choosy," which prompts firms to offer niche products to soften price competition. In a duopoly, our main model shows that differentiation in product relevance becomes more pronounced when search costs are either low or high but tapers off at intermediate values of search costs.

it is possible that the context is different because the relevance here enters as a vertical attribute not a horizontal attribute

Our current paper naturally relates to models of targeted marketing. In the duopoly of Narasimhan (1988), Chen et al. (2001) studies targeted *pricing* when each firm can not perfectly distinguish its loyal consumers from switchers. They show that improvements in targeting accuracy can increase both firms' profits, and that the firm with a larger (smaller) loyal customer base invests more in targetability when the cost of acquiring targetability is high (low). Both Iyer et al. (2005) and Johnson (2013) show that targeted *advertising* increases the firms' profits. In Iyer et al. (2005), product pricing is endogenous. They assume that (after incurring a fixed cost) each firm can perfectly distinguish its loyal consumers and switchers. They show that each firm advertises more to its loyal consumers than to switchers. Johnson (2013) treats product pricing as exogenous and allows imperfect targetability.[4] Zhang and Katona (2012) study contextual advertising, where a consumer's content preference is imperfectly aligned with her product preference. In these models, consumers do not search: Once exposed to an ad, the consumer costlessly observes whether the advertised product is a match. In contrast, we account for the reality that when product relevance is imperfect, consumers often have to incur an evaluation cost to discern whether the product matches her taste.

endogenize relevance(accuracy) is an important factor. Here is no spillover like Shin,Yu. Other endogenous paper: Shin, Yu; Zhong(platform design)

check the vertical differentiation literature and see the difference. I remember there is hypothesis for maximal/minimal differentiation, but we find relevance differentiation is moderated by the level of search cost

Our paper finds that the ex ante symmetric firms choose to differentiate in product relevance. When consumers incur search costs to ascertain match, product relevance becomes a quality-type attribute because all else equal they prefer to first sample the product with a greater relevance. Our model thus also enriches the literature on vertical differentiation (Shaked and Sutton 1982, Moorthy 1988). To our knowledge, ours is the first paper to demonstrate that product relevance can be an effective dimension along which competing firms differentiate themselves in a market with consumer search costs.

2 A Model with Homogeneous Search Costs

Two firms, 1 and 2, compete in this market. Consumers of unit mass have idiosyncratic needs and each demand at most one unit of the product. We assume that each firm offers a single product (or service) to each consumer. The value of each product is v to the consumer if it matches her need and is zero otherwise. Firm i's ($i = 1, 2$) product meets a consumer's need with probability λ_i. Subsequently, we will call λ_i the *product relevance* of firm i. As noted above, product relevance may reflect the extent of appeal of the product's

[4] Related, Bergemann and Bonatti (2011) study the impacts of targeted advertising on the price of advertisements (not the products advertised).

design characteristics or the accuracy of the firm's targeted communication. We assume that firm i's product relevance is common knowledge. For example, consumers may learn about each product's relevance from its design features or third-party reviews. In the case of targeted communication, it is well known that Amazon and Netflix provide more accurate recommendations than their peers. Ex ante, the consumer does not know if either product matches her idiosyncratic need. To determine if a product matches her need, the consumer has to incur an evaluation cost s. For now, we assume that all consumers have the same evaluation cost s. Each consumer seeks a product that matches with her need via sequential search.

We assume that the firms face a common technology to develop product relevance: There is a fixed cost $g(\lambda)$ for devising a product (or service) with relevance λ, where g is increasing and convex in λ. That is, there is an increasing marginal cost to improve product relevance. We also assume that the firms have the same constant marginal production cost, which are normalized to zero without further loss of generality.

The game unfolds in two stages. In stage one, both firms simultaneously choose and announce their product relevance, λ_i. In stage two, after observing each other's product relevance, the firms simultaneously set and announce prices, p_i. The consumers then observe both firms' product relevance and prices and make evaluation and purchase decisions. Specifically, the consumer decides which product to evaluate first and when to stop search. When search stops, she decides whether and which product to buy. Each firm wishes to maximize its expected profits and each consumer her expected surplus.

Note that to highlight consumer search for product match, we are treating price information as transparent for simplicity. Today, prices are relatively easy to convey (e.g., on the Internet), but evaluating product match often still requires non-trivial efforts. Indeed, consumers often check out the digital attributes (including price) of a product online before deciding whether to visit the store to touch and feel it. The assumption that price is ex ante freely observable but match is not has appeared in Anderson and Renault (2009) and Gu and Xie (2013).

To derive the sub-game perfect equilibrium, we proceed backwards and start with the second stage pricing sub-game. We seek a pure-strategy equilibrium if it exists and otherwise seek a mixed-strategy equilibrium.

2.1 Stage Two: The Sub-game Price Equilibrium

In stage 2, the firms' product relevance λ_i $(i = 1, 2)$ are already fixed, and they set prices to maximize their own profits (gross of any fixed costs of developing relevance). To derive each firm's demand function, we need to characterize the consumers' optimal search behavior. The optimal search rule is a special case of that in Weitzman (1979). Weitzman (1979) considers the following setting: A risk-neutral agent faces n $(n \geq 2)$ objects. Each object i carries a reward of x_i, which follows an independent distribution F_i, and it costs the agent s_i to learn the exact value of x_i. When search stops, the agent collects the highest reward uncovered. Weitzman (1979) shows that the agent's optimal search rule takes a reservation price form. Object i's *reservation price* r_i is uniquely given by $r_i = -s_i + r_i \int_0^{r_i} dF_i(x) + \int_{r_i}^{\infty} x dF_i(x)$, where the left-hand side is a known reward r_i and the right-hand side the expected net surplus of sampling object i. Therefore, object i's reservation price equals the known reward that makes the agent indifferent between accepting it and sampling i. The optimal search sequence is in descending order of reservation prices and search stops when the highest uncovered reward equals or exceeds the highest reservation price of the unsampled objects.

In our model, product i's *reservation price* r_i is given by the indifference condition $r_i = -s + \lambda_i(v - p_i) + (1 - \lambda_i)r_i$. That is, if a consumer is presented with a known and certain reward r_i, she will be indifferent between accepting this reward and continuing to sample product i. Thus, r_i is the certainty-equivalent reward of sampling product i. We can rewrite the reservation price r_i as

$$r_i \equiv a_i - p_i, \text{ where } a_i \equiv v - \frac{s}{\lambda_i}. \tag{1}$$

In equilibrium, firm i always sets its price low enough that $p_i \leq a_i$ (for $i = 1, 2$); Otherwise no consumer would sample product i. Without loss of generality, suppose $a_i - p_i > a_j - p_j \geq 0$ $(i, j = 1, 2,\ i \neq j)$. Then, all consumers will first sample product i, and will purchase i if it yields a good match (with probability λ_i). If product i turns out to be a poor match (with probability $1 - \lambda_i$), the consumer will proceed to sample j, and will purchase j if it yields a good match. Firm i's demand is thus λ_i and firm j's $(1 - \lambda_i)\lambda_j$. Below, we also call $(1 - \lambda_i)\lambda_j$ the number of firm j's captive customers. Each firm's demand is thus discontinuous, depending on whether its product is the first or second being evaluated. Because of this demand discontinuity, a pure strategy price equilibrium cannot be sustained. Consequently, we resort to a mixed-strategy price equilibrium.

For $i = 1, 2$, we let P_i denote the price strategy set of firm i, $\underline{p_i} = \inf P_i$ its lower bound, and $\overline{p_i} = \sup P_i$ its upper bound. We summarize the properties of the price equilibrium below.

Lemma 1. *(1). There is no pure-strategy price equilibrium; (2). P_1 and P_2 are convex sets; (3). $\underline{p_1} > 0$, $\underline{p_2} > 0$, $a_1 - \underline{p_1} = a_2 - \underline{p_2}$, $\overline{p_1} = a_1$, $\overline{p_2} = a_2$; (4). Firm i has no atom at prices below a_i.*

Part (1) of the Lemma is a consequence of the discontinuity in the firms' demand functions as noted earlier. Part (2) states that neither firm's strategy set contains a hole. The argument behind is as follows. Suppose one of the strategy sets, say P_1, contains a hole (b_1, b_2). Then, firm 2's expected demand remains constant over the price sub-interval $(b_1 - (a_1 - a_2), b_2 - (a_1 - a_2))$. Therefore, P_2 must contain a hole $(b_1 - (a_1 - a_2), b_2 - (a_1 - a_2))$. This, however, implies that firm 1 makes higher profit at a price in (b_1, b_2) than at price b_1. A contradiction.

To understand part (3) of the Lemma, note that firm i prices strictly above zero, i.e., $\underline{p_i} > 0$, because firm i has a positive demand $(1 - \lambda_j)\lambda_i$ even when pricing at a_i. Part (3) further states that both firms randomize their prices over intervals of identical length. Profit maximization ensures that $\overline{p_i} = a_i$. Part (3) also implies that as search cost increases, each firm's price strategy set shifts downwards. Finally, behind Part (4) of the Lemma is the familiar undercutting argument in the literature. These features of the mixed-strategy price equilibrium are similar to those in Varian (1980) and Narasimhan (1988), and the proof of Lemma 1 is omitted.

From Lemma 1, we easily characterize the price equilibrium. Proposition 1 shows how product relevance drives the firms' price strategies.

explain why?

Proposition 1 *Suppose $\lambda_1 \leq \lambda_2$. The unique mixed-strategy price equilibrium has two cases.*

(1). When $s \leq s_0 \equiv \frac{\lambda_1\lambda_2 v}{\lambda_1+\lambda_2}$, the equilibrium price distributions are $F_1(p_1) = \frac{1}{\lambda_1}\left[1 - \frac{(1-\lambda_1)a_2}{p_1-(a_1-a_2)}\right]$ and $F_2(p_2) = \frac{1}{\lambda_2}\left[1 - \frac{a_1-\lambda_1 a_2}{p_2+(a_1-a_2)}\right]$ for $a_1 - \lambda_1 a_2 \leq p_1 < a_1$ and $(1-\lambda_1)a_2 \leq p_2 < a_2$. Firm 2 has an atom $1-\frac{\lambda_1 a_2}{\lambda_2 a_1}$ at a_2. The firms make profits $R_1 = \lambda_1(a_1-\lambda_1 a_2)$ and $R_2 = \lambda_2(1-\lambda_1)a_2$.

(2). When $s > s_0$, the equilibrium price distributions are $F_1(p_1) = \frac{1}{\lambda_1}\left[1 - \frac{a_2-\lambda_2 a_1}{p_1-(a_1-a_2)}\right]$ and $F_2(p_2) = \frac{1}{\lambda_2}\left[1 - \frac{(1-\lambda_2)a_1}{p_2+(a_1-a_2)}\right]$ for $(1-\lambda_2)a_1 \leq p_1 < a_1$ and $a_2 - \lambda_2 a_1 \leq p_2 < a_2$. Firm 1 has an atom $1 - \frac{\lambda_2 a_1}{\lambda_1 a_2}$ at a_1. The firms make profits $R_1 = \lambda_1(1-\lambda_2)a_1$ and $R_2 = \lambda_2(a_2 - \lambda_2 a_1)$.

To understand the impact of search cost on price competition, we start with the benchmark case where search cost is zero ($s = 0$). In this case, each consumer freely observes her valuation of both products prior to purchase. Consumers in the number of $\lambda_1(1-\lambda_2)$ value product 1 at v and 2 at 0, and thus purchase only from firm 1. Consumers in the number of $\lambda_2(1-\lambda_1)$ value product 2 at v and 1 at 0, and thus purchase only from firm 2. Consumers in the number of $\lambda_1\lambda_2$ value both products at v and are thus price sensitive. Our model then degenerates into Narasimhan (1988). Since $\lambda_1 \leq \lambda_2$ (by assumption), the firm with a greater product relevance (firm 2 as assumed) has a larger captive customer segment, employs an atom at the upper bound of its price range, and makes the same profits as serving only its captive customers.

When $s > 0$, firm 2 (which has a greater product relevance by assumption) still enjoys a competitive advantage. From equation (1), we have $a_1 \leq a_2 \Leftrightarrow \lambda_1 \leq \lambda_2$. Thus, by part (3) of Lemma 1, firm 2 has a higher price range. Proposition 1 further shows that the firm with a greater product relevance makes higher profits than its competitor.

Proposition 1 reveals an interesting feature of the price equilibrium. *When search cost is low enough ($s \leq s_0$), the firm with a greater product relevance (firm 2) employs an atom in its price strategy. However, when search cost is sufficiently high ($s > s_0$), it is the firm with a lower product relevance (firm 1) that employs an atom in its price strategy.* Therefore, search cost moderates the relation between the firms' product relevance and their price strategies. The reason for this result is as follows. As search cost increases, all else equal consumers are more willing to first sample the product with a higher product relevance (firm 2). This enhances firm 2's competitive advantage, as the difference between the products' reservation prices, $a_2 - a_1 = \frac{(\lambda_2 - \lambda_1)s}{\lambda_1 \lambda_2}$, also increases in s. When search cost is relatively low ($s \leq s_0$), firm 2 has a limited competitive advantage, in that firm 1 attracts all consumers to sample its product first with a relatively small price cut. (All consumers will first sample product 1 when $p_2 - p_1 > a_2 - a_1$.) In this case, firm 2 maintains an atom at a_2 and is content with the profit of selling only to its captive customers (in the number of $\lambda_2(1 - \lambda_1)$) at price a_2. However, when search cost is high enough ($s > s_0$), firm 2's competitive advantage is large, prompting it to price more aggressively. In this case, firm 2 no longer employs an atom at its maximal price and makes more profits than serving only its captive customers.[5] In response, firm 1 (which has a lower product relevance) is now forced to maintain an atom at its price upper bound.

From Proposition 1, we easily obtain the effects of search cost on each firm's profits, as summarized in the next proposition.

Proposition 2 *Suppose $\lambda_1 \leq \lambda_2$. (1). When $s \leq s_0 \equiv \frac{\lambda_1 \lambda_2 v}{\lambda_1 + \lambda_2}$ and s increases, both firms' profits decrease. (2). When $s > s_0$ and s increases, firm 1's profits always decrease, firm 2's profits increase if $\lambda_2 > \sqrt{\lambda_1}$ and decrease if $\lambda_1 \leq \lambda_2 < \sqrt{\lambda_1}$ (see Figures 1 and 2).*

The profits of the product with lower relevance strictly decrease with search cost. *Interestingly, for the product with higher relevance, its profits may not be monotone in search*

[5] We can easily verify that $\lambda_2(a_2 - \lambda_2 a_1) > \lambda_2(1 - \lambda_1)a_2 \Leftrightarrow \lambda_2 a_1 < \lambda_1 a_2 \Leftrightarrow s > s_0$.

interesting! difference indicates competitive advantage

cost. The underlying rationale is as follows. As search cost increases, both products' reservation prices (a_1 and a_2) decrease, but their difference $a_2 - a_1 = \frac{(\lambda_2 - \lambda_1)s}{\lambda_1 \lambda_2}$ increases. That is, as s increases, both firms' price ranges decrease, but firm 1's price range decreases at a faster pace than firm 2's. A higher search cost not only lowers product 1's absolute appeal to consumers, but also enlarges its competitive disadvantage, naturally lowering its profits. In contrast, a higher search cost has mixed effects on product 2. While a higher search cost lowers product 2's absolute appeal to consumers, it strengthens its competitive advantage: All else equal, as search cost increases, consumers are more willing to first sample product 2 (which has a higher product relevance). When the former effect dominates the latter, firm 2's profits decreases in search cost s. However, when the firms' product relevance are sufficiently differentiated ($\lambda_2 > \sqrt{\lambda_1}$) and search cost is high enough ($s > s_0$), the latter effect dominates the former, causing firm 2's profits to increase in s.

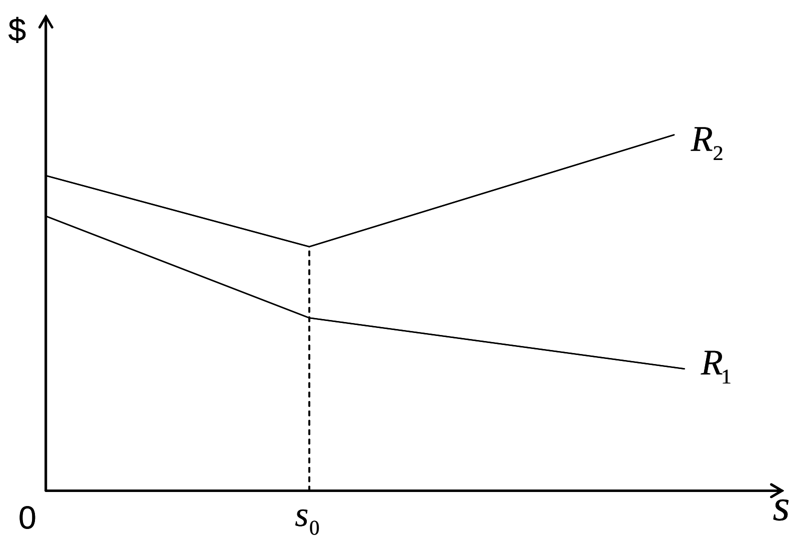

Figure 1. Firms' profits as functions of search cost: When $\sqrt{\lambda_1} < \lambda_2 < 1$

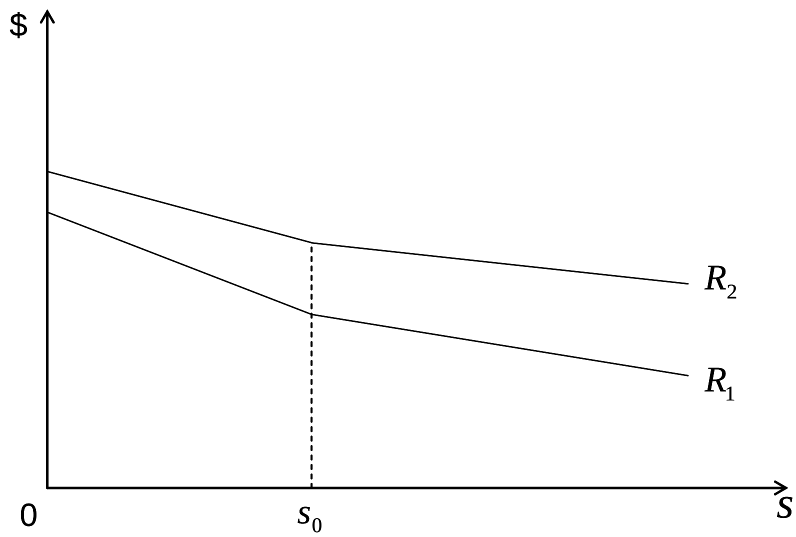

Figure 2. Firms' profits as functions of search cost: When $\lambda_1 \leq \lambda_2 < \sqrt{\lambda_1}$

Next, we turn to stage 1 to identify the equilibrium product relevance.

2.2 Stage One: Equilibrium Product Relevance

In stage one, each firm i $(i = 1, 2)$ chooses the degree of product relevance λ_i to maximize its net profits, anticipating the unique price equilibrium characterized above. By Proposition 1, when $s \leq s_0$, for $\lambda_1 \leq \lambda_2$ the firms' stage-1 profit functions are

$$\pi_1 = \lambda_1 \left[v - \frac{s}{\lambda_1} - \lambda_1 \left(v - \frac{s}{\lambda_2} \right) \right] - g(\lambda_1) \tag{2}$$

and

$$\pi_2 = \lambda_2 (1 - \lambda_1) \left(v - \frac{s}{\lambda_2} \right) - g(\lambda_2). \tag{3}$$

Note that $s_0 \equiv \frac{\lambda_1 \lambda_2 v}{\lambda_1 + \lambda_2}$ is a function of λ_1 and λ_2. We shall determine the equilibrium value of s_0 shortly. The first-order conditions jointly characterize the Sub-game Perfect Equilibrium (SPE):

$$v - 2\lambda_1^* \left(v - \frac{s}{\lambda_2^*} \right) = g'(\lambda_1^*), \tag{4}$$

and

$$(1-\lambda_1^*)v = g'(\lambda_2^*). \tag{5}$$

As s increases, λ_1^* increases (from (4)) and λ_2^* decreases (from (5)).

When $s > s_0$, the firms' stage-1 profit functions are

$$\pi_1 = \lambda_1 (1-\lambda_2) \left(v - \frac{s}{\lambda_1} \right) - g(\lambda_1) \tag{6}$$

and

$$\pi_2 = \lambda_2 \left[v - \frac{s}{\lambda_2} - \lambda_2 \left(v - \frac{s}{\lambda_1} \right) \right] - g(\lambda_2). \tag{7}$$

The following first-order conditions characterize the SPE:

$$(1-\lambda_2^*)v = g'(\lambda_1^*), \tag{8}$$

and

$$v - 2\lambda_2^* \left(v - \frac{s}{\lambda_1^*} \right) = g'(\lambda_2^*). \tag{9}$$

As s increases, λ_2^* increases (from (9)) and λ_1^* decreases (from (8)). Inspecting (4), (5), (8) and (9) reveals that when $s = s_0 \equiv \frac{v\lambda^*}{2}$, the firms will choose the same product relevance λ^*, where $(1-\lambda^*)v = g'(\lambda^*)$. The equilibrium values of λ_1^* and λ_2^* are illustrated in Figure 3.

Proposition 3 *Suppose firm 2 chooses a (weakly) higher product relevance than firm 1. Then, the unique SPE is λ_1^* and λ_2^* as characterized by equations (4), (5), (8), and (9). When $s \leq s_0 \equiv \frac{v\lambda^*}{2}$, where $(1-\lambda^*)v = g'(\lambda^*)$, λ_2^* decreases and λ_1^* increases in search cost s. When $s > s_0$, λ_2^* increases and λ_1^* decreases in search cost s.*[6]

In general, the firms choose differentiated product relevance (except at $s = s_0$). When search cost is positive, each firm wishes to draw consumers to sample its product first.

[6] Our model has two Sub-game Perfect Equilibria, which are essentially identical. Proposition 3 states the one where firm 2 has the higher product relevance. We obtain the other equilibrium by swapping the identities of the firms.

With an identical product relevance, the firms have to compete on the sole basis of price to influence consumers' search sequence. Indeed, price competition is most fierce when the firms have the same product relevance. To see this, when $\lambda_1 = \lambda_2$, we have $a_1 = a_2$ and $a_i - p_i > a_j - p_j$ reduces to $p_i < p_j$, i.e., even with a slightly lower price than the rival's a firm attracts all consumers to sample its product first. In contrast, when $\lambda_1 \neq \lambda_2$ so that $a_1 \neq a_2$, all consumers will first sample product i only if its price is sufficiently lower than j $(p_j - p_i > a_j - a_i)$. *Differentiation in product relevance thus weakens the incentives of price undercutting and relaxes competition.*

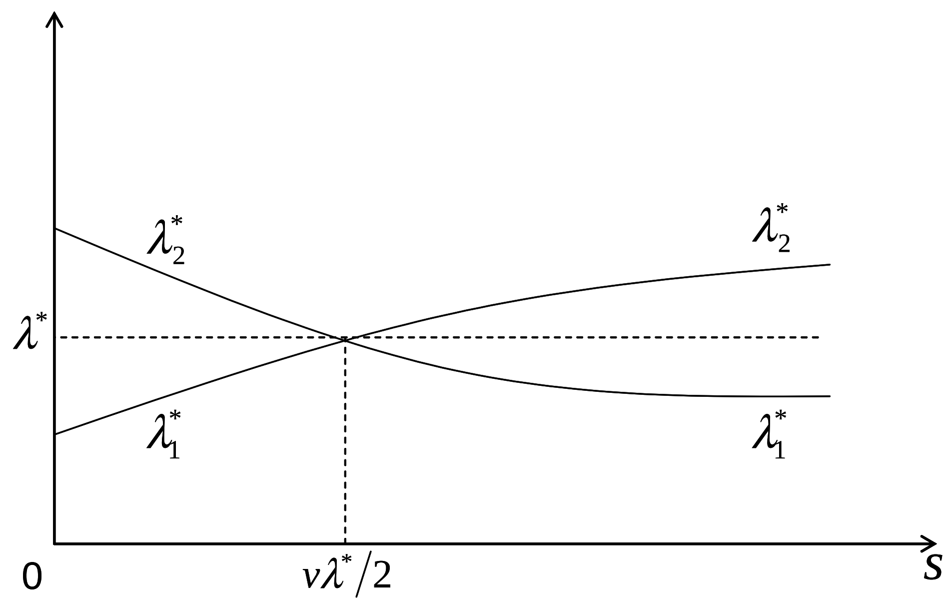

Figure 3. Product relevance as functions of search cost

Interestingly, *as search cost rises differentiation in product relevance narrows when $s \leq s_0$ and enlarges when $s > s_0$.* When search cost is low ($s \leq s_0$), $a_2 - a_1 = \frac{(\lambda_2 - \lambda_1)s}{\lambda_1 \lambda_2}$ is low for any given λ_1 and λ_2 ($\lambda_1 < \lambda_2$). With a limited competitive advantage, firm 2 maintains an atom in its price strategy, and its profits equal those of serving only its captive customers at price a_2. As search cost rises, firm 1's marginal return from investing in product relevance increases (see (4)) but firm 2's marginal return decreases (see (5)), causing λ_1^* to increase and λ_2^* to decrease. When search cost is high ($s > s_0$), firm 2 has a sufficiently large competitive advantage. It prices more aggressively and does not employ any atom. In this case, firm 2's profits exceed that of serving only its captive customers. As search cost rises, firm 2's marginal return from investing in product relevance increases (see (9)) and firm 1's marginal return decreases (see (8)), causing λ_1^* to decrease and λ_2^* to increase.

what about the relationship between profit and search cost?

3 A Model with Heterogeneous Search Costs

So far we have analyzed a model where consumers have an identical evaluation cost s. We now consider the case with heterogeneous evaluation costs. For simplicity, we assume that search cost s is uniformly distributed over $[0, 1]$. Each consumer knows her own search cost s. As in the basic model, the firms first choose product relevance and, after observing each other's product relevance, set prices. For tractability, we assume that the feasible range of product relevance is $[\lambda_L, \lambda_H]$, where $0 < \lambda_L < \lambda_H < 1$. The lower bound λ_L may be viewed as the benchmark level of relevance without extra investment, such as targeting accuracy based on random recommendation at an online platform or the product design following industry standard but without additional innovation efforts. After observing both products' relevance and prices, each consumer decides which product to sample first and when to stop search. When search stops, she decides whether and which product to buy.

Again, we first characterize the stage-2 price equilibrium for given product relevance levels.

3.1 Stage 2: Price Competition

The stage-2 price equilibrium depends on whether the two firms provide the same level of product relevance. We first consider the case with different product relevance.

The case where $\lambda_1 \neq \lambda_2$

Without loss of generality, suppose $\lambda_1 < \lambda_2$, so that $a_1 < a_2$ for each consumer s, where $a_i = v - \frac{s}{\lambda_i}$. Again, consumer s is willing to sample product i ($i = 1, 2$) only if it has a non-negative reservation price for the consumer, i.e., $a_i - p_i \geq 0$ or equivalently

$$s \leq s_i \equiv \lambda_i(v - p_i). \tag{10}$$

For tractability, we suppose v is sufficiently high so that all consumers (including those with the highest search cost $s = 1$) are willing to sample up to both products, i.e., $s_1 \geq 1$ and $s_2 \geq 1$. That is, if the first product sampled turns out to be a poor match, the consumer will continue to sample the other product. By Weitzman (1979), a consumer s will first evaluate product 1 if it has a higher reservation price for her, i.e., $a_1 - p_1 \geq a_2 - p_2$ or equivalently

$$s \leq s_3 \equiv \frac{\lambda_1\lambda_2(p_2 - p_1)}{\lambda_2 - \lambda_1}. \tag{11}$$

If the first evaluation yields a good match, the consumer will purchase product 1. Otherwise, she will continue to sample product 2, and will purchase it if it is a good match. On the other hand, a consumer with search cost $s > s_3$ will first sample product 2, and will purchase it if realizing a good match. Otherwise, she will then sample product 1, and purchase it if it is a good match. If neither product matches with her need, the consumer will not buy.

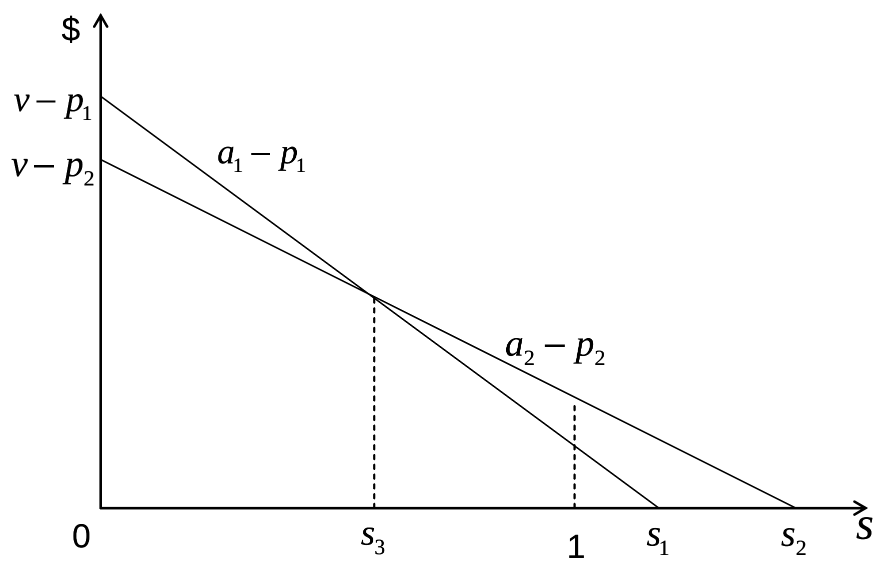

Figure 4. Market segmentation with asymmetric product relevance

We illustrate the above discussions in Figure 4, where firm 1 has a lower relevance and a lower price ($\lambda_1 < \lambda_2$ and $p_1 < p_2$). The segment of consumers with low search costs $[0, s_3)$ will first sample product 1, while the segment with high search costs $[s_3, 1]$ will first sample product 2. Each firm's demand comprises consumers from both segments. Specifically, firm 1's demand is

$$\begin{aligned} D_1 &= \lambda_1 s_3 + \lambda_1(1-\lambda_2)(1-s_3) \\ &= \lambda_1(1-\lambda_2+\lambda_2 s_3), \end{aligned} \tag{12}$$

and firm 2's demand is

$$\begin{aligned} D_2 &= \lambda_2(1-s_3) + \lambda_2(1-\lambda_1)s_3 \\ &= \lambda_2(1-\lambda_1 s_3). \end{aligned} \tag{13}$$

In stage two, the firms' product relevance λ_i are fixed. The firms simultaneously set prices p_i to maximize their respective profits $R_i = D_i p_i$. Their first-order conditions w.r.t. own prices jointly lead to:

$$p_1 = \frac{(\lambda_2-\lambda_1)\left[\lambda_2+2(1-\lambda_2)\lambda_1\right]}{3\lambda_1^2\lambda_2^2} \tag{14}$$

and

$$p_2 = \frac{(\lambda_2-\lambda_1)\left[2\lambda_2+(1-\lambda_2)\lambda_1\right]}{3\lambda_1^2\lambda_2^2}. \tag{15}$$

We can easily verify that $p_2 > p_1 \Leftrightarrow \lambda_2 > (1-\lambda_2)\lambda_1$, which holds since $\lambda_2 > \lambda_1$ by assumption. When $\lambda_2 - \lambda_1 < 2\lambda_1\lambda_2$ (or equivalently $\lambda_1 > \frac{\lambda_2}{1+2\lambda_2}$), we have $s_3 = \frac{\lambda_2-\lambda_1+\lambda_1\lambda_2}{3\lambda_1\lambda_2} < 1$, the above prices are indeed the unique equilibrium. We can easily verify that both p_1 and p_2 decrease in λ_1 and that p_2 increases in λ_2.[7] Therefore, provided $\lambda_1 > \frac{\lambda_2}{1+2\lambda_2}$, a lower λ_1

[7] We have $p_1 = \frac{1}{3\lambda_2^2}\left(\frac{\lambda_2}{\lambda_1}-1\right)\left[\frac{\lambda_2}{\lambda_1}+2(1-\lambda_2)\right]$ and $p_2 = \frac{1}{3\lambda_2^2}\left(\frac{\lambda_2}{\lambda_1}-1\right)\left[\frac{2\lambda_2}{\lambda_1}+(1-\lambda_2)\right]$, both of which

further differentiates the two products and relaxes price competition. The firms' equilibrium profits are

$$R_1 = \frac{(\lambda_2 - \lambda_1)\left[\lambda_2 + 2(1-\lambda_2)\lambda_1\right]^2}{9\lambda_1^2\lambda_2^2} \tag{16}$$

and

$$R_2 = \frac{(\lambda_2 - \lambda_1)\left[2\lambda_2 + (1-\lambda_2)\lambda_1\right]^2}{9\lambda_1^2\lambda_2^2}. \tag{17}$$

Since $\lambda_2 > \lambda_1$, we have $R_2 > R_1$, i.e., the firm with a higher product relevance earns greater profits. When choosing an optimal search strategy, each consumer takes into consideration her search cost and the products' relevance and prices. The segment with relatively high search costs ($s > s_3$) will sample product 2 first, and the segment with relatively low search costs ($s < s_3$) will first sample product 1. Intuitively, those consumers with high search costs care more about minimizing the expected number of searches and thus prefer to first sample the product with a greater relevance (product 2), despite its higher price. Consumers with low search costs care more about price, and hence prefer to first sample the product with a lower relevance (product 1) and lower price.

The firms' equilibrium profits can be rewritten as $R_1 = \frac{(\lambda_2-\lambda_1)}{9\lambda_2^2}\left[\frac{\lambda_2}{\lambda_1} + 2(1-\lambda_2)\right]^2$ and $R_2 = \frac{(\lambda_2-\lambda_1)}{9\lambda_2^2}\left[\frac{2\lambda_2}{\lambda_1} + (1-\lambda_2)\right]^2$, both of which decrease in λ_1. A lower λ_1 relaxes price rivalry and hence increases firm 2's profits. However, *given λ_2, it is somewhat surprising that a lower λ_1 also benefits firm 1.* The reason is that a lower λ_1 lowers firm 1's demand but increases its price due to relaxed competition, and that its price increase dominates its demand loss. We can easily verify that R_2 increases in λ_2.[8] This implies that when $\lambda_L > \frac{\lambda_H}{1+2\lambda_H}$ and $g(\lambda)$ is sufficiently low, firm 1 (2) has an incentive to lower (raise) its product relevance.

The case where $\lambda_1 = \lambda_2 = \lambda$

When the firms have the same product relevance, there is no pure-strategy price equilibrium. To see this, suppose there is a pure-strategy equilibrium (p_1, p_2). If $p_1 = p_2$, each firm

---

decrease in λ_1. We also have $p_2 = \frac{1}{3\lambda_1^2}\left[(2-\lambda_1) - (1-\lambda_1)\frac{\lambda_1}{\lambda_2} - \left(\frac{\lambda_1}{\lambda_2}\right)^2\right]$, which increases in λ_2.

[8] Note that firm 2's revenue can be rewritten as $R_2 = \frac{1}{9\lambda_1^2}\left[(2-\lambda_1)\lambda_2 + \lambda_1\right]\left[(2-\lambda_1) - (1-\lambda_1)\frac{\lambda_1}{\lambda_2} - \left(\frac{\lambda_1}{\lambda_2}\right)^2\right]$.

attracts an equal number of consumers to sample its product first. By unilaterally lowering its price slightly, however, either firm can attract all consumers to sample its product first and hence increase its demand by a discrete amount. If $p_i < p_j$, firm i can increase its profit by slightly raising its price while keeping $p_i < p_j$ (without altering consumers' search sequence and each firm's demand). Thus, a pure-strategy equilibrium can not hold.

Let P_i denote the price strategy set of firm i ($i = 1, 2$), F_i its cumulative price distribution function, $\underline{p_i} = \inf P_i$, $\overline{p_i} = \sup P_i$ and R_i its profits. Standard arguments show that $\underline{p_1} = \underline{p_2}$ and $\overline{p_1} = \overline{p_2}$, that P_i is a convex set, and that F_i has no atom. The following proposition characterizes the mixed-strategy price equilibrium. The technical details are available in the Appendix.

Proposition 4 *Suppose that $\lambda_1 = \lambda_2 = \lambda$ and $\frac{2}{\lambda_H(1+\sqrt{\lambda_H})} < v < \frac{2}{\lambda_L}$. The unique symmetric mixed-strategy price equilibrium is $F(p) = \frac{1}{\lambda}\left(1 - \frac{R}{\lambda p}\right)$ for $\underline{p} \leq p \leq v - \frac{1}{\lambda}$ and $F(p) = \frac{1}{\lambda}\left(1 - \frac{R}{\lambda^2(v-p)p}\right)$ for $v - \frac{1}{\lambda} < p \leq \overline{p}$, where $\underline{p} = \frac{\lambda(1-\lambda)v^2}{4}$, $\overline{p} = \frac{v}{2}$, and each firm earns expected profits $R = \lambda^2(1-\lambda)\frac{v^2}{4}$.*

Each firm's equilibrium profits approach zero when their common product relevance is either too large ($\lambda \to 1$) or too small ($\lambda \to 0$). When $\lambda \to 1$, the first product sampled by a consumer more likely captures the sale, which intensifies price rivalry. When $\lambda \to 0$, both products' absolute appeal decreases, which suppresses the average price.

3.2 Stage 1: The Equilibrium Product Relevance

We now analyze the firms' choice of product relevance in stage 1. To ease analysis, we assume that a firm incurs a fixed cost $g(\lambda) = \beta\lambda^2$ to develop product relevance λ.

Proposition 5 *Suppose $\frac{2}{\lambda_H(1+\sqrt{\lambda_H})} < v < \frac{2}{\lambda_L}$, $\lambda_H < 2\lambda_L$ and β is sufficiently low. Then, $\lambda_1 = \lambda_L$ and $\lambda_2 = \lambda_H$ are the unique SPE if $(\lambda_H - \lambda_L)\left[\lambda_H + 2(1-\lambda_H)\lambda_L\right]^2 > 9\lambda_H^4(1-\lambda_H)$ and $\lambda_H^2(1-\lambda_H) > \lambda_L^2(1-\lambda_L)$.*

how about comparative analysis regarding beta? does the difficulty lie in multiplicity in equilibrium?

The proof is given in the Appendix. The two conditions of Proposition 5 hold when λ_H is sufficiently low.[9] Proposition 5 shows that in equilibrium asymmetric product relevance can still arise with heterogenous search costs. Moreover, *there can be maximum differentiation in product relevance when the cost of developing relevance is sufficiently low*. One firm chooses the lowest permissible or standard level of product relevance (λ_L) and expends no additional investment, while the other firm invests to acquire the highest possible product relevance (λ_H).[10] Since a lower (higher) product relevance entails a lower (higher) price, the market is separated into two segments based on search cost. The two segments follow distinctive search strategies: The low-search-cost segment first evaluates the product with low relevance and a low price, and the high-search-cost segment first evaluates the product with high relevance and a high price. Product relevance is then analogous to quality in vertical differentiation (e.g., Moorthy 1988), and consumers with higher search costs have a stronger preference for relevance.

Interestingly, firm 1 may choose the lowest possible product relevance, λ_L. As firm 1 decreases its product relevance λ_1, firm 2 enjoys a greater competitive advantage. As a result, firm 2's price increases, which then allows firm 1 to raise its price. Furthermore, as λ_1 decreases, the size of the low-search cost segment (s_3) increases because more consumers want to first check out the opportunity of obtaining a match at a lower price. Therefore, firm 1's profit may increase as its product relevance decreases.

4 Conclusion

When consumers incur search costs to evaluate product match, the first product sampled by the consumer is more likely to capture the sale. We have analyzed a duopoly where the firms

[9] Note that the left-hand side of $(\lambda_H - \lambda_L)[\lambda_H + 2(1-\lambda_H)\lambda_L]^2 > 9\lambda_H^4(1-\lambda_H)$ is a cubic function of λ_H, and that $\lambda_H^2(1-\lambda_H) > \lambda_L^2(1-\lambda_L)$ is equivalent to $(\lambda_L+\lambda_H)[1-(\lambda_L+\lambda_H)] > -\lambda_L\lambda_H$, which clearly holds when $\lambda_L + \lambda_H < 1$.

[10] When the cost to improve product relevance (β) is not low, firm 2's equilibrium product relevance λ_2 can be lower than the upper bound λ_H.

employ both product relevance and price to influence consumer search. Interestingly, even when consumers have homogeneous search costs, the two ex ante symmetric firms choose differentiated product relevance. The reason is that differentiation in product relevance helps relax price rivalry. Moreover, differentiation in product relevance is greater when search cost is either large or small, but smaller when search cost is intermediate. The intuition behind is as follows. When search cost is low, the difference between the products' absolute appeal is low, which intensifies price rivalry as the firms rely more heavily on price to influence consumer search. Consequently, the firms are forced to further differentiate their product relevance. When search cost is high, the difference between the products' absolute appeal is large, which relaxes price rivalry. In this case, the firms will also differentiate more in product relevance because the return from differentiation is high. In comparison, the firms have less incentive to differentiate at intermediate search costs.

The evolving Internet and mobile technologies reduce consumers' search costs. Our above result suggests that predicting the effects of lower search cost on firms' marketing strategies can be subtle. The effect on differentiation in product relevance can be either positive or negative, depending on the range of search costs. Since the firms' price strategies depend on their product relevance, the effect of declining search costs on equilibrium prices is not straightforward, either. Any empirical testing should therefore start with a careful assessment of current market characteristics including the magnitudes of search costs (e.g., Kim, Albuquerque and Bronnenberg 2010).

When consumers have heterogeneous search costs, the firms may still choose different product relevance, and the firm choosing a higher relevance charges a higher price. The equilibrium demonstrates self-selection, and each product is prominent to a different segment of consumers. The consumers with relatively high (low) search costs first sample the product with higher (lower) relevance and a higher (lower) price.

Our stylized model has its limitations. First, we have assumed that in case of good match each product yields the same value to consumers. Future research may also treat product value (beside product relevance) as a decision variable. Second, our model is static. Product relevance has more profound implications for repeated competition, where a firm can offer different levels of product relevance to its repeat and new customers. In particular, the firm may offer greater product relevance to its repeat customers based on insights from previous transactions. Therefore, another direction for future work is to examine relevance competition in a dynamic setting.

5 Appendix

Proof of Proposition 4. A consumer with search cost $s \in [0,1]$ may sample product i only if it has a non-negative reservation price for him or her, i.e., $v - p_i - \frac{s}{\lambda} \geq 0$.

For $p \leq v - \frac{1}{\lambda}$, product i has a non-negative reservation price for all consumers. At price p, firm i makes expected profits

$$
\begin{aligned}
E\Pi_i(p) &= \{F_j(p)\lambda(1-\lambda) + [1-F_j(p)]\lambda\}\, p \\
&= \lambda\,[1-\lambda F_j(p)]\, p.
\end{aligned}
$$

That $E\Pi_i(p) = R_i$ leads to $F_j(p) = \frac{1}{\lambda}\left(1 - \frac{R_i}{\lambda p}\right)$.

For $v - \frac{1}{\lambda} < p < v$, at price p product i has a non-negative reservation price for consumer s if $v - p - \frac{s}{\lambda} \geq 0$ or $s \leq \lambda(v-p)$. At price p, firm i makes expected profits

we have assumed that v is large enough such that all players will engage in search for both products
but here it discusses the possibility of some consumers do not search on products
it is possible that lambda*v/2>1

$$
\begin{aligned}
E\Pi_i(p) &= \lambda(v-p)\,\{F_j(p)\lambda(1-\lambda) + [1-F_j(p)]\lambda\}\, p \\
&= \lambda^2\,[1-\lambda F_j(p)]\,(v-p)p.
\end{aligned}
$$

That $E\Pi_i(p) = R_i$ leads to $F_j(p) = \frac{1}{\lambda}\left(1 - \frac{R_i}{\lambda^2(v-p)p}\right)$.

We focus on the case where $\underline{p} < v - \frac{1}{\lambda} < \overline{p}$. When $p = \overline{p}$, we have $E\Pi_i(\overline{p}) = \lambda^2\,(1-\lambda)\,(v-\overline{p})\overline{p}$. Profit maximization yields $\overline{p} = \frac{v}{2}$ and $R_i = R_j = R = \lambda^2(1-\lambda)\frac{v^2}{4}$. We then have $F_i(p) = F_j(p) = F(p)$. Since $E\Pi_i(\underline{p}) = \lambda\underline{p} = R$, we have $\underline{p} = \frac{\lambda(1-\lambda)v^2}{4}$.

We can verify that $\overline{p} > v - \frac{1}{\lambda} \Leftrightarrow v < \frac{2}{\lambda}$, and that $\underline{p} < v - \frac{1}{\lambda} \Leftrightarrow \frac{2}{\lambda_H(1+\sqrt{\lambda_H})} < v < \frac{2}{\lambda_L}$. Therefore, when $\frac{2}{\lambda_H(1+\sqrt{\lambda_H})} < v < \frac{2}{\lambda_L}$, $F(p)$ above is indeed the unique price equilibrium. Q.E.D.

Proof of Proposition 5. When the firms choose differentiated product relevances λ_1 and λ_2 ($\lambda_1 < \lambda_2$), we have

$$\frac{\partial \pi_1}{\partial \lambda_1} = -\frac{[\lambda_2 + 2(1-\lambda_2)\lambda_1]}{9\lambda_1^3\lambda_2^2}\left\{\lambda_1\left[\lambda_2 + 2(1-\lambda_2)\lambda_1\right] + 2\lambda_2(\lambda_2 - \lambda_1)\right\} - 2\beta\lambda_1 < 0$$

and

$$\frac{\partial \pi_2}{\partial \lambda_2} = \frac{[2\lambda_2 + (1-\lambda_2)\lambda_1]}{9\lambda_1^2\lambda_2^3}\left\{(2\lambda_1 - \lambda_2)\left[2\lambda_2 + (1-\lambda_2)\lambda_1\right] + 2\lambda_2(\lambda_2 - \lambda_1)(2-\lambda_1)\right\} - 2\beta\lambda_2.$$

Since $\frac{\partial \pi_1}{\partial \lambda_1} < 0$, we have $\lambda_1^* = \lambda_L$. When $\lambda_H < 2\lambda_L$ and β is sufficiently low, we have $\frac{\partial \pi_2}{\partial \lambda_2} > 0$ and thus $\lambda_2^* = \lambda_H$.

To show that $\lambda_1 = \lambda_L$ and $\lambda_2 = \lambda_H$ are the SPE, we still need to ensure that neither firm will unilaterally deviate to the rival's product relevance. If firm 1 deviates to $\lambda_1' = \lambda_H$, by Proposition 4 firm 1 makes profits $\pi_1' = \lambda_H^2(1-\lambda_H)\frac{v^2}{4} - \beta\lambda_H^2$. Since $v < \frac{2}{\lambda_L}$ (by assumption), we can verify that $\frac{(\lambda_H-\lambda_L)[\lambda_H+2(1-\lambda_H)\lambda_L]^2}{9\lambda_L^2\lambda_H^2} > \lambda_H^2(1-\lambda_H)\frac{v^2}{4}$ holds when $(\lambda_H - \lambda_L)\left[\lambda_H + 2(1-\lambda_H)\lambda_L\right]^2 > 9\lambda_H^4(1-\lambda_H)$.

If firm 2 unilaterally deviates to $\lambda_2' = \lambda_L$, by Proposition 4 firm 2 makes profits $\pi_2' = \lambda_L^2(1-\lambda_L)\frac{v^2}{4} - \beta\lambda_L^2$. When $\lambda_1 = \lambda_L$ and $\lambda_2 = \lambda_H$, firm 2's profits are $\frac{(\lambda_H-\lambda_L)[2\lambda_H+(1-\lambda_H)\lambda_L]^2}{9\lambda_L^2\lambda_H^2} > \frac{(\lambda_H-\lambda_L)[\lambda_H+2(1-\lambda_H)\lambda_L]^2}{9\lambda_L^2\lambda_H^2}$. Under the conditions of the Proposition, we have just shown that $\frac{(\lambda_H-\lambda_L)[\lambda_H+2(1-\lambda_H)\lambda_L]^2}{9\lambda_L^2\lambda_H^2} > \lambda_H^2(1-\lambda_H)\frac{v^2}{4}$. Therefore, if $\lambda_H^2(1-\lambda_H) > \lambda_L^2(1-\lambda_L)$, we have $\frac{(\lambda_H-\lambda_L)[2\lambda_H+(1-\lambda_H)\lambda_L]^2}{9\lambda_L^2\lambda_H^2} > \lambda_L^2(1-\lambda_L)$. This proves that $\pi_2 > \pi_2'$ when β is sufficiently low. Q.E.D.

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