

# Performance-Aware Self-Configurable Multi-Agent Networks: A Distributed Submodular Approach for Simultaneous Coordination and Network Design



Zirui Xu

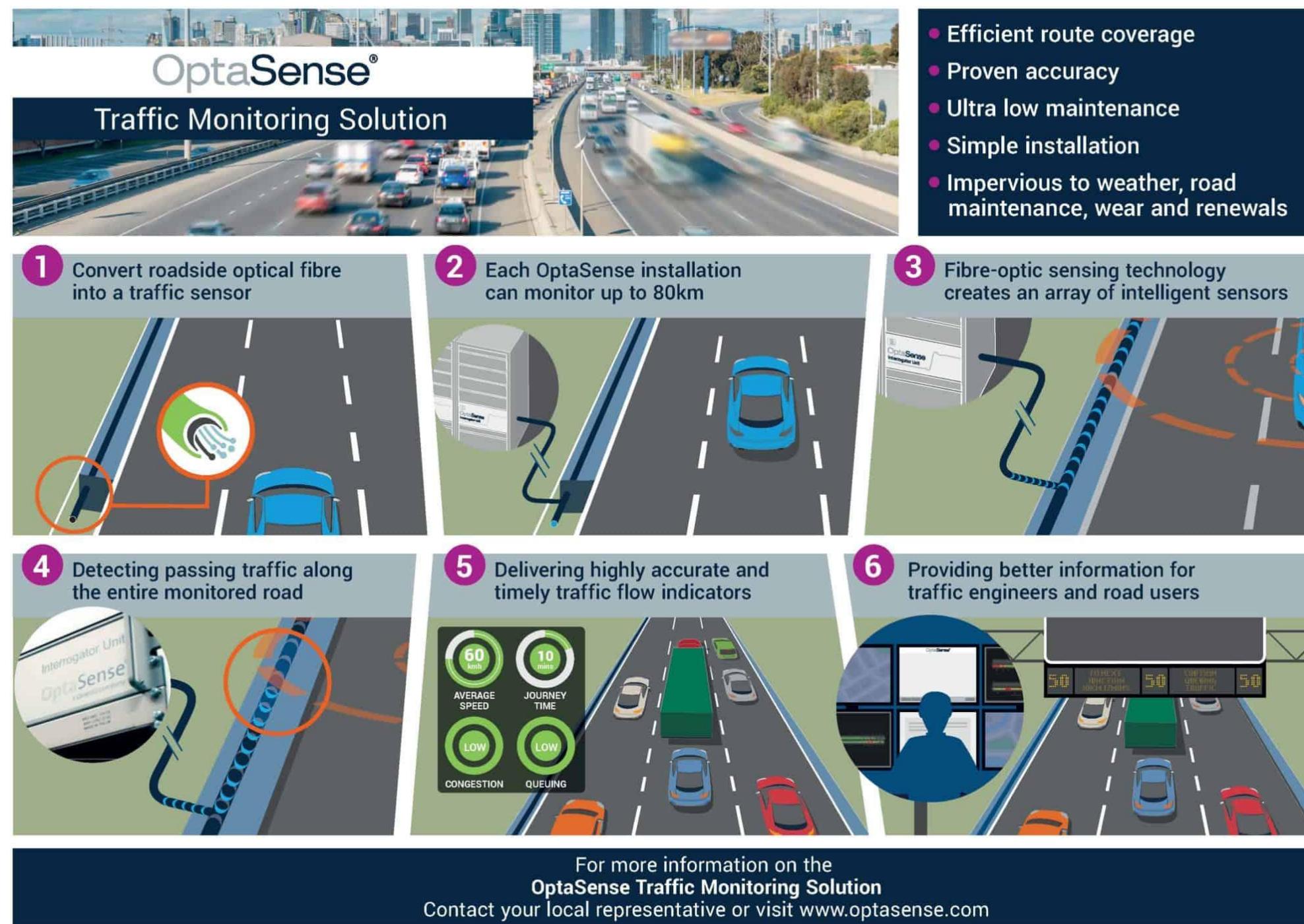


Vasileios Tzoumas

# Multi-Agent Decision-Making Problems

## Traffic Monitoring

Goal: Maximize knowledge of traffic conditions based on live data collected by distributed traffic sensors



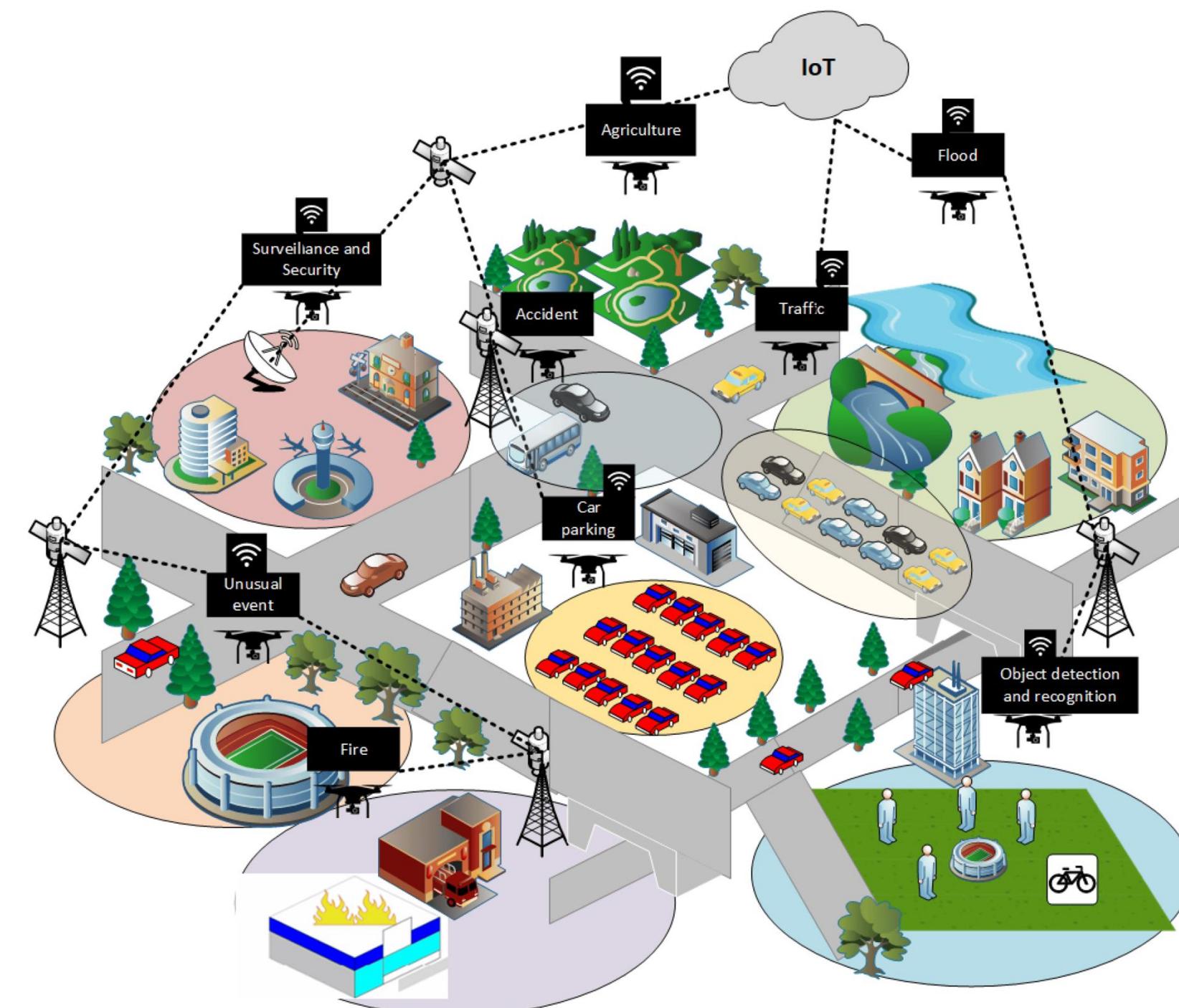
Problem: How should the sensors coordinate where to observe to achieve the goal?<sup>1</sup>

<sup>1</sup>Li, Mehr, Horowitz, Transportation Research B '23

# Multi-Agent Decision-Making Problems

## Event Detection

Goal: Maximize number of detected events based on live data collected by wireless sensor networks



Problem: How should the sensors coordinate where to observe to achieve the goal?<sup>2</sup>

<sup>2</sup>Kumar, Rus, Singh, IEEE Pervasive Computing '04

# Multi-Agent Decision-Making Problems

## Wildfire Management

Goal: Maximize knowledge of wildfire influence based on live data collected by distributed remote sensors



Problem: How should the sensors coordinate where to observe to achieve the goal?<sup>3</sup>

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<sup>3</sup>Afghah, Razi, Chakareski, Ashdown, IEEE INFOCOM '19

# Multi-Agent Coordination Problems

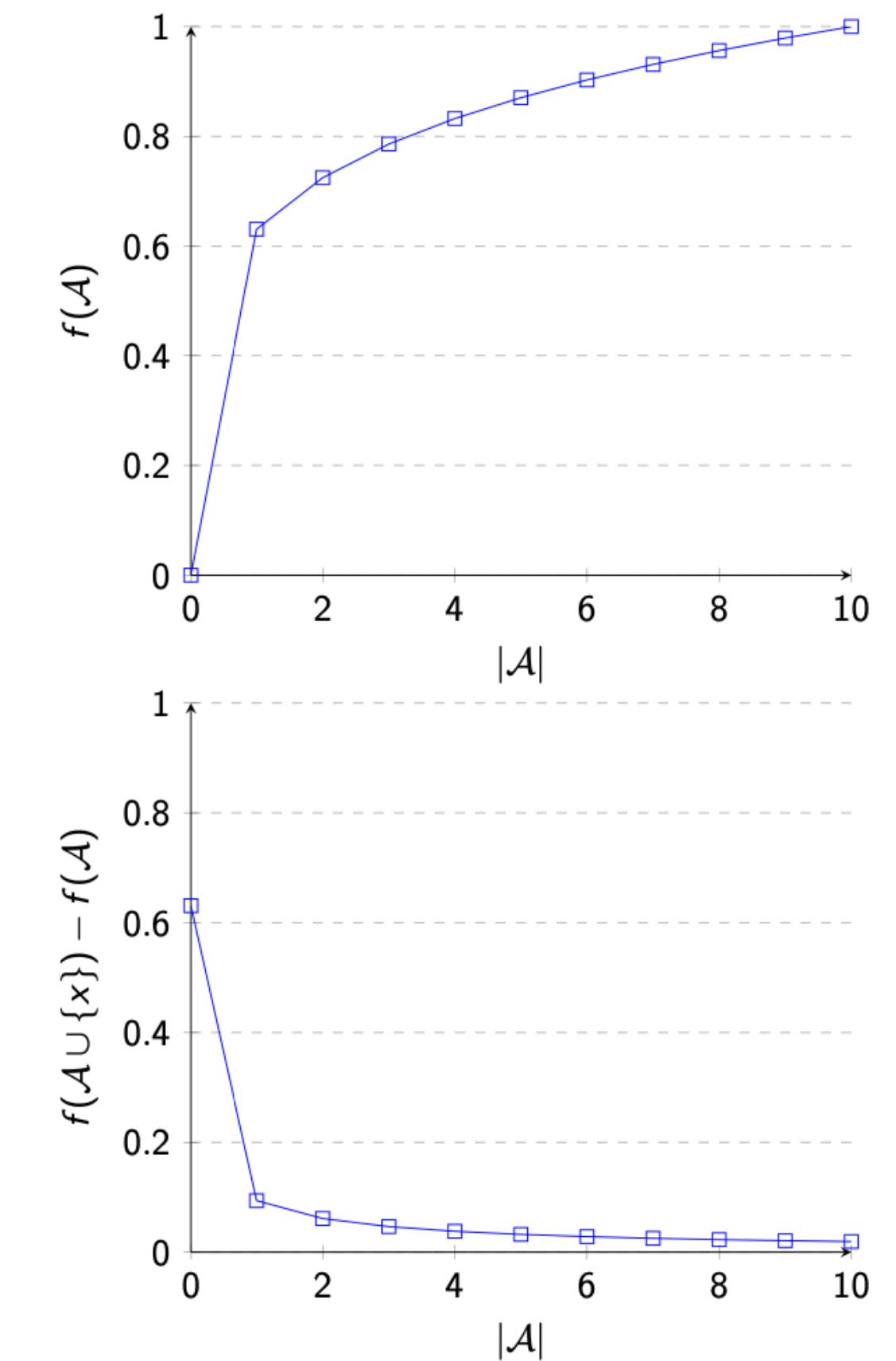
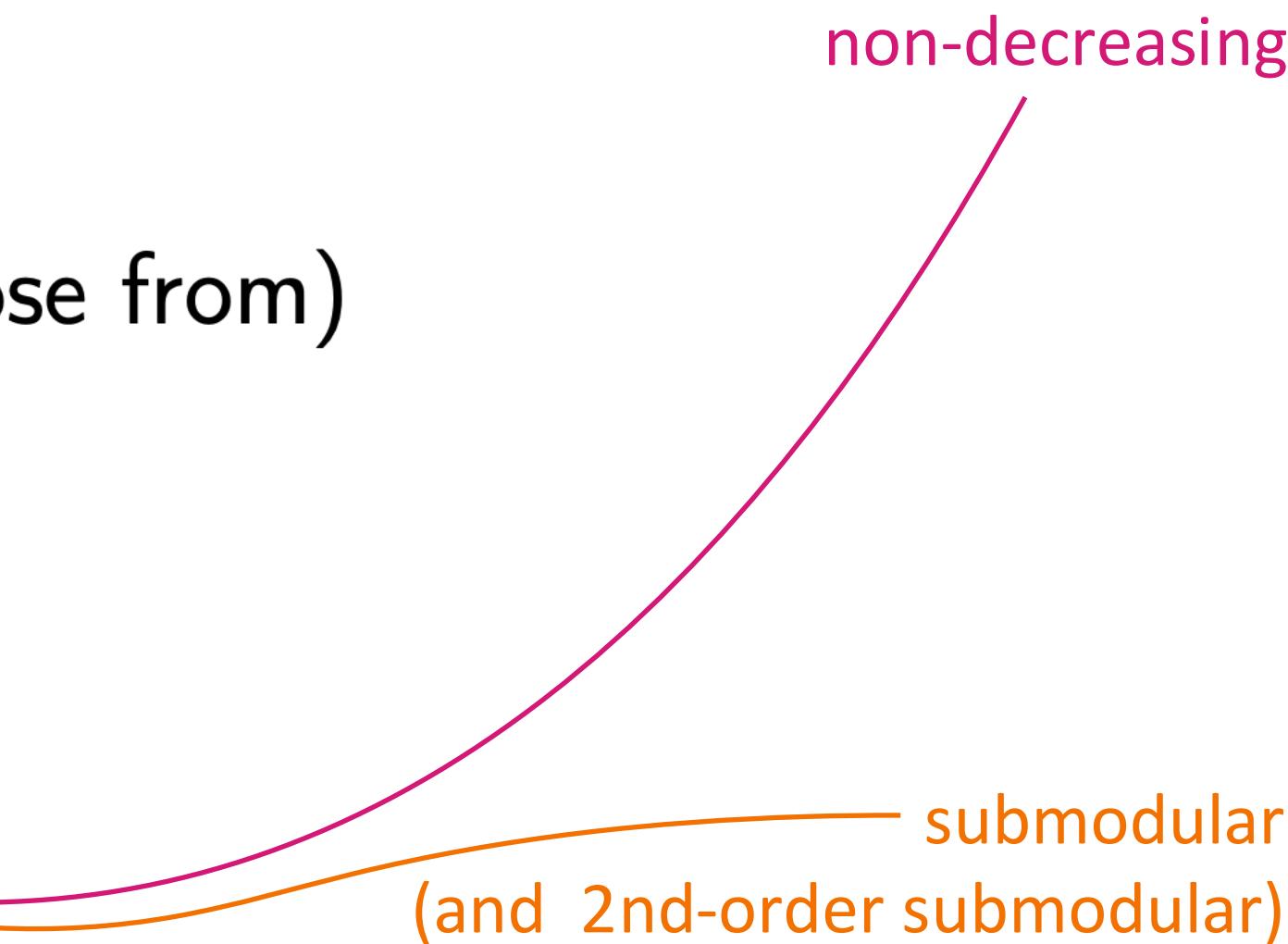
All above are submodular maximization problems<sup>1</sup>

**Given:**

- agents  $\mathcal{N}$
- finite available action sets  $\mathcal{V}_i, \forall i \in \mathcal{N}$   
(e.g., a set of FOV's for a camera to choose from)
- set function  $f: 2^{\prod_{i \in \mathcal{N}} \mathcal{V}_i} \mapsto \mathbb{R}$

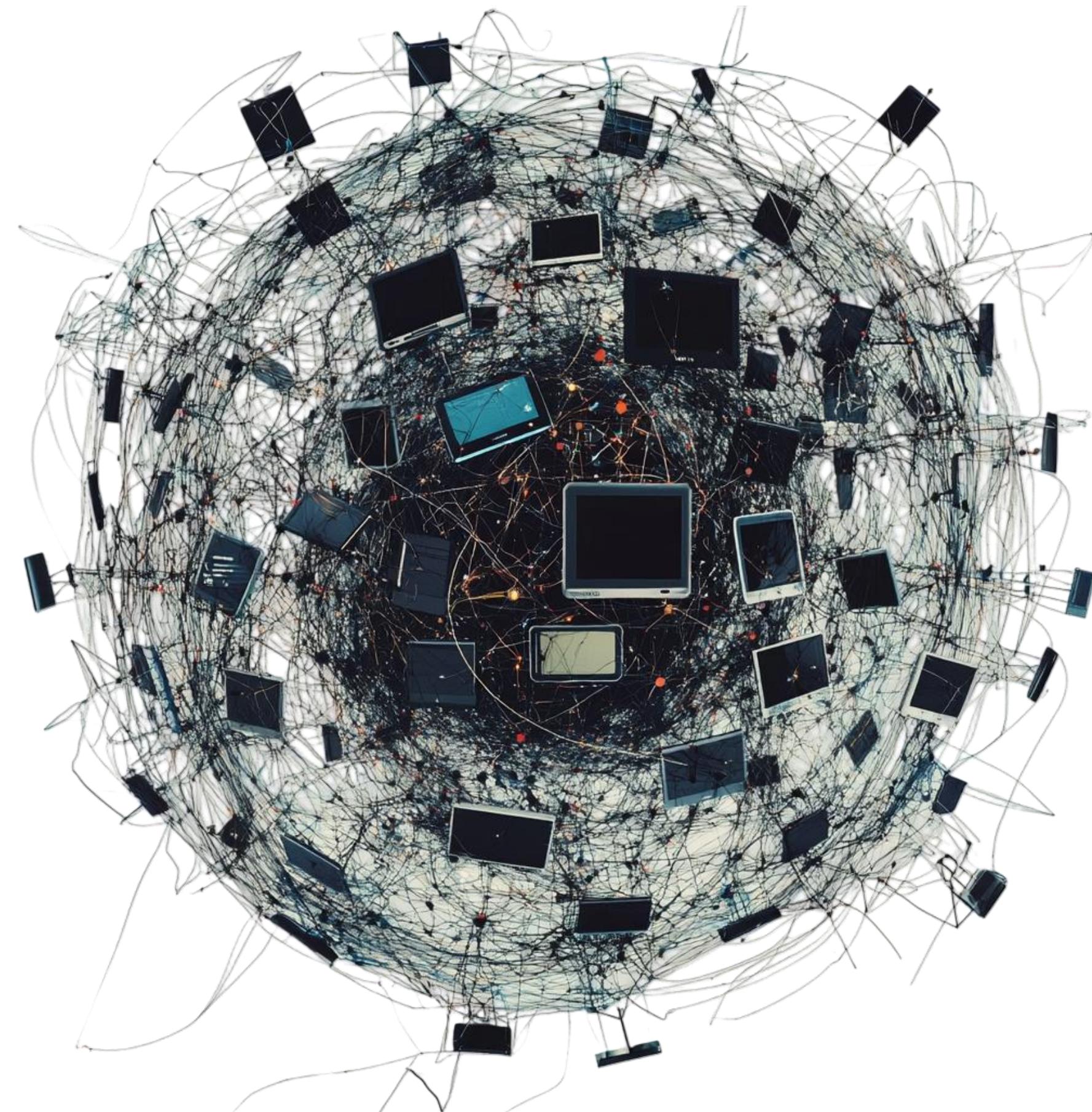
the agents  $\mathcal{N}$  select actions  $\{a_i\}_{i \in \mathcal{N}}$  to solve

$$\max_{a_i \in \mathcal{V}_i, \forall i \in \mathcal{N}} f(\{a_i\}_{i \in \mathcal{N}})$$

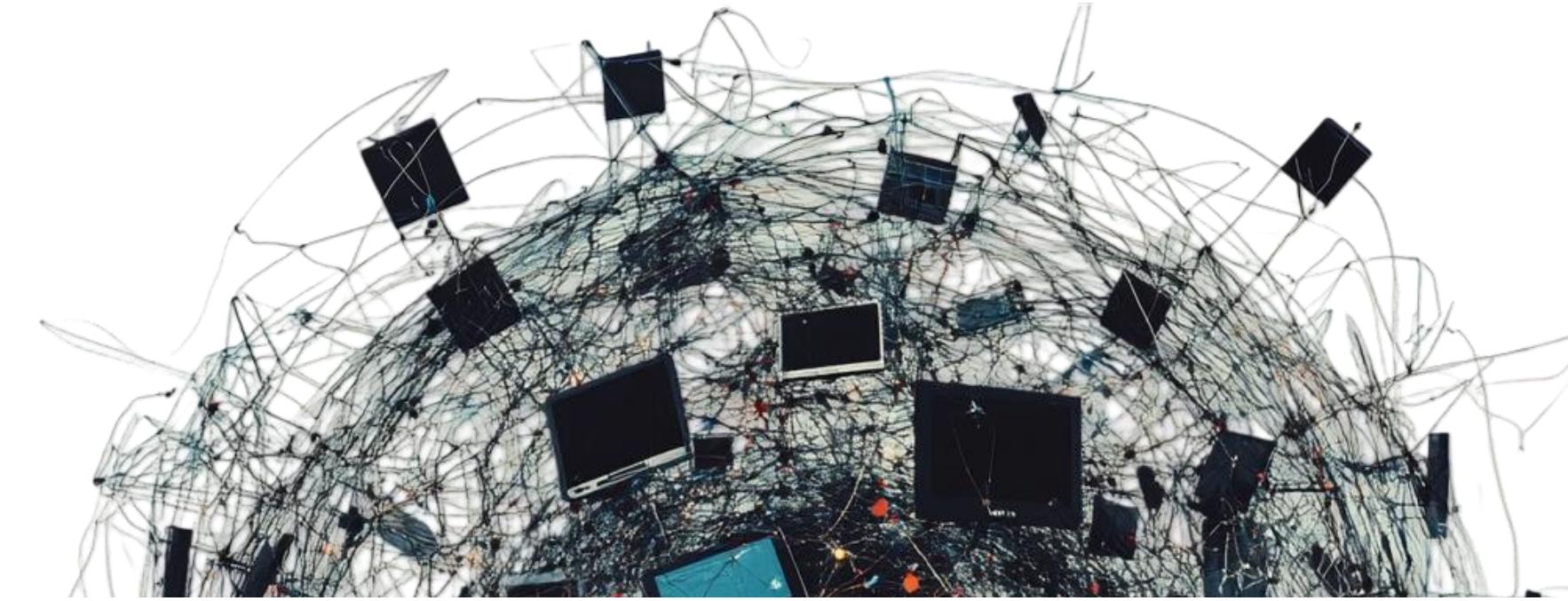


<sup>1</sup>Atanasov; Bilmes; Bushnell; Calinescu; Chekuri; Clark; Corah; Gharesifard; Hassani; Hespanha; Iyer; Karbasi; Kia; Konda; Krause; Li; Marden; Martinez; Michael; Mirzasoleiman; Mokhtari; Poovendran; Rezazadeh; Robey; Smith; Sundaram; Tokekar; ...

# Agents Have Limited Communication Bandwidth and Data Rate



# Agents Have Limited Communication Bandwidth and Data Rate



Having all sensors coordinating with all other is often impractical:

- Impractical communication and computation loads/requirements
- Impractical decision time



# Agents Have Limited Communication Bandwidth and Data Rate

Research Question:

Given the agents' communication constraints, what is the best network configuration?

## EMERGING INDUSTRIES (continued)

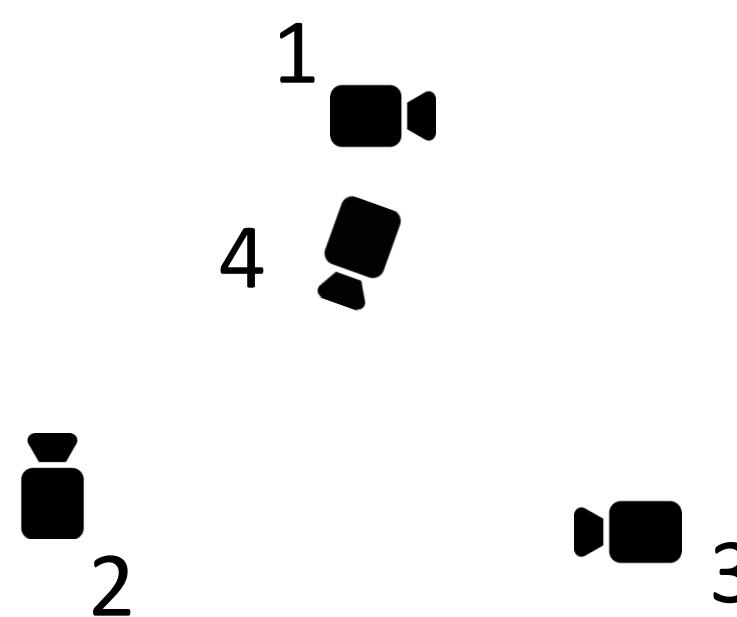
**Artificial intelligence:** Data-intensive science – the interaction between people and technology – and the use of a convergent approach to research come together in the research area of AI. For example, imagine a future in which autonomous vehicles fill the roads and the sky, all the while constantly communicating with each other, the roadway and traffic control signals. Understanding the behavior of this “swarm” and ensuring that terrestrial and aerial traffic flow evolves safely and efficiently is a research challenge that requires insights from biology, mathematics, engineering, human psychology and computer science. The research addresses the problem of how to integrate large flows of data from sensors in vehicles and embedded in the roadway and visual information from cameras. The traffic flow of highways and skyways of the future is just one example of the ways in which research that provides the ability to deploy AI on a large scale will transform our lives. Other potential outcomes from research on AI and cloud computing include robot assistants for the home-bound, diagnostic systems to aid physicians, improved factory automation and, when coupled with novel approaches to the analysis of large data-streams, new tools for the intelligence community.

NSF Strategic Plan 2022-2026

# Network Configuration Matters to Optimization Performance

## Area Monitoring Example:

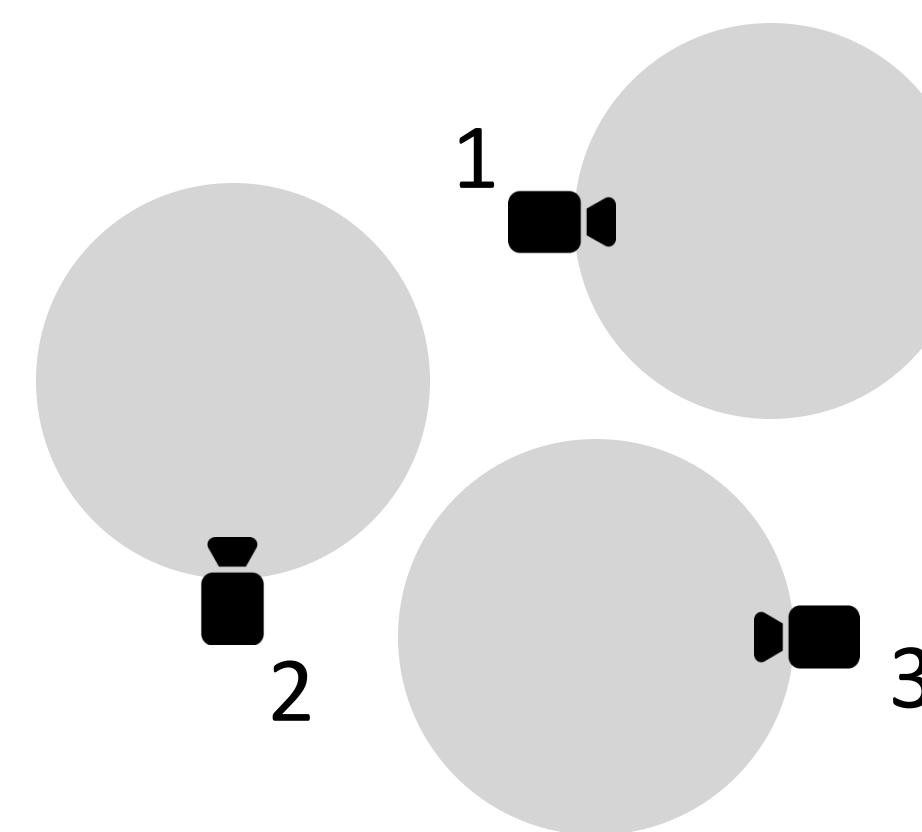
- 4 cameras select their FOV's to collaboratively maximize total covered area
- Each camera is able to receive information from only **up to 2 others**



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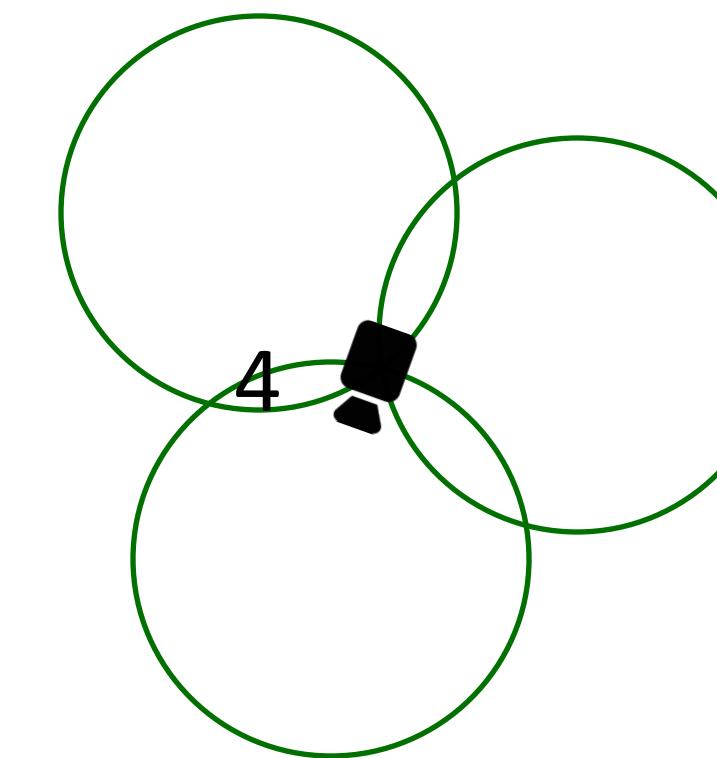


cameras 1-3 already decided

# Network Configuration Matters to Optimization Performance

Area Monitoring Example:

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and camera 4 has 3 options

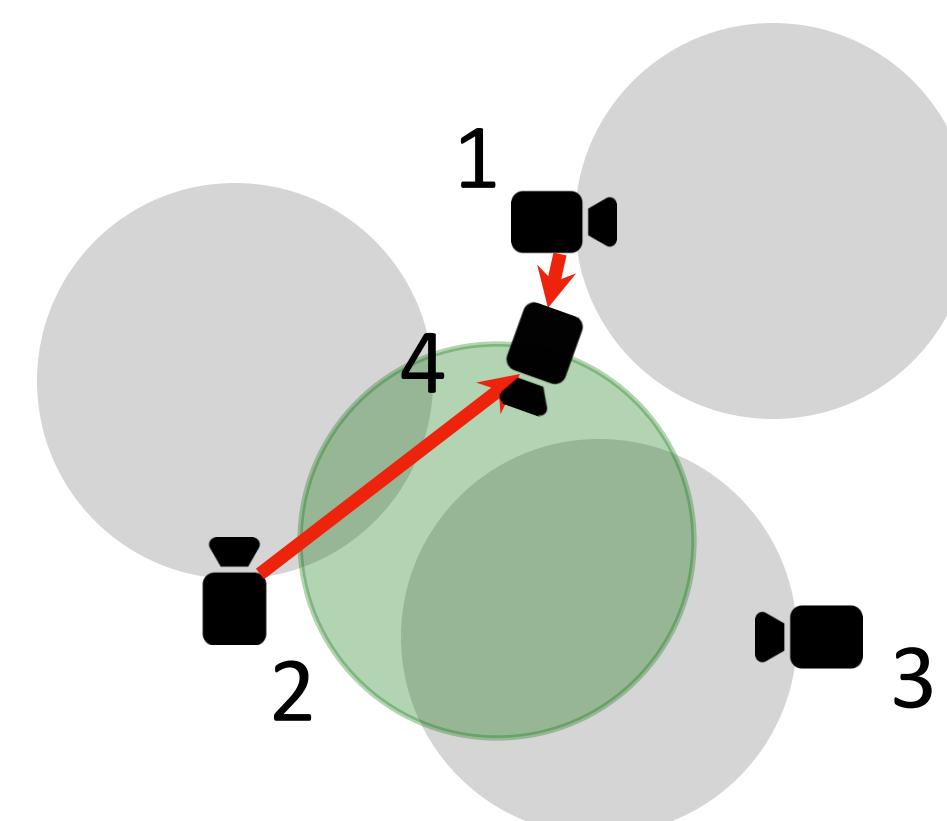
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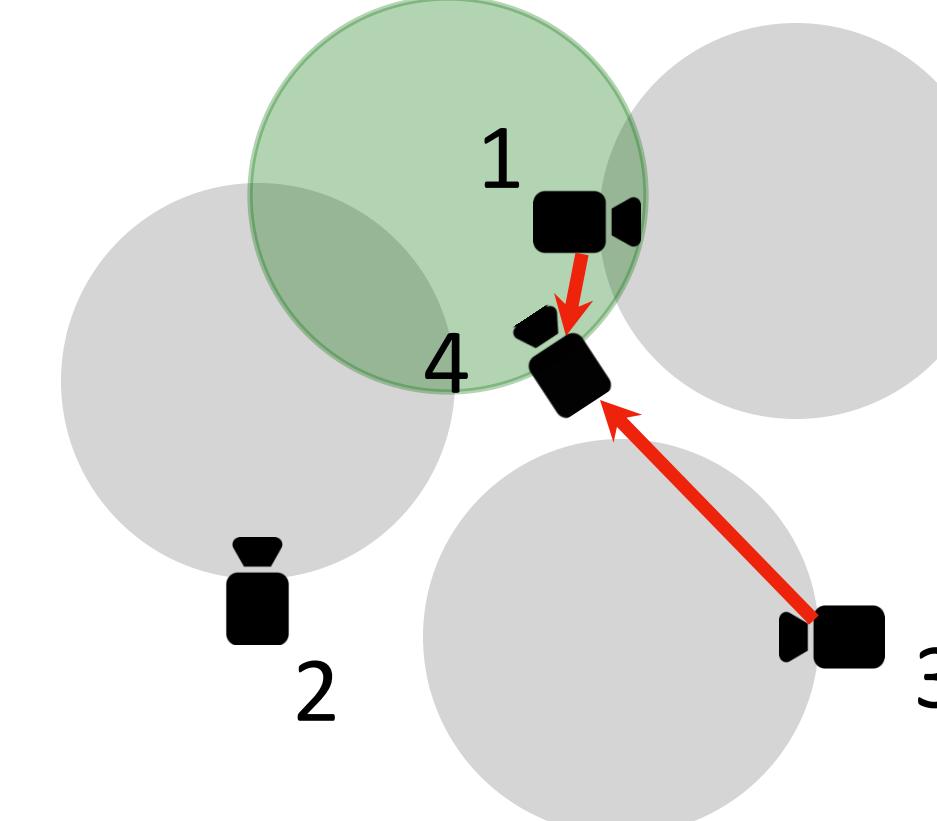
- 4 cameras select their FOV's to collaboratively maximize total covered area
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All possible local network configurations for camera 4:

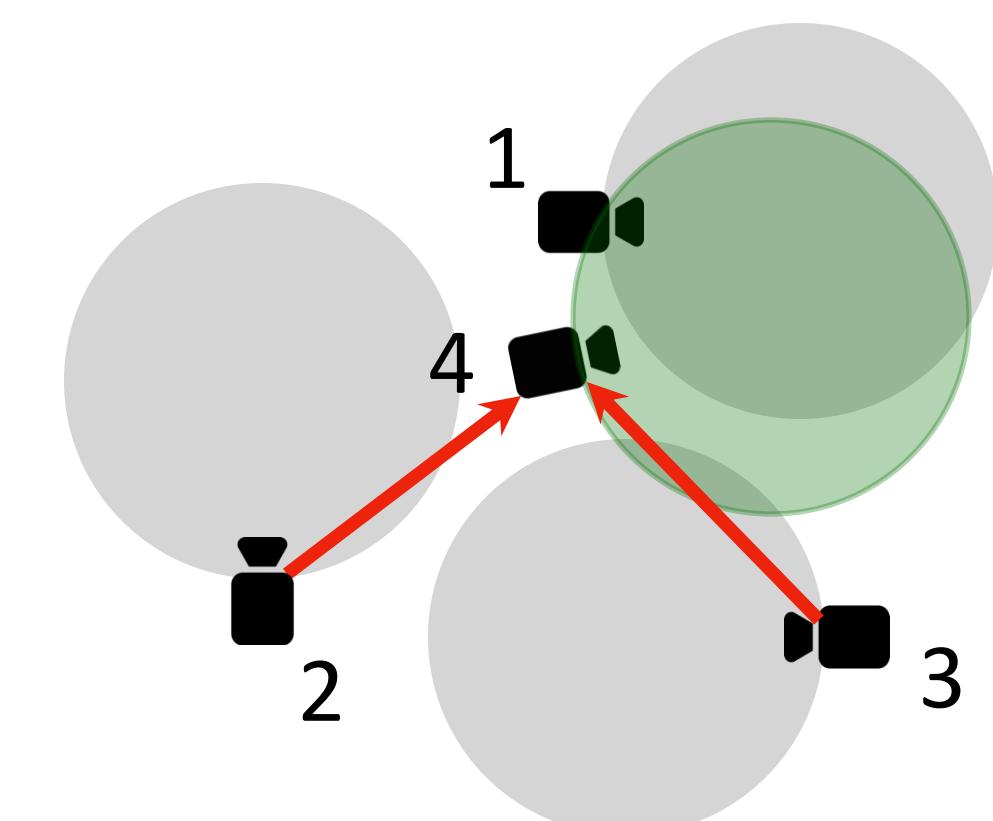
Scenario 1:  
Listen to 1 & 2, no info about 3



Scenario 2:  
Listen to 1 & 3; no info about 2



Scenario 3:  
Listen to 2 & 3; no info about 1



Best network: gives largest total covered area

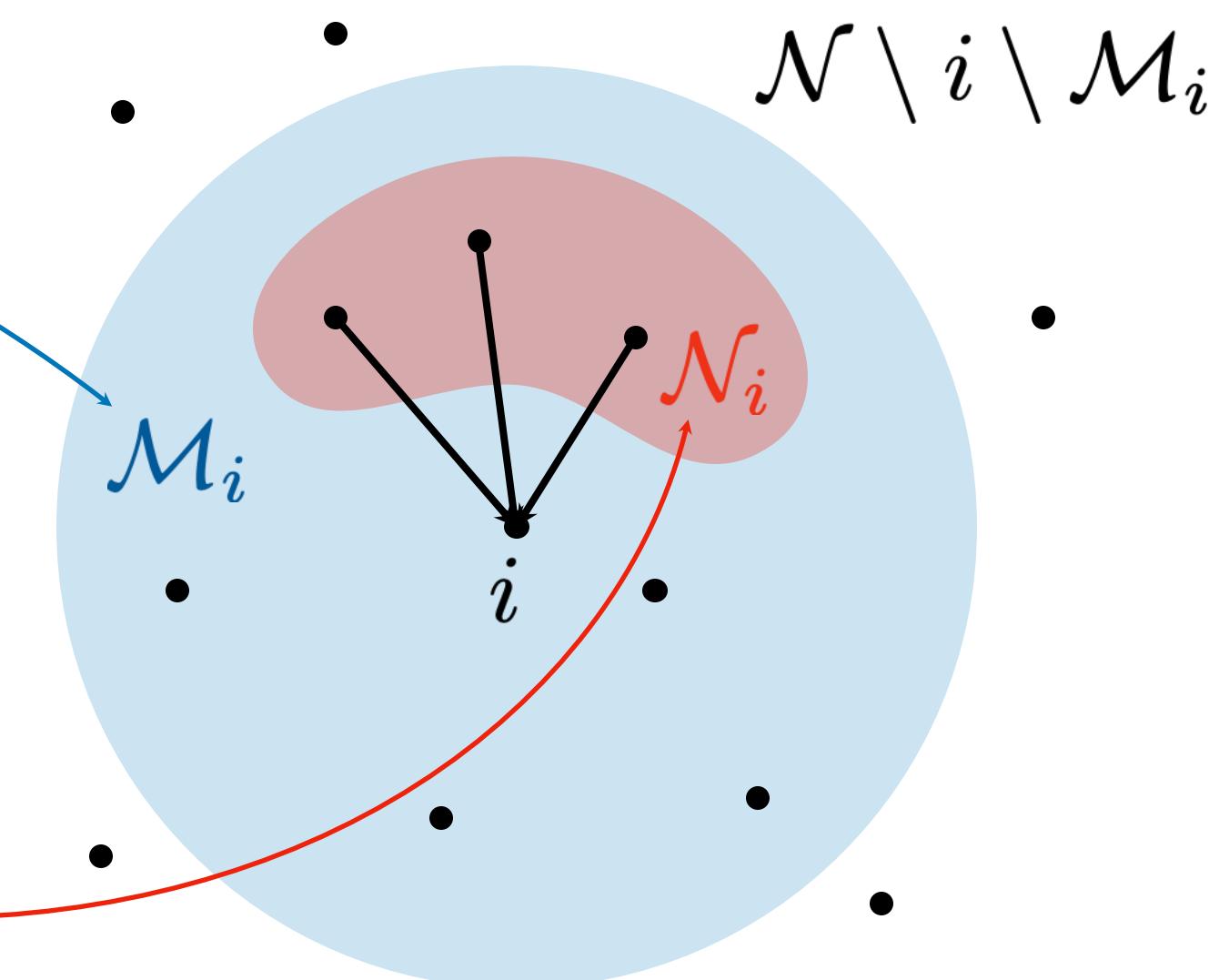
# Distributed Simultaneous Action Coordination & Network Self-Configuration

For each agent  $i \in \mathcal{N}$ , given:

- reachable neighbor candidates  $\mathcal{M}_i \subseteq \mathcal{N} \setminus i$
- communication bandwidth budget  $\alpha_i$
- finite action sets  $\mathcal{V}_i, \forall i \in \mathcal{N}$
- set function  $f: 2^{\prod_{i \in \mathcal{N}} \mathcal{V}_i} \mapsto \mathbb{R}$

agent  $i$  needs to select neighborhood  $\mathcal{N}_i$  and action  $a_i$  to collaboratively solve

$$\mathcal{N}_i \subseteq \mathcal{M}_i, |\mathcal{N}_i| \leq \alpha_i, \forall i \in \mathcal{N} \quad a_i \in \mathcal{V}_i, \forall i \in \mathcal{N} \quad f(\{a_i\}_{i \in \mathcal{N}})$$



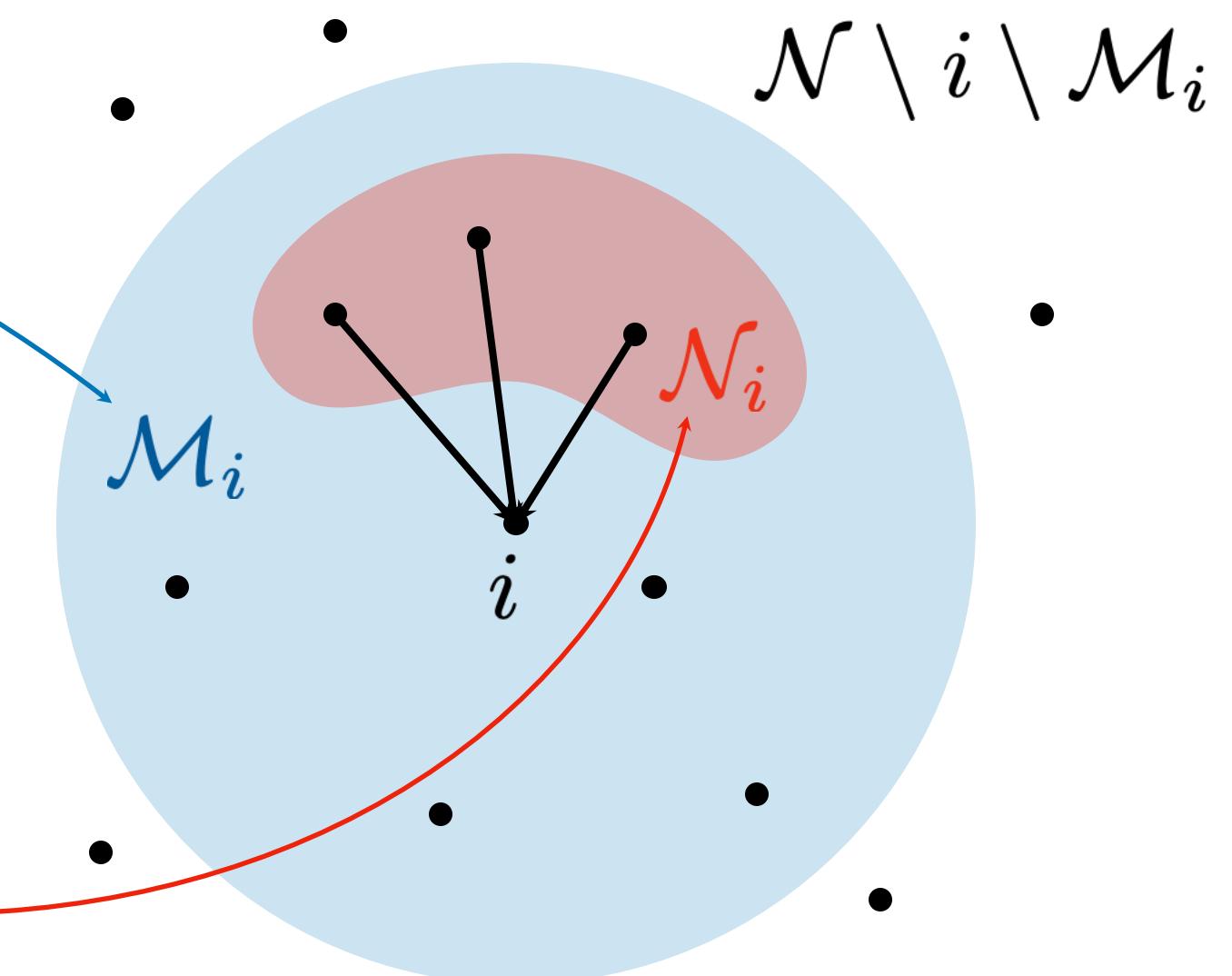
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if  $\mathcal{M}_i = \mathcal{N} \setminus i$  and  $\alpha_i = \infty$ , then becomes the original maximization problem

# Current Distributed Submodular Maximization Approaches

$$\max_{\mathcal{N}_i \subseteq \mathcal{M}_i, |\mathcal{N}_i| \leq \alpha_i, \forall i \in \mathcal{N}} \max_{a_i \in \mathcal{V}_i, \forall i \in \mathcal{N}} f(\{a_i\}_{i \in \mathcal{N}}) \text{ has not been studied before}$$

Distributed algorithms for

$$\max_{a_i \in \mathcal{V}_i, \forall i \in \mathcal{N}}: f(\{a_i\}_{i \in \mathcal{N}})$$

- Sequential Greedy and SOTA variants:<sup>1–6</sup>

- Suboptimality guarantee  $1 - 1/e$  or  $1/2$

- Decision time including communication delays:

- $O(|\mathcal{N}|^2 \text{diam}(\mathcal{G}))$  time for connected, directed graphs<sup>2</sup>

- $O(|\mathcal{N}|^3)$  time for connected, directed graphs |  $O(|\mathcal{N}|^3)$  for strongly connected, undirected graphs<sup>3</sup>

agents must relay info  
across the network

<sup>1</sup>Fisher, Nemhauser, Wolsey, Math Prog Studies '78

<sup>2</sup>Liu, Zhou, Tokekar, RAL '20

<sup>3</sup>Konda, Grimsman, Marden, ACC '22

<sup>4</sup>Robey, Adibi, Schlotfeldt, Hassani, Pappas, L4DC '21

<sup>5</sup>Du, Qian, Claudel, Sun, TAC '22

<sup>6</sup>Rezazadeh, Kia, Automatica '23

# Current Distributed Submodular Maximization Approaches

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    - $O(|\mathcal{N}|^2)$  time for connected, directed graphs |  $O(|\mathcal{N}|^3)$  for strongly connected, undirected graphs<sup>3</sup>
- Works that examine impact of limited information access to suboptimality bound:<sup>7–9</sup>
  - Quantification for **worst-case  $f$** : Suboptimality guarantee  $1/(\alpha + 1)$ <sup>7–8</sup>

agents must relay info across the network

degrades proportionally to number of agents that select actions independently

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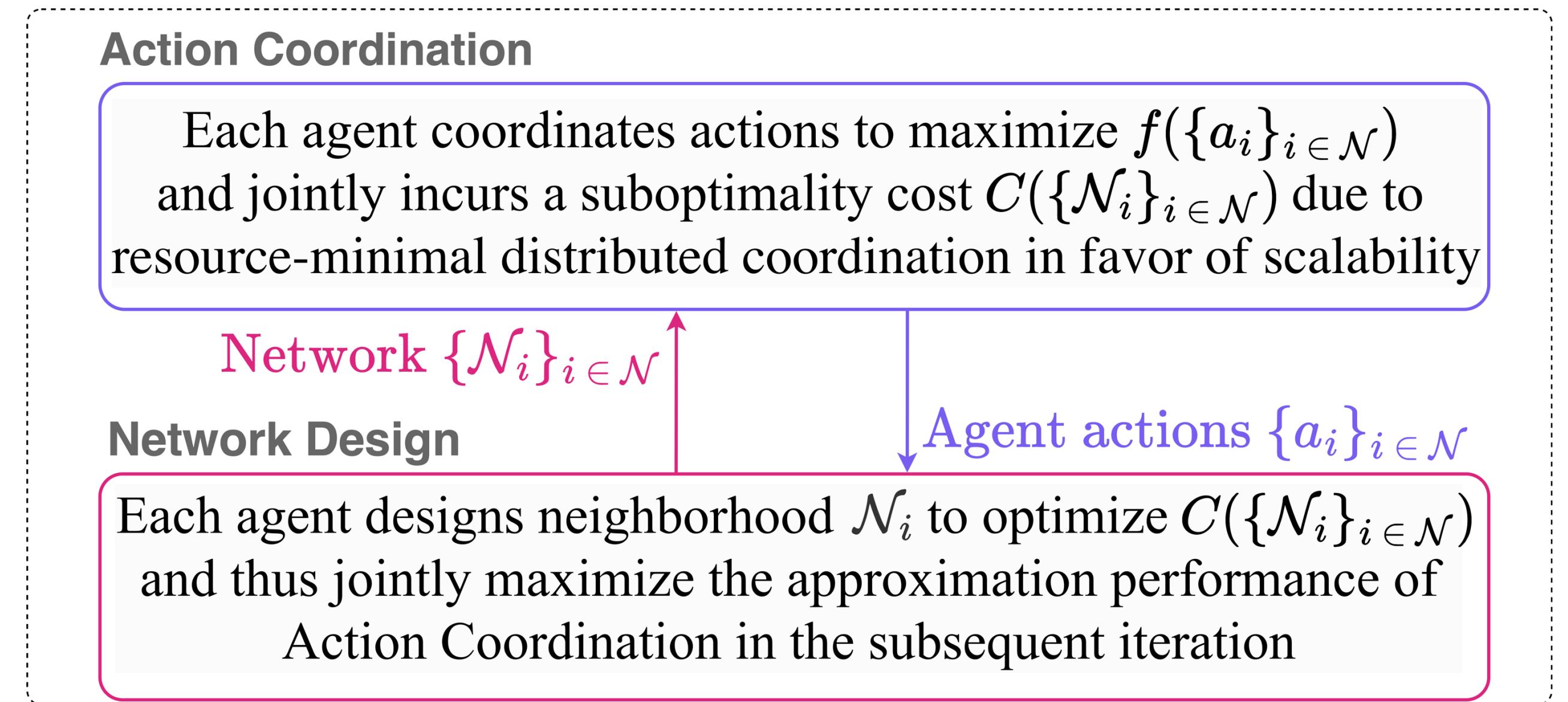
<sup>7</sup>Gharesifard and Smith, TCNS '18

<sup>8</sup>Grimsman, Ali, Hespanha, Marden, TCNS '19

<sup>9</sup>Corah, Michael, CDC '18

# Our Contributions

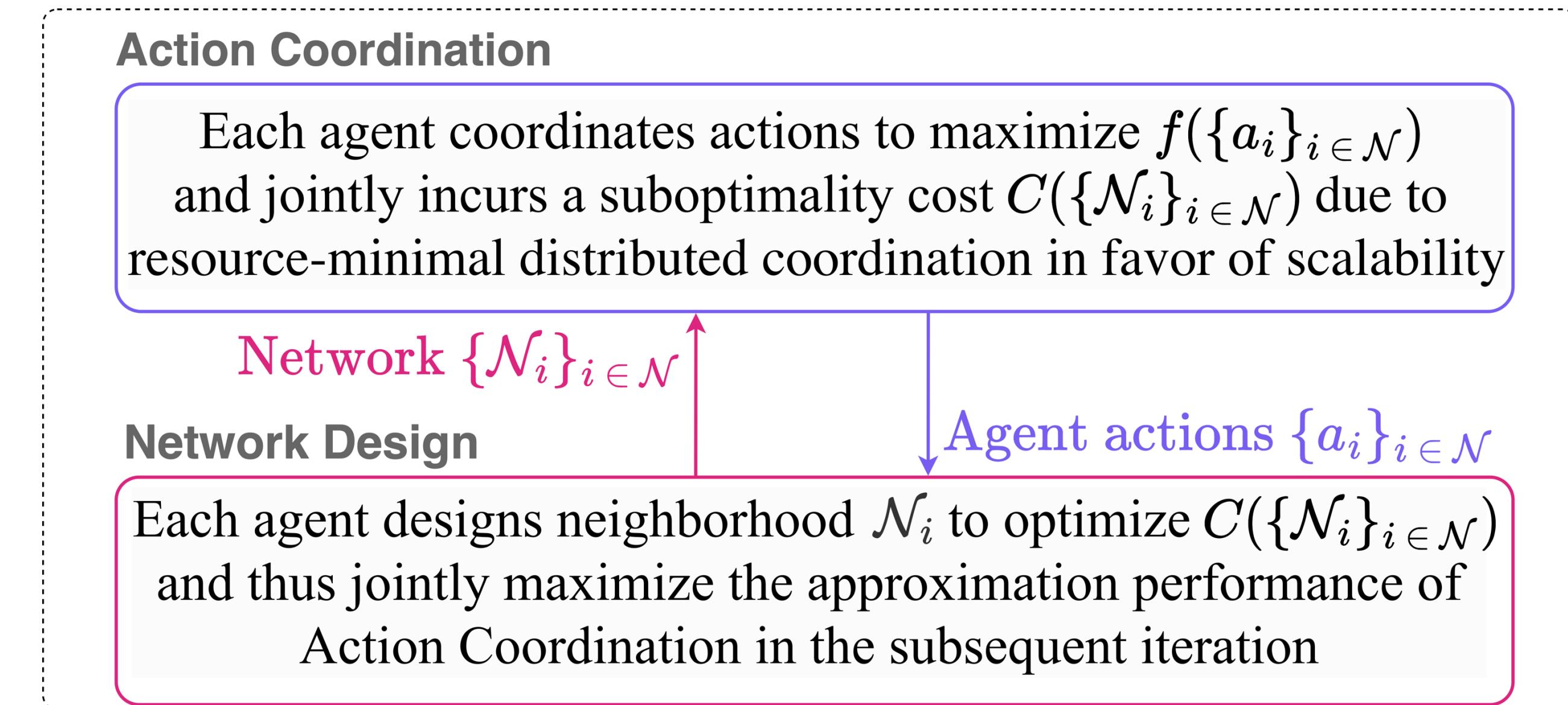
Introduced distributed optimization algorithm:



**Algorithm 1:** AlterNAting COordination and Network-Design Algorithm (Anaconda)

# Our Contributions

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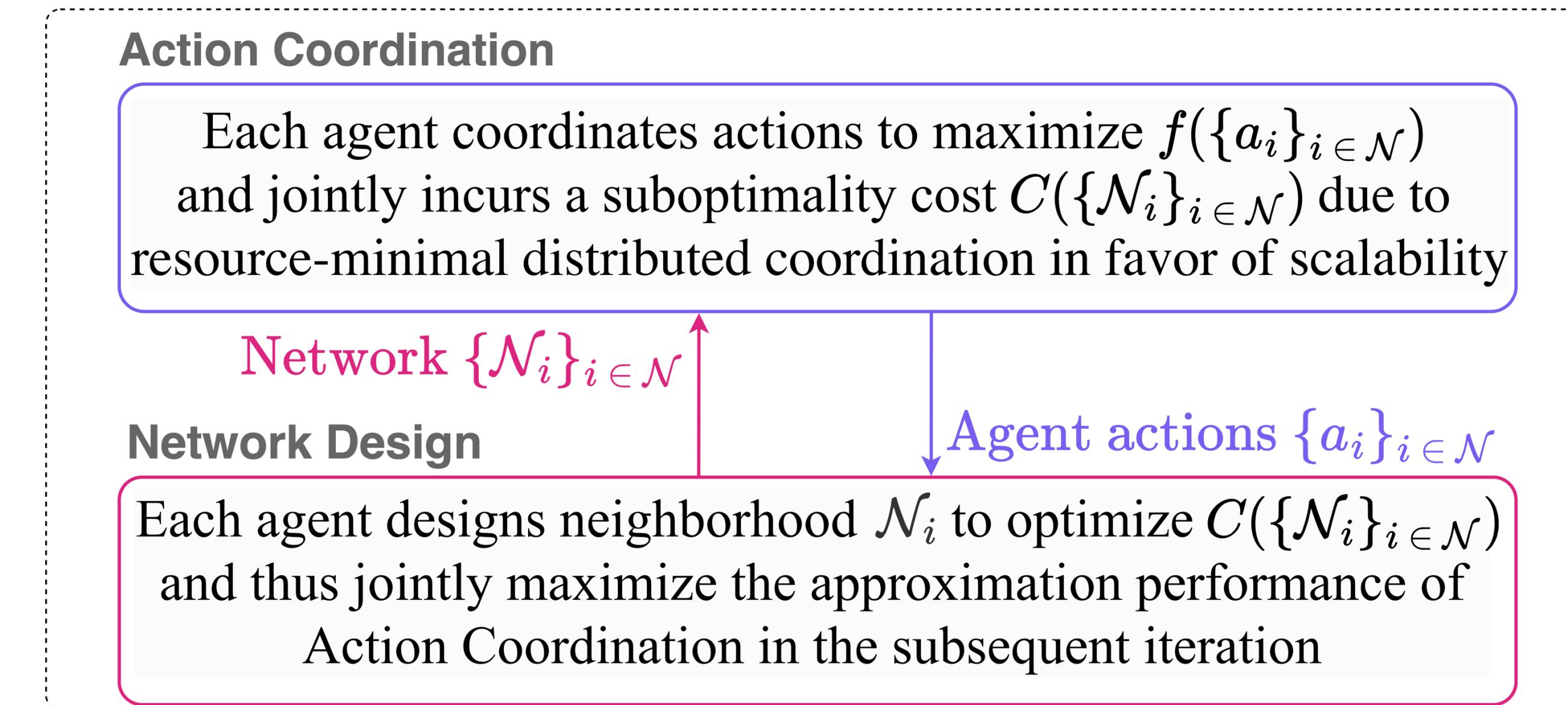
**Algorithm 1:** AlterNAting COordination and Network-Design Algorithm (Anaconda)

Key ideas in making Algorithm 1 scalable and near-optimal:

- Design Action Coordination and Network Design steps to require minimal communications
- Quantify impact of network configuration  $\{\mathcal{N}_i\}_{i \in \mathcal{N}}$  to suboptimality of action coordination
- Use quantification to optimize network configuration via alternating optimization

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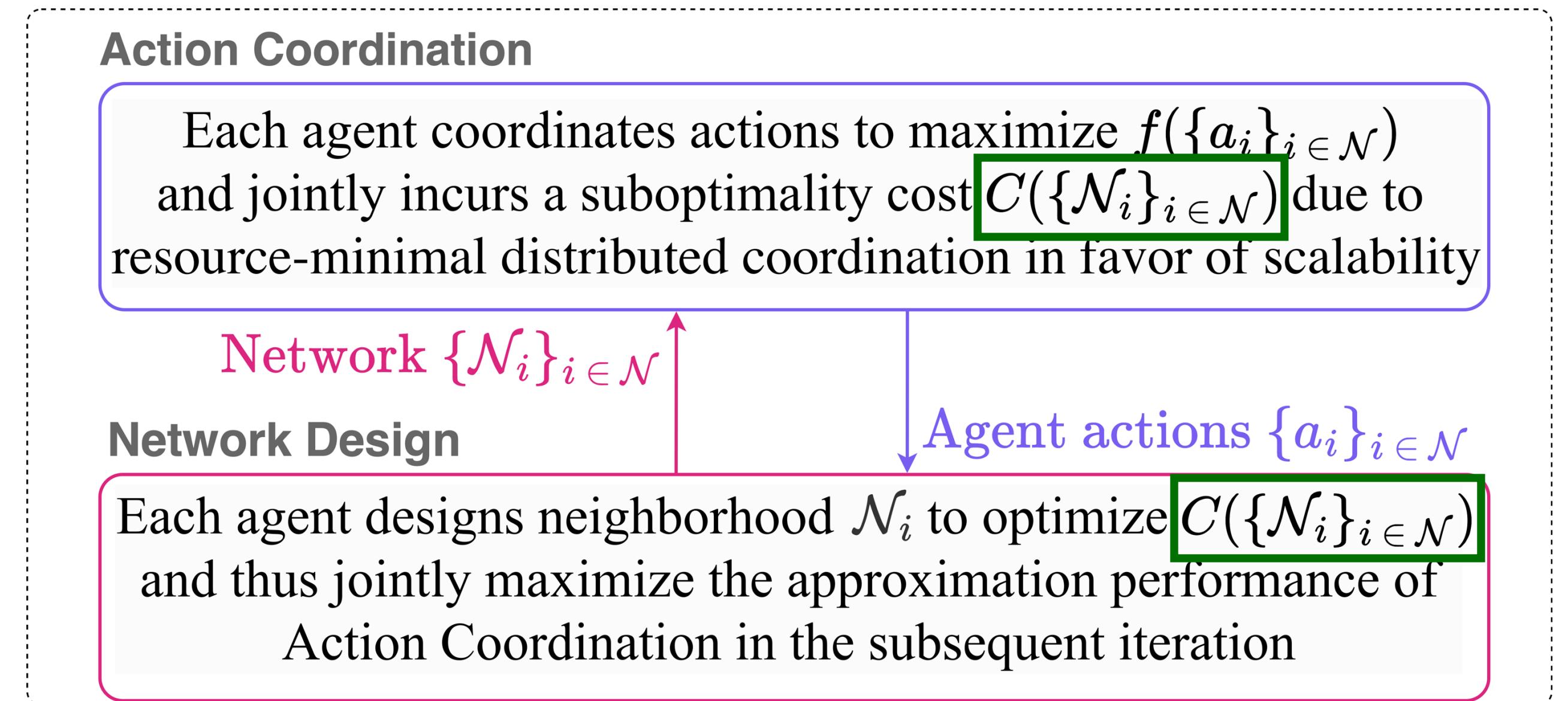
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denote it  
 $C(\{\mathcal{N}_i\}_{i \in \mathcal{N}})$

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# A Closer Look to Algorithm 1

At iteration  $t$ :

**Stage 1 (ActionCoordination).** *Simultaneously* sample action  $a_{i,t} \in \mathcal{V}_i$  based on past rewards to solve

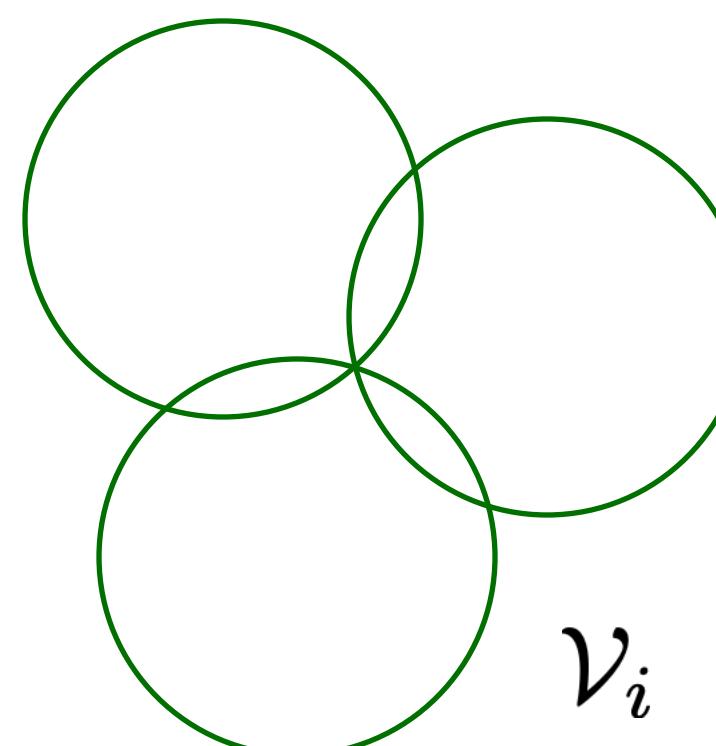
$$\max_{a_{i,t} \in \mathcal{V}_i, \forall i \in \mathcal{N}} f(\{a_{i,t}\}_{i \in \mathcal{N}})$$

**Stage 2 (NeighborSelection).** *Simultaneously* sample neighborhood  $\mathcal{N}_{i,t} \subseteq \mathcal{M}_i$  based on past rewards to solve

$$\max_{\mathcal{N}_{i,t} \subseteq \mathcal{M}_i, |\mathcal{N}_{i,t}| \leq \alpha_i, \forall i \in \mathcal{N}} \max_{a_{i,t} \in \mathcal{V}_i, \forall i \in \mathcal{N}} f(\{a_{i,t}\}_{i \in \mathcal{N}})$$

Given neighborhood  $\mathcal{N}_{i,t-1}$  selected in **Stage 2 of time step**  $t - 1$ , each agent  $i$ :

1. Compute rewards  $r_{a,t-1} = f(a \mid \{a_{j,t-1}\}_{j \in \mathcal{N}_{i,t-1}}), \forall a \in \mathcal{V}_i$  ————— marginal gain in  $f$  of selecting action  $a$  given neighborhood  $\mathcal{N}_{i,t-1}$
  2. Update probability distribution  $p_{i,t}$  over  $\mathcal{V}_i$  using  $\{r_{a,t-1}\}_{a \in \mathcal{V}_i}$
  3. Sample action  $a_{i,t} \in \mathcal{V}_i$  from  $p_{i,t}$
- Multiplicative Weights Update<sup>1</sup>**

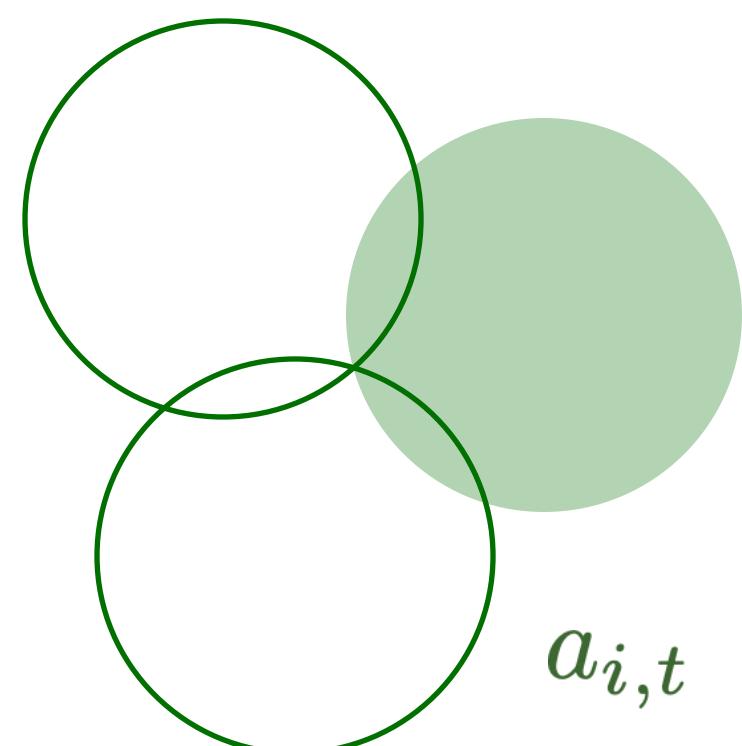



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# Suboptimality Performance of ActionCoordination

## Definition (Suboptimality Cost of ActionCoordination)

The suboptimality cost due to network decentralization is  $C(\{\mathcal{N}_{i,t}\}_{i \in \mathcal{N}}) = \sum_{i \in \mathcal{N}} C_{i,t}(\mathcal{N}_{i,t})$ , where

$$C_{i,t}(\mathcal{N}_{i,t}) \triangleq f(a_{i,t} \mid \{a_{j,t}\}_{j \in \mathcal{N}_{i,t}}) - f(a_{i,t})$$

# Suboptimality Performance of Action Coordination

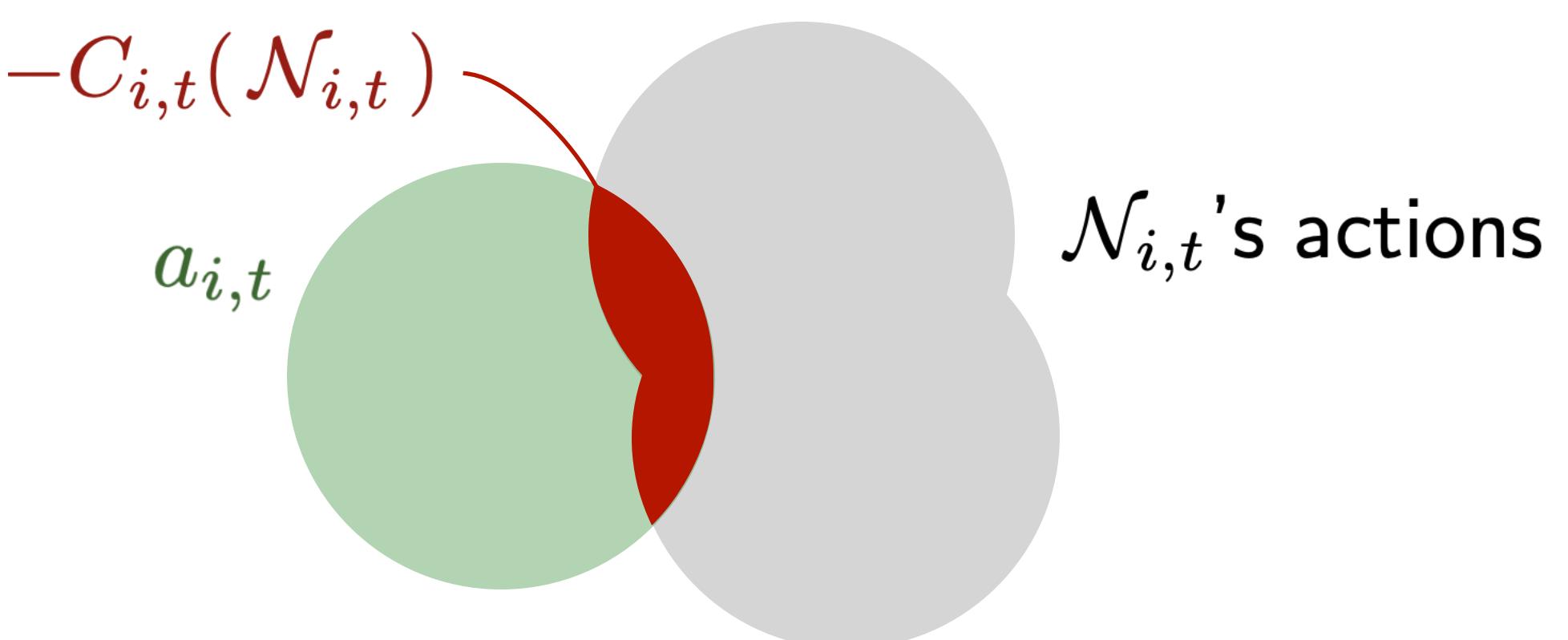
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Intuition:

$-C_{i,t}$  captures the overlap of  $i$ 's action and  $\mathcal{N}_{i,t}$ 's actions:



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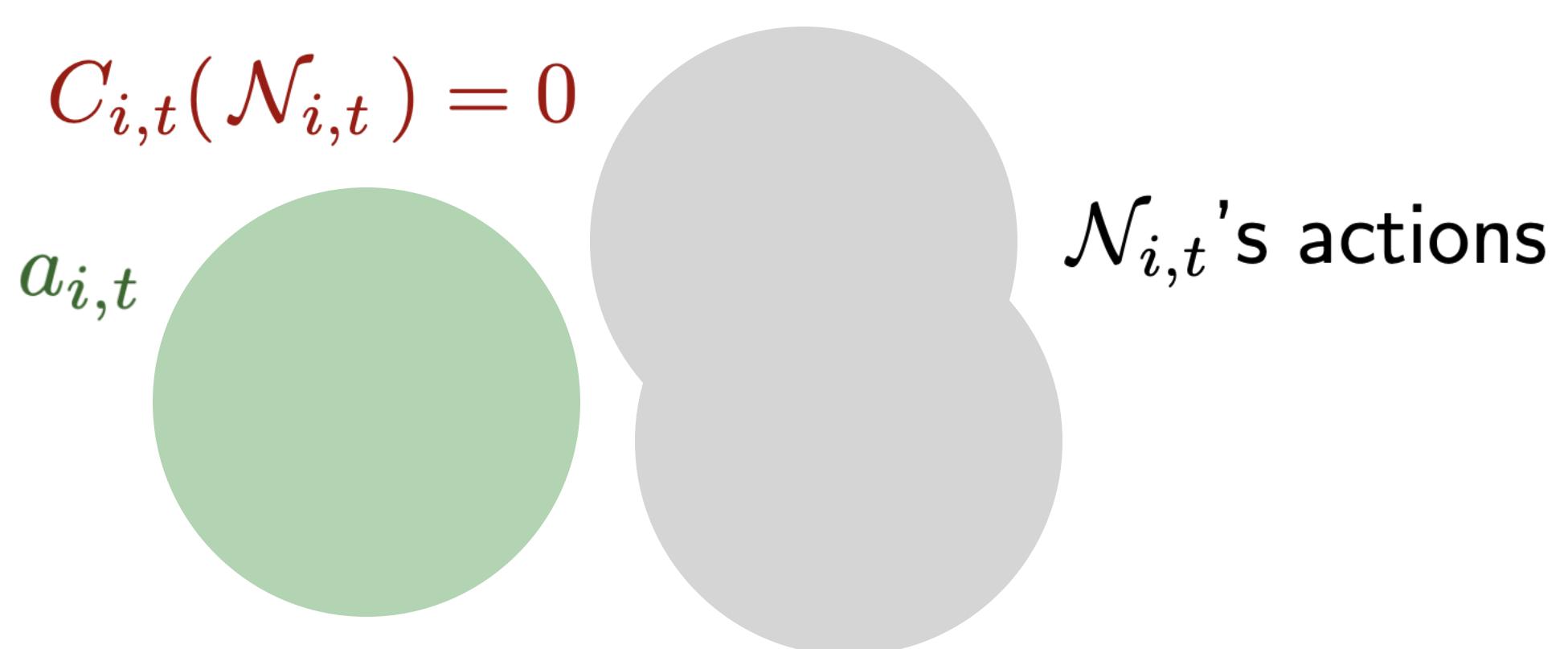
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# Suboptimality Performance of ActionCoordination

## Proposition 1

At each time step  $t$ , ActionCoordination instructs each agent  $i \in \mathcal{N}$  to select action  $a_{i,t}$  such that when  $t \geq \tilde{O}(1/\epsilon^2)$ ,  $C_{i,t}(\mathcal{N}_{i,t})$  is minimized in expectation with respect to  $a_{i,t}$ , i.e.,

$$\mathbb{E}[C_{i,t}(\mathcal{N}_{i,t})] \leq \min_{a \in \mathcal{V}_i} [f(a | \{a_{j,t}\}_{j \in \mathcal{N}_{i,t}}) - f(a)] + \epsilon$$

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and that when  $t \geq \tilde{O}(|\mathcal{N}|^2/\epsilon^2)$ , for all agents' actions  $\mathcal{A}_t \triangleq \{a_{i,t}\}_{i \in \mathcal{N}}$ ,

$$\mathbb{E}[f(\mathcal{A}_t)] \geq (1 - \kappa_f) \left[ f(\mathcal{A}^{\text{OPT}}) - \sum_{i \in \mathcal{N}} \mathbb{E}[C_{i,t}(\mathcal{N}_{i,t})] \right] - \epsilon$$



$f$ 's **curvature**  $\kappa_f \triangleq 1 - \min_{z \in \mathcal{V}} \frac{f(\mathcal{V}) - f(\mathcal{V} \setminus z)}{f(z)} \in [0, 1]$ .  
 $\kappa_f$  measures how  $\mathcal{V}$ 's elements *substitute* each other:

- $\kappa_f = 0 \Leftrightarrow f(\mathcal{A}) = \sum_{z \in \mathcal{A}} f(z)$ ;
- $\kappa_f = 1 \Leftrightarrow \exists z \in \mathcal{V} \text{ s.t. } f(\mathcal{V}) = f(\mathcal{V} \setminus z)$ .

# Neighbor Selection: A Bandit Supermodular Minimization Approach

## Problem (Neighbor Selection)

At each time step  $t$ , each agent  $i \in \mathcal{N}$  needs to select neighborhood  $\mathcal{N}_{i,t} \subseteq \mathcal{M}_i$  *online* to solve

$$\min_{\mathcal{N}_{i,t} \subseteq \mathcal{M}_i, |\mathcal{N}_{i,t}| \leq \alpha_i} C_{i,t}(\mathcal{N}_{i,t}),$$

being able to evaluate  $C_{i,t}(\mathcal{M})$ ,  $\forall \mathcal{M} \subseteq \mathcal{M}_i$  only after having selected  $\mathcal{M}$  as neighbors and received actions  $\{a_{j,t}\}_{j \in \mathcal{M}}$  from them.

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## Lemma (Monotonicity and Supermodularity of $C_{i,t}$ )

Given non-decreasing and 2nd-order submodular function  $f$  and action  $a_{i,t}$ ,  $C_{i,t}(\mathcal{N}_{i,t})$  is non-increasing and supermodular in  $\mathcal{N}_{i,t}$

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**bandit feedback** 
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**Difficulty:** **NP-Hard** to achieve approximation bound better than  $\kappa_f^{-1}(1 - e^{-\kappa_f})(\geq 1 - 1/e)$  even when  $C_{i,t}(\mathcal{M})$ ,  $\forall \mathcal{M} \subseteq \mathcal{M}_i$  can be evaluated before  $\mathcal{M}$  is selected

# A Closer Look to Algorithm 1

At iteration  $t$ :

**Stage 2 (NeighborSelection).** Simultaneously sample neighborhood  $\mathcal{N}_{i,t} \subseteq \mathcal{M}_i$  based on past rewards to solve

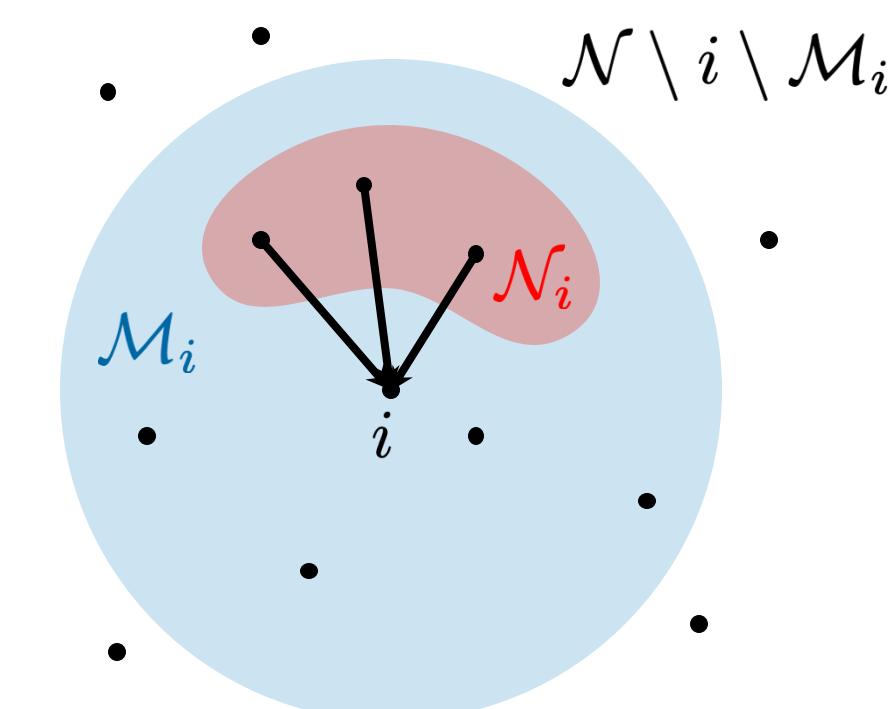
$$\max_{\mathcal{N}_{i,t} \subseteq \mathcal{M}_i, |\mathcal{N}_{i,t}| \leq \alpha_i, \forall i \in \mathcal{N}} \max_{a_{i,t} \in \mathcal{V}_i, \forall i \in \mathcal{N}} f(\{a_{i,t}\}_{i \in \mathcal{N}}) \longrightarrow \min_{\mathcal{N}_{i,t} \subseteq \mathcal{M}_i, |\mathcal{N}_{i,t}| \leq \alpha_i} C_{i,t}(\mathcal{N}_{i,t})$$

# NeighborSelection

Given action  $a_{i,t}$  selected in **Stage 1 of time**  $t$  and  $\mathcal{N}_{i,t} \leftarrow \emptyset$ , for  $k = 1, \dots, \alpha_i$ , each agent  $i$ :

1. Update probability distribution  $q_{i,t}^{(k)}$  over  $\mathcal{M}_i$  using past reward  $r_{j_{t-1}^{(k)}, t-1}$  — EXP3-IX algorithm<sup>1</sup>
2. Sample  $k$ -th neighbor  $j_t^{(k)} \in \mathcal{M}_i$  from  $q_{i,t}^{(k)}$  and receive its action  $a_{j_t^{(k)}, t}$
3. Compute reward  $r_{j_t^{(k)}, t} = C_{i,t}(\mathcal{N}_{i,t}) - C_{i,t}(\mathcal{N}_{i,t} \cup j_t^{(k)})$  and  $\mathcal{N}_{i,t} \leftarrow \mathcal{N}_{i,t} \cup j_t^{(k)}$

→ marginal loss in  $C_{i,t}$  of adding neighbor  $j_t^{(k)}$  to neighborhood  $\mathcal{N}_{i,t}$




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<sup>1</sup>Neu, NeurIPS, '15

# Suboptimality Performance of NeighborSelection

## Proposition

At each time step  $t$ , NeighborSelection instructs each agent  $i \in \mathcal{N}$  to select neighborhood  $\mathcal{N}_{i,t}$  such that when  $t \geq \tilde{O}(|\mathcal{M}_i| \alpha_i^2 / \epsilon^2)$ ,

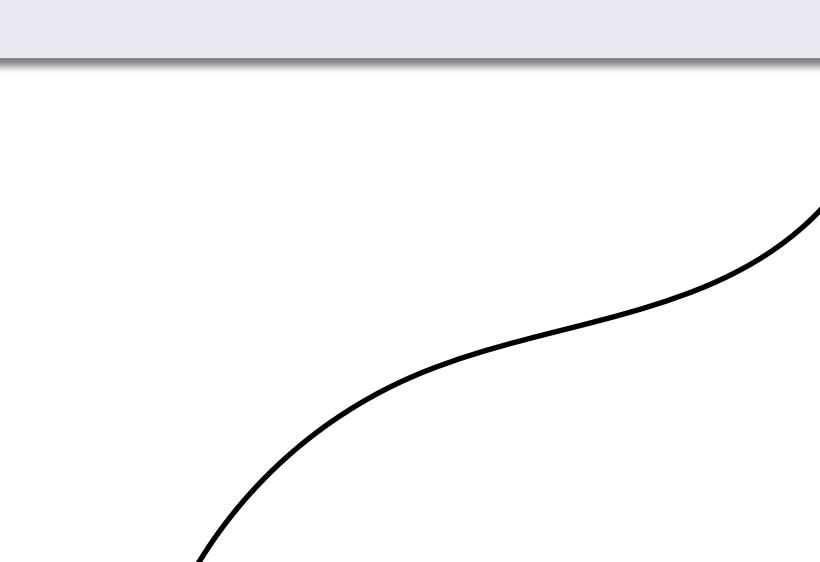
$$\mathbb{E}[C_{i,t}(\mathcal{N}_{i,t})] \leq \kappa_f^{-1}(1 - e^{-\kappa_f}) \mathbb{E}[C_{i,t}(\mathcal{N}_i^*)] + \epsilon$$

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$\mathcal{N}_i^*$  is agent  $i$ 's optimal neighborhood given actions  $\{a_{i,t}\}_{t \in [T]}$  selected by ActionCoordination during horizon  $T$ :

$$\mathcal{N}_i^* \in \arg \min_{\mathcal{N}_{i,t} \subseteq \mathcal{M}_i, |\mathcal{N}_{i,t}| \leq \alpha_i} \mathbb{E}[C_{i,t}(\mathcal{N}_{i,t})]$$

# Approximation Performance of Anaconda

## Theorem 1 (Fully Centralized Networks)

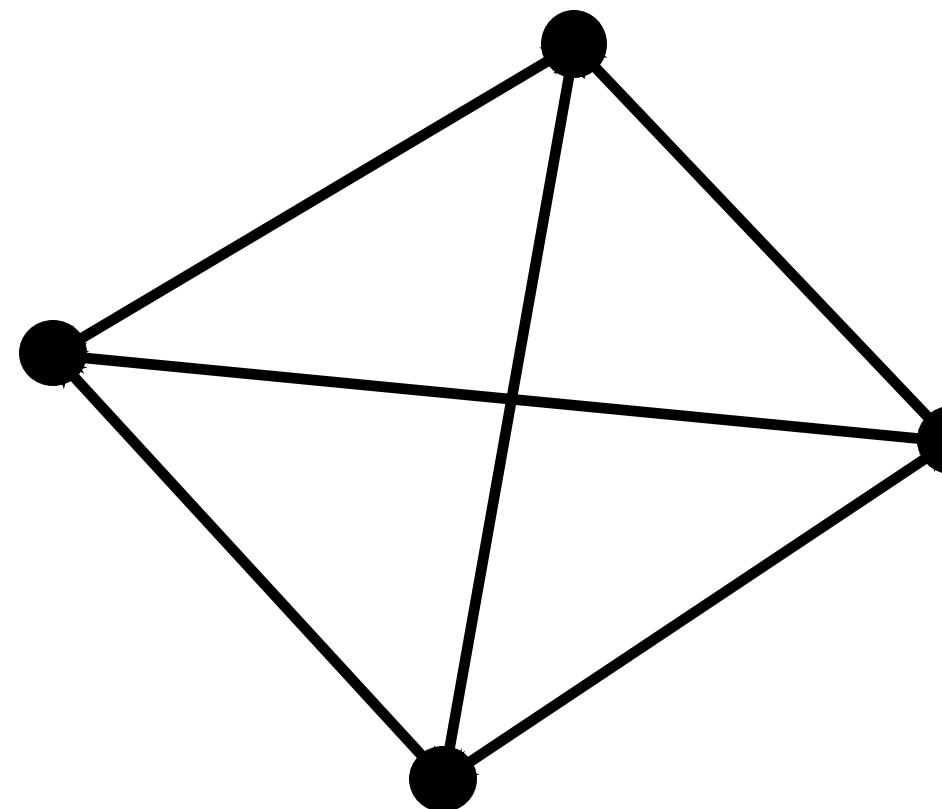
At each time step  $t$ , Anaconda instructs each agent  $i \in \mathcal{N}$  to select action  $a_{i,t}$  and neighborhood  $\mathcal{N}_{i,t}$  such that

- If the emergent network is *fully centralized* with  $\mathcal{N}_{i,t} \equiv \mathcal{N} \setminus i$ , when  $t \geq \tilde{O}(|\mathcal{N}|^2 / \epsilon^2)$ ,

$$\mathbb{E} [f(\mathcal{A}_t)] \geq \frac{1}{1 + \kappa_f} f(\mathcal{A}^{\text{OPT}}) - \epsilon$$



Match the tight bound of Sequential Greedy [Conforti and Cornuejols, '78]



No need to select neighbors since every agent has enough bandwidth budget to hear all others

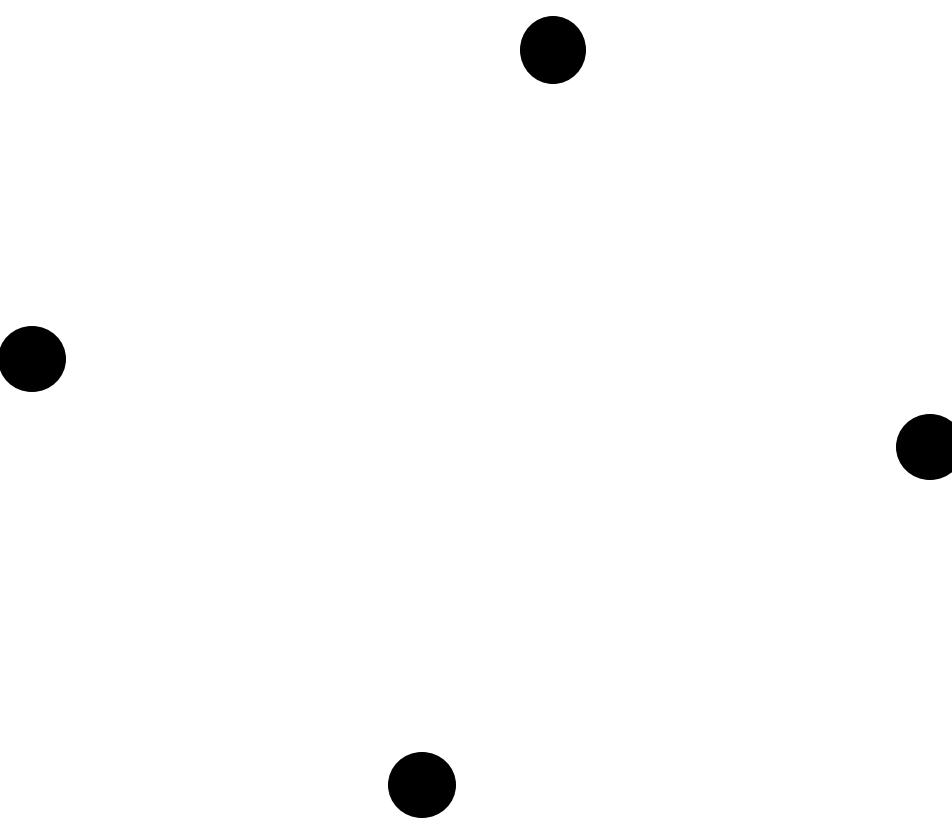
# Approximation Performance of Anaconda

## Theorem 1 (Fully Disconnected Networks)

- If the emergent network is *fully decentralized* with  $\mathcal{N}_{i,t} \equiv \emptyset$ , when  $t \geq \tilde{O}(|\mathcal{N}|^2 / \epsilon^2)$ ,

$$\mathbb{E} [f(\mathcal{A}_t)] \geq (1 - \kappa_f) f(\mathcal{A}^{\text{OPT}}) - \epsilon$$

Match the bound [Sviridenko et al. '17]



No agent can hear others so all actions are myopically selected

# Approximation Performance of Anaconda

## Theorem 1 (Arbitrary Decentralized Network)

- If the emergent network is *anything in between*, when  $t \geq \tilde{O}(|\bar{\mathcal{M}}| |\mathcal{N}|^2 \bar{\alpha}^2 / \epsilon^2)$ ,

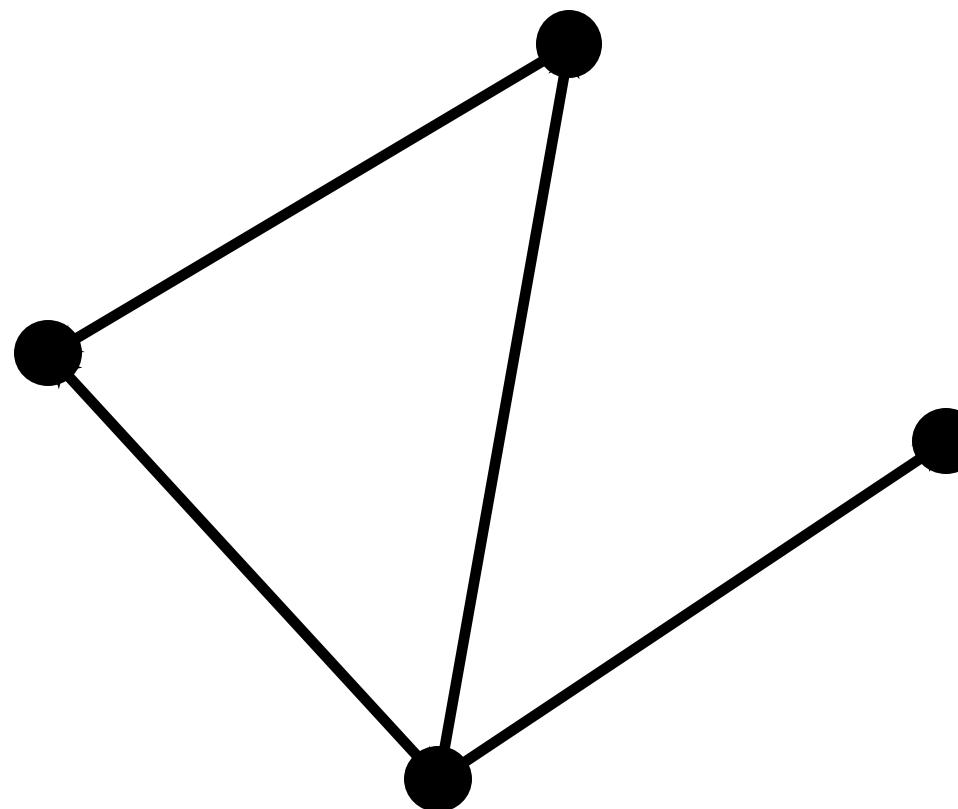
$$\mathbb{E}[f(\mathcal{A}_t)] \geq (1 - \kappa_f) f(\mathcal{A}^{\text{OPT}}) - \frac{1 - \kappa_f}{\kappa_f} (1 - e^{-\kappa_f}) \sum_{i \in \mathcal{N}} \mathbb{E}[C_{i,t}(\mathcal{N}_i^*)] - \epsilon$$

$$\geq \beta f(\mathcal{A}^{\text{OPT}}), \quad \beta \in \left[1 - \kappa_f, \frac{1 - \kappa_f}{1 - \kappa_f(1 - e^{-\kappa_f})}\right]$$

$\beta$  increases with larger  $\alpha_i$  and “better”  $\mathcal{M}_i$  [Xu and Tzoumas, arXiv '24]

$$|\bar{\mathcal{M}}| = \max_{i \in \mathcal{N}} |\mathcal{M}_i|$$

$$\bar{\alpha} = \max_{i \in \mathcal{N}} \alpha_i$$



Network that is not fully connected emerges after agents select neighbors individually

# Decision Speed of Anaconda

## Theorem 2

Anaconda terminates in  $O \left[ (\tau_f \overline{|\mathcal{V}_i| + \alpha_i} + \tau_c) |\bar{\mathcal{M}}| |\mathcal{N}|^2 / \epsilon \right]$  time.

function evaluation delay  $\tau_f$

$$\overline{|\mathcal{V}_i| + \alpha_i} = \max_{i \in \mathcal{N}} (|\mathcal{V}_i| + \alpha_i)$$

communication delay  $\tau_c$

## Theorem 2

Anaconda terminates in  $O \left[ (\tau_f \overline{|\mathcal{V}_i| + \alpha_i} + \tau_c) |\bar{\mathcal{M}}| |\mathcal{N}|^2 / \epsilon \right]$  time.

Comparison to SOTA decision time: In sparse directed networks where  $|\mathcal{M}_i| = o(|\mathcal{N}|)$ ,

- Anaconda:  $O(|\mathcal{N}|^2 / \epsilon)$
- Sequential Greedy with depth-first search:<sup>1</sup>  $O(|\mathcal{N}|^3)$

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<sup>1</sup>Konda, Grimsman, Marden, ACC '22

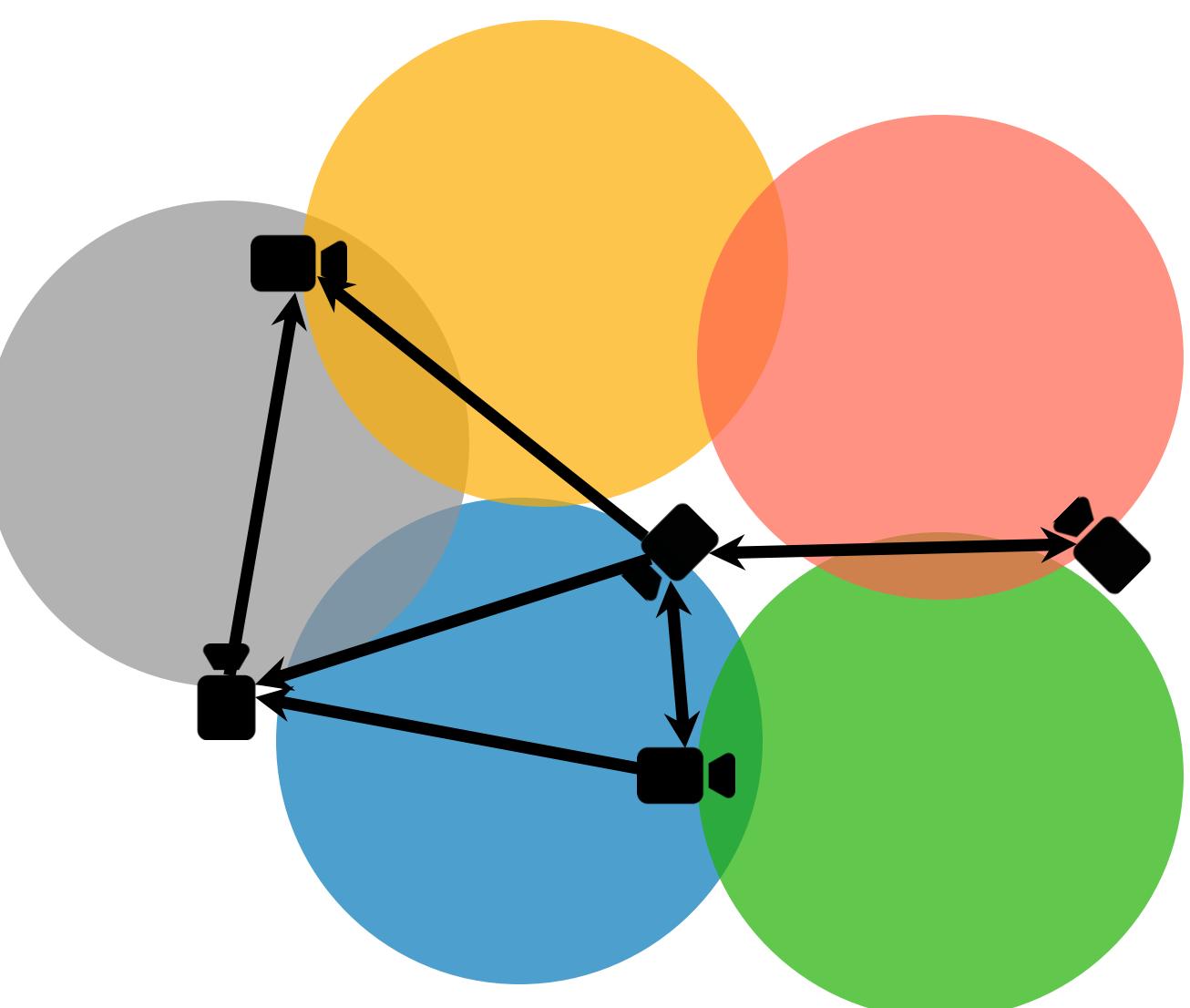
# Simulation: Area Monitoring with Multiple Cameras

**Agents:** 60 downward-facing cameras  $\mathcal{N}$  randomly located above a  $100 \times 100$  static environment, each with a circular FOV of radius 7

**Actions:** Directions  $\mathcal{V}_i$  to place FOV ( $i$  has no knowledge of  $\mathcal{V}_j$ ,  $\forall j \neq i$ )

**Communication:** Each camera  $i \in \mathcal{N}$  can receive images of FOV from other cameras within its communication range, denoted as  $\mathcal{M}_i$

**Objective function:** Area  $f(\{a_{i,t}\}_{i \in \mathcal{N}})$  covered by all cameras' FOV's, each in direction  $a_{i,t} \in \mathcal{V}_i$ ,  $\forall t$



# Simulation: Area Monitoring with Multiple Cameras

## Compared algorithms:

- a. **Anaconda with different uniform bandwidth budgets:**  $|\mathcal{N}_{i,t}| \leq \alpha_i$ ,  $\alpha_i = \alpha \in \{0, 1, 3, 5\}$ ,  $\forall i \in \mathcal{N}$
- b. **DFS-SG** [Konda et al., ACC '22]:  $\mathcal{N}_{i,t} \equiv \mathcal{M}_i$ 
  - Sequential Greedy [Fisher et al., Math Prog Studies '78] on a **pre-defined connected network**
  - Agents' action-selection order determined by depth-first search (DFS)

# Simulation: Area Monitoring with Multiple Cameras

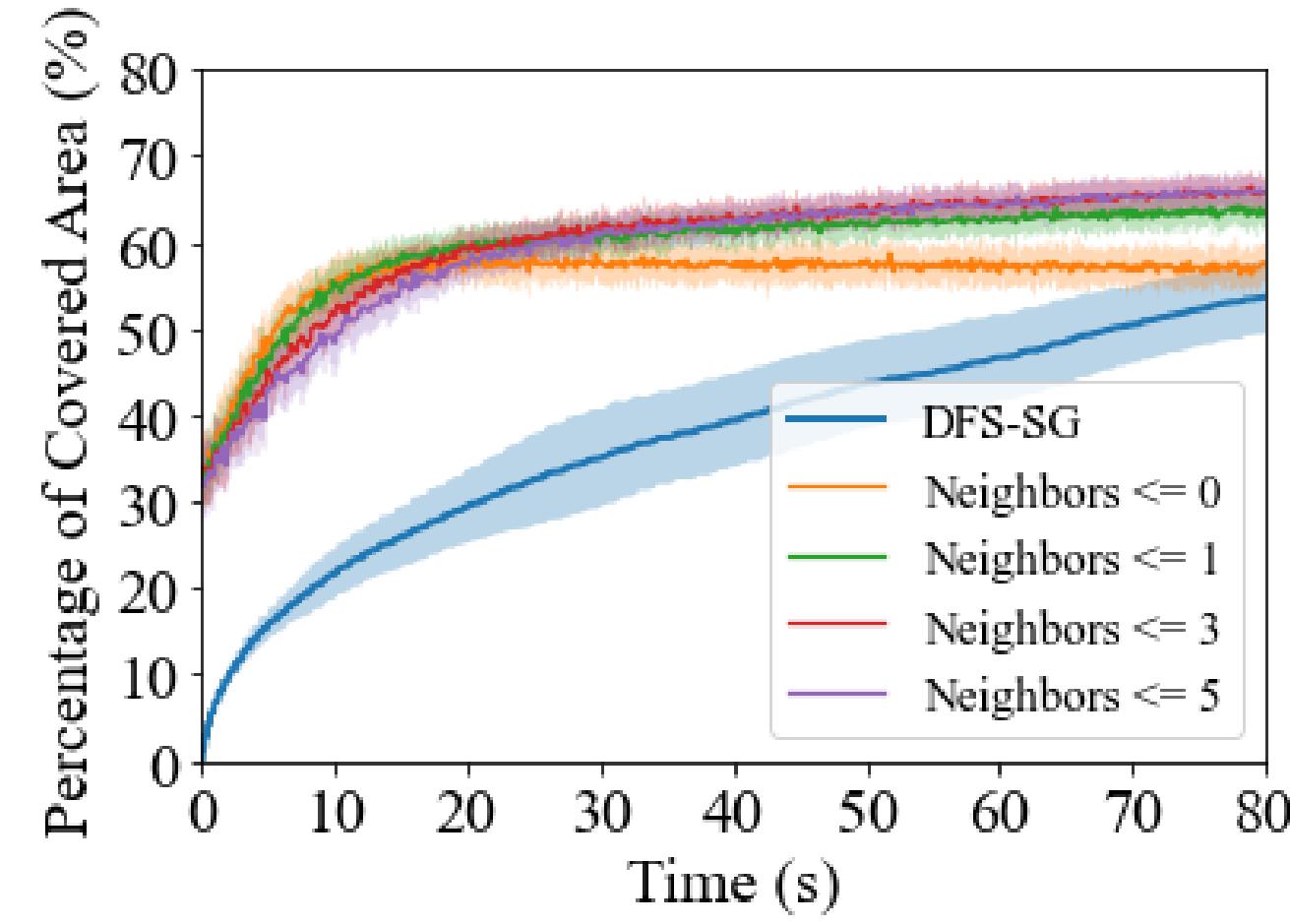
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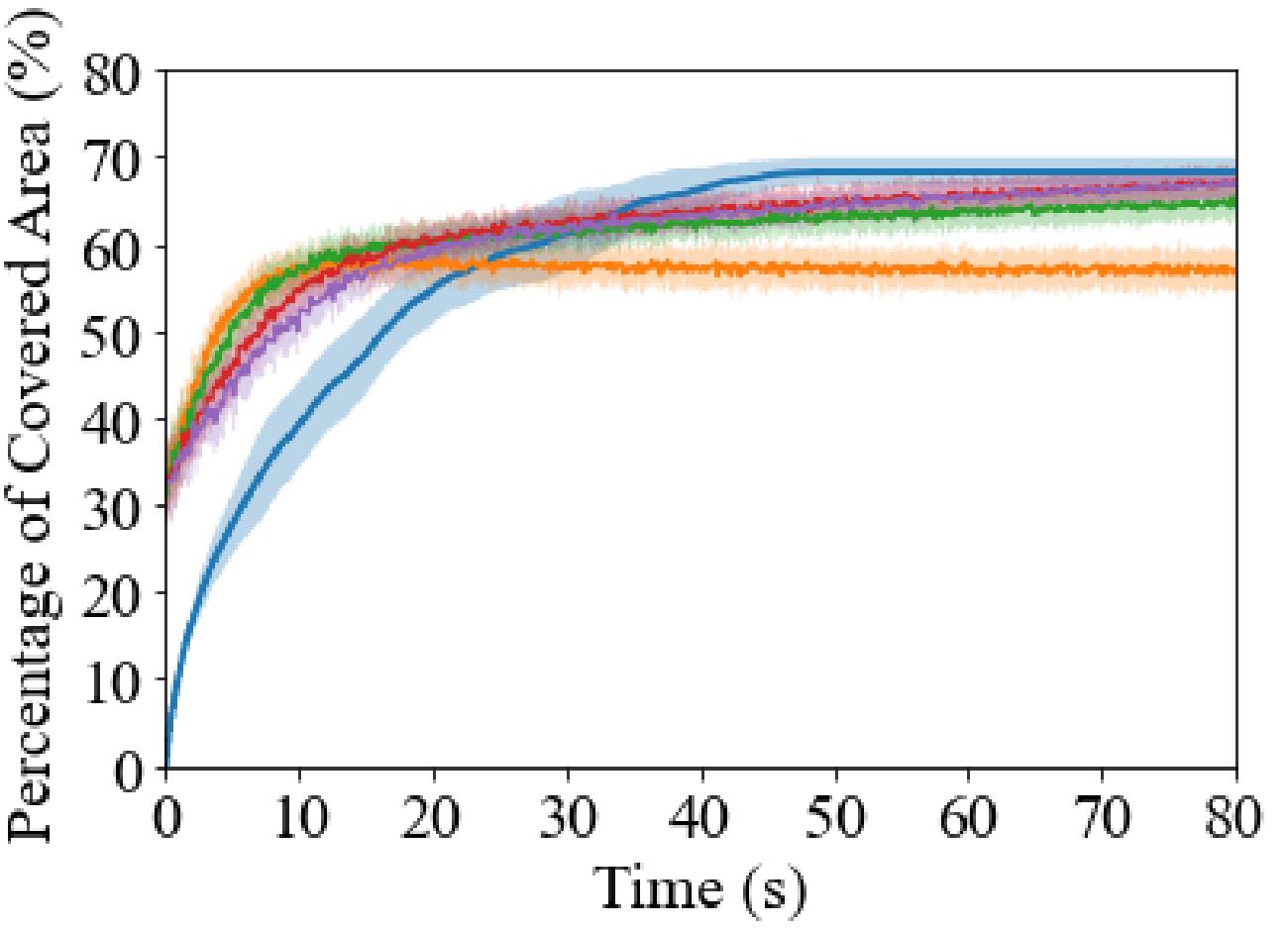
## Monte-Carlo simulation setup:

- a. **Scenarios with different computation & communication loads:** 30 trials  $\times$  5 algorithms  $\times$  3 combinations of **function evaluation delay  $\tau_f$  & communication delay  $\tau_c$**
- b. **Network connectivity:** for all trials, sample communication range  $c_i \in [15, 20]$ ,  $\forall i \in \mathcal{N}$  for network connectivity (required by DFS-SG only)

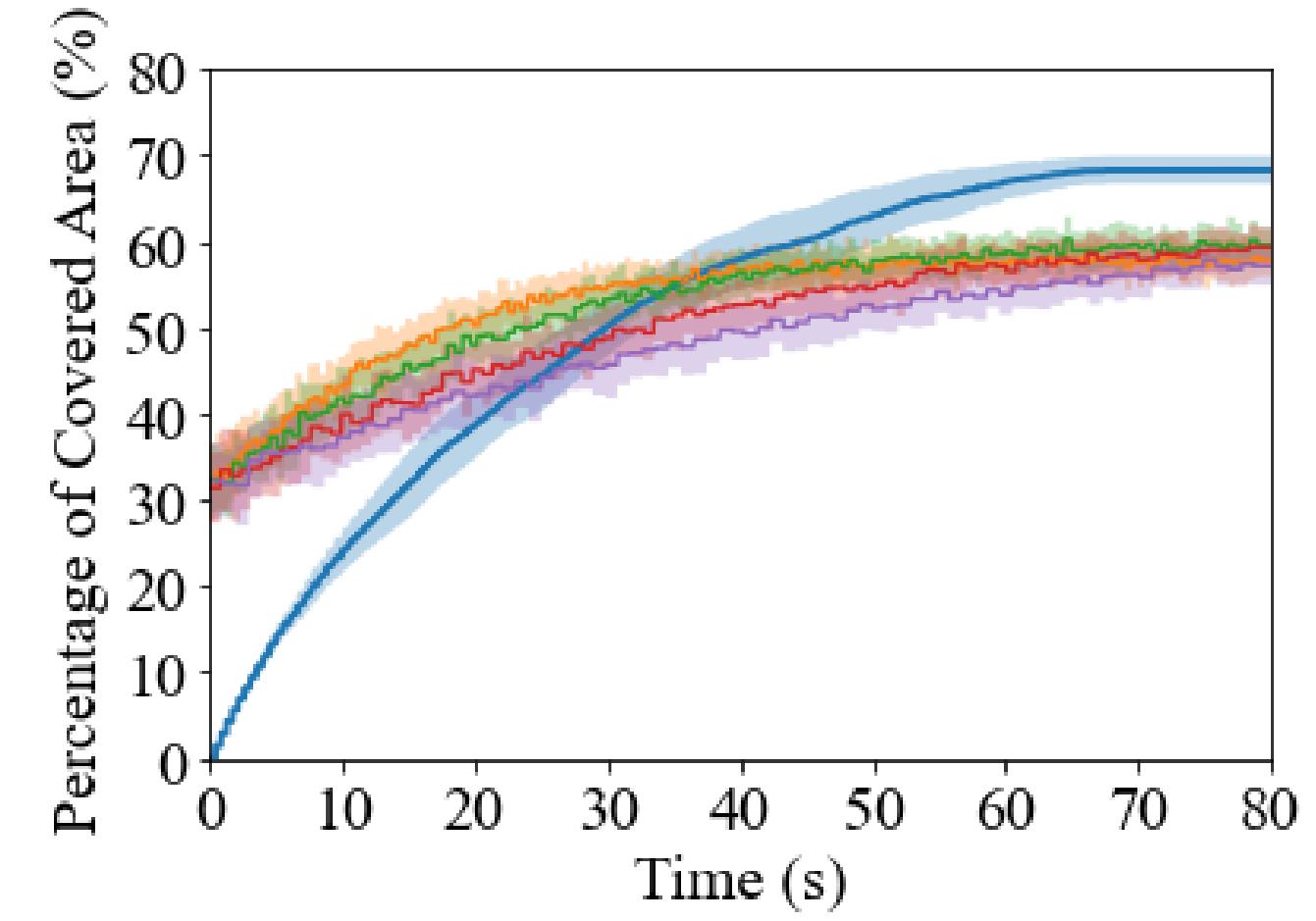
# Simulation Results



$$\tau_f = 0.01s, \tau_c = 0.05s$$



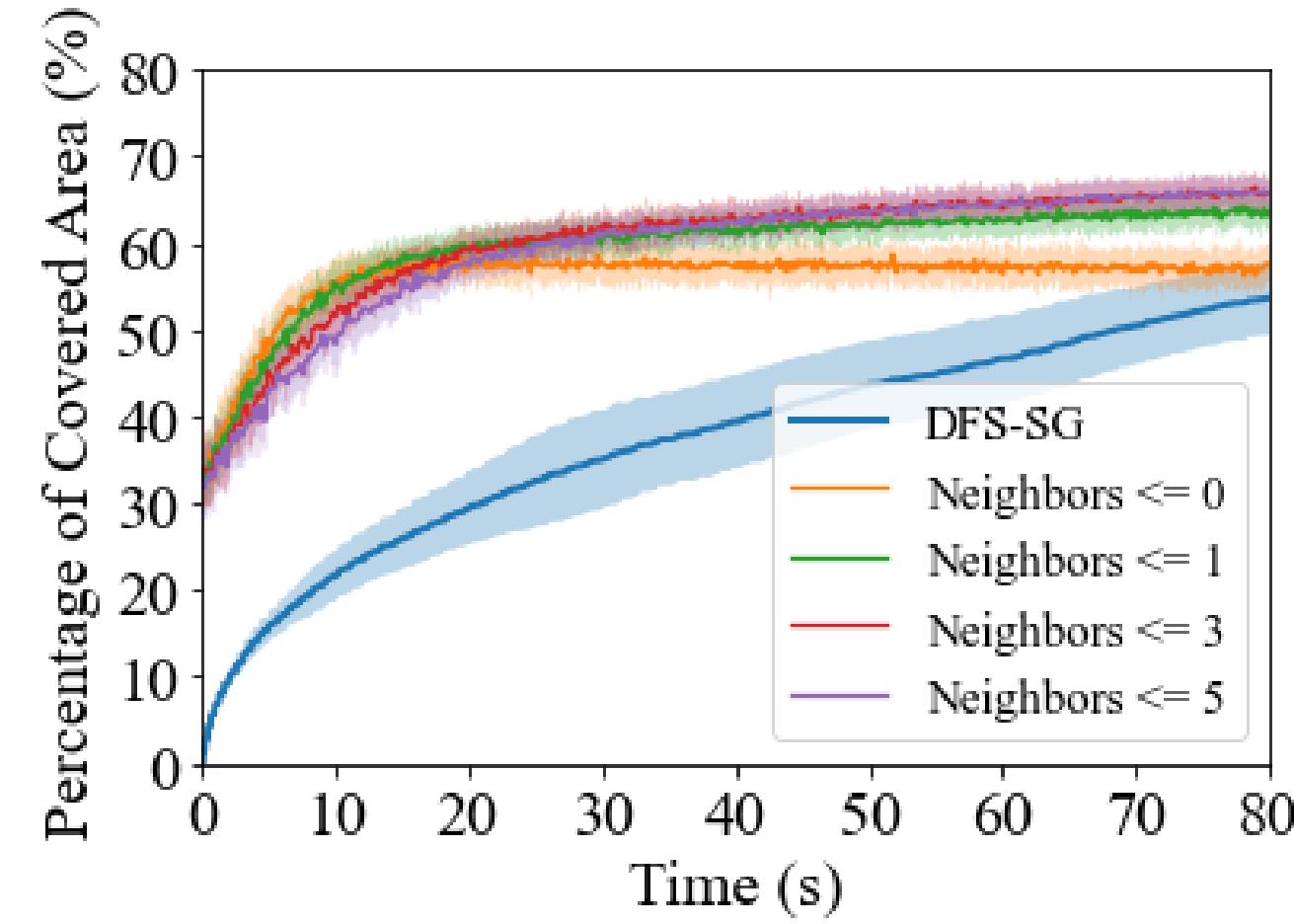
$$\tau_f = 0.01s, \tau_c = 0.01s$$



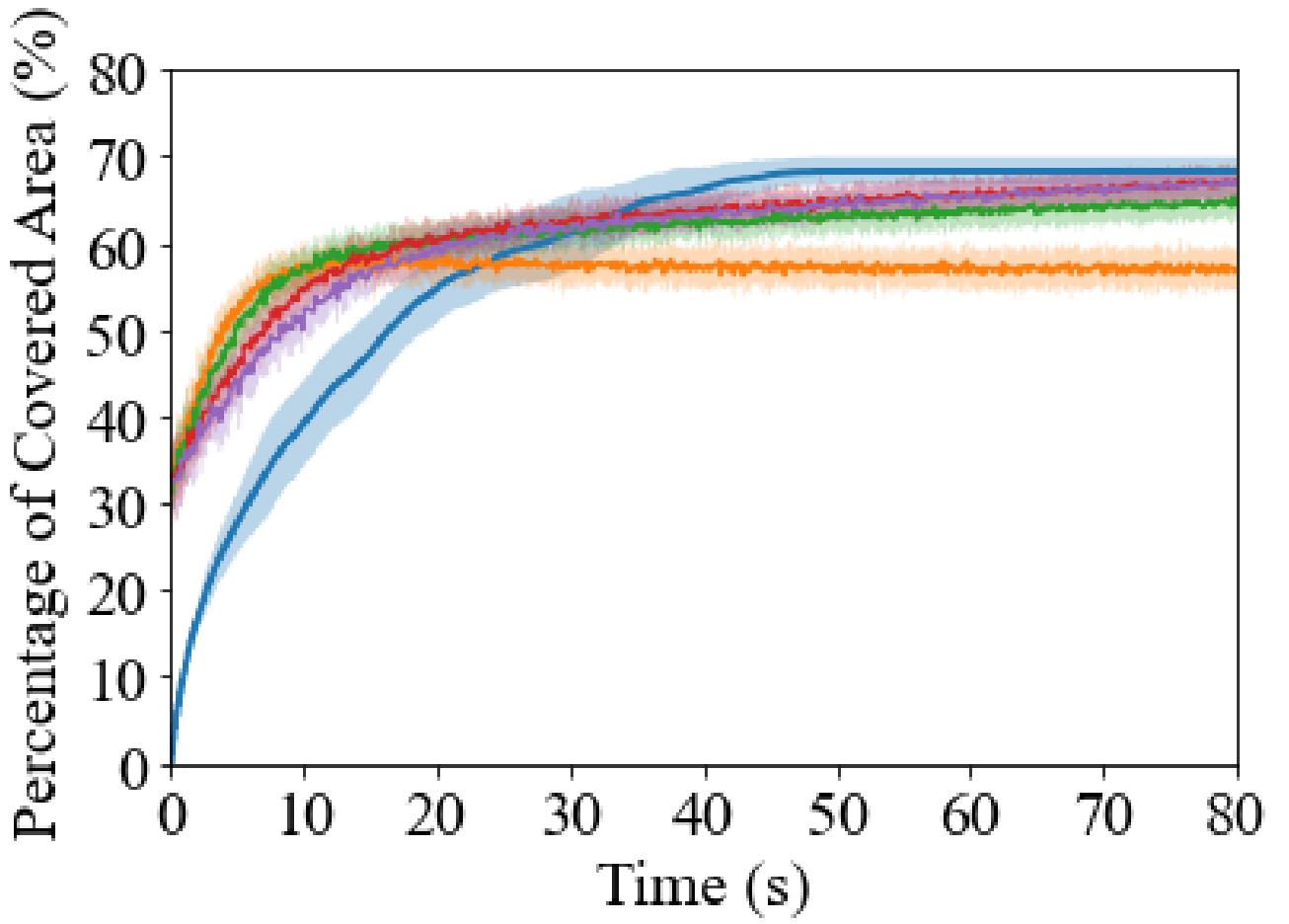
$$\tau_f = 0.05s, \tau_c = 0.01s$$

Anaconda with different uniform bandwidth budgets: As bandwidth budget increases, Anaconda's coverage performance increases (Theorem 1) and convergence speed decreases (Proposition 2)

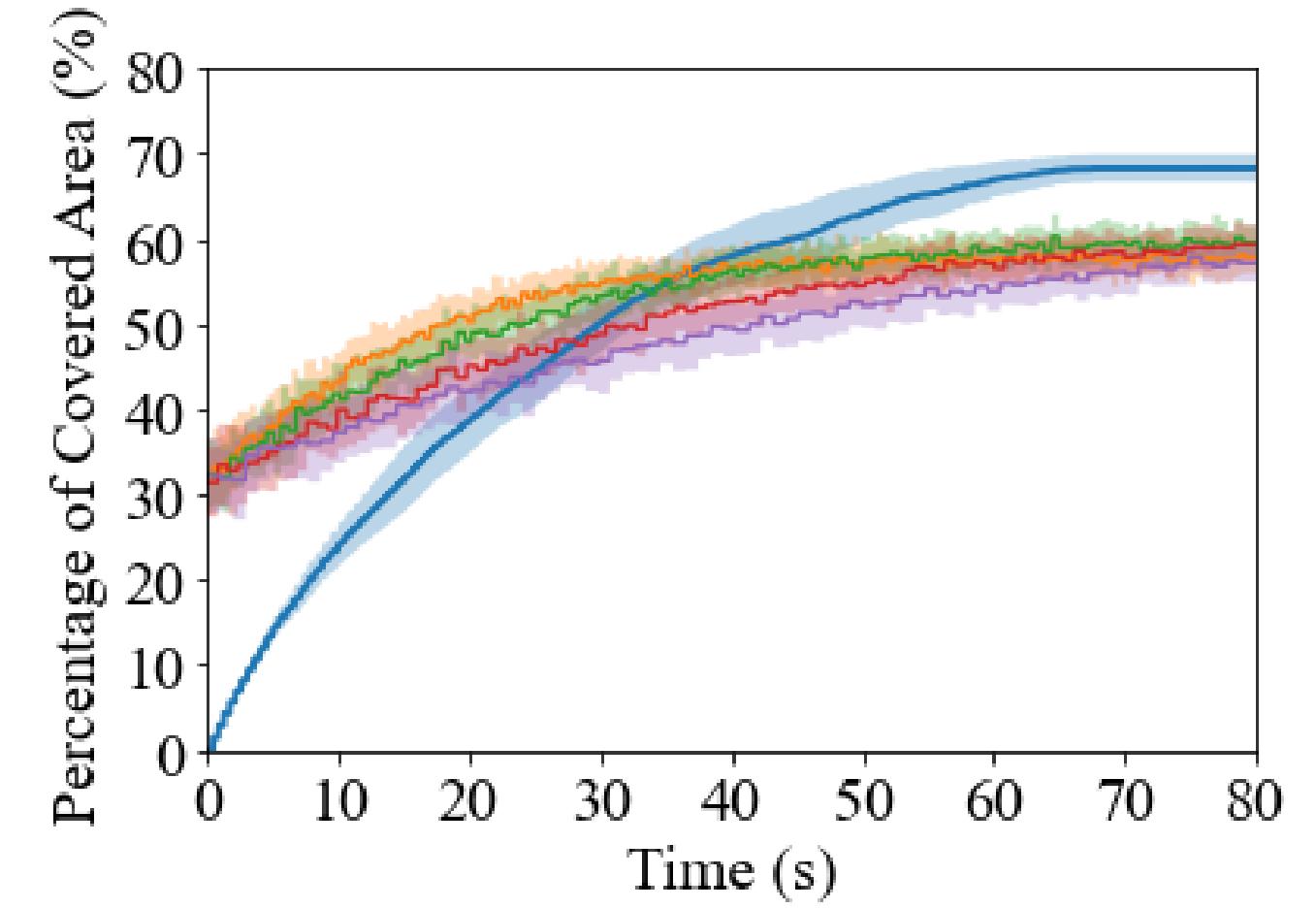
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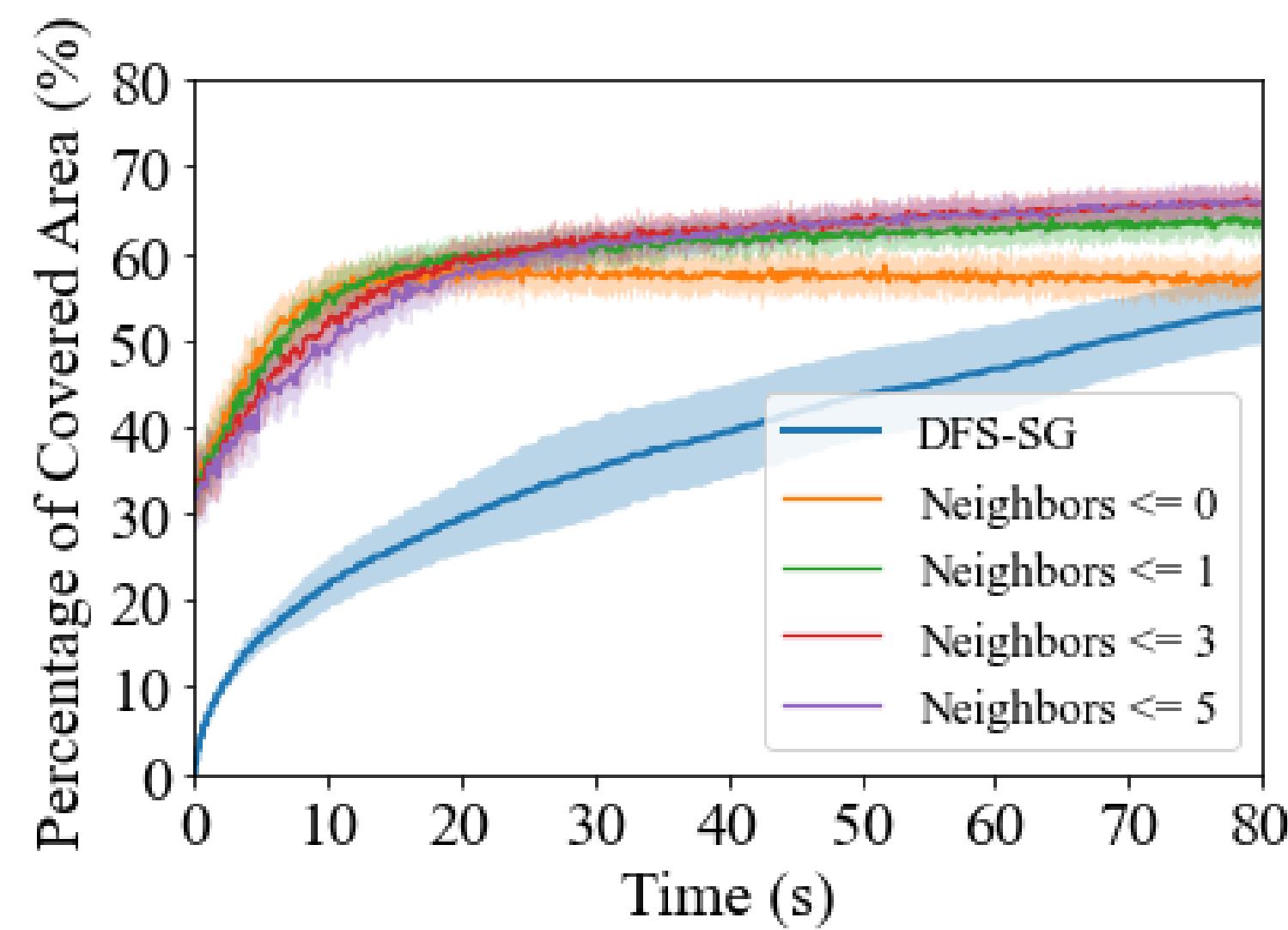
Anaconda with different uniform bandwidth budgets: As bandwidth budget increases, Anaconda's coverage performance increases (Theorem 1) and convergence speed decreases (Proposition 2)

**Comparison with DFS-SG:** achieve comparable coverage performance after convergence

- **Anaconda starts with higher coverage performance:** cameras all select FOV's from the start per Anaconda but sequentially per DFS-SG
- **Anaconda's coverage performance converges faster when communication delay  $\tau_c$  is high:** e.g., when  $\tau_c > \tau_f$

# Summary and Extensions

- Introduce  $\mathcal{N}_i \subseteq \mathcal{M}_i, |\mathcal{N}_i| \leq \alpha_i, \forall i \in \mathcal{N}$   $a_i \in \mathcal{V}_i, \forall i \in \mathcal{N}$   $f(\{a_i\}_{i \in \mathcal{N}})$
- Provide distributed alternating optimization algorithm
  - Neighborhood self-configuration
  - Quantify impact of network configuration to suboptimality
  - Scalability vs. optimality trade-off
- One-order faster than SOTA when accounting for inter-agent communication delay



## Extensions:

- Unknown a priori  $f$
- Linear decision time
- Tight approximation bound