06-32167 & 06-32212 Neural Computation (Extended)

CW1: Backpropagation and Softmax

Released: November 2nd, 2020
Deadline: November 20th at 5pm, 2020.
Weight: 25 %

Figure 1: Visualisation of training examples from the Fashion MNIST dataset and examples of predicted classes (with probabilities in parentheses). Incorrect classifications are highlighted in red.

> You should complete this coursework in groups as specified on Canvas. To account for different contributions between group members, we will ask for a peer evaluation on Canvas at the end of the coursework. The individual marks will be adjusted in the interval [–5%,5%] (out of a total mark of 25%) based on this evaluation. Discrepancies and anomalies in the peer evaluation will be considered on a case-by-case basis.
>
> Each group should make one submission consisting of one tarball `cw1-2020-files.tar.gz`. The contents of the tarball should be one directory `cw1-2020-files` containing your completion of the provided Python files (see below) and a report in PDF file format with the file name `report.pdf`. To facilitate automated assessment, **you should not change** the function names or function signatures in the provided file `backprop-softmax.py`.
>
> Each student should also submit a separate Canvas assignment with a description of their own contribution to the project (no more than 100 words), and the peer evaluation. This assignment is not marked, but will be used to determine peer marking for the coursework.

The goal of this coursework is to implement and train a feedforward neural network with a softmax layer to classify pictures of fashion items from the Fashion MNIST data set[1]. The dataset consists

[1] `https://github.com/zalandoresearch/fashion-mnist`

of a training set of 60,000 examples and a test set of 10,000 examples. Each example is a 28x28 grayscale image, associated with one of 10 labels (t-shirt, trouser, etc.), as shown in Figure 1.

To train the network, we will implement the backpropagation algorithm and mini-batch gradient descent. The pseudo-code of the algorithms are provided in the lectures notes on Backpropagation and Softmax. A skeleton implementation together with some utility functions for reading and plotting data are provided as a tarball file `cw1-2020-files.tar.gz` on Canvas. The tarball also contains the dataset, hence you do not need to download this separately.

To complete the coursework, you should use the Numpy and Matplolib in Python 3. For coursework 1, you should not use any neural network library which provides autograd functionality, such as PyTorch or TensorFlow.

Task 1 (1 %). *Extend the code by implementing the* `softmax` *function which computes a probability vector* $(p_1, \ldots, p_{10})$ *from the 10 activation units* $z_1^L, \ldots, z_{10}^L$ *in the softmax layer.*

Input argument:

- *a vector* $\mathtt{z} = (z_1^L, \ldots, z_{10}^L)$ *corresponding to the z units in the softmax layer*

Output:

- *the probability vector* $(p_1, \ldots, p_{10})$ *in the output layer*

Example:

In the following example, we first instantiate an object of the BackPropagation class (which causes the dataset to be loaded), and then call the softmax function with a simple test vector.

```
>>> bp = BackPropagation()
Extracting fashion-mnist/data/fashion/train-images-idx3-ubyte.gz
Extracting fashion-mnist/data/fashion/train-labels-idx1-ubyte.gz
Extracting fashion-mnist/data/fashion/t10k-images-idx3-ubyte.gz
Extracting fashion-mnist/data/fashion/t10k-labels-idx1-ubyte.gz
>>> z = np.array([0,1,2,3,4,5,6,7,8,9])
>>> bp.softmax(z)
array([7.80134161e-05, 2.12062451e-04, 5.76445508e-04, 1.56694135e-03,
       4.25938820e-03, 1.15782175e-02, 3.14728583e-02, 8.55520989e-02,
       2.32554716e-01, 6.32149258e-01])
```

Task 2 (3 %). *Extend the code by implementing the* `forward` *function which sets the activation units in the input layer to an input image x, then feeds forward the data through the network, and finally returns the probability vector corresponding to the 10 probability units in the output layer.*

Input arguments:

- `x` $\in [-1, 1]^{784}$ *the image as a vector of 784 elements*

Output:

- *the probability vector $(p_1, \dots, p_{10})$ in the output layer*

Example:

Here, we call the `forward` *function with the first image in the training set as input. Depending on the current weights and biases in your network, you may see a different output than the probability vector below.*

```
>>> bp.forward(bp.trainX[0])
array([0.10425727, 0.09743926, 0.0998025 , 0.09768295, 0.1084072 ,
       0.09732901, 0.08471379, 0.09933064, 0.09877855, 0.11225882])
```

Task 3 (1 %). *We used the maximum likelihood principle to derive a loss function*

$$C^{(i)} := -\log p_{y^{(i)}},$$

where $p_j \in [0, 1]$ is the network's predicted probability of class j, and $y^{(i)} \in \{1, \dots, 10\}$ is the correct class for the i-th training item. Implement the softmax loss function called `loss` *in the provided file* `backprop-softmax.py`.

Input argument:

- *a vector* `pred`$= (p_1, \dots, p_m)$ *corresponding to the* 10 *probability units in the output layer*
- `y` $\in \{0, 1\}^{10}$ *a "one-hot encoding" of the correct class of the image. E.g., if the first class is correct, vector* `y` *is (1,0,0,0,0,0,0,0,0,0). Hint: Use* `np.argmax(y)` *to obtain the class index from the one-hot encoded vector.*

Output:

- *the loss value*

Example:

We forward data from the first image, then compute the loss. The loss values may depend on the state of your network.

```
>>> bp.forward(bp.trainX[0])
array([0.09371258, 0.10556965, 0.09432195, 0.10503915, 0.12660278,
       0.10768817, 0.08212865, 0.10272175, 0.07511968, 0.10709562])
>>> bp.loss(bp.a[bp.L-1], bp.trainY[0])
2.0667007822344643
```

Task 4 (5 %)**.** *Extend the code by implementing the* `backward` *function which first computes the local gradients* `self.delta[l]` *for each layer in the network, then evaluates the partial derivatives of the cost relative to the weights* `self.dw[l]` *and the biases* `self.db[l]` *for each layer in the network.*

Before the backward function is called, you can assume that the `forward` *function has been called with an appropriate input vector x.*

Input arguments:

- `x` $\in [-1, 1]^{784}$ *the image as a vector of 784 elements*
- `y` $\in \{0, 1\}^{10}$ *a "one-hot encoding" of the correct class of the image. E.g., if the first class is correct, vector* `y` *is (1,0,0,0,0,0,0,0,0,0).*

Output:

- *the "function" produces no output*

Example:

Here, we first inspect the partial derivatives in a given layer, e.g. layer 1, to check that they have been initialised to 0. We then call the the `forward` *function with the first image in training set as input, and then the* `backward` *function with the corresponding label of the corresponding training example. Finally, we notice that the partial derivatives have been updated by the* `backward` *function.*

```
>>> bp.dw[1]
array([[0., 0., 0., ..., 0., 0., 0.],
       [0., 0., 0., ..., 0., 0., 0.],
       [0., 0., 0., ..., 0., 0., 0.],
       ...,
       [0., 0., 0., ..., 0., 0., 0.],
       [0., 0., 0., ..., 0., 0., 0.],
       [0., 0., 0., ..., 0., 0., 0.]])
>>> bp.forward(bp.trainX[0])
array([0.0967909 , 0.07826556, 0.11721793, 0.10145362, 0.09939592,
       0.10454699, 0.11850906, 0.09381184, 0.09803316, 0.09197501])
>>> bp.backward(bp.trainX[0],bp.trainY[0])
>>> bp.dw[1]
array([[ 0.        ,  0.        ,  0.        , ...,  0.        ,
```

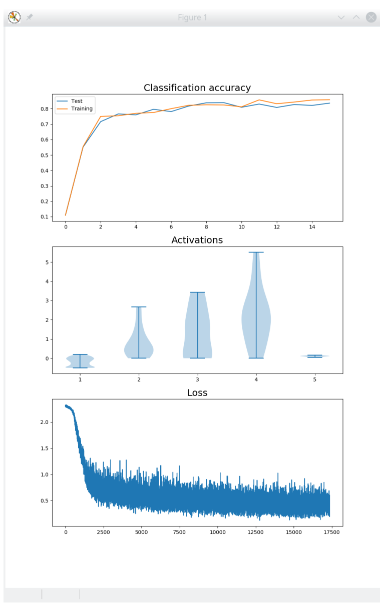
```
          0.        ,  0.        ],
        [-0.00243765, -0.00243765, -0.00243765, ..., -0.00243765,
         -0.00243765, -0.00243765],
        [-0.02531953, -0.02531953, -0.02531953, ..., -0.02531953,
         -0.02531953, -0.02531953],
        ...,
        [ 0.        ,  0.        ,  0.        , ...,  0.        ,
        [ 0.        ,  0.        ,  0.        , ...,  0.        ,
          0.        ,  0.        ]])
```

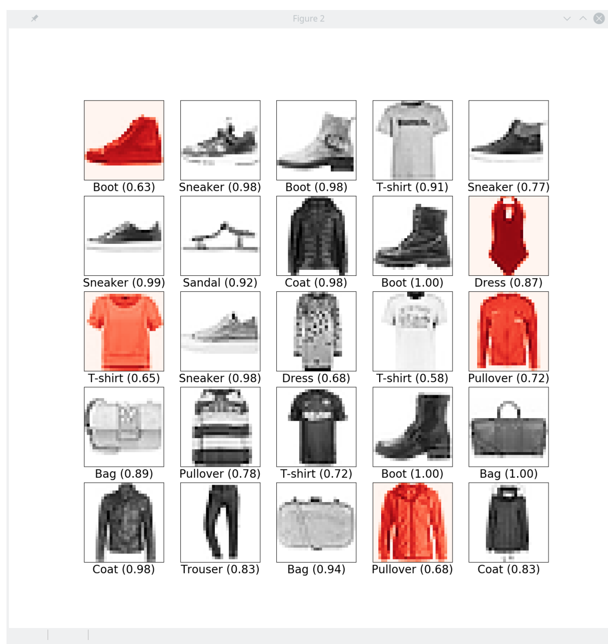
Task 5 (5 %). *Complete the code in* `backprop-softmax.py` *by implementing the places marked* `TODO`.

Example:

We launch the code from the command line

```
[pkl@phi backprop]£ python3 backprop-softmax.py
Extracting fashion-mnist/data/fashion/train-images-idx3-ubyte.gz
Extracting fashion-mnist/data/fashion/train-labels-idx1-ubyte.gz
Extracting fashion-mnist/data/fashion/t10k-images-idx3-ubyte.gz
Extracting fashion-mnist/data/fashion/t10k-labels-idx1-ubyte.gz
```





Task 6 (10 %). *Design a set of experiments to investigate how the following hyper-parameters affect classification accuracy: training time, learning rate, minibatch size, network topology, and activation function.*

What is the best classification accuracy you can achieve on the test data? Summarise your findings in a report of at most 5 pages (A4 page, 11pt font). The report should clearly state the experiments you have carried out, including the method, the results, and your interpretation of the results.