GuP: Fast Subgraph Matching by Guard-based Pruning

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My Intuitive Understanding of Guards

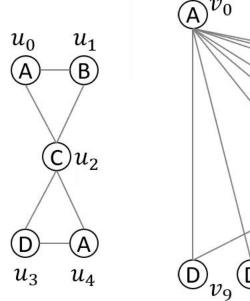
- R(u,v) stores data vertices of which at least one must be "reserved" in order for a full-embedding to be possible. If all vertices of R(u,v) are assigned before (u,v), then we need not continue extending M.
- NV(u,v) stores assignments (u',v') that guarantee a deadend, so GuP can avoid fruitless searches earlier on. We want as small a nogood guard as possible, so more fruitless partial embeddings can be detected when they are still small.

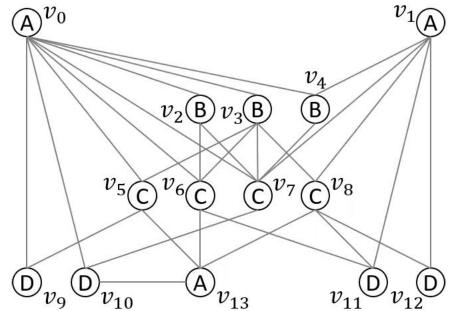
Candidate filtering



Extract data vertices that can be destinations of a specific query vertex

• Compare local structures (e.g., label, degree, adjacent labels)





• $C(u_0) = \{v_0, v_1\}$

•
$$C(u_1) = \{v_2, v_3, v_4\}$$

•
$$C(u_2) = \{v_5, v_6, v_7, v_8\}$$

•
$$C(u_3) = \{v_9, v_{10}, v_{11}, v_{12}\}$$

•
$$C(u_4) = \{v_0, v_1, v_{13}\}$$

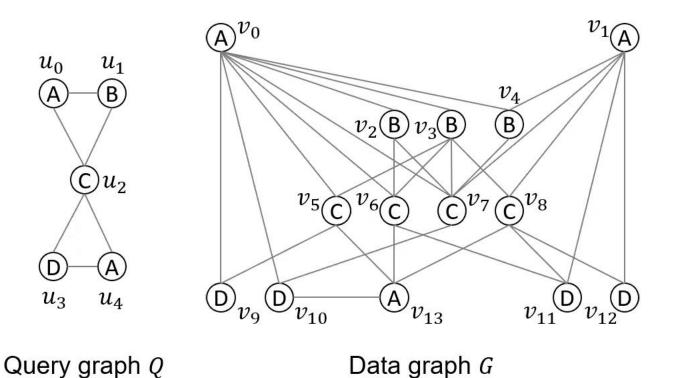
Query graph Q

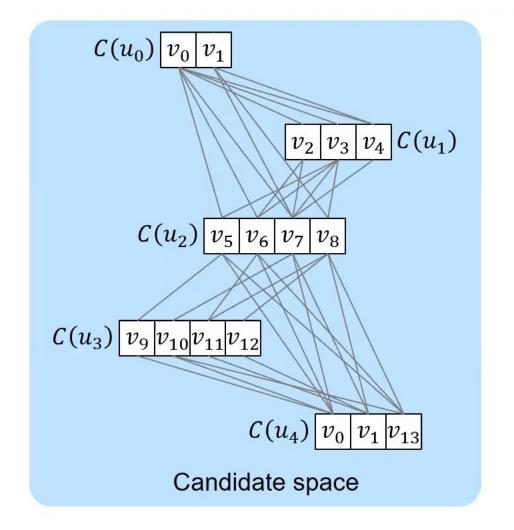
Data graph G

Candidate space [Han+ SIGMOD'19]



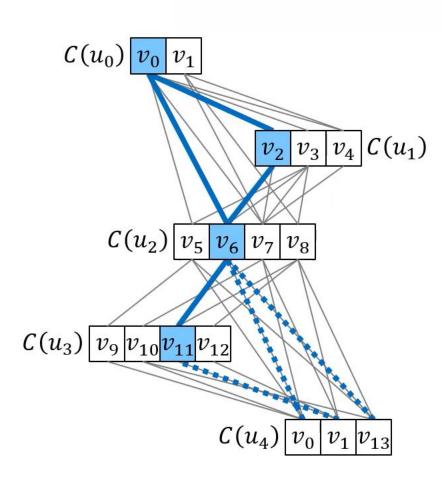
Candidate vertices + edges between them





Backtracking search





Recursively extend **partial embedding** *M* until it becomes a **full embedding**

$$M = \emptyset$$

$$M = \{(u_0, v_0)\}$$

$$M = \{(u_0, v_0), (u_1, v_2)\}$$

$$M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6)\}$$

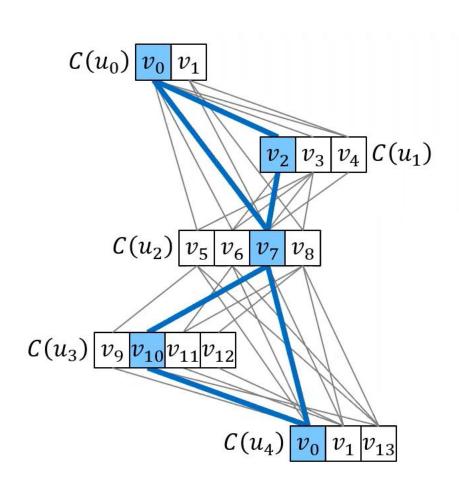
$$M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6), (u_3, v_{11})\}$$

$$M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6), (u_3, v_{11}), (u_4, _)\} \times$$

- v_6 and v_{11} lack a common neighbor for u_4
- "Deadend"

Backtracking search



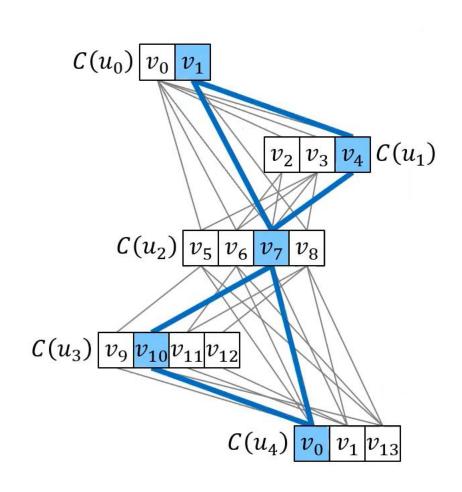


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M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6), (u_3, v_{11}), (u_4, \_)\} \times M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6)\} 
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• Non-injective mapping (using v_0 twice)

Backtracking search



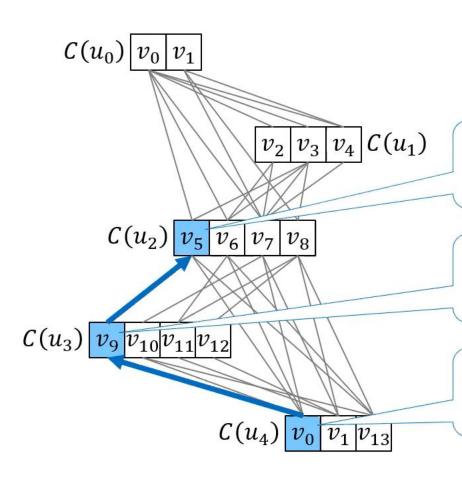


$$M = \{(u_0, v_0), (u_1, v_2), (u_2, v_7), (u_3, v_{10}), (u_4, v_0)\} \times \dots$$
 $M = \{(u_0, v_0)\}$ No full embedding found $M = \emptyset$ $M = \{(u_0, v_0)\}$ is a deadend $M = \{(u_0, v_1)\}$...
 $M = \{(u_0, v_1), (u_1, v_4), (u_2, v_7), (u_3, v_{10}), (u_4, v_0)\} \checkmark$

All the recursions from $M = \{(u_0, v_0)\}$ were fruitless

Reservation guard: basic idea





Propagate the injectivity constraint to earlier backtracking steps

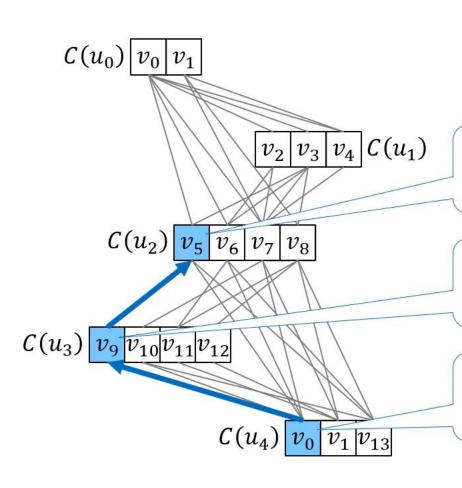
Inconsistent if v_0 is used in M because v_9 is the only adjacent candidate for u_3

Inconsistent if v_0 is used in M because v_0 is the only adjacent candidate for u_4

Inconsistent if v_0 is used in M because of the injectivity constraint

Reservation guard: formalization





Reservation guard $R(u_i, v) \subseteq V_G$

Filter out v if all the vertices in $R(u_i, v)$ is used in M

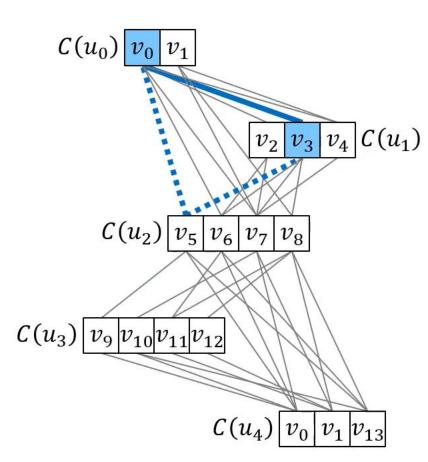
$$R(u_2, v_5) = R(u_3, v_9) = \{v_0\}$$

$$R(u_3, v_9) = R(u_4, v_0) = \{v_0\}$$

$$R(u_4, v_0) = \{v_0\}$$

Reservation guard: running example

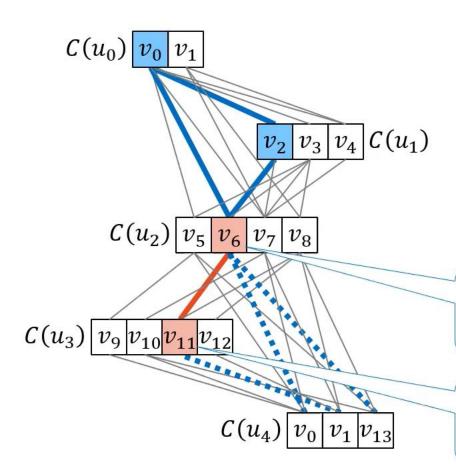




Filtered out because $R(u_2, v_5) = \{v_0\}$ and v_0 is already used for u_0 $M = \{(u_0, v_0), (u_1, v_3)\}\$ $M = \{(u_0, v_1, v_3), (u_2, v_5)\}$ $M = \{(u_0, v_1, v_3), (u_2, v_5), (u_3, v_9)\}$ $M = \{(u_0, v_1), (u_1, v_2), (u_2, v_5), (u_3, v_9), (u_4, v_0)\} \times M = \{(u_0, v_1), (u_1, v_3), (u_2, v_5), (u_3, v_9)\}$ $M = \{(u_0, v_1, v_3), (u_2, v_5)\}$ $M = \{(u_0, \iota, (u_1, v_3))\}$ $M = \{(u_0, v_0), (u_1, v_3), (u_3, v_6)\}$

Nogood guard: basic idea





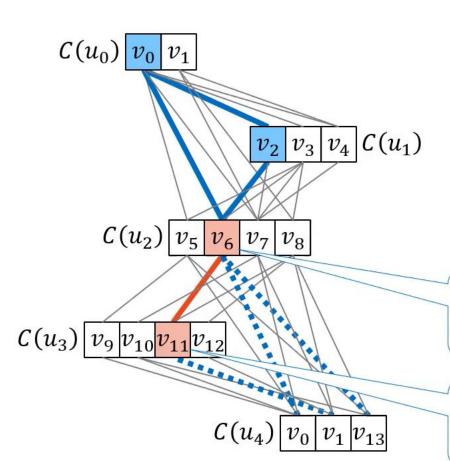
Learn inconsistent assignments from deadends encountered during backtracking

Inconsistent with arbitrary M because its only neighbor v_{11} is inconsistent with (u_2, v_6)

Inconsistent with M containing (u_2, v_6) because v_6 and v_{11} lack common neighbor for u_4

Nogood guard: formalization





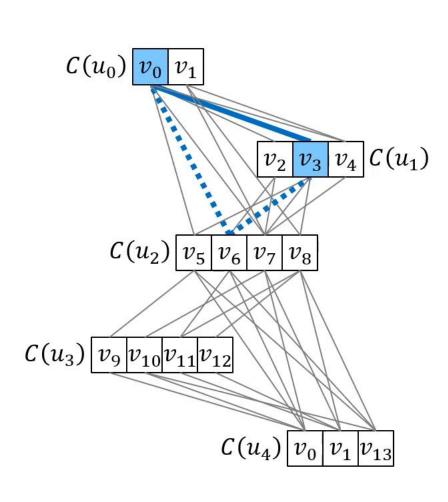
Nogood guard $NV(u_i, v) \subseteq V_Q \times V_G$ Filter out v for M s.t. $NV(u_i, v) \subseteq M$

$$NV(u_2, v_6) = \emptyset$$

$$NV(u_3, v_{11}) = \{(u_2, v_6)\}\$$

Nogood guard: running example





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NV(u_3, v_{11}) \leftarrow \{(u_2, v_6)\}\
                                                      NV(u_2, v_6) \leftarrow \emptyset
M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6), (u_3, v_{11})\}
M = \{(u_0, v_0), (u_1, v_2), (u_2, v_6), (u_3, v_{11}), (u_4, \_)\} \times
                                                      Filtered out because
M = \{(u_0, v_0), (u_1, v_3)\}
                                                          NV(u_2, v_6) \subseteq M
M = \{(u_0, v_0, v_1, v_3), (u_2, v_6)\}
M = \{(u_0, v_0 | v_1, v_3), (u_2, v_6), (u_3, v_{11})\}
M = \{(u_0, v_0) | v_1, v_2, v_3, (u_3, v_{11}), (u_4, v_4) \} \times M = \{(u_0, v_0) | v_1, v_3, (u_2, v_6), (u_3, v_{11}) \}
M = \{(u_0, v_0, u_1, v_3), (u_2, v_6)\}
M = \{(u_0, v_0, \lambda_1, v_3)\}
M = \{(u_0, v_0), (u_1, v_3), (u_2, v_7)\}\
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