Bayesian Multilevel Modeling of Intensive Longitudinal Data with brms

Please download the materials:
download zip from green *Code*-button under
https://github.com/zitamayer/sgpbayes

Part I:
Time-related trends:
Change over time and individual differences in change

Short break

Part II:
Bivariate within-person associations and individual differences in associations (exercise)

The data: 3-month longitudinal diary study

- 251 students (220 female, M_{age} = 23.14) who recently shelved a personal goal to focus on their academic goal

- Shelved goals: Mainly leisure goals
  - Sport hobby (35 %)
  - Leisure hobby (25 %)
  - Music hobby (17 %)

- Diaryphase: For 3 months, on every 3rd evening, questionnaire on phone ($N_{occasions}$ = 32)

First look at the data: Descriptives

Change over time and individual differences in change

Model convergence
Model interpretation
Model selection & fit
Prior setting
Hypothesis testing

Research question

To what extent does shelved goal importance vary within and between persons?

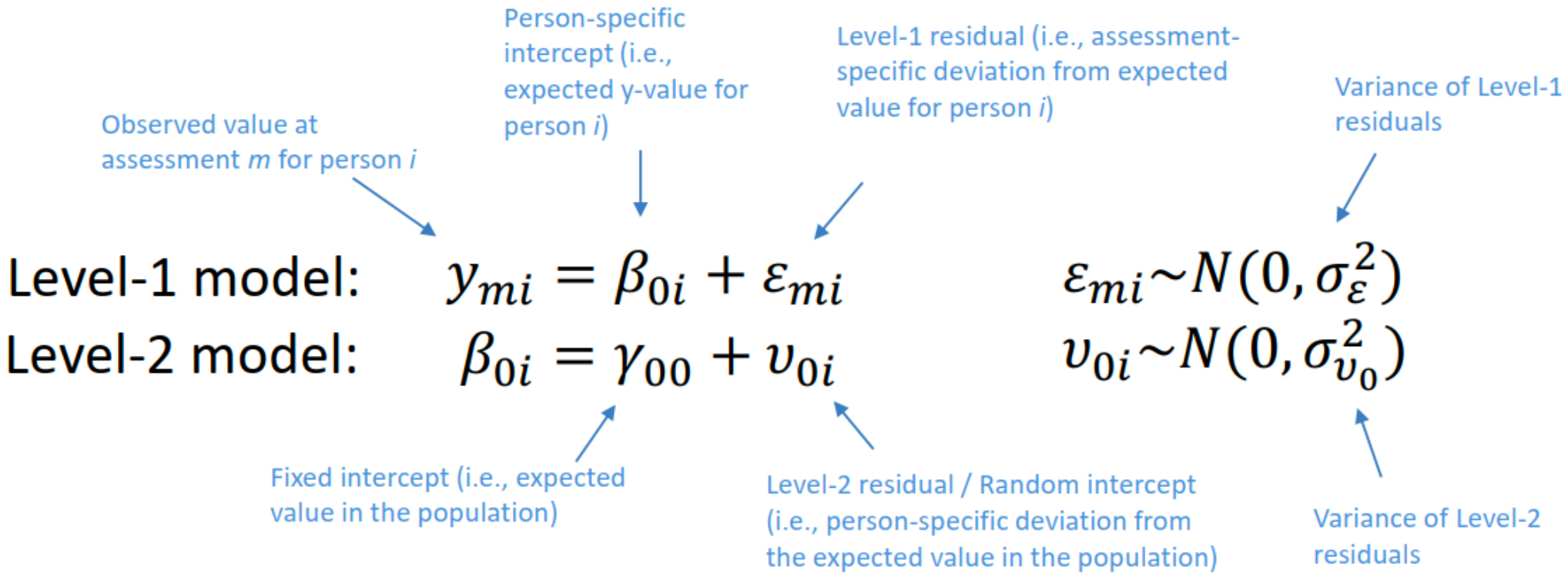
Momentary goal importance

Goal importance item (0 – 100): “Right now, how important is this goal to you?”

Random intercept-only ("unconditional means") model

- Stipulates no change over time
- Are there between-person differences in the intercept?
- Is used for separating variance of variable into level 1 (within-person) and level 2 (between-person) variance

Equation



Full model: $i \quad ni = \boxed{\gamma_{00}} + \boxed{\upsilon_{0i} + \varepsilon_{mi}}$

Fixed part (captures constants)

Random part (captures variation)

Intraclass correlation

- The intraclass correlation (ICC) is the ratio of the random intercept variance (between-person) to the total variance (between + within)

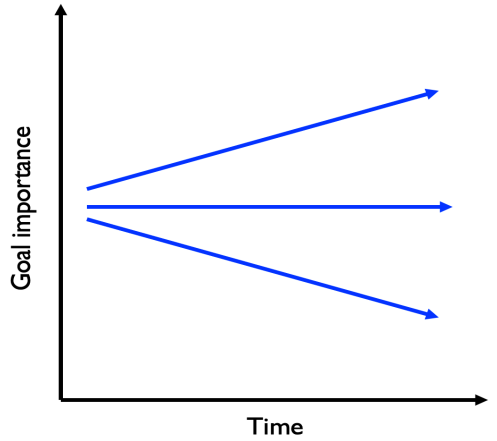
$$\rho = \frac{\sigma_{v_0}^2}{\sigma_Y^2} = \frac{\sigma_{v_0}^2}{\sigma_{v_0}^2 + \sigma_\varepsilon^2}$$

Research question

Does shelved goal importance change over time?

Does this change differ between persons?

Momentary goal importance



Linear growth model

Random intercept and fixed slope model

- Includes predictors (time at level 1)
- The fixed regression coefficient captures the effect of the predictor on the outcome
- The random intercept captures unexplained variation between persons in the outcome variable (at »baseline» t0)

Equation

Observed value at assessment *m* for person *i*

Person-specific intercept (i.e., expected y-value for person *i* given $x_{mi} = 0$)

Person-specific slope (capturing the effect of X for person *i*)

Level-1 residual

Level-1 model: $y_{mi} = \beta_{0i} + \beta_{1i} \cdot x_{mi} + \varepsilon_{mi}$

Level-2 model: $\beta_{0i} = \gamma_{00} + \upsilon_{0i}$

Random intercept

$\beta_{1i} = \gamma_{10}$

Fixed intercept

Fixed slope

Full model: $y_{mi} = \boxed{\gamma_{00} + \gamma_{10} \cdot x_{mi}} + \boxed{\upsilon_{0i} + \varepsilon_{mi}}$

Fixed part

Random part

Random intercept and slope model

- Compared to the random intercept model, it allows for:
  - Variation in person-specific regression slopes
  - Covariation between person-specific intercepts and slopes

Equation

Level-1 model: $y_{mi} = \beta_{0i} + \beta_{1i} \cdot x_{mi} + \varepsilon_{mi}$

Level-2 model: $\beta_{0i} = \gamma_{00} + \upsilon_{0i}$

$\beta_{1i} = \gamma_{10} + \upsilon_{1i}$

Random slope

$$\Sigma_{\upsilon} = \begin{bmatrix} \sigma^2_{\upsilon_0} & \sigma_{\upsilon_1\upsilon_0} \\ \sigma_{\upsilon_1\upsilon_0} & \sigma^2_{\upsilon_1} \end{bmatrix}$$

Covariance of person-specific deviations with regard to intercept and slope

Variance-covariance-matrix of level-2 residuals (i.e., of deviations from fixed effects)

Full model: $y_{mi} = \boxed{\gamma_{00} + \gamma_{10} \cdot x_{mi}} + \boxed{\upsilon_{0i} + \upsilon_{1i} \cdot x_{mi} + \varepsilon_{mi}}$

Fixed part

Random part

The shape of change

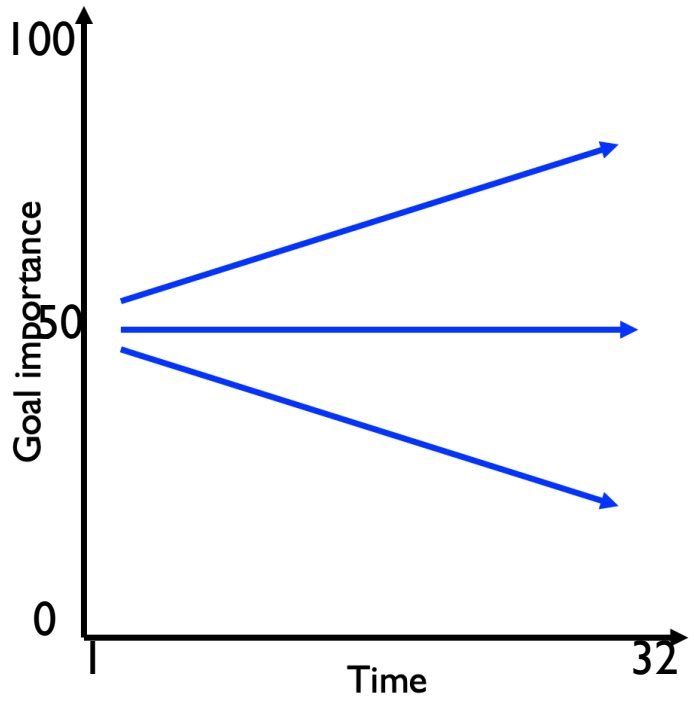
- Linear?
- Quadratic?
- Non-linear?
  - Splines / GAM(M)

Considerations

- Assumptions of multilevel models: Level-1 and Level-2 residuals are independent and identically distributed random variables following a (multivariate) normal distribution (commonly violated, see https://psyarxiv.com/qudr6/download?format=pdf)
  - Dealing with non-normally distributed Level-1 residuals: Use generalized multilevel models with link-functions (or mixtures)
  - Dealing with non-independent Level-1 residuals: Specify the Level-1 residual correlation matrix across repeated measurements

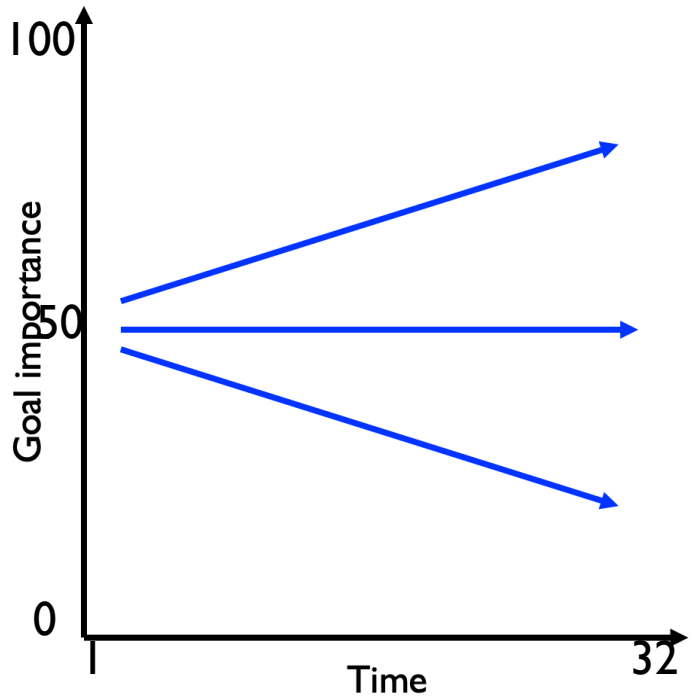
Priors

- Priors in brms: Defined by Paul Bürkner and his community
- Advantages of adapting priors: Better convergence, more realistic results



Priors

- Our expert knowledge: More than 20 points change across all days/0.66 per day unrealistic
- Weakly informative: Prior density peaking across this area; normal(0, 2)
- Moderately informative: Most prior mass within this area; normal (0, 0.33)
- Strongly informative: All prior mass within area -0.66; 0.66: normal(0, 0.11)
- Attention: A priori (theoretical) vs a posteriori (empirical) informativity

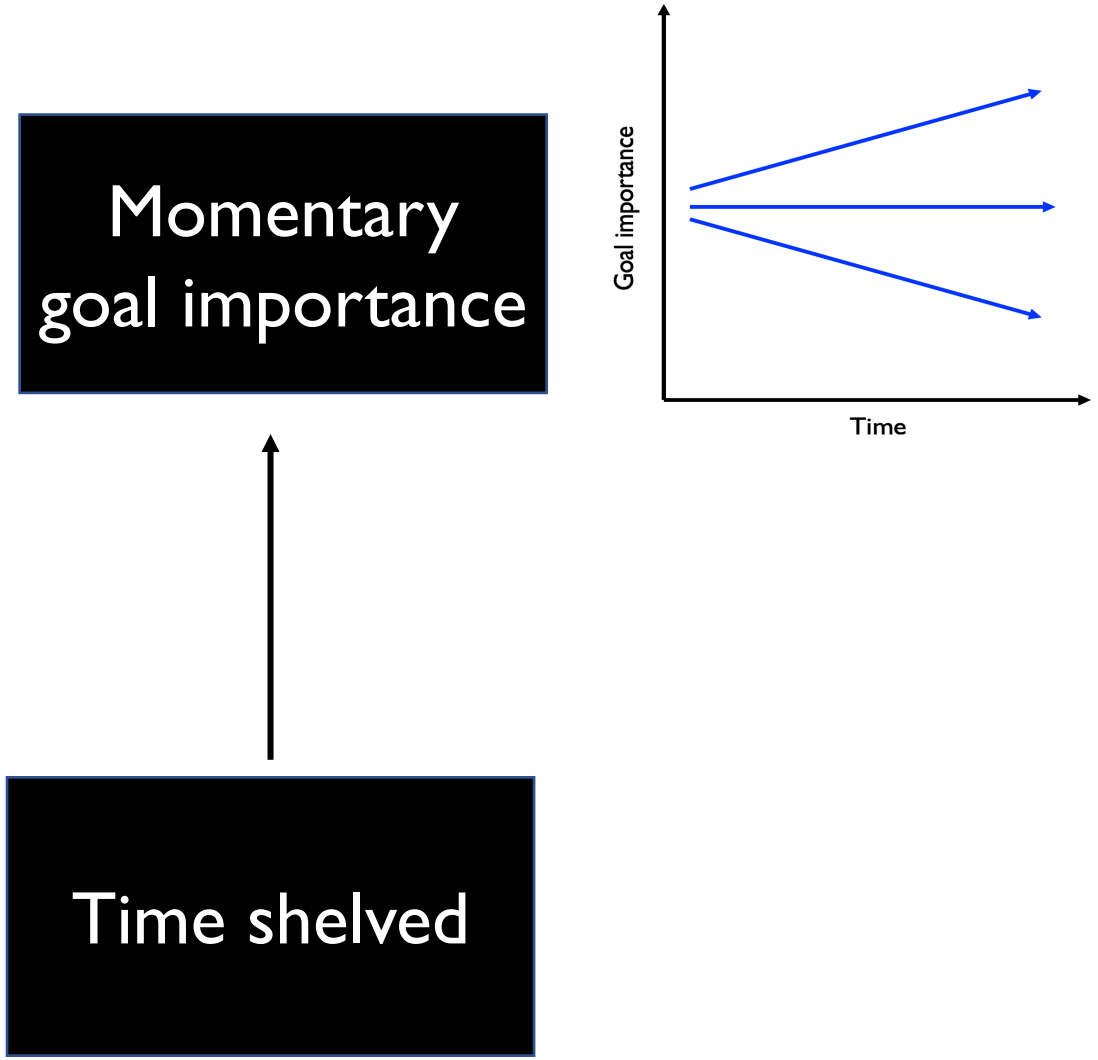


Research question

Does the length of shelving predict:

I. between-person differences in average shelved goal importance (random intercepts)

II. between-person differences in within-person change in shelved goal importance (random slopes)?



Linear growth model with a level-2 predictor

- We now include a level-2 predictor that may explain differences in person-specific intercepts and slopes
- Differences in how long the goal has been shelved may explain differences in shelved goal importance across days (random intercepts)
- Differences in how long the goal has been shelved may explain differences in change shelved goal importance across days (random slopes)

Equation

Level-1 model: $y_{mi} = \beta_{0i} + \beta_{1i} \cdot x_{mi} + \varepsilon_{mi}$

Level-2 model: $\beta_{0i} = \gamma_{00} + \gamma_{01} \cdot z_i + \upsilon_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11} \cdot z_i + \upsilon_{1i}$

Full model:

Fixed regression coefficient capturing the moderating influence of *Z* on the effect of *X* on *Y*

$$y_{mi} = \boxed{\gamma_{00} + \gamma_{10} \cdot x_{mi} + \gamma_{01} \cdot z_i + \gamma_{11} \cdot z_i \cdot x_{mi}} + \boxed{\upsilon_{0i} + \upsilon_{1i} \cdot x_{mi} + \varepsilon_{mi}}$$

Fixed part

Random part

Break ☺

Within-person covariation and between-person differences in within-person covariation

Exercise

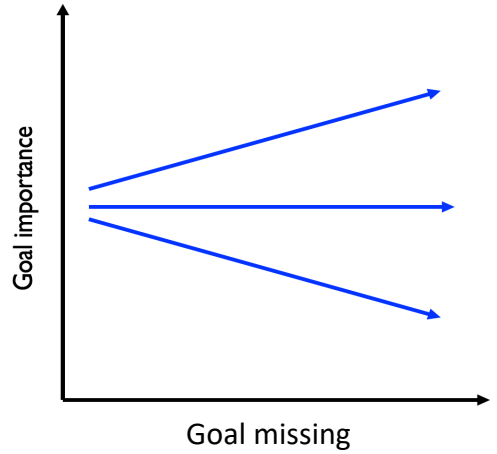
Exercise

- Couple up with a few people around you!
- Say hello and introduce yourself!
- Is momentary goal importance higher when people report to have missed the goal more in their everyday life?
- Is there a concurrent within-person association between goalmissing and goalimportance?
- Does this within-person association differ between persons, and can we explain between-person differences through the level-2 variable ”timeshelved” (months since goal shelved)?

One possible approach

Research question

Momentary goal missing → Momentary goal importance



Between-person: Do people who, on average, report greater missing of the goal report greater importance of the shelved goal?

Within-person: Is the momentary sense of missing of the shelved goal pursuit linked to the momentary importance of the shelved goal? Does this within-person association differ across persons?

Random intercept and slope model

- Includes level-2 and level-1 predictor
- The fixed regression coefficient captures the association between predictor and outcome variable
- The random intercept captures unexplained variation between persons in the outcome variable
- The random slope captures unexplained variation in the association between the predictor and the outcome variable

Considerations: Detrending

- The effect of a level-1 predictor may be confounded by predictor and outcome sharing a common time-trend
  - Include time as (fixed) level-1 predictor
- Correlations between composite residuals may depend on time
  - Include time as a (random) level-1 predictor

Equation

Level-1 model: $y_{mi} = \beta_{0i} + \beta_{1i} \cdot x_{mi} + \beta_{2i} \cdot t_m + \varepsilon_{mi}$

Level-2 model: $\beta_{0i} = \gamma_{00} + \gamma_{01} \cdot z_i + \upsilon_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11} \cdot z_i + \upsilon_{1i}$

$\beta_{2i} = \gamma_{20} + \upsilon_{2i}$

Fixed and random effects of time

Full model:

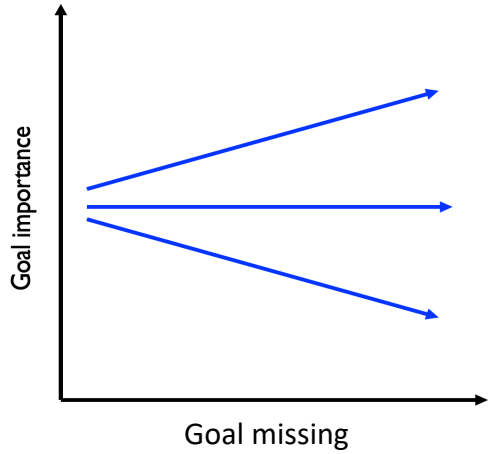
$$y_{mi} = \boxed{\gamma_{00} + \gamma_{10} \cdot x_{mi} + \gamma_{01} \cdot z_i + \gamma_{11} \cdot z_i \cdot x_{mi} + \gamma_{20} \cdot t_m} + \quad \text{Fixed part}$$

$$\boxed{\upsilon_{0i} + \upsilon_{1i} \cdot x_{mi} + \upsilon_{2i} \cdot t_i + \varepsilon_{mi}} \quad \text{Random part}$$

Research question

Momentary goal missing

Momentary goal importance



Time shelved

Does the length of shelving predict:

I. between-person differences in average shelved goal importance (random intercepts)

II. between-person differences in the contingency of momentary goal importance on momentary goal missing (random slopes)?

Model with a cross-level interaction

- We now include a level-2 predictor that may explain differences in person-specific intercepts and slopes
- Differences in how long the goal has been shelved may explain differences in shelved goal importance across days (random intercepts)
- Differences in how long the goal has been shelved may explain differences in the association between momentary goal missing and momentary goal importance (random slopes)

Considerations: Dealing with heteroscedasticity

- Mixed-effects location-scale models allow for heterogeneity in both within-subject variance and between-subject variance
- The logarithms of these two variances (or standard deviations) are modeled as a function of Level-1 and Level-2 predictors