

# Improving the repositioning operation for bike sharing system

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# Motivation: Background

- Dock based bike
  - repositioning is driven by the demand on bikes & docks
  - limited by the facility's locations
- Dockless bikes
  - repositioning is only driven by the demand on bikes
  - increased cost of finding a bike
- Mixed bike service
  - Nice Ride



## Motivation: Research Questions

Due to a potential mismatch of rental/return demand versus availability in dock-based/dockless system, operating an inventory rebalancing strategy could be beneficial.

- Find out the difference in the optimal rebalancing strategy and the optimal cost between two bike systems
- Utilizing empirical information to evaluate proposed policy performance

## [1] Inventory Repositioning in On-Demand Product Rental Networks

- Motivating example: car2go.
- State variable:  $(\mathbf{x}, \gamma)$
- Key assumption 1: return probability consists of "rental length" and "return location"

$$p_{t,ij} = p_t q_{t,ij}$$

Rental period can be greater than one.

- Key assumption 2: lost sales cost outweighs the repositioning cost

$$\rho c_{\max} - c_{\min} \leq p_{\min}(\beta - c_{\min})$$

- Utilize approximate dynamic programming approach to approximate the convex objective function.
  - Finding lower bounding hyperplanes to a convex function.

### [2] Robust Repositioning for Vehicle Sharing

- Motivating example: car2go.
- State variable:  $\mathbf{x}$
- Key assumption: lost sales cost outweighs the repositioning cost

$$\bar{p}_{it} \geq \sum_{j \neq i} s_{ji(t+1)} \alpha_{ijt}$$

- A stochastic dynamic programming can be solved optimally with 2 region and the assumption of temporal independence among demands.
- Introduce distributionally robust optimization approach (DRO) to deal with uncertainty in the distribution, resulting in a multi-period robust optimization problem.
- An enhanced linear decision rule (ELDR) approximation is used to promote computational tractability.

[3] BRAVO: Improving the Rebalancing Operation in Bike Sharing with Rebalancing Range Prediction.

- Motivating example: Nice ride
- Rebalancing interval: Utilizing the data sets to predict the demand on bikes/docks for each station.
- Routing: approximate algorithm for TSP where each path's weight is the Euclidean distance
- Rebalancing Amount Algorithm
  - reduces the total amount of loading/unloading bikes
  - "robbing Peter to pay Paul" problem

- To compare the performance/repositioning policy between dock-based/dockless system, we adopt the stochastic dynamic programming model in [2] to analyze results for 2-region system.
- Modeling dockless system: the original model in [2]
- Modeling the dock-based system: adding respective capacity limit at each location in the formulation.



## Model Settings for two locations

- Total bike numbers:  $N$ .
- State variable: number of bikes in location 1:  $x_t$  ( $N - x_t$  for location 2).
- Reposition quantity  $r_t$ , where  $r_t \geq 0$  represents reposition from location 1 to 2, and  $r_t \leq 0$  represents otherwise.
- Return probability  $\alpha_{ijt}$ , where  $\sum_j \alpha_{ijt} = 1$  implies **one-period rental**.
- Unit repositioning cost per trip:  $s_{12t}$  and  $s_{21t}$ .
- Average lost sales cost in location 1 and 2:

$$\bar{p}_{1t} = p_{11t}\alpha_{11t} + p_{12t}\alpha_{12t}$$

$$\bar{p}_{2t} = p_{22t}\alpha_{22t} + p_{21t}\alpha_{21t}$$

## Model Settings for two locations: Capacity limits

- Capacity limits:  $C = C_1 + C_2$ .
- Holding cost:  $h_{1t}$  and  $h_{2t}$ 
  - Instead of using hard constraint, we add it as an additional cost when return capacity is violated.

## DP formulation for dock based counterpart

$$V_t(x_t) = \min_{x_t - N \leq r_t \leq x_t} \left\{ \underbrace{s_{12t}r_t^+ + s_{21t}r_t^-}_{\text{Repositioning cost}} + \mathbb{E}_{\mathbb{P}}[J_t(y_t, \mathbf{d}_t)] \right\}$$

where

$$J_t(y_t, \mathbf{d}_t) = \min_{w_{1t}, w_{2t}} \left\{ \underbrace{\bar{p}_{1t}(d_{1t} - w_{1t}) + \bar{p}_{2t}(d_{2t} - w_{2t})}_{\text{The lost sale cost}} + \right. \\ \left. \underbrace{h_{1t}(x_{t+1} - C_1)^+ + h_{2t}(N - x_{t+1} - C_2)^+}_{\text{The holding cost}} + V_{t+1}(x_{t+1}) \right\}$$

$$\begin{aligned} \text{s.t. } x_{t+1} &= y_t - \alpha_{12t}w_{1t} + \alpha_{21t}w_{2t}, \\ w_{1t} &\leq \min(y_t, d_{1t}) \\ w_{2t} &\leq \min(N - y_t, d_{2t}) \end{aligned}$$

where  $a^+ = \max(0, a)$  and  $a^- = -\min(0, a)$

- Terminal cost  $V_{T+1}(x_{T+1}) = 0$ .

# Optimal Policy Structure

- Assumption:  $\bar{p}_{it} \geq \alpha_{ijt}(h_{jt} + s_{ji(t+1)})$ , for  $j \neq i$  and  $i, j \in \{1, 2\}$ .
- Policy structure: a two threshold policy  $[\underline{x}_t, \bar{x}_t]$ :

$$\begin{aligned}y_t^*(x_t) &= \underline{x}_t, & x_t &\in [0, \underline{x}_t) \\y_t^*(x_t) &= x_t, & x_t &\in [\underline{x}_t, \bar{x}_t] \\y_t^*(x_t) &= \bar{x}_t, & x_t &\in (\bar{x}_t, N].\end{aligned}$$

where  $y_t(x_t) = x_t - r_t$  is the target inventory level after repositioning, and  $\underline{x}_t, \bar{x}_t$  is the optimal reposition up-to and down-to level solved by the following convex programs:

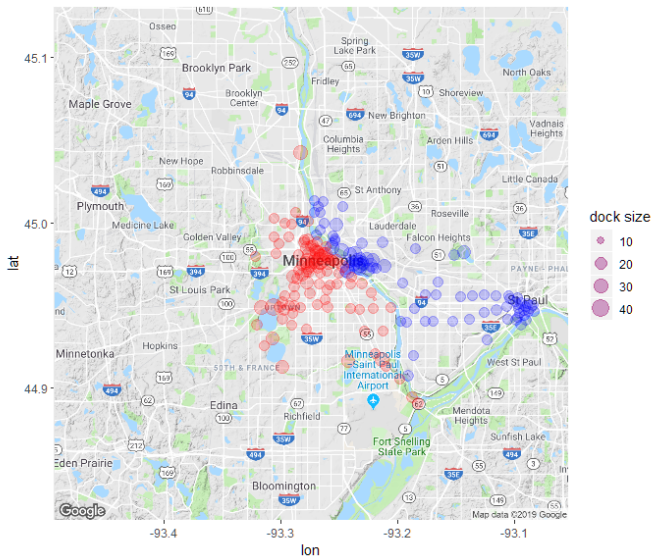
$$\begin{aligned}\underline{x}_t &= \arg \min_{0 \leq y \leq N} \{s_{21t}y + \mathbb{E}_{\mathbb{P}}[J_t(y, \mathbf{d}_t)]\} \\ \bar{x}_t &= \arg \min_{0 \leq y \leq N} \{-s_{12t}y + \mathbb{E}_{\mathbb{P}}[J_t(y, \mathbf{d}_t)]\}\end{aligned}$$

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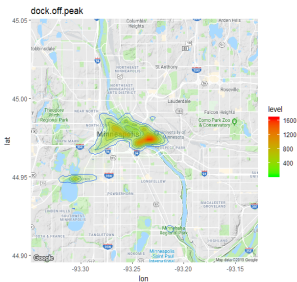
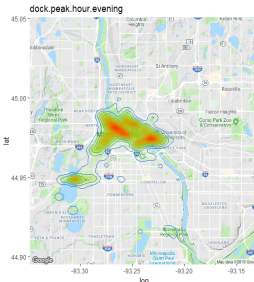
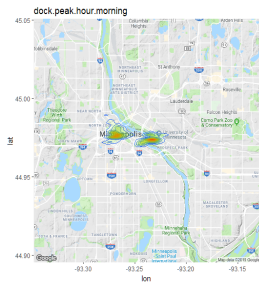
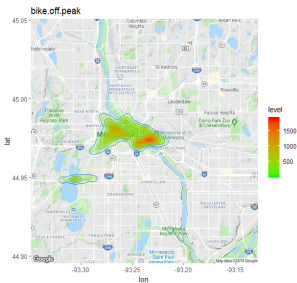
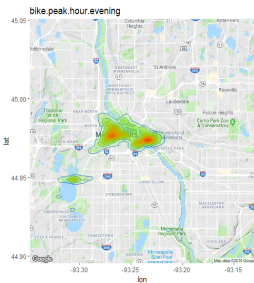
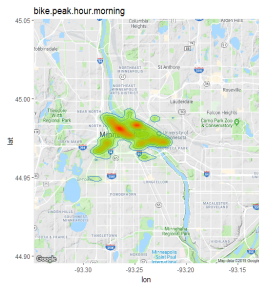
## Optimal Policy Structure (contd.)

- The structure is the same as the original dockless settings, by seeing that adding terms  $h_{1t}(x_{t+1} - C_1)^+ + h_{2t}(N - x_{t+1} - C_2)^+$  to  $J_t$  doesn't change the convexity of the function.
- However, the upper and lower threshold do change comparing to its dockless counterpart.

# Data



# Data: Bike demand & Dock demand



- Nice ride checkin/checkout records for September, October and November
- Spatial feature: The active region can be divided into 2 parts
- Time & Direction feature:
  - In the morning, the bike move from residential area (low bike density) to workplace (high bike density)
  - In the morning, the bike move from workplace (high bike density) to residential area (low bike density)
  - Off-peak time doesn't have clear trend



# Simulation Settings

- Study: The impact of different (total capacity)/(total bike number) ratio on the thresholds and optimal cost.
- Ratio: 0.2,0.4,0.6,...,2. Balanced capacity in two locations.
- Total number of bikes: 178. (scaled by 10)
- Total period: 3.
- $\alpha$ : one period

$$\alpha_t = \begin{pmatrix} 0.82 & 0.18 \\ 0.22 & 0.78 \end{pmatrix}$$

- $p$ : Lost sales cost

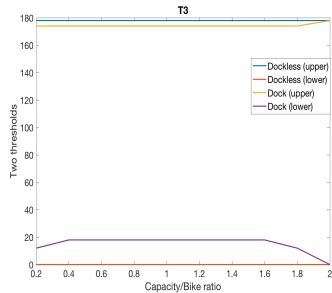
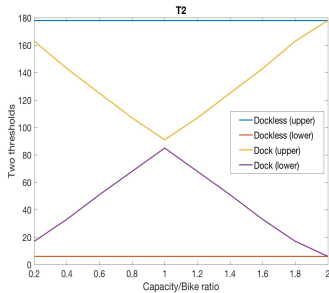
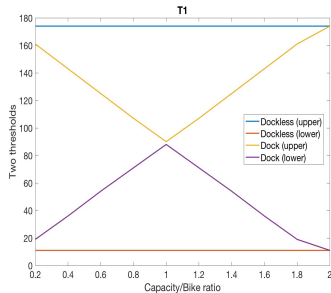
$$p = \begin{pmatrix} 0.8176 & 0.9412 \\ 0.9956 & 0.53 \end{pmatrix}$$

## Simulation Settings (contd.)

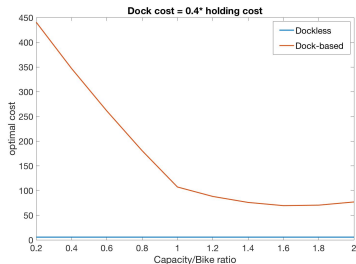
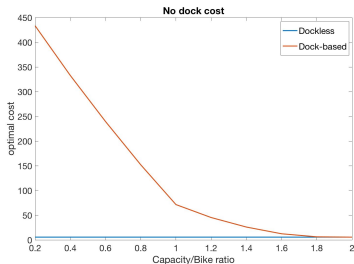
- Repositioning cost: 1.5
- Holding cost: 1
- Demand mean :

$$\mu = \begin{pmatrix} 12.02 & 31.25 & 22.94 \\ 7.81 & 29.61 & 16.54 \end{pmatrix}$$

# Results - Two thresholds



# Results - Optimal cost



### Threshold policy:

- Dockless:
  - Large no-repositioning region: Large return-to-same-place probability ( $\alpha_{11}, \alpha_{22}$ )
- Dock-Based:
  - Similar thresholds for extreme capacity ratio  $\Rightarrow$  large repositioning region
  - Repositioning becomes efficient when capacity ratio  $\approx 1 \Rightarrow$  small no-repositioning region
  - T3: Larger no-repositioning region in last period  $\Rightarrow$  similar to dockless system.

### Optimal cost

- Due to the lack of holding cost, the dockless system outperforms the dock-based one.
- By adding the dock cost, there exists an optimal capacity ratio

# Sensitivity Analysis - Settings

Holding cost  $h$ :

- Ratio: 0.5, 0.6, ..., 2. Holding cost = ratio \* [1, 1].

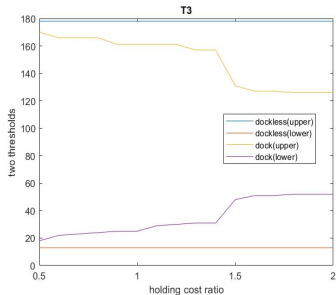
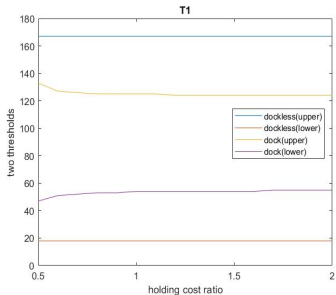
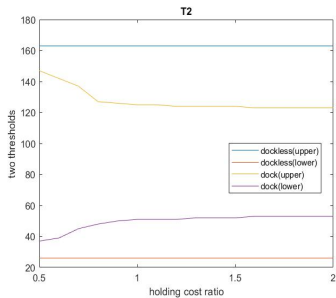
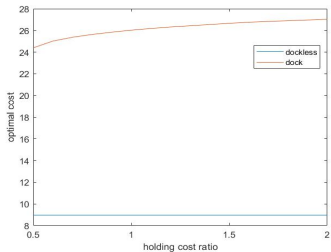
Repositioning cost  $s$ :

- Ratio: 0.5, 0.6, ..., 2. Repositioning cost = ratio \* 1.5.

Unbalanced dock numbers in two locations:

- Fixed total docks = 1.4 \* total bikes.
- $C_1/C_2 = 0.5, 0.6, \dots, 2$ .

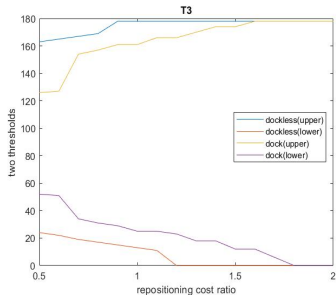
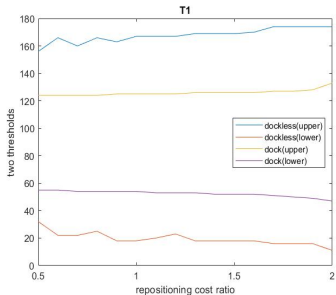
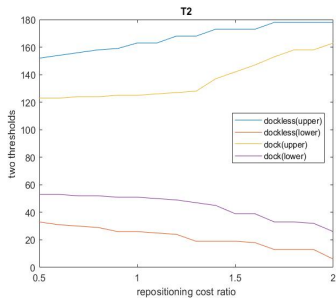
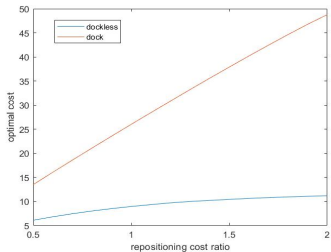
# Sensitivity analysis - holding cost



- Higher holding cost  $\Rightarrow$  more incentive to reposition  $\Rightarrow$  small no-repositioning region.
- By assumption  $\bar{p}_{it} \geq \alpha_{ijt}(h_{jt} + s_{ji(t+1)})$ , for  $j \neq i$  and  $i, j \in \{1, 2\}$ , the holding cost needs to be bounded, thus limiting the impact on the system.
- The thresholds in the later period are less sensitive to the increasing holding cost.

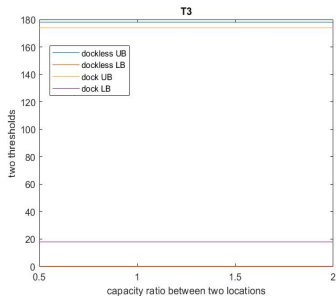
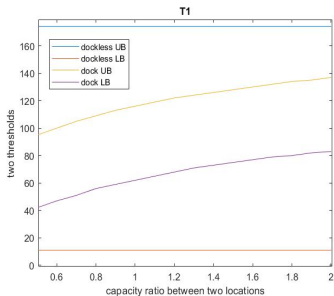
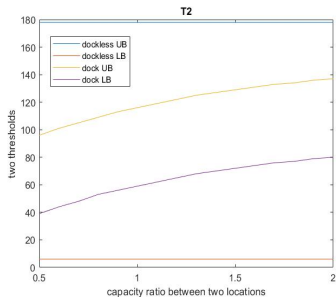
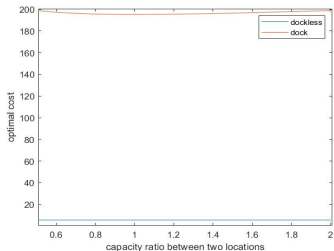


# Sensitivity analysis - Reposition cost



- Higher repositioning cost  $\Rightarrow$  larger no-repositioning region.
- The total cost of the dock-based system is more sensitivity to the repositioning cost
- As opposed to  $t = 3$ , where the system tends to not reposition, in  $t = 1$  the system tends to reposition more to account for the cost in future periods, hence the increased repositioning cost has less impact on the thresholds policy.

# Sensitivity analysis - Imbalanced docks between stations



- The optimal cost is less sensitive to the change of the ratio  $C_1/C_2$ .
- The size of no-repositioning region doesn't change, but the target inventory level in location 1 increases as the ratio  $C_1/C_2$  increases.

# Conclusion

- The project serves as an initial step to analyze the impact of capacity constraints on the repositioning policy in bike-rental system.

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- Due to large state/action space, a repositioning policy with more theoretical basis (such as in [1] or [2]) becomes unsolvable when problem size grows larger. To apply in real scenario, certain aggregation methods need to be tested. However, some crucial effects could be cancelled out in a aggregated system.

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- Due to large state/action space, a repositioning policy with more theoretical basis (such as in [1] or [2]) becomes unsolvable when problem size grows larger. To apply in real scenario, certain aggregation methods need to be tested. However, some crucial effects could be cancelled out in a aggregated system.
- In recent years, the "capacity constraints" in similar systems vanishes away as evolving to "dockless" settings. Needs to find new motivating examples for such research. For example: electric charging station.



- [1]Benjaafar, Saif and Jiang, Daniel and Li, Xiang and Li, Xiaobo, Inventory Repositioning in On-Demand Product Rental Networks (December 18, 2018). Available at SSRN: <https://ssrn.com/abstract=2942921> or <http://dx.doi.org/10.2139/ssrn.2942921>
- [2]He, Long and Hu, Zhenyu and Zhang, Meilin, Robust Repositioning for Vehicle Sharing (March 31, 2018). Forthcoming in Manufacturing Service Operations Management. Available at SSRN: <https://ssrn.com/abstract=2973739> or <http://dx.doi.org/10.2139/ssrn.2973739>
- [3]Wang, S., He, T., Zhang, D., Shu, Y., Liu, Y., Gu, Y., Liu, C., Lee, H., Son, S.H. (2018). BRAVO: Improving the Rebalancing Operation in Bike Sharing with Rebalancing Range Prediction. IMWUT, 2, 44:1-44:22.