Advanced Control and Dynamics

COURSEWORK

Chapter 1

System Introduction and Analysis

1. Plant description

In this section, an engineering control system is proposed and analyzed, featuring the theoretical dynamic behavior. The very flexible circuit shown in Fig. 1.1 is called an operational amplifiers (Op-amp) biquadratic circuit. Its transfer function can be made to be the ratio of two second-order or quadratic polynomials. By selecting different values for R_a; R_b; R_c; and R_d the circuit can realize a low-pass, band-pass, high-pass, or band-reject (notch) filter.

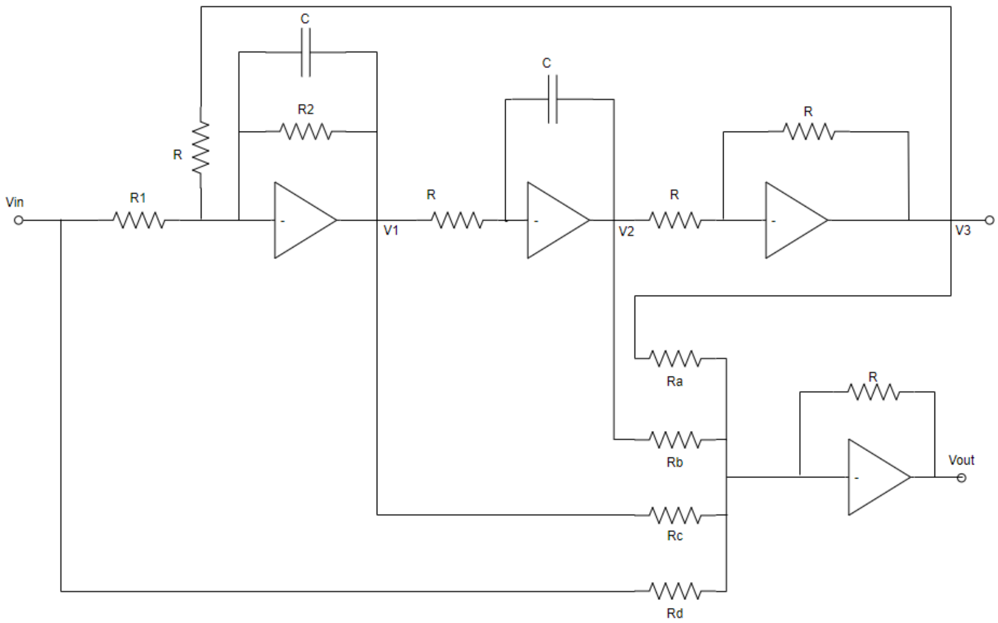

Fig. 1.1 Op-amp biquadratic circuit

The following is a brief description of the operation of the circuit. Based on the circuit structure and Kirchhoff's principle, the following equations can be obtained:

$$\frac{V_{in}}{R_1}+\frac{V_3}{R}=-(\frac{V_1}{R_2}+C\dot{V}_1)$$

$$\frac{V_1}{R} = -C\dot{V}_2$$

$$V_3 = -V_2$$

$$\frac{V_3}{R_a} + \frac{V_2}{R_b} + \frac{V_1}{R_c} + \frac{V_{in}}{R_d} = -\frac{V_{out}}{R}$$

If $R_a = R$; and $R_a = R_b = R_c = \infty$; the transfer function from V_{in} to V_{out} can be written as the low-pass filter. And the transfer function can be described as following equation.

$$\frac{V_{out}}{V_{in}} = \frac{\frac{R}{R_1}}{(RC)^2 s^2 + \frac{R^2 C}{R_2} \mathrm{s} + 1} \qquad (1-1)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{\frac{s^2}{\omega_n^2} + 2\xi \frac{s}{\omega_n} + 1} \qquad (1-2)$$

Where:

$$A_0 = \frac{R}{R_1}$$

$$\omega_n = \frac{1}{RC}$$

$$\xi = \frac{R}{2R_2}$$

The theoretical dynamics is:

$$\frac{Y(s)}{U(s)} = g_p(s) = \frac{1}{s^2 + 0.6s + 4}$$

The theoretical transfer function is corresponding to the Op-amp biquadratic circuit transfer function as in formula (1-2). As a result, following values can be derived in our plant:

$$A_0 = \frac{R}{R_1} = \frac{1}{4}$$

$$\omega_n = \frac{1}{RC} = 2$$

$$\xi = \frac{R}{2R_2} = 0.15$$

$$R_1 = 4R;\ R = \frac{10}{3} R;\ C = \frac{1}{2R}$$

Then, we can obtain the state space model $S(A, B, C, D)$ which can be described as:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

$$\frac{Y(s)}{U(s)} = G(s) = C(sI_n - A)^{-1}B + D$$

Where:

$$A = \begin{bmatrix} 0 & 1 \\ -4 & -0.6 \end{bmatrix} \quad (1-3)$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (1-4)$$

$$C = [1 \quad 0] \quad (1-5)$$

$$D = 0 \quad (1-6)$$

2. Feedback system analysis

In this section, the idea of using output feedback and the state feedback to shape the dynamic behavior is proposed and analyzed in detail. According to the formula (1-6), the controlled system is as followed:

$$\Sigma_0 = (A, B, C)$$

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

2.1 Output feedback control

The output feedback is a feedback method in which the output variables of the system y are transferred to the input via a proportional loop. The control system block diagram featuring the output feedback is shown in Fig. 1.2.

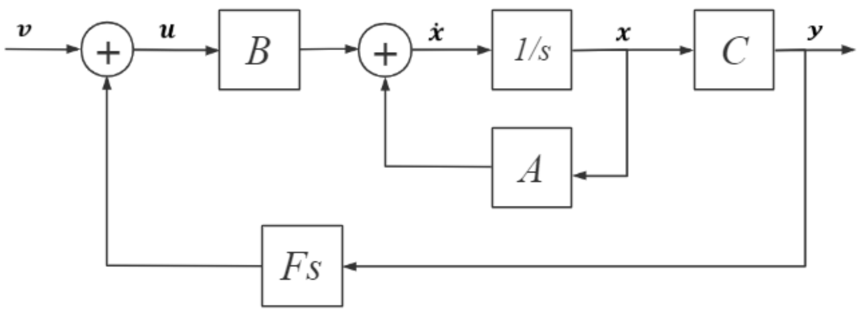

Fig. 1.2 Block diagram output feedback control

The modified control system can be described as:

$$u = F_s y + v$$

$$\Sigma_H = (A + BF_s C), B, C)$$

Analysis: By selecting the output feedback gain array F_s, the eigenvalues of the closed loop system can be changed, thus altering the control characteristics of the system. Output feedback does not affect the controllability and observability of the system. While it can only change the location of the system poles within a limited range.

2.1 State feedback control

State feedback is a feedback method in which each state variable is multiplied by a corresponding feedback coefficient and then fed back to the input and summed to the reference input to form a control law that serves as the control input to the controlled system. The control system block diagram featuring the state feedback is shown in Fig. 1.3.

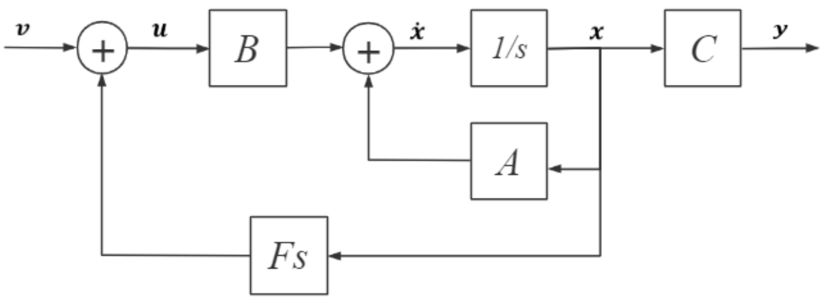

Fig. 1.3 Block diagram state feedback control

The state linear feedback control law is:

$$u = F_s x + v$$

$$\Sigma_K = (A + BF_s), B, C)$$

Analysis: Through comparing open-loop system $\Sigma_0 = (A, B, C)$ and closed-loop system $\Sigma_K = (A + BF_s), B, C)$, we can conclude that the introduction of the state feedback array F_s does not increase the dimensionality of the system but allows the eigenvalues of the closed-loop system to be freely changed by the choice of F_s, thus enabling the system to achieve the required performance.

Discussion:

Similarities:

(1) Both output feedback and state feedback can be used to modify system performance

through pole-placement.

(2) For $\Sigma_H = \Sigma_K$, The feedback from the output to the reference input is equivalent to the state feedback. That is, for any output feedback system, an equivalent state feedback can always be found.

Differences:

(3) Output feedback takes the output variable as feedback. State feedback uses the state variable as feedback.

(4) For state feedback, given a value of F_s in $\Sigma_H = \Sigma_K$, it is not always possible to solve for F_s. Therefore, the output feedback can be considered as partial state feedback. What is contained in the output information is not necessarily the full state variables of the system.

(5) Without the addition of a compensator, output feedback is not as effective as a state feedback system. However, the ease of technical implementation of output feedback is its advantage [1].

(6) The output feedback does not affect the controllability and observability of the system. The state feedback does not change the controllability of the system but may change the observability of the system.

3. Plant Performance Analysis

In this section, the performance of the system will be analyzed from the following perspectives: observability, stability, controllability, and time response.

3.1 Stability

According to the first Lyapunov criterion, for the systems $\Sigma: (A, B, C)$, a sufficient condition for the output of the system to be stable is that the poles of its transfer function are all located on the negative real axis side of the complex plane.

Characteristic equation of the plant:

$$s^2 + 0.6s + 4 = 0$$

This system has no (finite) zeros and two poles:

$$s = \alpha_1 = -0.3 + 1.98i$$

and

$$s = \alpha_2 = -0.3 - \ 1.98i$$

The pole-zero map is shown in Fig. 1.4.

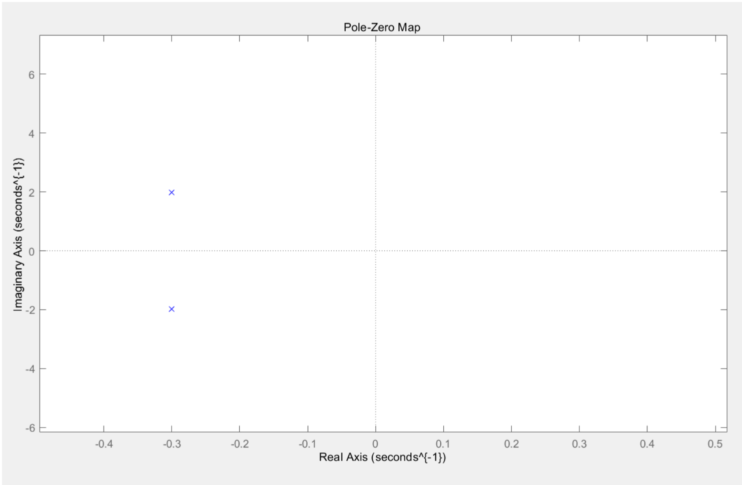

Fig. 1.4 Pole-Zero Map

To conclude, the pole position meets the criterion, and the system is stable.

3.2 Controllability

a) State controllability

A dynamic system is said to be completely state controllable if for any time t_0, it is possible to construct an unconstrained control vector $u(t)$ that will transfer any given initial state $x(t_0)$ to any final state $x(T)$ in a finite time interval $t_0 < t < T$.

A criterion is introduced to check the state controllability:

For 2nd order dynamic,

$$P = [B \ \ AB]$$

According to the formula (1-3) to (1-6), we obtain:

$$P = \begin{bmatrix} 0 & 1 \\ 1 & -0.6 \end{bmatrix}$$

where $n = 2$ is the dimension of matrix A. P is called the state controllability matrix. A system is completely state controllable if the rank of matrix P equals to n. Because $det(P) \neq 0$, P is a full rank matrix, and $rank(P) = 2$. The system has complete state controllability.

b) Output controllability

A system is said to be completely state controllable if, for any time t_0, it is possible to construct an unconstrained control vector $u(t)$ that will transfer any given initial output $y(t_0)$ to any final output $y(T)$ in a finite time interval $t_0 < t < T$.

A criterion is introduced to check the output controllability:

For 2nd order dynamic,

$$Q = [CB \quad CAB]$$

According to the formula (1-3) to (1-6), we obtain:

$$Q = [0 \quad 1]$$

A system is completely output controllable if the rank of matrix Q equals to 1, where 1 is the dimension of output. The Q matrix has rank 1 and $rank(Q)$ is equal to the rank of output. So, the system has complete output controllability.

3.3 Observability

A system is said to be completely observable on $t_0 < t < T$ if, for every t_0 and some T, every state vector $x(t_0)$ can be determined from the knowledge of the output vector $y(t)$ on $t_0 < t < T$. In physical terms, a system is completely observable if every transition of the system state eventually affects the output. A criterion to check the observability is introduced in this section.

$$R = [C \quad CA]^T$$

The observability of the system can be determined directly from the A and C matrices. According to the formula (1-3) and (1-5), the following matrices can be obtained:

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A system is completely observable if the rank of matrix R equals to $n = 2$. Because the $det(R) \neq 0$, R is a full rank matrix, and $rank(R) = 2$, the system has complete state observability.

3.4 Time response

Firstly, the fundamental matrix can be obtained by following formula,

$$\phi(t, t_0) = e^{(A(t-t_0))} = e^{(At)}\Big|_{t_0=0} = L^{-1}[(sI - A)^{-1}]$$

$$\phi(t,t_0) = L^{-1}\begin{bmatrix} \frac{5s+3}{s^2+3s+20} & \frac{5}{s^2+3s+20} \\ \frac{-20}{s^2+3s+20} & \frac{5s}{s^2+3s+20} \end{bmatrix}$$

$$= \begin{bmatrix} -0.15e^{-0.3t}\,sin(1.98t) & 0.5e^{-0.3t}\,sin(1.98t) \\ -2e^{-0.3t}\,sin(1.98t) & -0.15e^{-0.3t}\,sin(1.98t) \end{bmatrix}$$

3.4.1 State response

The state response can be obtained as:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \phi(t,t_0)x(t_0) + \int_{t_0}^{t} \phi(t-\tau)Bu(\tau)d\tau$$

$$x(t) = \begin{bmatrix} 0.25 - 0.25e^{-0.3t}\,cos(1.98t) - 0.04e^{-0.3t}\,sin(1.98t) \\ 0.5e^{-0.3t}\,sin(1.98t) \end{bmatrix}$$

3.4.2 Output response

$$y(t) = C\phi(t,t_0)x(t_0) + C\int_{t_0}^{t} \phi(t-\tau)Bu(\tau)d\tau + Du(t) = Cx(t)$$

$$= 0.25 - 0.25e^{-0.3t}\,cos(1.98t) - 0.04e^{-0.3t}\,sin(1.98t)$$

3.4.3 Time response to a unit step

Input a unit step signal to the system. The response of the system is shown in Fig. 1.5.

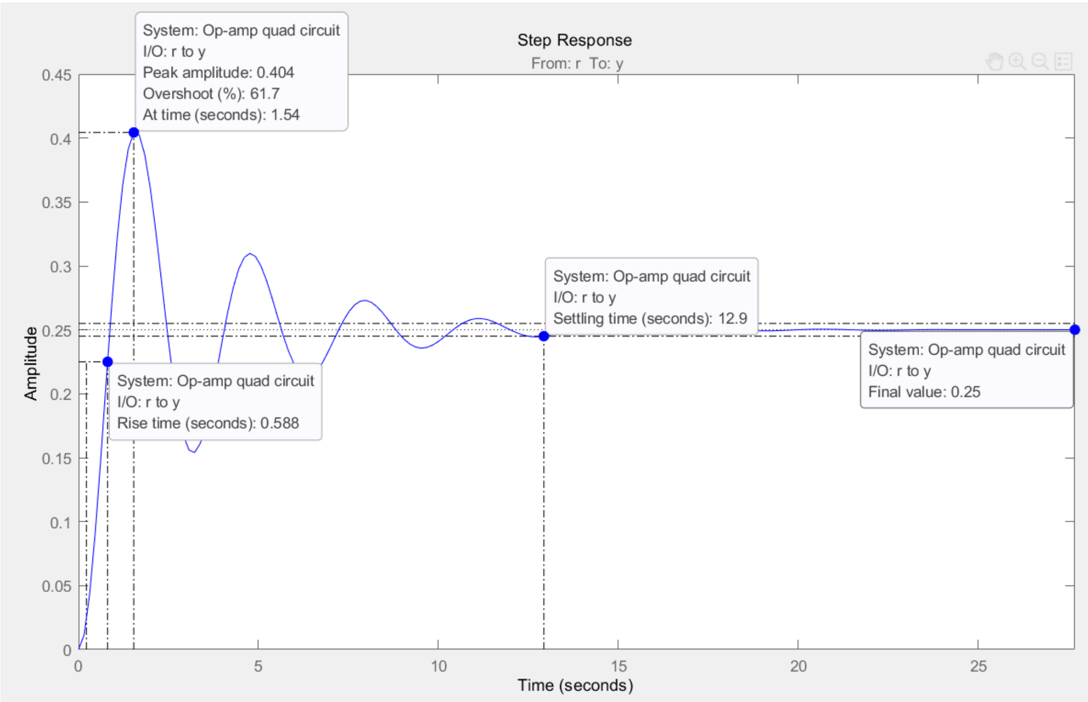

Fig. 1.5 Step Response

The rise time is 0.588s. The peak amplitude is 0.404 at 1.54s. The percentage overshoot (PO) is 61.7%. The settling time is 12.9s. The final value is 0.25.

Chapter 2

System Design

4. Design a state feedback controller.

In this section, a state feedback controller is designed for the plant. In order to make the op-amp quadratic circuit operation more stable, the designed state feedback controller needs to enable the system to meet the following requirements:

(a) Faster response time

(b) Smaller overshoot

(c) Steady state requirements

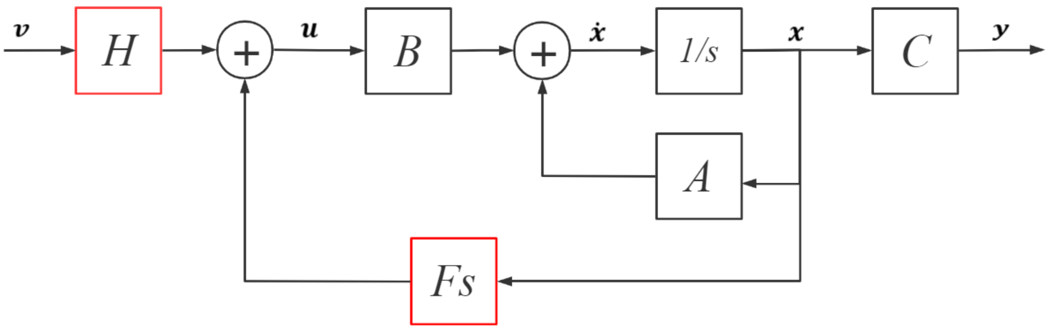

Fig. 2.1 Structured control system

$$u(t) = F_s x(t) + Hv(t)$$

The design of the state feedback controller which is shown in Fig. 2.1 is structured in two steps as follows.

1) Design procedure for determining $< Fs >$ from TF

(1) Assign desired poles.

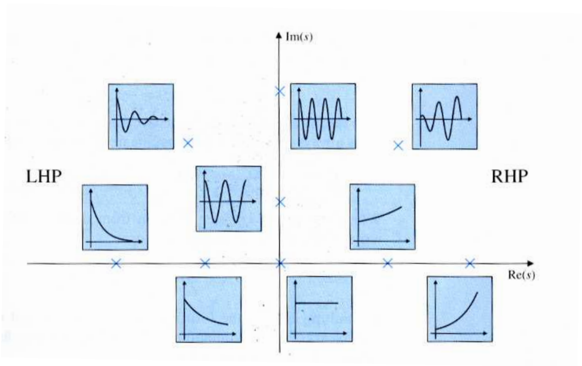

Fig. 2.2 pole location on the "s" plane

Based on the rules that pole position contributes to response and the above requirements for system design, the desired poles for this system are:

$$s_1 = -4$$

$$s_2 = -5$$

Then,

$$\Delta(s) = \prod_{i}^{n}(s + a_i) = \sum_{i=0}^{n} \alpha_i s^{n-i}$$

The corresponding characteristic equation is:

$$s^2 + 9s + 20 = 0$$

(2) Determine dynamic gain matrix from TF denominator with formula (1-3) to (1-5).

$$\det[sI - A - BF_s] = s^2 + (0.6 - f_2)s + (4 - f_1)$$

(3) Find Fs by equalling:

$$\Delta(s) = \prod_{i}^{n}(s + a_i) = \sum_{i=0}^{n} \alpha_i s^{n-i} = \det[sI - A - BF_s]$$

$$s^2 + (0.6 - f_2)s + (4 - f_1) = s^2 + 9s + 20$$

$< Fs >$ is obtained as followed:

$$Fs = \begin{bmatrix} -16 \\ -8.4 \end{bmatrix} \qquad (2-1)$$

2) Design procedure for determining $< H >$ from TF

The criteria for determining $< H >$ is:

$$y(t = \infty) = \lim_{s \to 0} s\,Y(s) \Leftrightarrow \left.\frac{Y(s)}{V(s)}\right|_{s=0} = 1$$

Steady state for a unit step input $\frac{1}{s}$, by final value theorem, the CLTF gives:

$$-\{[C + DF_s][A + BF_s]^{-1}B + D\}H = 1$$

$$H = -\{[C + DF_s][A + BF_s]^{-1}B - D\}^{-1}$$

Then we obtain:

$$H = 20 \qquad (2-2)$$

5. Observer

Design a full order identity observer to estimate the system states, which the poles of the observer are required to lie at s = -4 and s = -5.

As poles of the observer are specified by the determinant of the characteristic equation, we have,

$$det[sI - L] = (s+4)(s+5) = s^2 + 9s + 20$$

1) Determine matrix N

Assuming $N = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$, we have,

$$det[sI - L] = det[sI - (A - NC)]$$

$$= det\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ -4 & -0.6 \end{bmatrix} - \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}[1 \quad 0]\right)\right]$$

$$= s^2 + (0.6 + n_1)s + (4 + 0.6n_1 + n_2)$$

Corresponding coefficients are equal. We have,

$$s^2 + 9s + 20 = s^2 + (0.6 + n_1)s + (4 + 0.6n_1 + n_2)$$

$$n_1 = 8.4,\ n_2 = 10.96$$

Then matrix N is obtained as

$$N = \begin{bmatrix} 8.4 \\ 10.96 \end{bmatrix} \qquad (2-3)$$

2) Determine matrix L

$$L = A - NC = \begin{bmatrix} 0 & 1 \\ -4 & -0.6 \end{bmatrix} - \begin{bmatrix} 8.4 \\ 10.96 \end{bmatrix}[1 \quad 0] = \begin{bmatrix} -8.4 & 1 \\ -14.96 & -0.6 \end{bmatrix} \qquad (2-4)$$

3) Determine matrix M

$$M = B - ND = B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (2-5)$$

Finally, the observer can be written as:

$$\dot{x}(t) = Lx(t) + Mu(t) + Ny(t) = \begin{bmatrix} -8.4 & 1 \\ -14.96 & -0.6 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 8.4 \\ 10.96 \end{bmatrix} y(t)$$

Usage:

The detection and analysis of faults is particularly important due to the existence of a large number of circuit devices such as capacitors and reactors in this system. The designed observer allows the state vectors to be fed back to the display instrument, and the staff can be kept informed of the working of the circuit so that the system can be operated and employed correctly. At the same time, the operator can make a judgement as to whether the circuit system is operating consistently, and replace components when necessary, ensuring the long-term stable operation of the system.

Chapter 3

Simulation and Discussion

6. Relevant performance data and analysis from the simulated studies.

6.1 Feedback controller simulation

According to the designed state feedback controller in Chapter 2 and the parameters in formula (2-1) and (2-2), the system simulation is shown in Fig. 3.1, which presents the responses shown in Fig. 3.2 between the original system and modified system.

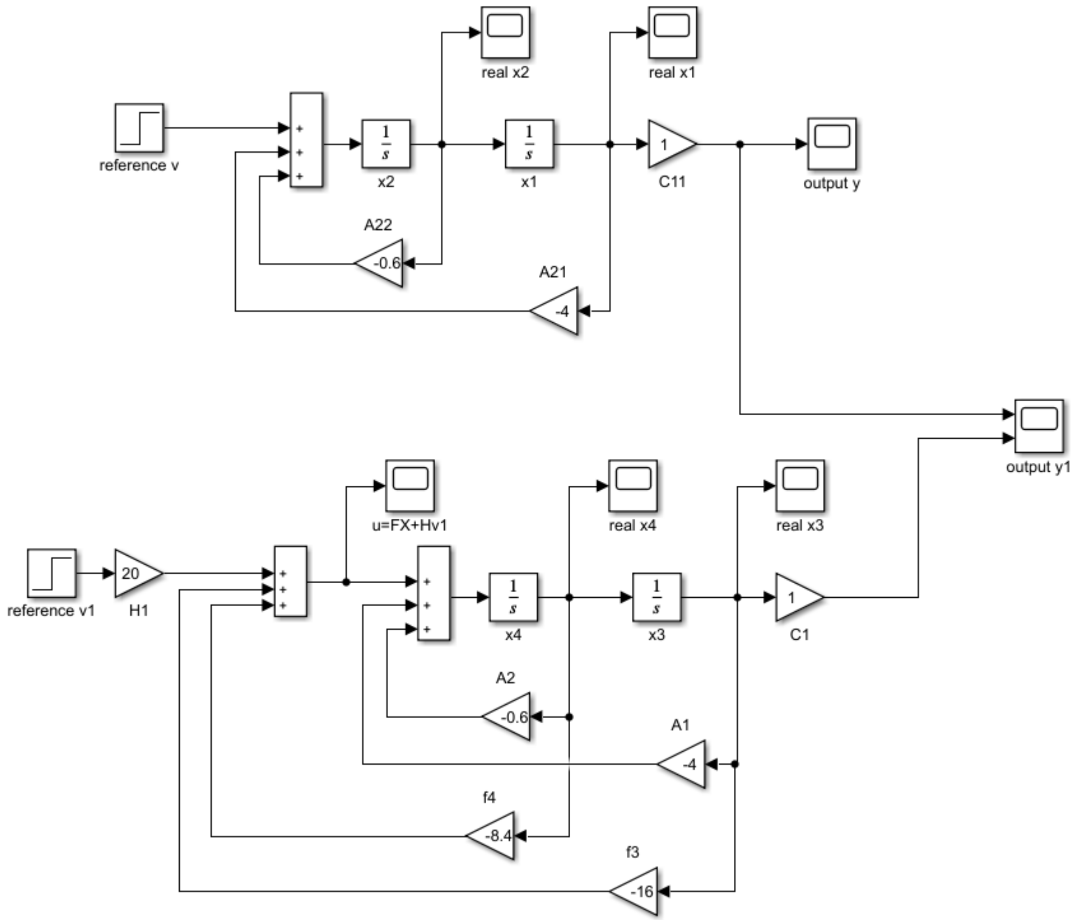

Fig.3.1 Feedback controller

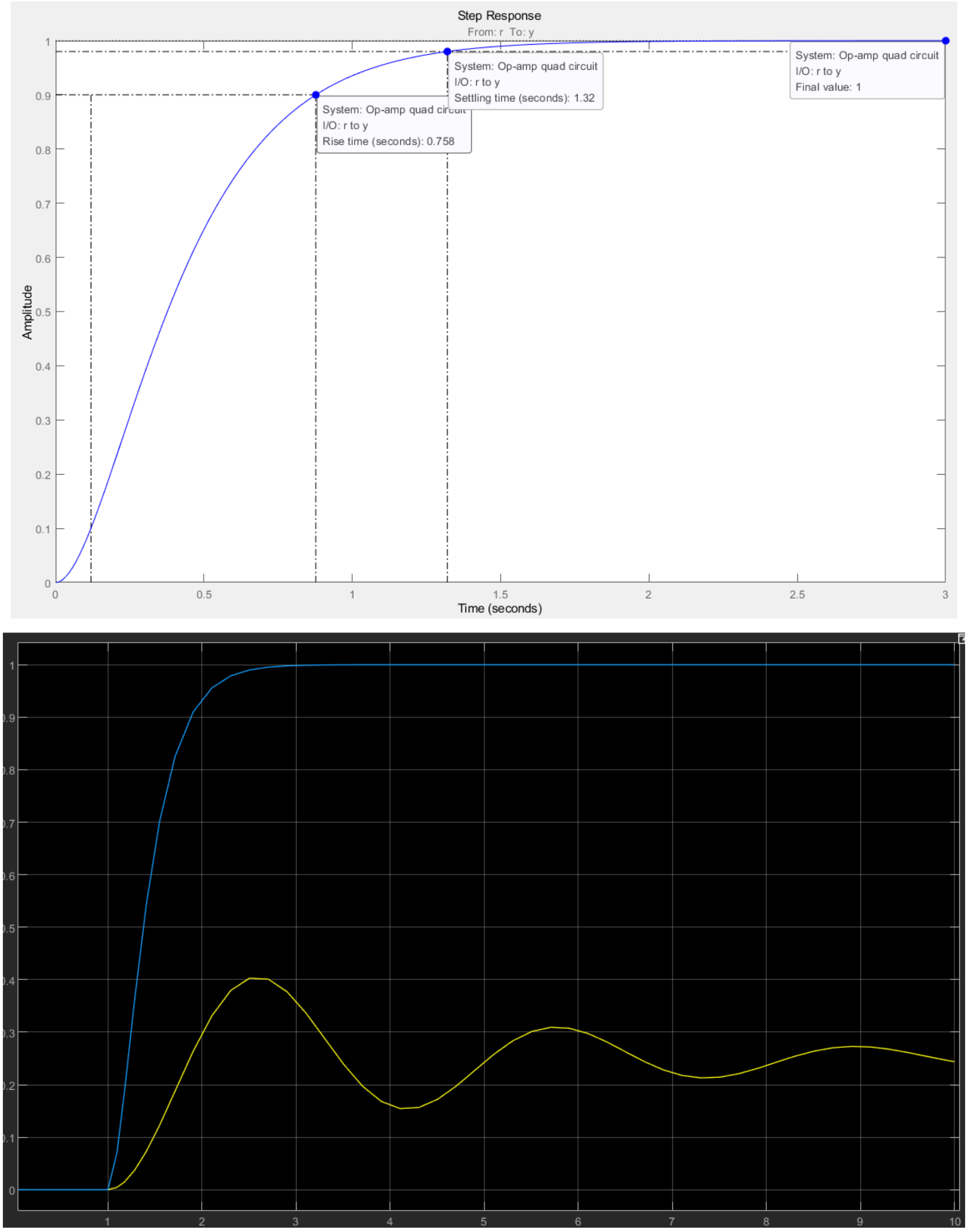

Fig.3.2 Responses and comparison

Compared to the original response curve, the response of modified system has shorter rise time which is 0.758s. The peak amplitude is 1 after 3s. The percentage overshoot (PO) is zero. The settling time is 1.32s. The final value is 1. And there is virtually no steady-state error.

In the low-pass filter circuit system under discussion, the modified system is far superior to the original one and the latter is unable to track the signal well. It is essential to have a robust ability to follow the input voltage.

6.2 Observer simulation

According to the designed system observer in Chapter 2 and the parameters in formula (2-3) to (2-5), the system simulation is shown in Fig. 3.3, which presents the observer components in detail.

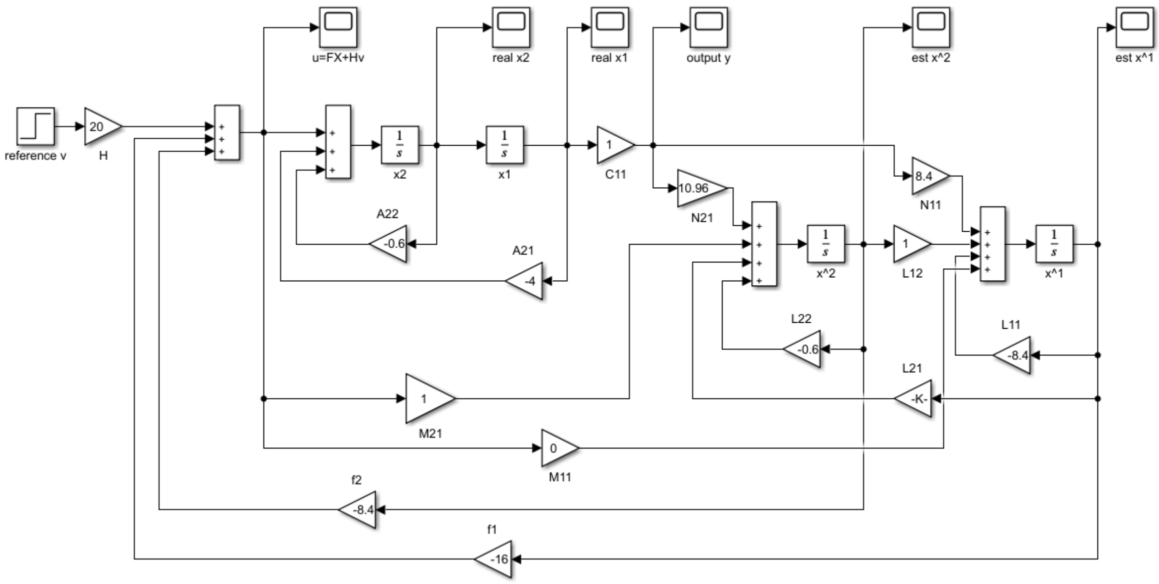

Fig. 3.3 Plant with observer

Extract the state volumes which are shown in Fig. 3.4 to 3.7 and analyze them.

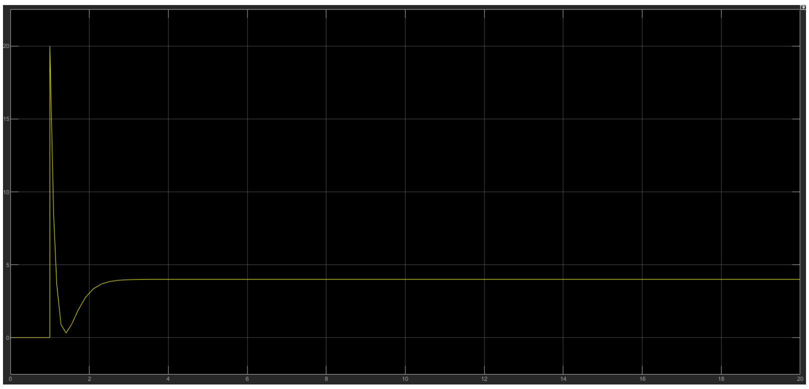

Fig. 3.4 u curve

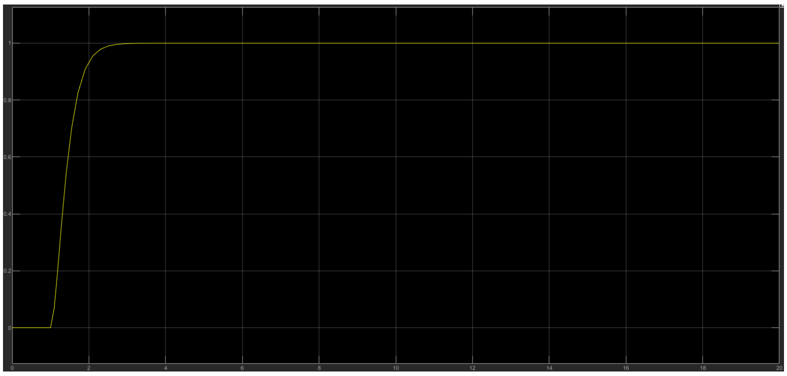

Fig. 3.5 y curve

Affected by the amplified signal, the curve of u which is shown in Fig. 3.4 decreases rapidly monotonically from a peak of 20 from 1s, then rises back up at 1.3s and reaches a steady state after 3s. The output curve y shown in Fig. 3.5 also shows a monotonically increasing trend as the beginning and tends to a steady state at 1 after 3s.

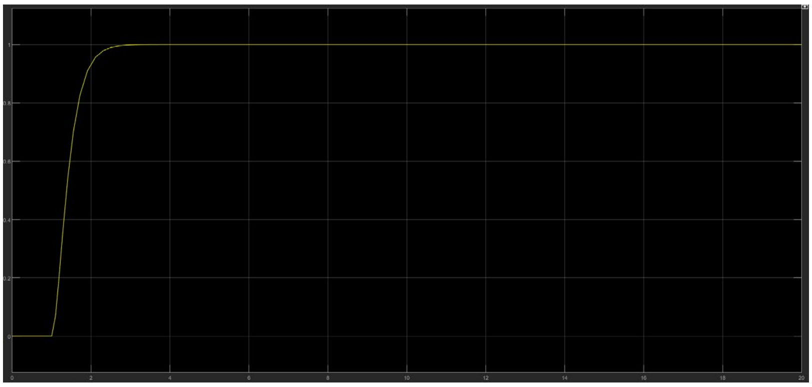

Fig. 3.6 $\hat{x}_1$ from estimator

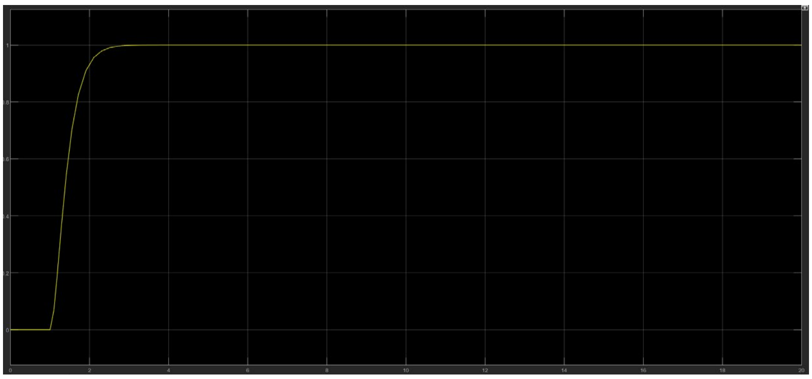

Fig. 3.7 real x_1

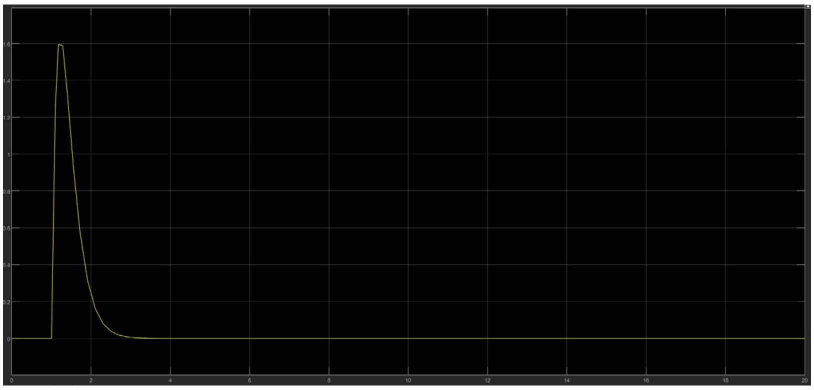

Fig. 3.8 $\hat{x}_2$ from estimator

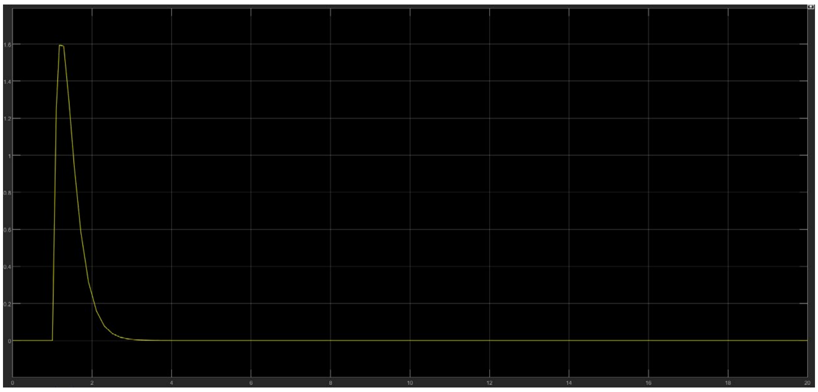

Fig. 3.9 real x_2

According to the Fig. 3.6-3.9, the curve of x_1 shows a monotonically increasing trend as the beginning and tends to a steady state at 1 after 3s. The curve of x_2 decreases monotonically from a peak of 1.6 starting at 1s and also reaches a steady state after 3s, with the value dropping to zero.

By observing and comparing the estimated and observed values of state volumes, the present state observer works very efficiently and achieves a high degree of accuracy in the observation of the real values.

7. Computer based digital control system.

In this section, the computer based digital control system including digital controller algorithm and major hardware components is analysed. And the sampling time required for the computer control system is determined.

7.1 Digital controller

According to the designed system in chapter 2, the continuous time plant is as followed:

$$g(s) = \frac{1}{s^2 + 9s + 20}$$

The structure of the digital control system is shown in Fig. 3.8.

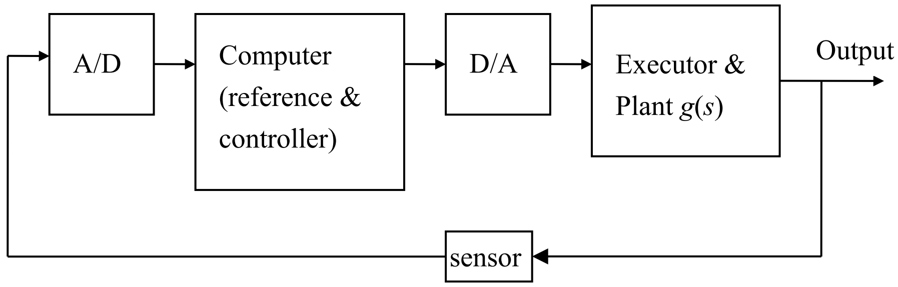

Fig. 3.8 Digital control system

(1) A/D converter

The A/D converter is designed to guarantee the quantization level. The higher the number of bits, the longer it takes to perform the convert operation and the price of the A/D converter increases with the speed and number of bits. Combining precision and hardware resource usage, the quantization level in this design part is less than 0.005 under signal range of –10 to 10 volts for selecting the A/D bits [2].

In this case, we have,

the number of the bits is obtained:

$$x_{max} = 10,\ x_{min} = -10,\ \Delta = 0.005$$

$$m = \frac{x_{\max} - x_{\min}}{\Delta} + 1 = \frac{20}{0.005} + 1 = 4001$$

the number of the bits is obtained:

$$b \geq log_2 m = 11.966$$

Therefore, the A/D converter is selected with 12 bits.

(2) D/A converter

The function of the D/A converter is to convert digital quantities from the computer into voltage quantities, playing the role of sampling and holding. They are relatively inexpensive compared with A/D converter.

7.2 Sampling time

Sampling interval T or sampling rate 1/T can be determined by several rules of thumb:

(a) Sampling rate must be at least twice the bandwidth of the signal.

The bandwidth is not specified in this design.

(b) Sampling interval must be five to ten time shorter than the fastest time constant in a system.

The transfer function is written equivalently as:

$$G(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)} = \frac{\frac{1}{20}}{\left(\frac{1}{4}s + 1\right)\left(\frac{1}{5}s + 1\right)}$$

Time constant of $g(s)$ is:

The sample interval is obtained:

$$T_1 = 0.25\,, \qquad T_2 = 0.2$$

The sample interval is obtained:

$$T = \frac{0.2}{10} \sim \frac{0.2}{5} = 0.02 \sim 0.04$$

And the sampling rate is:

$$f = \frac{1}{0.04} \sim \frac{1}{0.02} = 25 \sim 50$$

References

[1] Franklin G F, Powell J D, Emami-Naeini A, et al. Feedback control of dynamic systems[M]. Upper Saddle River, NJ: Prentice hall, 2002.

[2] Franklin G F, Powell J D, Workman M L. Digital control of dynamic systems[M]. Reading, MA: Addison-wesley, 1998.