Lab 11

1. Starting with the values 1, 2, 4, 4, 5, 6, 9, 11, 12, 12, 17, do the following:
    a. Create a heap H in which these values are the keys.
    b. Perform the insertItem algorithm to insert the value 7 into H. Show all steps.
    c. Perform the removeMin algorithm on H and show all steps.
    d. Represent H in the form of an array A.
    e. Perform the array-based insertItem algorithm to insert 14 into A – show all steps.
    f. Perform the array-based removeMin algorithm on A – show all steps.

2. Carry out the array-based version of HeapSort on the input array

    [1, 4, 3, 9, 12, 2, 4]

    Show steps and outputs along the way. Make sure to distinguish between Phase I and Phase II of the algorithm.

3. The recursive version of BottomUpHeap relies on the Proposition given below. For this exercise, prove the Proposition.

    Proposition. Suppose n is a positive integer of the form 2^h-1 for some h. Then n may be written in the form $n = 1 + m + m$ for some m. Moreover, m must equal $2^{h-1} - 1$.

4. Carry out the steps of the recursive algorithm BottomUpHeap for the input sequence 11, 5, 2, 3, 17, 24, 1

5. A. Prove the following facts about numbers:
    i. $3 \mid x^2 - 1$ for any x that is not a multiple of 3
    ii. $5 \mid x^4 - 1$ for any x that is not a multiple of 5

    Rewrite these statements using mod notation. Then make a guess about the general result: What general fact are these two problems special cases of?

    B. Let $Z_3 = \{0, 1, 2\}$ and $Z_5 = \{0, 1, 2, 3, 4\}$. Which numbers in Z_3 are perfect squares? That is, for which numbers $a \in Z_3$ does there exist an $x \in Z_3$ such that $x^2 \equiv a \bmod 3$? Which numbers a in Z_5 are perfect squares? These numbers a are called *quadratic residues mod 3 (or 5)*. For any odd prime p, how many quadratic residues are there mod p?