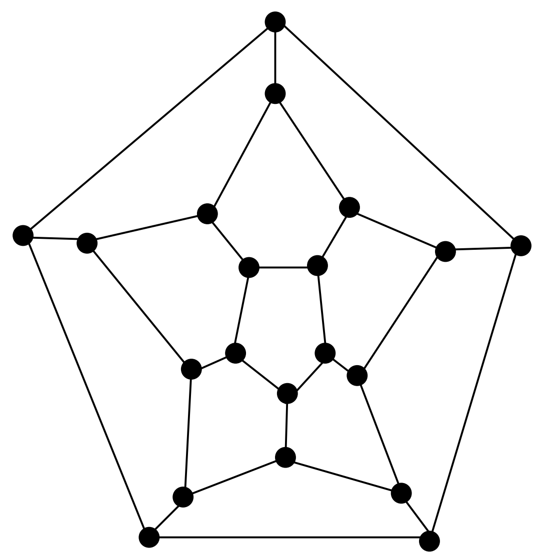
Lab 12

1. *Hamiltonian Graphs.* The following graph has a Hamiltonian cycle. Find it.



2. *Vertex Covers.* Create an algorithm for computing the smallest size of a vertex cover for a graph. The input of your algorithm is a set V of vertices along with a set E of edges. Assume you have the following functions available (no need to implement these):
    - computeEndpoints(edge) – returns the vertices that are at the endpoints of the input edge
    - belongsTo(vertex, set) – returns true if the input vertex is a member of the given set

    Hint: Loop through all subsets of V. For each subset W, check to see if W is a vertex cover. Do this by looping through all edges; for each edge e, check to see if at least one of its endpoints lies in W.

3. The decision problem formulation of the Vertex Cover problem is this: Given a positive integer k, and a graph G, is there a vertex cover for G having size $\leq k$? Show that this decision problem belongs to *NP.* (The definition of the class NP is given in Lecture 1 (Lesson 1) in (approximately) slide 42.)

4. The goal of this exercise is to devise a feasible algorithm that decides whether an input integer is prime. The key fact that you will make use of is the following:

    <u>Fact</u>: There is a function f, which runs in O(log n) (that is, O(length(n))), such that for any odd positive integer n and any a chosen randomly in $[1, n - 1]$, if $f(a, n) = 1$, then

n is composite, but if $f(a,n) = 0$, n is "probably" prime, but is in fact composite with probability < ½.
A first try at such an algorithm would be:

Algorithm **FirstTry**:
Input: A positive integer n
Ouptut: TRUE if n is prime, FALSE if n is composite

```
if n % 2 = 0 return FALSE
a ← random number in [1, n-1]
if f(a,n) = 1
   return FALSE
return TRUE
```

Notice that **FirstTry** runs in O(log n). It also produces a correct result more than half the time.

What could be done to improve the degree of correctness of **FirstTry** but still preserve a reasonably good running time? Explain.

5. Show that if a graph G has |V| -1 edges and has no cycle, then G is connected.
   Hint: Assume G is disconnected, with connected components H_1, H_2, .. H_k. What can you say about each of these components? Do a computation to show that G must in this case have fewer than |V| - 1 edges (giving a contradiction).