Lab 6

1. Show all steps of QuickSort in sorting the array [1, 6, 2, 4, 3, 5]. Use leftmost values as pivots at each step.

2. In our average case analysis of QuickSort, we defined a *good self-call* to be one in which the pivot x is chosen so that number of elements $< x$ is less than $3n/4$, and also the number of elements $> x$ is less than $3n/4$. We call an x with these properties a *good pivot.* When n is a power of 2, it is not hard to see that at least half of the elements in an n-element array could be used as a good pivot (exactly half if there are no duplicates). For this exercise, you will verify this property for the array A = [5, 1, 4, 3, 6, 2, 7, 1, 3] (here, n = 9). Note: For this analysis, use the version of QuickSort in which partitioning produces 3 subsequences *L, E, R* of the input sequence *S.*
    a. Which x in A are good pivots? In other words, which values x in A satisfy:
        i. the number of elements $< x$ is less than $3n/4$, and also
        ii. the number of elements $> x$ is less than $3n/4$
    b. Is it true that at least half the elements of A are good pivots?

3. Explain in your own words what it means to say that "QuickSort runs in $O(n\log n)$ time with high probability". Then explain in two or three sentences the steps for proving this is true (no need to include any math – just a high level discussion).

4. Devise a *fast* algorithm for determining whether a sorted array A of distinct integers contains an element m for which A[m] = m. In this case "fast" means that the algorithm runs faster than $\Theta(n)$ time (in other words, it runs in $O(n)$ time but not $\Theta(n)$ time). You must prove that your algorithm is fast in this sense.

5. In the slides several pivot-selection strategies were mentioned: random selection, selecting leftmost (or rightmost) element, or "median-of-three." Consider the following alternative pivot-selection strategy: Whenever another pivot is needed, use QuickSelect to locate the median value for the array being considered (the median value is the value x for which the number of values in the array that are less than x is (approximately) equal to the number of values greater than x, For instance, 4 is the median value for {1,3,4,7,8} and also for {1,4,5,6}.) What would the expected running time be for QuickSort if this new pivot selection strategy is used? Explain.

6. Show the steps performed by QuickSelect as it attempts to find the median of the array [1, 12, 8, 7, -2, -3, 6]. (The median is the element that is less than or equal to n/2 of the elements in the array. Since n is odd in this case, it is the element whose position lies exactly in the middle. Hint: The median is 6.) For pivots, always use the leftmost element of the current array.