

This code will work for directed graphs too:

An eg input would be:

```
int graph[V][V] = {  
    {0, 4, 0, 0, 0, 0, 0, 8, 0},  
    {0, 0, 8, 0, 0, 0, 0, 0, 0},  
    {0, 0, 0, 7, 0, 4, 0, 0, 2},  
    {0, 0, 0, 0, 9, 14, 0, 0, 0},  
    {0, 0, 0, 0, 0, 10, 0, 0, 0},  
    {0, 0, 4, 0, 0, 0, 2, 0, 0},  
    {0, 0, 0, 0, 0, 0, 0, 1, 6},  
    {0, 0, 0, 0, 0, 0, 0, 0, 7},  
    {0, 0, 2, 0, 0, 0, 6, 0, 0}};
```

The time complexity of Dijkstra's algorithm is  $O(V^2)$  for a dense graph and  $O(E \log V)$  for a sparse graph, where  $V$  is the number of vertices and  $E$  is the number of edges in the graph<sup>1</sup>. The auxiliary space required is  $O(V)$  to store the distance values of vertices<sup>1</sup>.

To improve the time complexity of Dijkstra's algorithm, we can use a min-priority queue instead of a linear search to find the vertex with the smallest tentative distance.

Below is the algorithm based on the above idea:

Initialize the distance values and priority queue.

Insert the source node into the priority queue with distance 0.

While the priority queue is not empty:

    Extract the node with the minimum distance from the priority queue.

    Update the distances of its neighbors if a shorter path is found.

    Insert updated neighbors into the priority queue.

Hence the Time Complexity will be  $O((V + E) \log V)$  as:

Initialization:  $O(V)$

    Setting up distance array (dist), priority queue, and other auxiliary arrays.

Main Loop:  $O(V \log V + E \log V)$

The main loop runs  $V$  times, and in each iteration, the minimum distance vertex is extracted from the priority queue, which takes  $O(\log V)$  time.

For each vertex, its adjacent vertices are relaxed (distance updated) if a shorter path is found.

This can happen at most once for each edge, leading to a total of  $E$  relaxation operations.

So, the time complexity of the main loop is  $O(V \log V + E \log V)$ , which can be simplified to  $O((V + E) \log V)$ .

The time complexity and auxiliary space complexity you've mentioned,  $O((V + E) \log V)$  and  $O(V)$ , respectively, correspond to an optimized version of Dijkstra's algorithm using a priority queue or a min-heap data structure.

Auxiliary Space Complexity:  $O(V)$

dist array:  $O(V)$  - To store the shortest distances from the source vertex to all other vertices.

priority queue:  $O(V)$  - In the worst case, all vertices can be added to the priority queue.

The relevant data structures for implementing Dijkstra's algorithm are:

Adjacency matrix: A 2D array of size  $V \times V$  where  $V$  is the number of vertices in the graph. If there is an edge between vertex  $i$  and vertex  $j$ , then the value of the matrix at  $(i, j)$  is the weight of the edge. Otherwise, it is 0.

Adjacency list: An array of linked lists where each element of the array represents a vertex and the linked list contains the adjacent vertices and their weights.

Min-priority queue: A data structure that supports the following operations:

Insertion of an element with a key

Deletion of the element with the minimum key

Decrease-key operation to decrease the key of an element<sup>1</sup>.