HW5 Signal   Matin Zivdar   97243037

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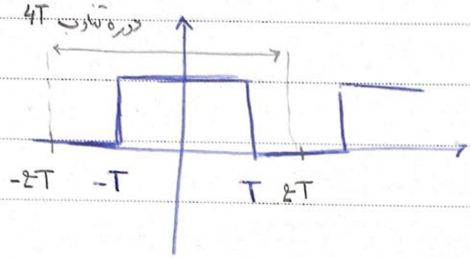
سری فوریه‌ی سیگنال‌ها، موج‌های زیر را بدست آورید.



جا موج مربعی :

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t).dt = \frac{1}{4T}\int_{-T}^{T} 1.dt = \frac{1}{2}$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t).e^{-jk\omega_0 t}dt = \frac{1}{4T}\int_{-T}^{T} 1.e^{-jk\omega_0 t}dt = \frac{1}{4T}\left[\frac{1}{-jk\omega_0}e^{-jk\omega_0 t}\right]_{-T}^{T}$$

$$= \frac{1}{4T}\cdot\frac{1}{-jk\omega_0}\Big[\underbrace{e^{-jk\omega_0 T} - e^{-jk\omega_0(-T)}}_{-2j\sin(k\omega_0 T)}\Big] = \frac{\sin(k\omega_0 T)}{2Tk\omega_0}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4T} = \frac{\pi}{2T} \Rightarrow a_k = \frac{1}{k\pi}.\sin\left(\frac{k\pi}{2}\right)$$

$$\hat{x}(t) = \sum_{-\infty}^{+\infty} a_k\, e^{jk\omega_0 t}$$

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Sin پ)

$$\sin(wt) = \frac{1}{2j}\left(e^{jw_0t} - e^{-jw_0t}\right) \xrightarrow{w_0 = \frac{2\pi}{2\pi} = 1} \frac{1}{2j}\left(e^{jt} - e^{-jt}\right)$$

$$\longrightarrow \sin t = \underbrace{\frac{1}{2j}}_{a_1} e^{j(1)t} \underbrace{- \frac{1}{2j}}_{a_{-1}} e^{j(-1)t}$$

$$a_k = \begin{cases} \frac{1}{2j} & k=1 \\ -\frac{1}{2j} & k=-1 \\ 0 & k=\# \end{cases} \qquad \hat{x}(t) = \sum_{-\infty}^{+\infty} a_k \cdot e^{jkt}$$

Cos ت)

$$\cos(wt) = \frac{1}{2}\left(e^{jt} + e^{-jt}\right) \xrightarrow{w_0 = \frac{2\pi}{2\pi} = 1} \frac{1}{2}\left(e^{jt} + e^{-jt}\right)$$

$$\longrightarrow \cos t = \underbrace{\frac{1}{2}}_{a_1} e^{j(1)t} + \underbrace{\frac{1}{2}}_{a_{-1}} e^{j(-1)t}$$

$$a_k = \begin{cases} \frac{1}{2} & k=-1,1 \\ 0 & k=\# \end{cases} \qquad \hat{x}(t) = \sum_{-\infty}^{+\infty} a_k \cdot e^{jkt}$$

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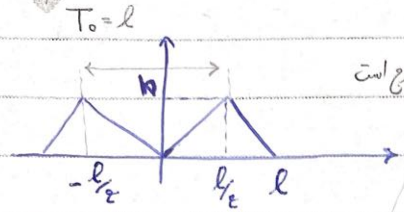
ش ۱) تابع مثلثی با بسط نیم بازه Cos



چون تابع زوج است

$$f(t)=\begin{cases}\frac{2h}{\ell}\cdot t & 0<t<\frac{\ell}{2}\\ -\frac{2h}{\ell}\cdot t & -\frac{\ell}{2}<t<0\end{cases}$$

$$a_0=\frac{1}{T_0}\int_{T_0}x(t)\,dt=\frac{2}{\ell}\int_0^{\ell/2}\frac{2h}{\ell}\cdot t=\frac{2}{\ell}\left[\frac{h}{\ell}\cdot t^2\right]_0^{\ell/2}=\frac{2}{\ell}\left[\frac{\ell^2}{4}\cdot\frac{h}{\ell}\right]=\frac{h}{2}$$

$$a_k=\frac{1}{T_0}\int_{T_0}x(t)\cdot e^{-jkw_0t}\,dt=\frac{1}{\ell}\left[\int_{-\frac{\ell}{2}}^{0}-\frac{2h}{\ell}\cdot t\cdot e^{-jkw_0t}\,dt+\int_0^{\ell/2}\frac{2h}{\ell}\cdot t\cdot e^{-jkw_0t}\,dt\right]$$

$$=\frac{2h}{\ell^2}\left[\int_0^{\ell/2}t\cdot e^{-jkw_0t}\,dt-\int_{-\frac{\ell}{2}}^{0}t\,e^{-jkw_0t}\cdot dt\right]$$

$$\int \underbrace{t}_{u}\cdot\underbrace{e^{-jkw_0t}\,dt}_{dv}=t\cdot\frac{e^{-jkw_0t}}{-jkw_0}-\int\frac{e^{-jkw_0t}}{-jkw_0}\,dt=\frac{t\cdot e^{-jkw_0t}}{-jkw_0}-\frac{e^{-jkw_0t}}{(jkw_0)^2}$$

$$=e^{-jkw_0t}\left(\frac{-tjkw_0-1}{(jkw_0)^2}\right)=\frac{-e^{-jkw_0t}\cdot(jkw_0t+1)}{(jkw_0)^2}$$

$$\longrightarrow a_k=\frac{2h}{\ell^2}\left[\frac{-e^{-jkw_0\frac{\ell}{2}}(jkw_0\cdot\frac{\ell}{2}+1)+e^0(1)+e^0(1)-e^{jkw_0\frac{\ell}{2}}(-jkw_0\frac{\ell}{2}+1)}{(jkw_0)^2}\right]$$

$$w_0=\frac{2\pi}{T_0}=\frac{2\pi}{\ell}\longrightarrow a_k=\frac{2h}{\ell^2}\left[\frac{e^{-jk\pi}(jk\pi+1)+e^{jk\pi}(-jk\pi+1)-2}{(kw_0)^2}\right]$$

$$\hat{x}(t)=\sum_{-\infty}^{+\infty}a_k\cdot e^{jkw_0t}$$

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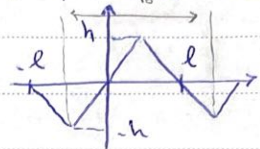
ج) تابع مثلثی با بسط نیم بازه Sin

$$f(t)\begin{cases}\frac{2h}{\ell}\left(t-\frac{\ell}{2}\right)+h=\frac{2h}{\ell}t & -\frac{\ell}{2}<t<\frac{\ell}{2}\\ -\frac{2h}{\ell}\left(t-\frac{\ell}{2}\right)+h=-\frac{2h}{\ell}t+2h & \frac{\ell}{2}<t<\frac{3\ell}{2}\end{cases}$$



۰ تابع فرد

$$a_0=\frac{1}{T_0}\int_{T_0}x(t)\,dt=\frac{1}{2\ell}\left[\cancel{\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}}\frac{2h}{\ell}t\,dt}+\int_{\frac{\ell}{2}}^{\frac{3\ell}{2}}-\frac{2h}{\ell}t+2h\,dt\right]$$

$$=\frac{h}{2\ell}\left[-\frac{t^2}{\ell}+2t\right]_{\ell/2}^{3\ell/2}=\frac{h}{2\ell}\left(-\frac{(2\ell^2)}{\ell}+2\ell\right)=0$$

$$a_k=\frac{1}{T_0}\int_{T_0}x(t)\cdot e^{-jk\omega_0 t}\,dt=\frac{1}{2\ell}\left[\frac{2h}{\ell}\underbrace{\int_{-\ell/2}^{\ell/2}t\cdot e^{-jk\omega_0 t}\,dt}_{A}+\int_{\ell/2}^{3\ell/2}(\underbrace{-\frac{2h}{\ell}t}_{B}+\underbrace{2h}_{C})e^{-jk\omega_0 t}\,dt\right]$$

(A) $$\int_{-\frac{\ell}{2}}^{\ell/2}\underbrace{t}_{u}\cdot\underbrace{e^{-jk\omega_0 t}\,dt}_{dv}=\cancel{\frac{t\cdot e^{-jk\omega_0 t}}{-jk\omega_0}}-\int\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\,dt=\frac{t\cdot e^{-jk\omega_0 t}}{-jk\omega_0}-\frac{e^{-jk\omega_0 t}}{(jk\omega_0)^2}$$

$$=\frac{1}{-jk\omega_0}\left(\frac{\ell}{2}\cdot e^{-jk\omega_0\ell/2}+\frac{\ell}{2}\cdot e^{jk\omega_0\ell/2}\right)+\frac{1}{-(jk\omega_0)^2}\cdot\left(e^{-jk\omega_0\ell/2}-e^{jk\omega_0\ell/2}\right)$$

(C) $$\int_{\ell/2}^{3\ell/2}e^{-jk\omega_0 t}\,dt=\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Bigg|_{\ell/2}^{3\ell/2}=\frac{1}{-jk\omega_0}\left(e^{-jk\omega_0\frac{3\ell}{2}}-e^{-jk\omega_0\ell/2}\right)$$

$$\omega_0=\frac{2\pi}{T_0}=\frac{2\pi}{2\ell}=\frac{\pi}{\ell}\longrightarrow\omega_0\ell=\pi$$

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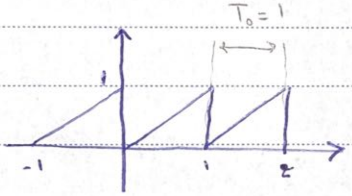
Ⓑ $\displaystyle\int_{\ell/2}^{3\ell/2} t\cdot e^{-jk\omega_0 t}\,dt = \left[\frac{t\cdot e^{-jk\omega_0 t}}{-jk\omega_0} - \frac{e^{-jk\omega_0 t}}{(jk\omega_0)^2}\right]_{\ell/2}^{3\ell/2}$

$$= \frac{1}{-jk\omega_0}\left(\frac{3\ell}{2}\cdot e^{-jk\omega_0 \frac{3\ell}{2}} - \frac{\ell}{2}\cdot e^{-jk\omega_0 \ell/2}\right) + \frac{1}{(jk\omega_0)^2}\left(e^{-jk\omega_0 \cdot 3\ell/2} - e^{-jk\omega_0 \ell/2}\right)$$

$$a_k = \frac{h}{\ell^2}\cdot Ⓐ - \frac{h}{\ell^2} Ⓑ + \frac{h}{\ell} Ⓒ \qquad , \quad \hat{x} = \sum_{-\infty}^{+\infty} a_k \cdot e^{-jk\omega_0 t}$$



ج) موج دندان اره‌ای

$$x(t) = t - \underbrace{\lfloor t \rfloor}_{floor(t)}$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \int_0^1 t\,dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \int_0^1 \underbrace{t}_{u}\cdot \underbrace{e^{-jk\omega_0 t}\,dt}_{dv} = \frac{t\cdot e^{-jk\omega_0 t}}{-jk\omega_0} - \int \frac{e^{-jk\omega_0 t}}{-jk\omega_0}\,dt$$

$$= \left[\frac{t\cdot e^{-jk\omega_0 t}}{-jk\omega_0} - \frac{e^{-jk\omega_0 t}}{(jk\omega_0)^2}\right]_0^1 = \left(\frac{e^{-jk\omega_0}}{-jk\omega_0} - 0\right) - \left(\frac{e^{-jk\omega_0} - e^{0}}{(jk\omega_0)^2}\right)$$

$$a_k = \frac{e^{-jk\omega_0}}{-jk\omega_0} + \frac{e^{-jk\omega_0} - 1}{(k\omega_0)^2}$$

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الف) $Sinc(n) = \frac{Sin(X\pi)}{X\pi}$

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ج) $F_{(n)} = \delta[N]$

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د) پالس مربع

تابع پیوسته نیست ⟸ با روش های فعلی نمی‌توان

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