Colloquium Attendance #1: Calculus and Prime Numbers
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Topic: Calculus is about continuous, smooth, differential functions, but functions describing primes typically are not. The talk explores the extent to which calculus can be used to the study of primes. It beings by defining fermat numbers $F_n = 2^{2^n} + 1$,which are conjectured to be prime. unfortunately, for $n > 4$, not prime!

Then, some open questions were explored:

1. Are any F_n prime?
2. Are the number of F_n prime finite?
3. Is F_{33} prime or composite?

Then, the Mersenne numbers, $M_p = 2^p - 1$ where p is prime, were defined. Some interesting facts were then explored. For example, the largest prime known is $M_{74,207,281}$! Then the notion of prime deserts where discussed. A proof for why prime deserts of arbitrary length, say k, necessarily exist. This is because $k! + 1, \cdots, k$ is necessarily a k prime desert.

The talk then pivoted to discuss the search for a smooth function to approximate $\pi(x)$, which results in the prime number theorem, which states:

$$\lim_{x \to \infty} \frac{\pi(x)}{x/\ln(x)} = 1$$

In this vein, the talk explored an experiment by Guass, where he counted 6272 primes between 2,600,000 and 2,700,000. He then computed

$$\int_{2,600,000}^{2,600,000} \frac{dt}{\ln t} = 6271.72$$

by hand. Very surprising how similar they are! The talk concluded by discussing applications to the Riemann hypothesis.

Strengths: Introduces topics well with examples and relating back to course material. Very clear, interesting and exciting!

Weaknesses: $\varnothing$