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Probability, Bayes theorem

Question 1A:

$$P(twins) = P(Fraternal\ twins \cap tiwns) + P(Identical\ twins \cap tiwns) = \frac{1}{125} + \frac{1}{300} = \frac{17}{1500}$$

$$P(Identical\ twins / twins) = \frac{P(Identical\ twins \cap tiwns)}{P(twins)} = \frac{\frac{1}{300}}{\frac{17}{1500}} = \frac{5}{17}$$

Given that Elvis was a twin, the probability of him being an identical twin is $\frac{5}{17}$.

Question 1B:

$$\begin{aligned} P(chocolate) &= P(chocolate \cap bowl\ 1) + P(chocolate \cap bowl\ 2) \\ &= 0.5 \cdot \frac{30}{30+10} + 0.5 \cdot \frac{20}{20+20} \\ &= \frac{30}{80} + \frac{20}{80} \\ &= \frac{5}{8} \end{aligned}$$

$$P(bowl\ 1 / chocolate) = \frac{P(chocolate \cap bowl\ 1)}{P(chocolate)} = \frac{0.5 \cdot \frac{30}{30+10}}{\frac{5}{8}} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

The probability that Arik chose bowl 1 when he got a chocolate cookie is $\frac{3}{5}$

Question 2:

$$P(G1996\ \cap Y1994) = \frac{20}{100} \cdot \frac{20}{100} = \frac{4}{100}$$

$$P(G1994 \cap Y1996) = \frac{10}{100} \cdot \frac{14}{100} = \frac{7}{500}$$

$$P(G\ and\ Y\ from\ 1994\ and\ 1996) = \frac{4}{100} + \frac{7}{500} = \frac{27}{500}$$

$$P(G1996 \cap Y1994 / G \text{ and } Y \text{ from } 1994 \text{ and } 1996) = \frac{\frac{4}{100}}{\frac{27}{500}} = \frac{20}{27}$$

Given that you got a yellow and green M&M, the probability that the yellow M&M came from the 1994 bag is $\frac{20}{27}$

Question 3A:

$$P(\text{positive test} / \text{healthy}) = \frac{1}{100}$$

$$P(\text{have flu}) = \frac{1}{10,000}$$

$$\begin{aligned} P(\text{positive test}) &= P(\text{healthy} \cap \text{positive test}) + P(\text{have flu} \cap \text{positive test}) \\ &= P(\text{positive test} / \text{healthy})P(\text{healthy}) + P(\text{positive test} / \text{have flu})P(\text{have flu}) \\ &= \frac{1}{100} \cdot \frac{9,999}{10,000} + \frac{1}{10,000} \cdot 1 \\ &= 0.009999 + 0.0001 \\ &= 0.010099 \end{aligned}$$

$$P(\text{have flu} / \text{positive test}) = \frac{\frac{1}{10,000}}{0.010099} = \frac{100}{10099} \approx 0.0099$$

The probability that you have swine flu is about 0.0099

Question 3B:

$$P(\text{have flu}) = \frac{1}{200}$$

$$\begin{aligned} P(\text{positive test}) &= \frac{1}{100} \cdot \frac{199}{200} + \frac{1}{200} \cdot 1 \\ &= 0.00995 + 0.005 \\ &= 0.01495 \end{aligned}$$

$$P(\text{have flu} / \text{positive test}) = \frac{\frac{1}{200}}{0.01495} = \frac{100}{299} \approx 0.3344$$

The probability you have swine flu given that you came from Thailand is about 0.3344.

Random Variables

Question 1:

Roi can only win if you get $12,\ 9,\ 6$ or 3

$$P(get:12,9,6,3) = \frac{1}{36} + \frac{4}{36} + \frac{5}{36} + \frac{2}{36} = \frac{12}{36} = \frac{1}{3}$$

$$E[Gain\ from\ 3\$\ bet] = 6\$ \cdot \frac{1}{3} - 3 \cdot \left(1 - \frac{1}{3}\right) = 2 - 2 = 0$$

Roi's expected value of playing the game is $0\$$.

Question 2:

Alex wins only if he gets a total of $13,\ 14,\ 15$

$$P(get:13,14,15) = \frac{3}{25} + \frac{2}{25} + \frac{1}{25} = \frac{6}{25}$$

$$P(get:12) = \frac{4}{25}$$

$$P(get:less\ than\ 12) = 1 - \frac{6+4}{25} = \frac{15}{25} = \frac{3}{5}$$

$$E[gain\ from\ 6\$\ bet] = 5 \cdot \frac{6}{25} + 0 \cdot \frac{4}{25} - 6 \cdot \frac{3}{5} = -2.4$$

Alex's expected value of playing the game is $-2.4\$$.

Question 3:

Since this is a Binomial distribution

mean:

$$\mu = 0.4 \cdot 8 = 3.2$$

std:

$$\begin{aligned} \sigma &= \sqrt{P(male) \cdot (1 - P(male)) \cdot 8} \\ &= \sqrt{0.4 \cdot 0.6 \cdot 8} \\ &= \frac{4\sqrt{5}}{5} \end{aligned}$$

The mean of the number of males is 3.2 and the standard deviation is $\frac{4\sqrt{5}}{5}$.

Question 4:

$$\mu = 26.000\$$$
$$\sigma = 2,000\$$$

Since the prices are normally distributed, and according to the 68 95 99.7 rule we have that

$$P(\mu - 1\sigma \leq X \leq \mu + 1\sigma) = P(24 \leq X \leq 28) = 0.68$$
$$P(24 \leq X \leq 26) = 0.34$$

$$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) = P(22 \leq X \leq 30) = 0.95$$
$$P(22 \leq X \leq 26) = 0.34 + 0.136 = 0.476$$

$$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) = P(20 \leq X \leq 32) = 0.997$$
$$P(20 \leq X \leq 26) = 0.476 + 0.021 = 0.497$$
$$P(30 \leq X \leq 32) = 0.021$$

$$\begin{aligned} P(26 < X < 30) &= P(20 \leq X \leq 32) - P(30 \leq X \leq 32) - P(20 \leq X \leq 26) \\ &= 0.997 - 0.497 - 0.021 \\ &= 0.479 \end{aligned}$$

Question 5:

Since the distribution is 2 linear functions, we have that

$$f(x) = \frac{0.4}{3}x = \frac{2}{15}x$$

for $0 \leq x \leq 3$.

We need to only find the area under the function

$$P(X \leq 3) = \int_0^3 \frac{2}{15}x\,dx = \frac{2}{15} \cdot \frac{3^2}{2} - \frac{2}{15} \cdot \frac{0^2}{2} = \frac{9}{15}$$
$$P(X > 3) = 1 - \frac{9}{15} = \frac{6}{15}$$

Question 6:

$$T = has\ child, F = doesn't\ have\ child$$
$$P(T) = 0.6$$
$$P(F) = 0.4$$
$$P(T,T,T,F) = 0.6^3 \cdot 0.4 = 0.0864$$

Since there are 4 combinations possibe

$$P(1\ out\ of\ 4\ doesn't\ have\ child) = 4 \cdot 0.0864 = 0.3456$$

Question 7:

$$E(x) = -10 \cdot 0.1 - 5 \cdot 0.35 + 0 \cdot 0.1 + 5 \cdot 0.35 + 10 \cdot 0.1 = 0$$

The expected value of x is 0