

SI100B
Introduction to Information
Science and Technology
(Electrical Engineering)

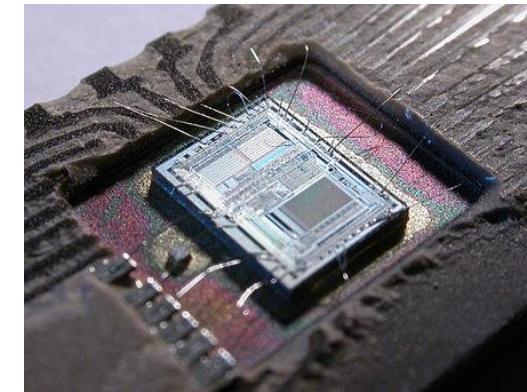
Lecture #4 (Digital)
Sequential Logic Circuits

Instructor: Junrui Liang (梁俊睿)
Sept. 30th, 2022

The Theme Story



Devices (1)



Circuits (2/4)

Systems (1)



(Pictures are from the Internet)

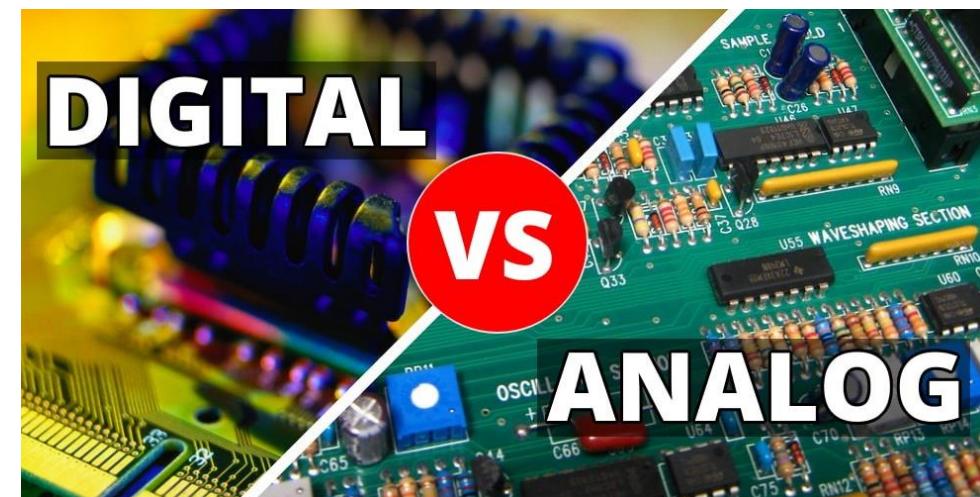
Study Purpose of Lecture #4

- 哲学 (bao'an) 三问
 - Who are you?
 - Where are you from?
 - Where are you going?

To answer those questions
throughout your life



- In this lecture, we ask
 - What is the fundamental difference between combinational and sequential logic circuits?
 - What is **latch** 锁存器? What is **flip-flop** 触发器?
 - How to use **finite state machine** 有限状态机 to build useful applications?

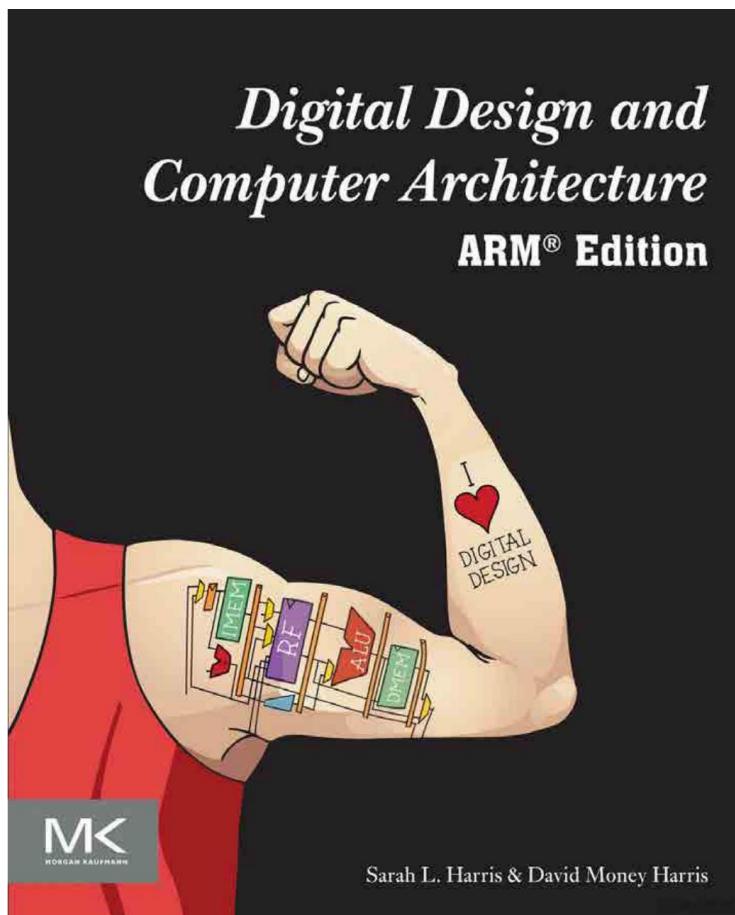


(Pictures are from the Internet)

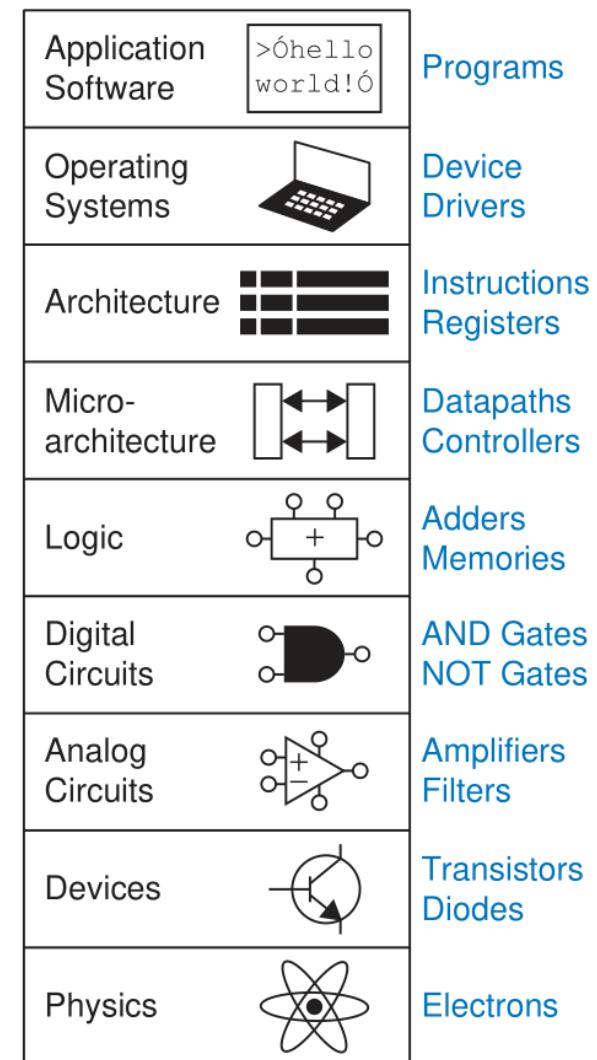
Lecture Outline

- Different levels of abstraction (complementary)
- Computer configuration (complementary)
- Sequential logic circuits 时序逻辑电路
 - definition
- Latches and flip-flop 锁存器与触发器
 - The evolution of latches and flip-flops
 - Synchronous and asynchronous circuits 同步与异步电路
 - Example: 4 × 3 memory
- Finite state machines (FSM) 有限状态机
 - Traffic light example

Different levels of abstraction

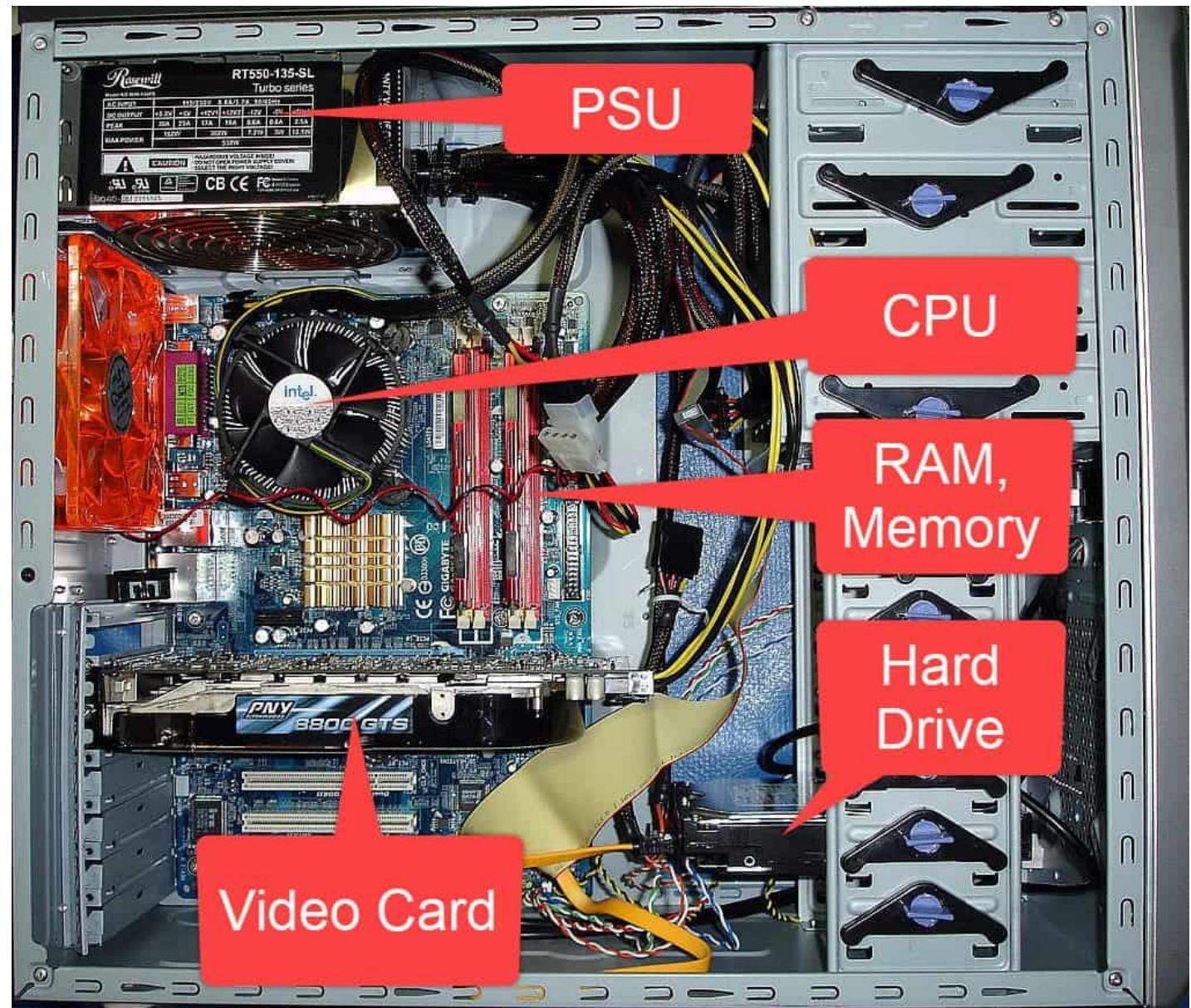
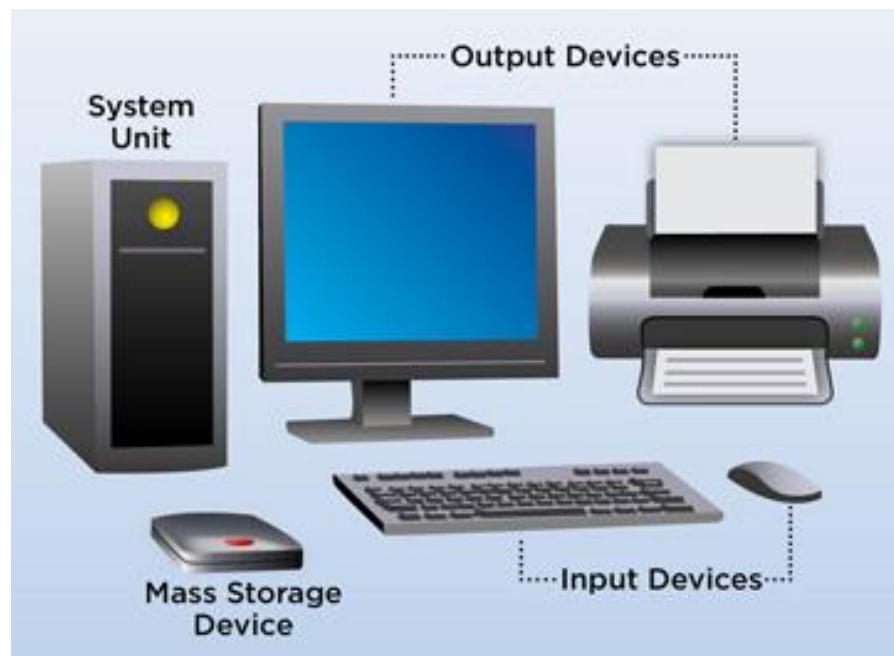


Grasp your spoon!

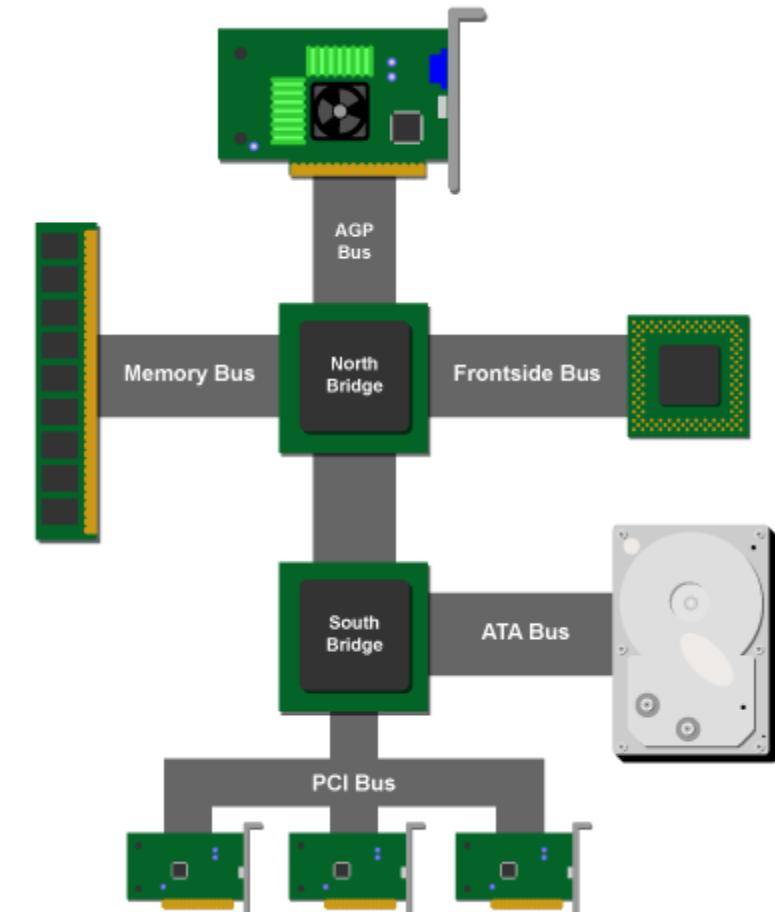
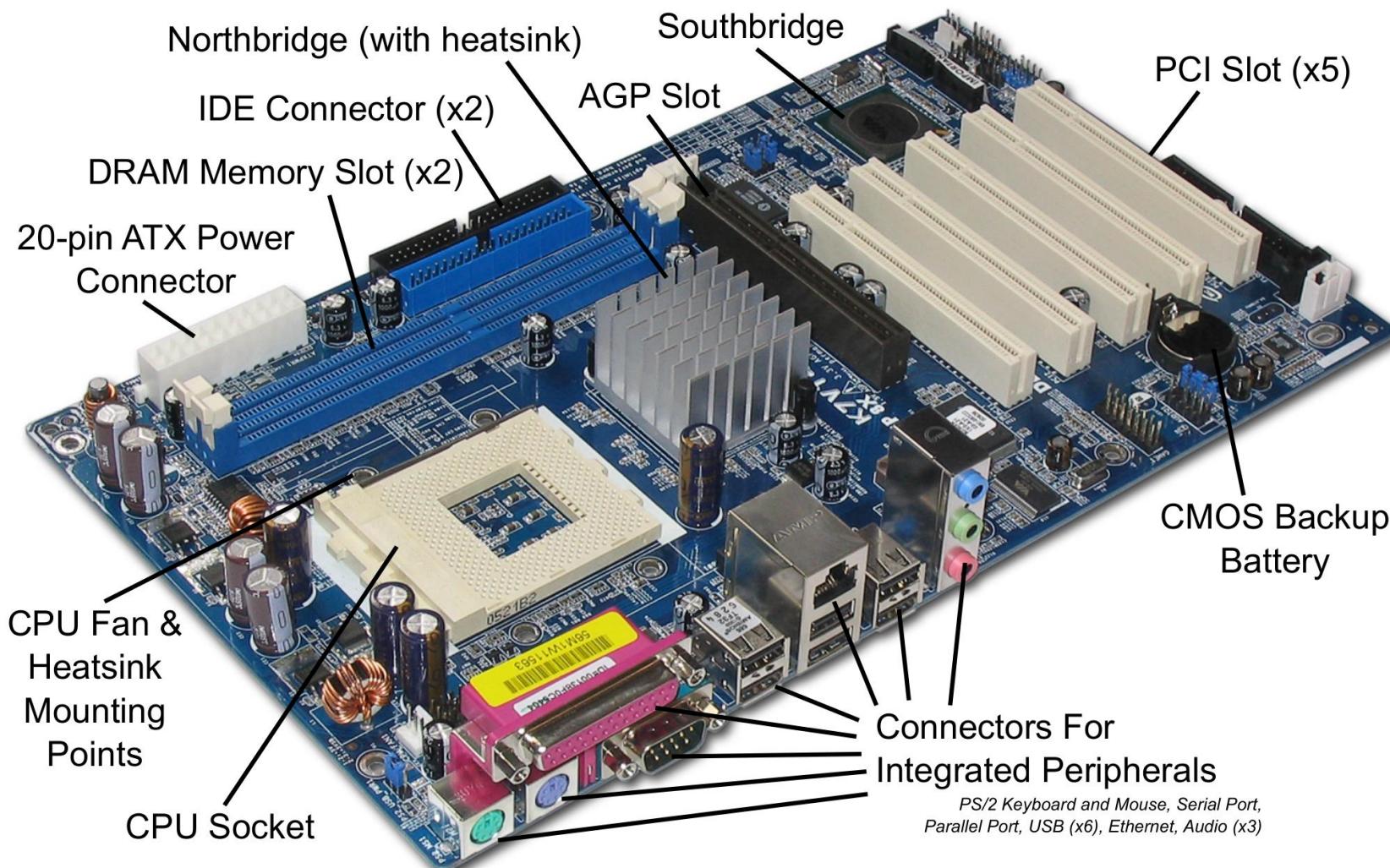


**Figure 1.1 Levels of abstraction
for an electronic computing system**

Inside a PC (personal computer)



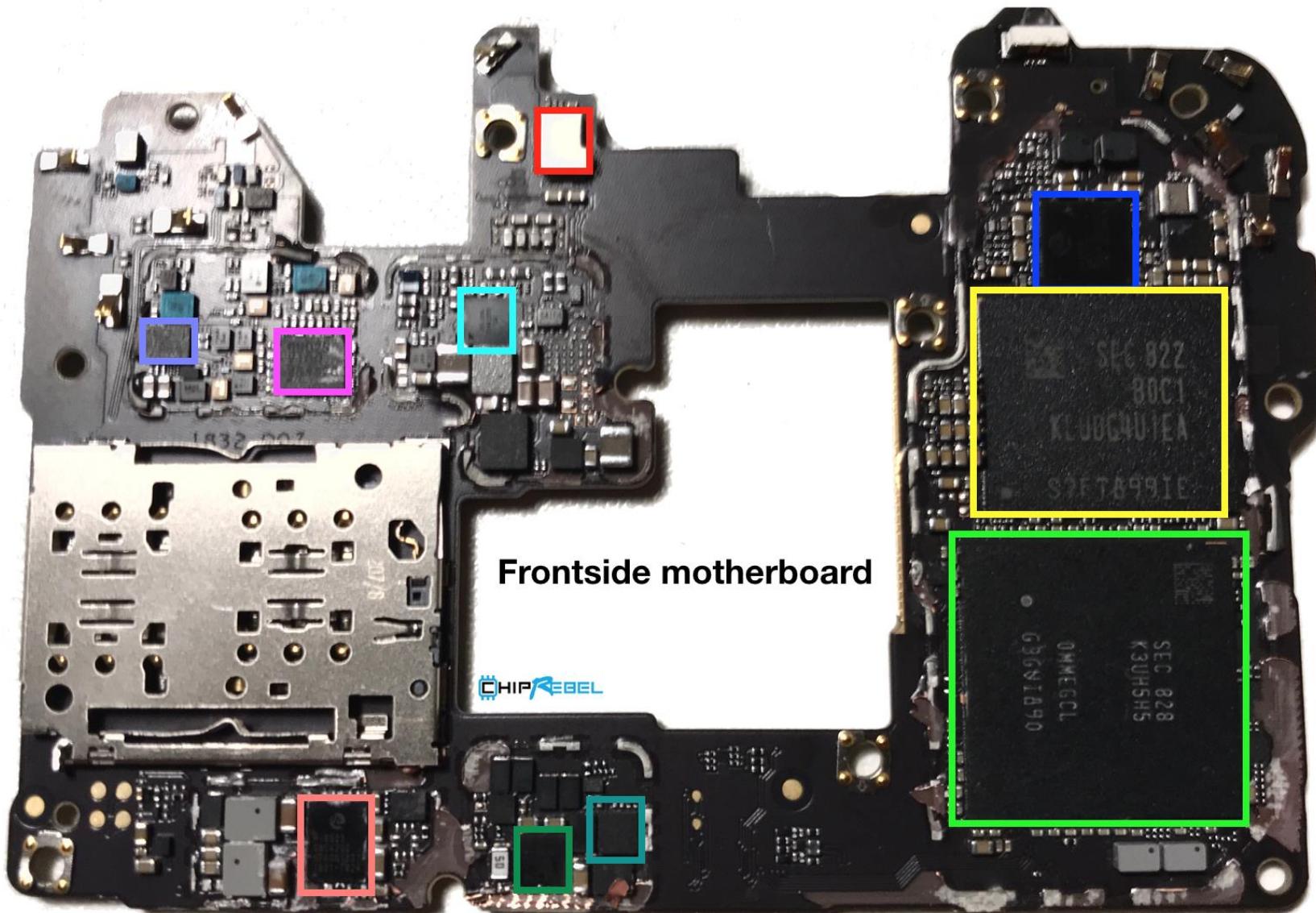
A PC motherboard 主板



Inside a cellphone (Huawei Mate 20 teardown)



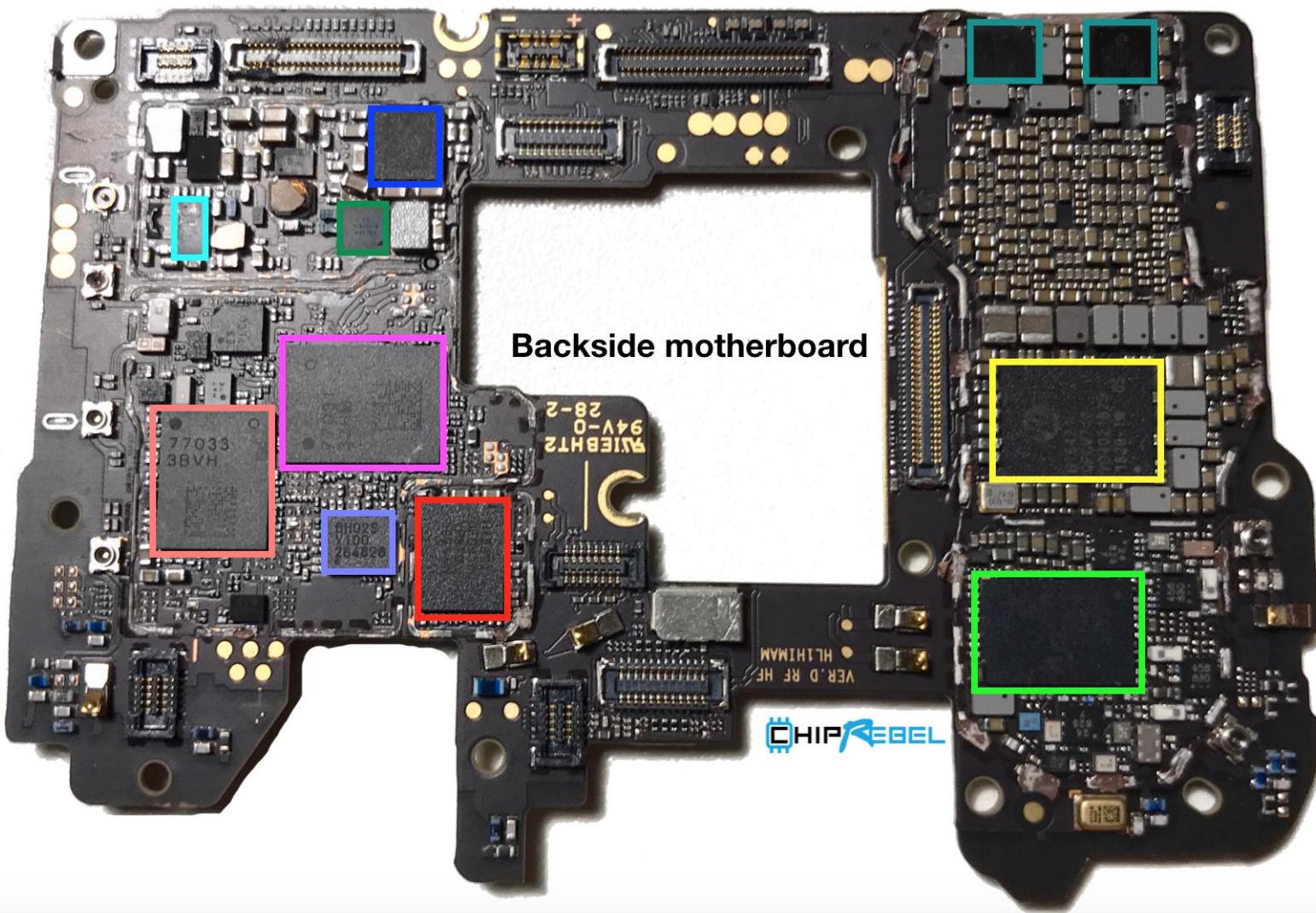
Inside a cellphone (frontside)



Huawei Mate 20 Chip package markings:

- SEC 828 K3UH5H5 which is Samsung's LPDDR4X SDRAM chip on top of the Kirin 980 AP. Package-on-Package
- SEC 822 BOC1 KLUDG4U1EA eUFS Samsung 128GB NAND Flash memory
- Hi 6403 Audio Codec
- 6H02s Y100
- 35L36A B1AE1815
- 9498UK T83206 Z1D808
- GJ9BK
- Hi 6523 Battery Charger
- M12x6Y2

Inside a cellphone (backside)

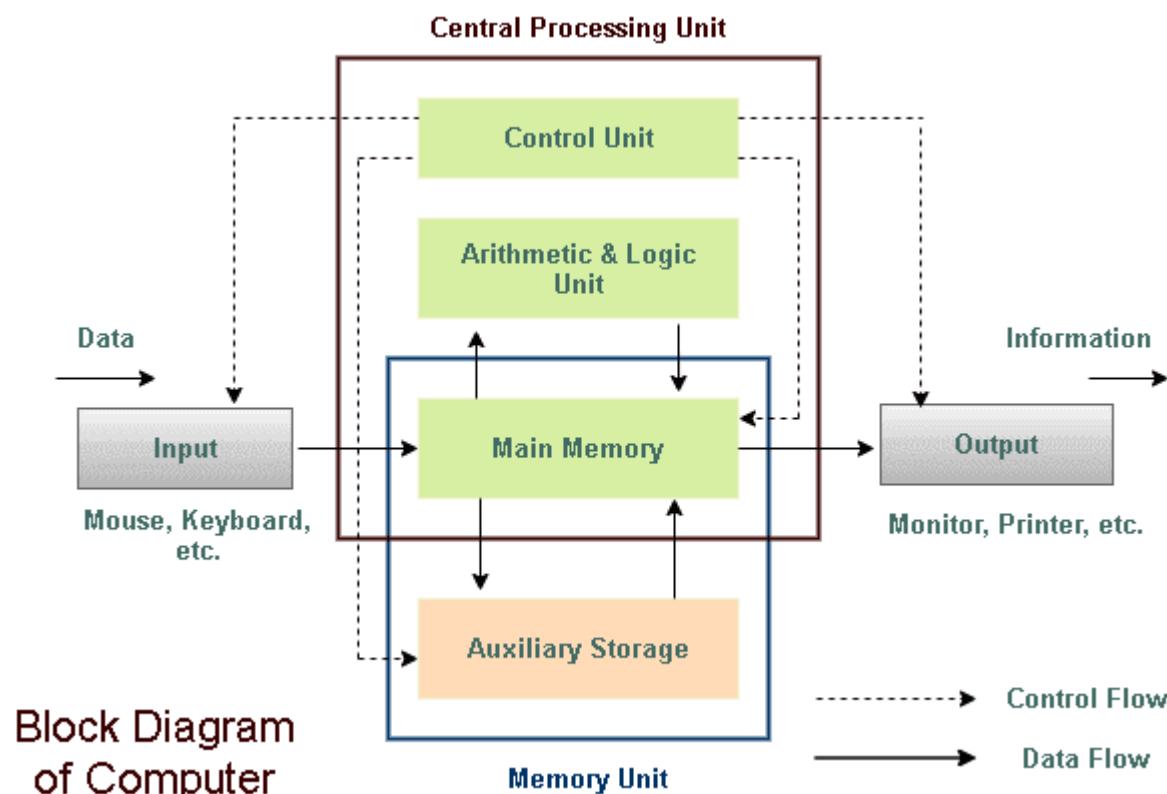


Huawei Mate 20 Chip package markings:

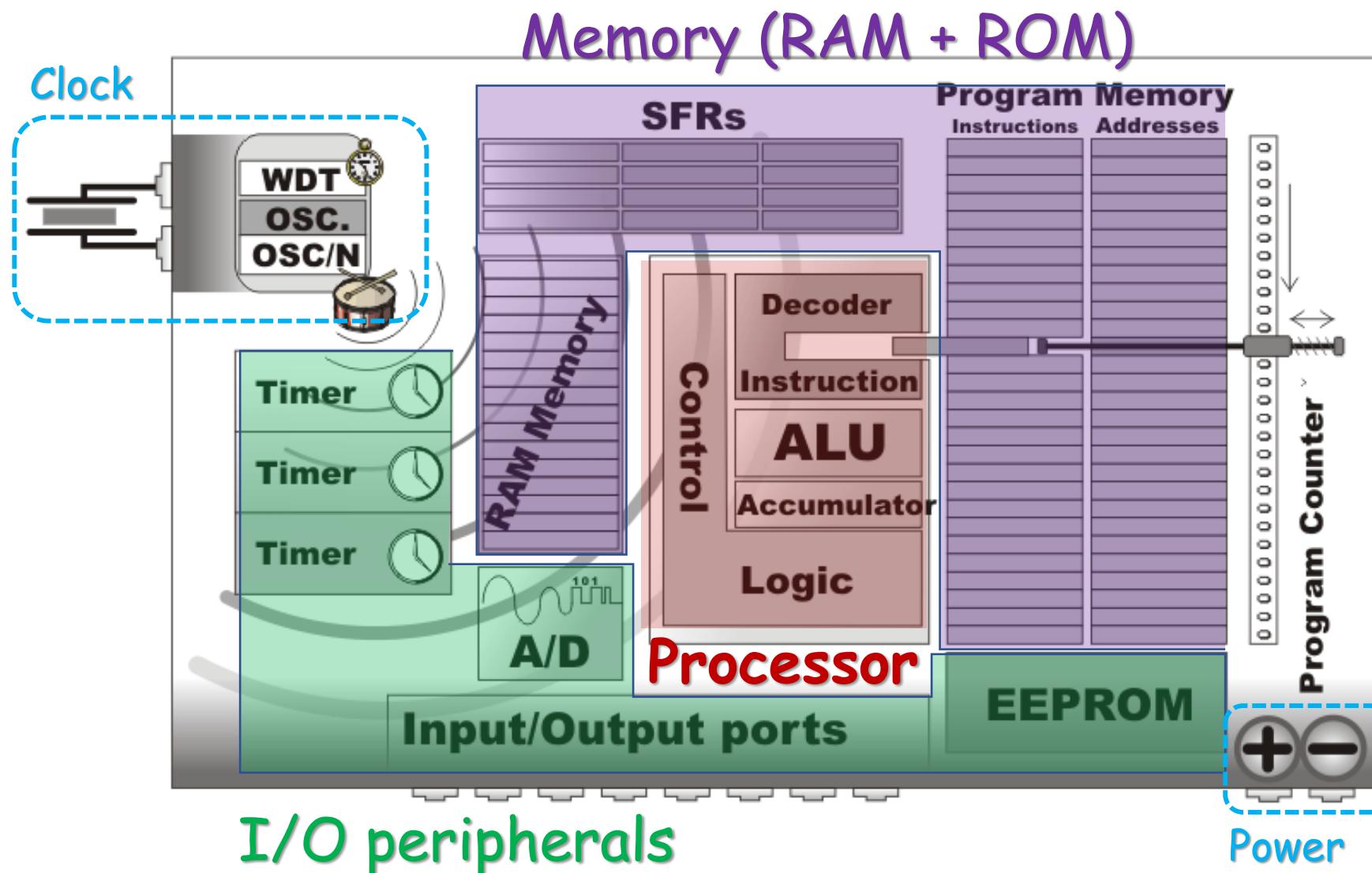
- Hi 1103 GFCV110
HDGTC1830 WiFi chipset
- Hi 6421 6FCV7-16
YD4021829 PMIC
- Hi 6422 Envelope tracker
- NXP 80T37 3403 SSD827
NFC controller
- 35L36A B1AE1815
- 77031 32HM Qorvo Front
End Module mid/high
band
- 77033 38HV Qorvo Front
End Module low/very low
band
- 8H02S V100 284828
- Hi 6363 GFC V100
YP8203830 RF
Transceiver
- Qorvo Rf 8129 envelope
tracker

Computer Systems - Von Neumann Architecture 冯诺依曼结构

- Three major parts
 - CPU 中央处理器 (Central Processing Unit)
 - CU 控制单元 (Control Unit)
 - ALU 算术逻辑单元 (Arithmetic Logical Unit)
 - Register 寄存器
 - Memory 存储器
 - I/O 输入输出设备 (Input/Output)

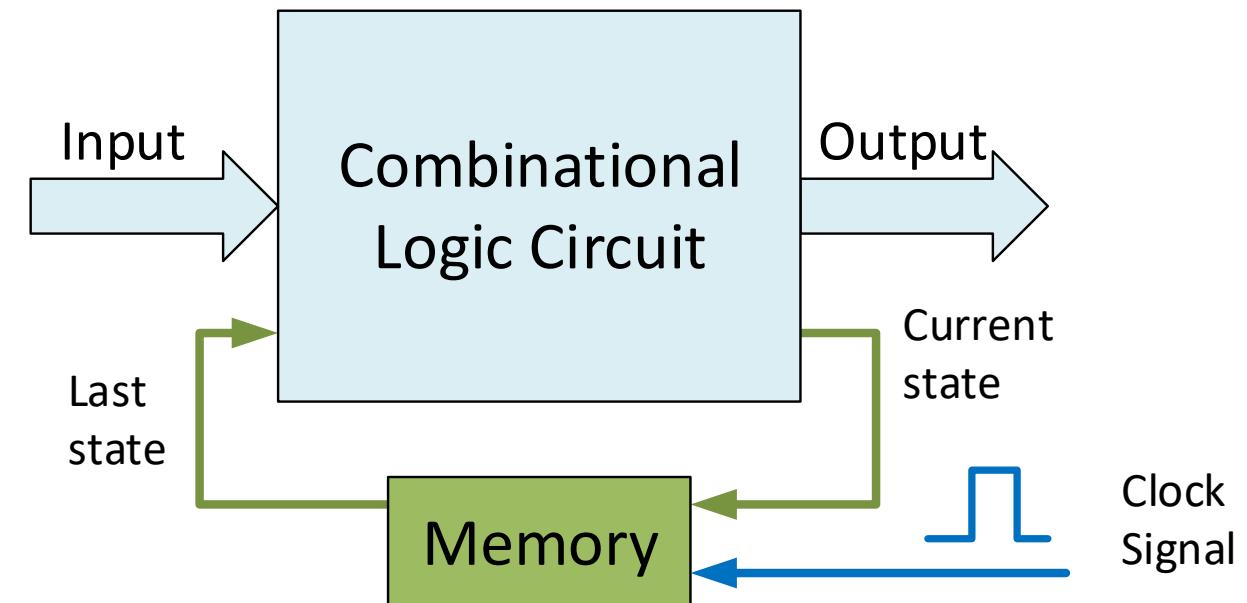
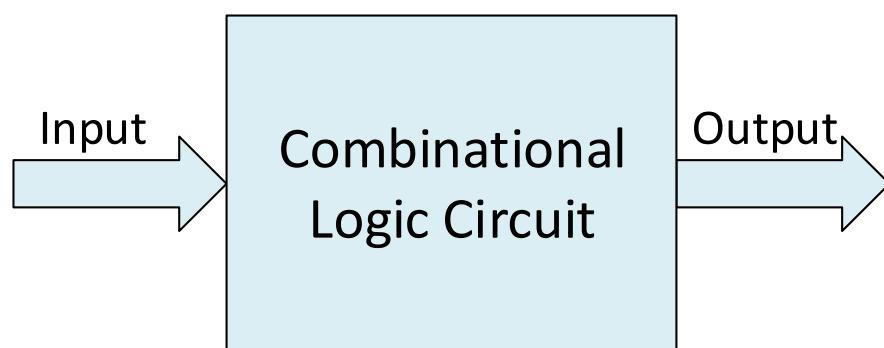


How does a CPU work?



Combinational Logic 组合逻辑 vs. Sequential Logic 时序逻辑

- Combinational logic circuit
 - Output depends on
 - Present input values
 - No memory
- Sequential Logic circuit
 - Output depends on
 - Present inputs
 - The history of past inputs
 - With memory

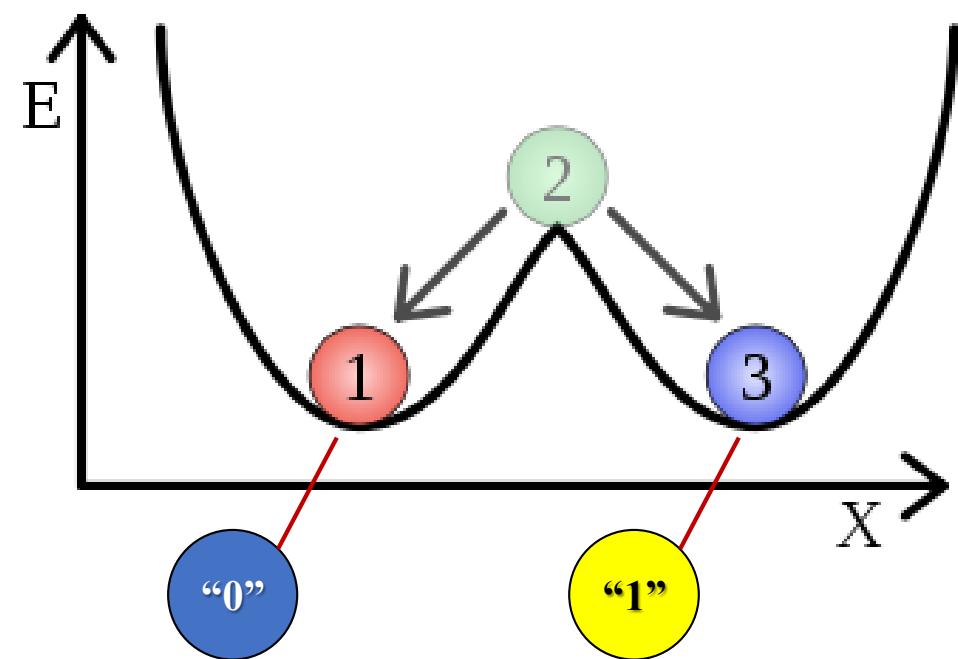


(Pictures are from the Internet)

The original latch 门闩



- Bistable 双稳态 element



Why designing latches?

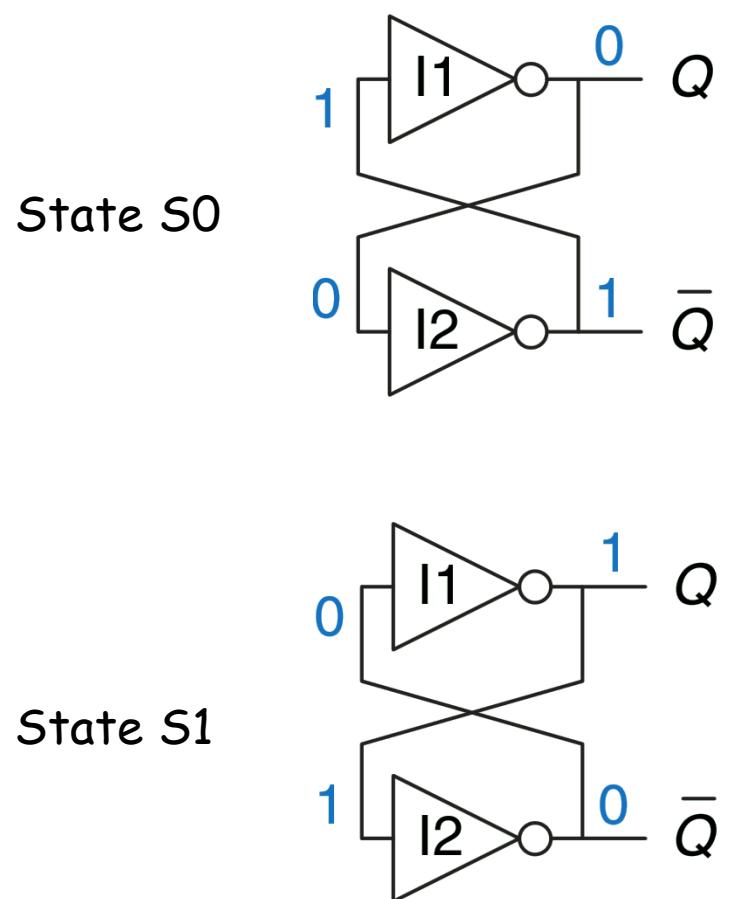
- The fundamental building block of memory is a bistable element, an element with two stable states (either "0" or "1").

Memorizing the state at either 1 or 3 position

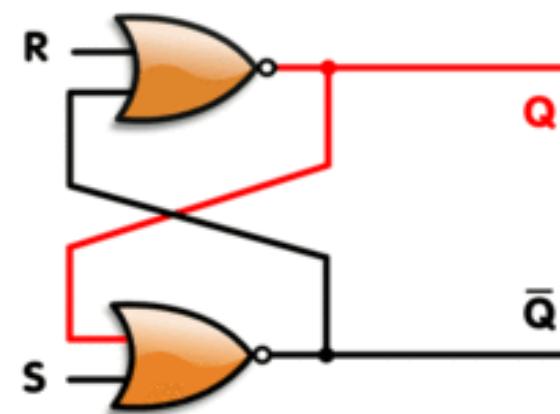
(Pictures are from the Internet)

Digital latch 数字锁存器

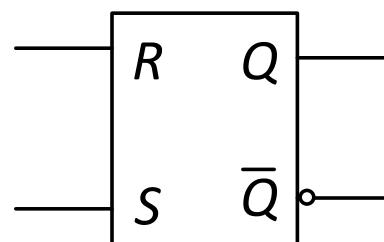
- Cross-coupled inverter pair



- SR latch (with input)
 - S: set (set output to 1) 置1
 - R: reset (set output to 0) 置0



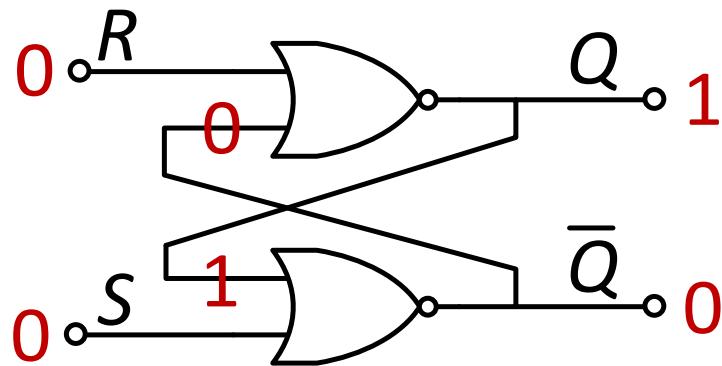
- Symbol



(Pictures are from the Internet)

SR latch (when S=0, R=0)

- $Q=1$

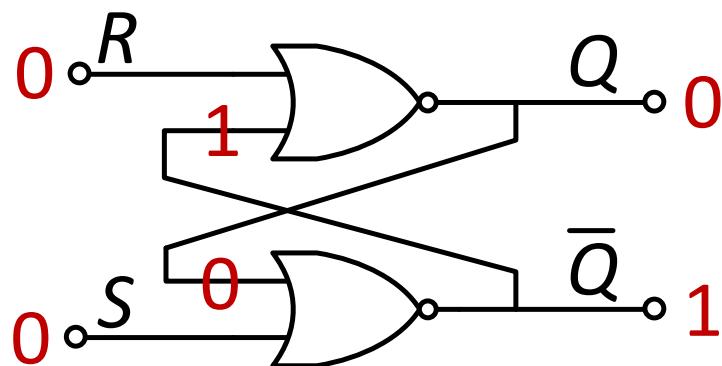


- Truth table

S	R	Q	\bar{Q}
0	0	Q_{prev}	\bar{Q}_{prev}

-- Hold state 保持状态

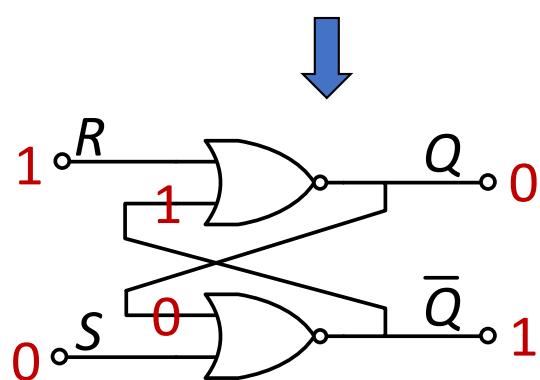
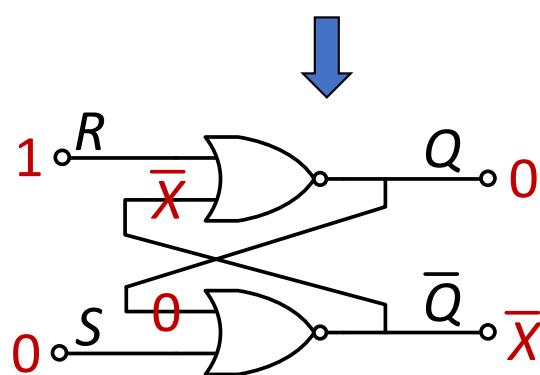
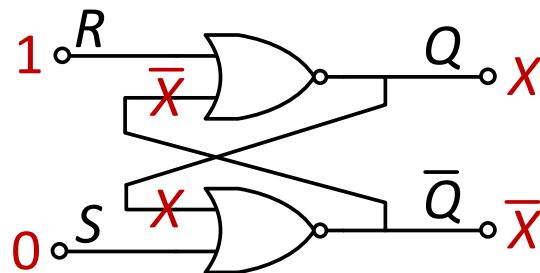
- $Q=0$



(Pictures are from the Internet)

SR latch (when S=0, R=1)

- Truth table



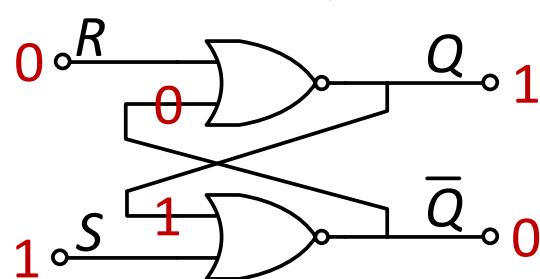
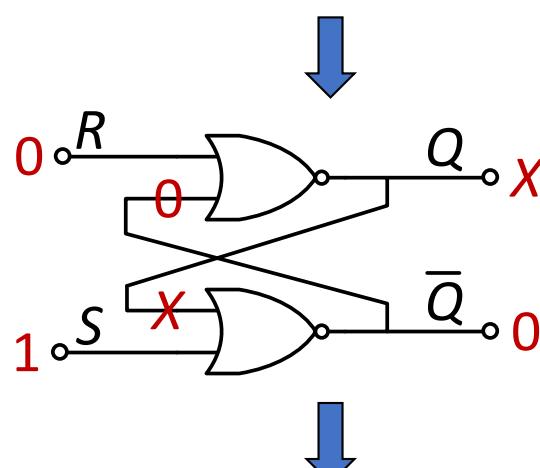
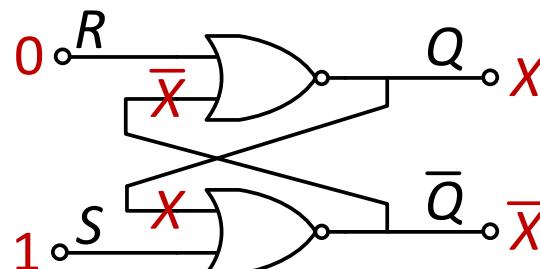
S	R	Q	\bar{Q}
0	0	Q_{prev}	\bar{Q}_{prev}
0	1	0	1

-- Hold state 保持状态
-- Reset 重置 (清零)

(Pictures are from the Internet)

SR latch (when S=1, R=0)

- Truth table

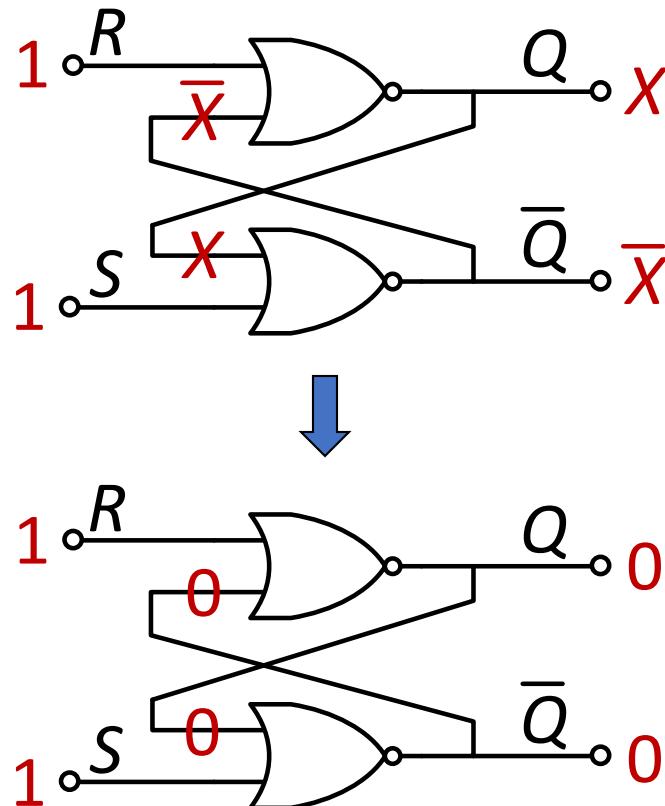


S	R	Q	\bar{Q}
0	0	Q_{prev}	\bar{Q}_{prev}
0	1	0	1
1	0	1	0

-- Hold state 保持状态
-- Reset 重置 (清零)
-- Set 置位 (置一)

SR latch (when $S=1, R=1$)

- Truth table



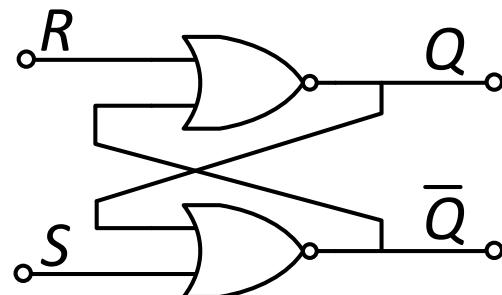
S	R	Q	\bar{Q}
0	0	Q_{prev}	\bar{Q}_{prev}
0	1	0	1
1	0	1	0
1	1	0	0

-- Hold state 保持状态
-- Reset 重置 (清零)
-- Set 置位 (置一)
-- Not allow!

(Pictures are from the Internet)

Timing control

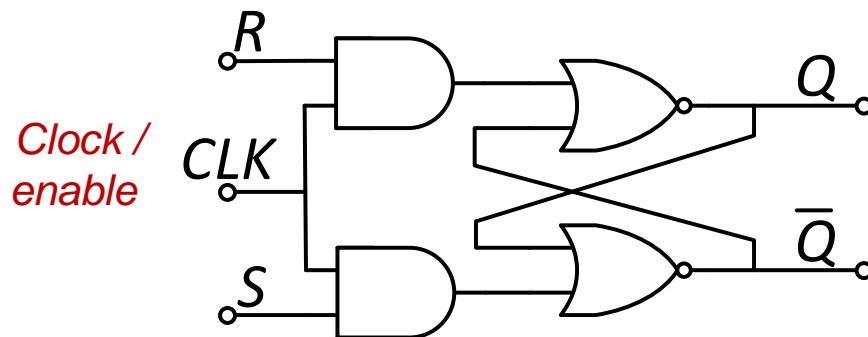
- Transparent latch



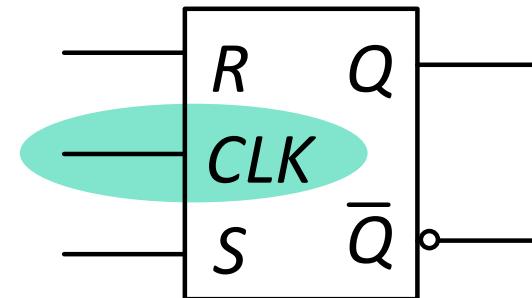
- Operation

E/C	Action
0	No action (keep state)
1	The same as non-clocked SR latch

- Gated SR Latch (level-sensitive)



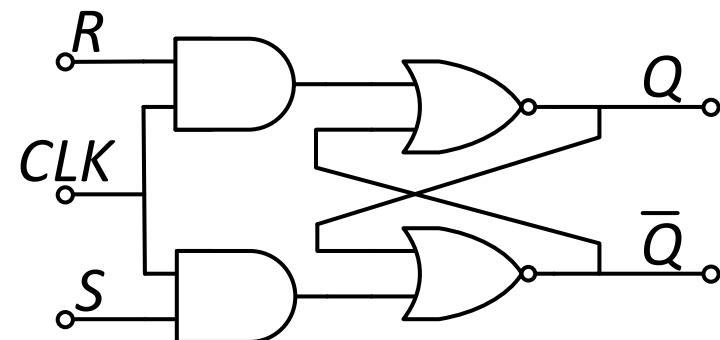
- Symbol



(Pictures are from the Internet)

State determination → D latch

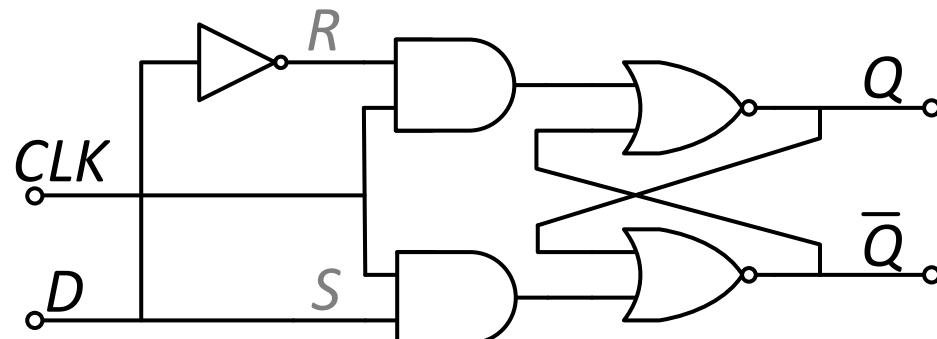
- Avoid the $R=1, S=1$ confusion



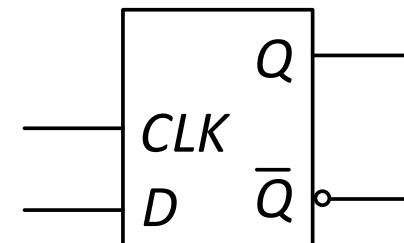
- Operation

CLK	D	\bar{D}	S	R	Q	\bar{Q}
0	X	X	0	0	Q _{prev}	\bar{Q}_{prev}
1	0	1	0	1	0	1
1	1	0	1	0	1	0

- Modification → D latch

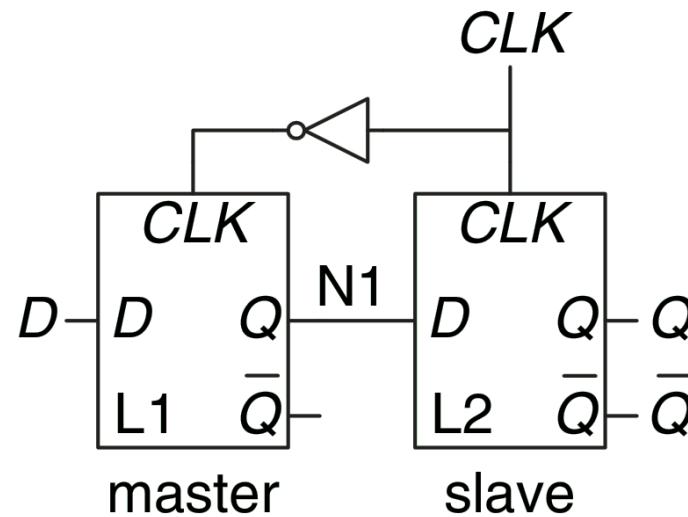


- Symbol

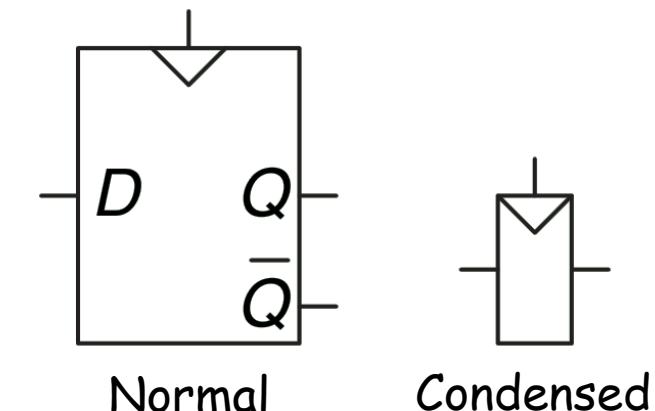


More accurate timing control → edge-triggering

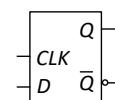
- D flip-flop 触发器



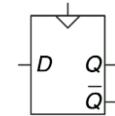
- Symbols



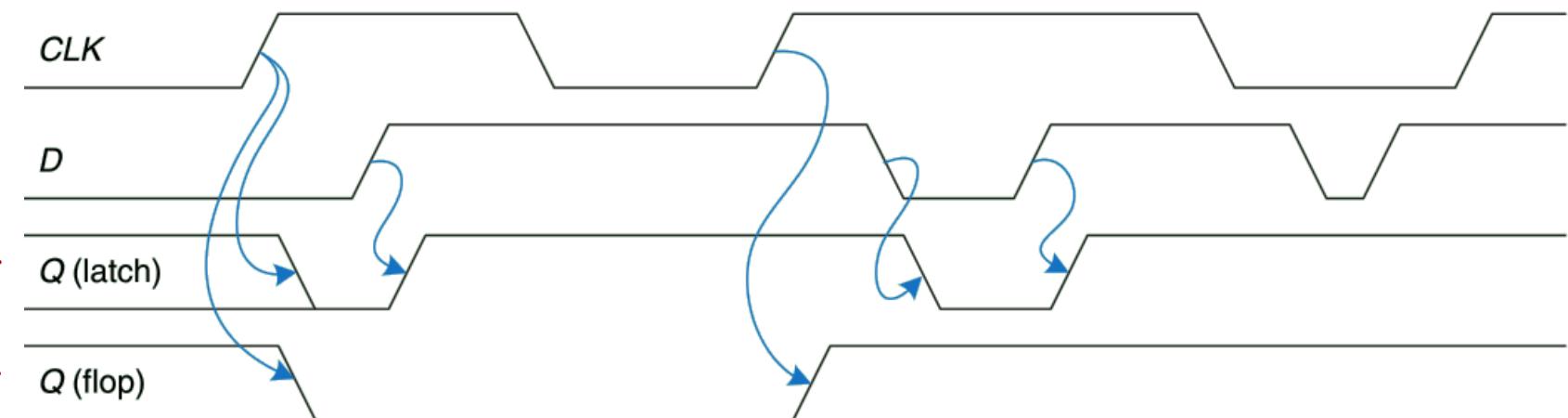
- Timing waveform



D latch output

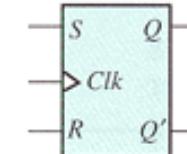


D flip-flop output

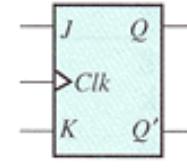


Other flip-flops

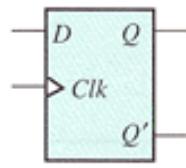
- SR (Set & Reset)

FLIP-FLOP NAME	FLIP-FLOP SYMBOL	CHARACTERISTIC TABLE	CHARACTERISTIC EQUATION	EXCITATION TABLE																																			
SR		<table border="1"> <thead> <tr> <th>S</th> <th>R</th> <th>Q(next)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>NA</td> </tr> </tbody> </table>	S	R	Q(next)	0	0	Q	0	1	0	1	0	1	1	1	NA	$Q(\text{next}) = S + R'Q$ $SR = 0$	<table border="1"> <thead> <tr> <th>Q</th> <th>Q(next)</th> <th>S</th> <th>R</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> <td>X</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>X</td> <td>0</td> </tr> </tbody> </table>	Q	Q(next)	S	R	0	0	0	X	0	1	1	0	1	0	0	1	1	1	X	0
S	R	Q(next)																																					
0	0	Q																																					
0	1	0																																					
1	0	1																																					
1	1	NA																																					
Q	Q(next)	S	R																																				
0	0	0	X																																				
0	1	1	0																																				
1	0	0	1																																				
1	1	X	0																																				

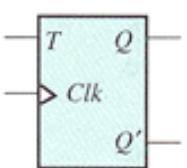
- JK (most versatile)

JK		<table border="1"> <thead> <tr> <th>J</th> <th>K</th> <th>Q(next)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>Q'</td> </tr> </tbody> </table>	J	K	Q(next)	0	0	Q	0	1	0	1	0	1	1	1	Q'	$Q(\text{next}) = JQ' + K'Q$	<table border="1"> <thead> <tr> <th>Q</th> <th>Q(next)</th> <th>J</th> <th>K</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> <td>X</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>X</td> </tr> <tr> <td>1</td> <td>0</td> <td>X</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>X</td> <td>0</td> </tr> </tbody> </table>	Q	Q(next)	J	K	0	0	0	X	0	1	1	X	1	0	X	1	1	1	X	0
J	K	Q(next)																																					
0	0	Q																																					
0	1	0																																					
1	0	1																																					
1	1	Q'																																					
Q	Q(next)	J	K																																				
0	0	0	X																																				
0	1	1	X																																				
1	0	X	1																																				
1	1	X	0																																				

- D (Data)

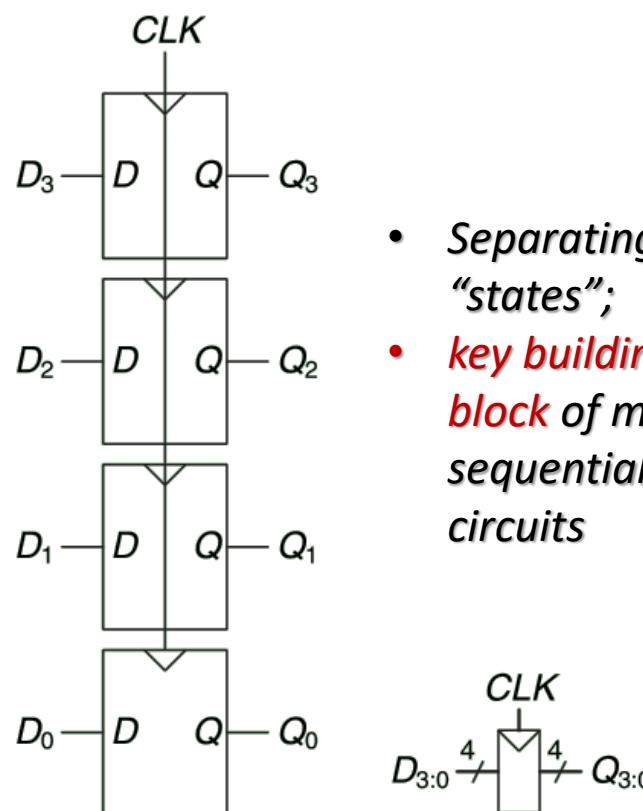
D		<table border="1"> <thead> <tr> <th>D</th> <th>Q(next)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> </tr> </tbody> </table>	D	Q(next)	0	0	1	1	$Q(\text{next}) = D$	<table border="1"> <thead> <tr> <th>Q</th> <th>Q(next)</th> <th>D</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>	Q	Q(next)	D	0	0	0	0	1	1	1	0	0	1	1	1
D	Q(next)																								
0	0																								
1	1																								
Q	Q(next)	D																							
0	0	0																							
0	1	1																							
1	0	0																							
1	1	1																							

- T (Triggered/Toggle)

T		<table border="1"> <thead> <tr> <th>T</th> <th>Q(next)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>Q</td> </tr> <tr> <td>1</td> <td>Q'</td> </tr> </tbody> </table>	T	Q(next)	0	Q	1	Q'	$Q(\text{next}) = TQ' + T'Q$	<table border="1"> <thead> <tr> <th>Q</th> <th>Q(next)</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	Q	Q(next)	T	0	0	0	0	1	1	1	0	1	1	1	0
T	Q(next)																								
0	Q																								
1	Q'																								
Q	Q(next)	T																							
0	0	0																							
0	1	1																							
1	0	1																							
1	1	0																							

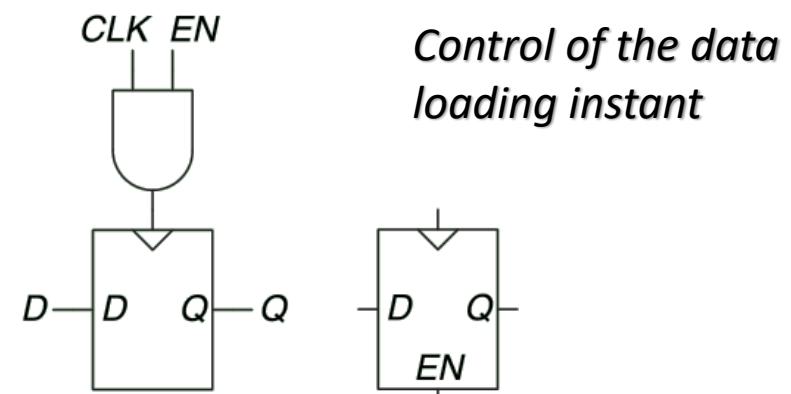
Flip-flop derivations

- Register 寄存器



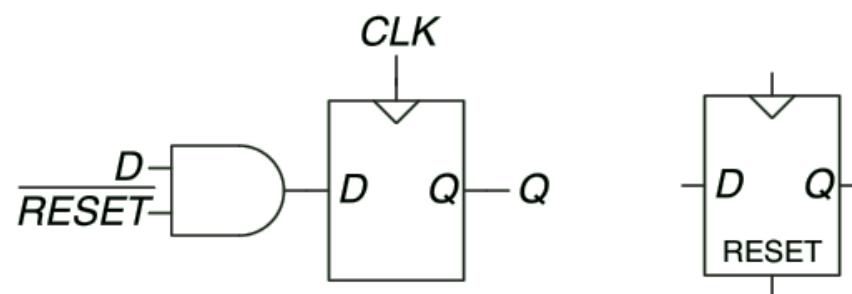
- Separating the “states”;
- key building block of most sequential circuits

- Enabled flip-flop



Control of the data loading instant

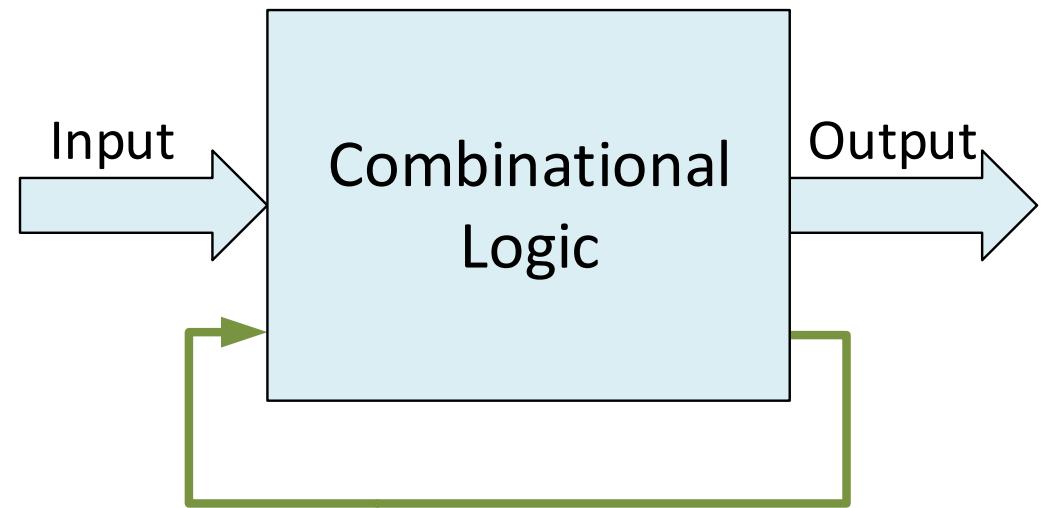
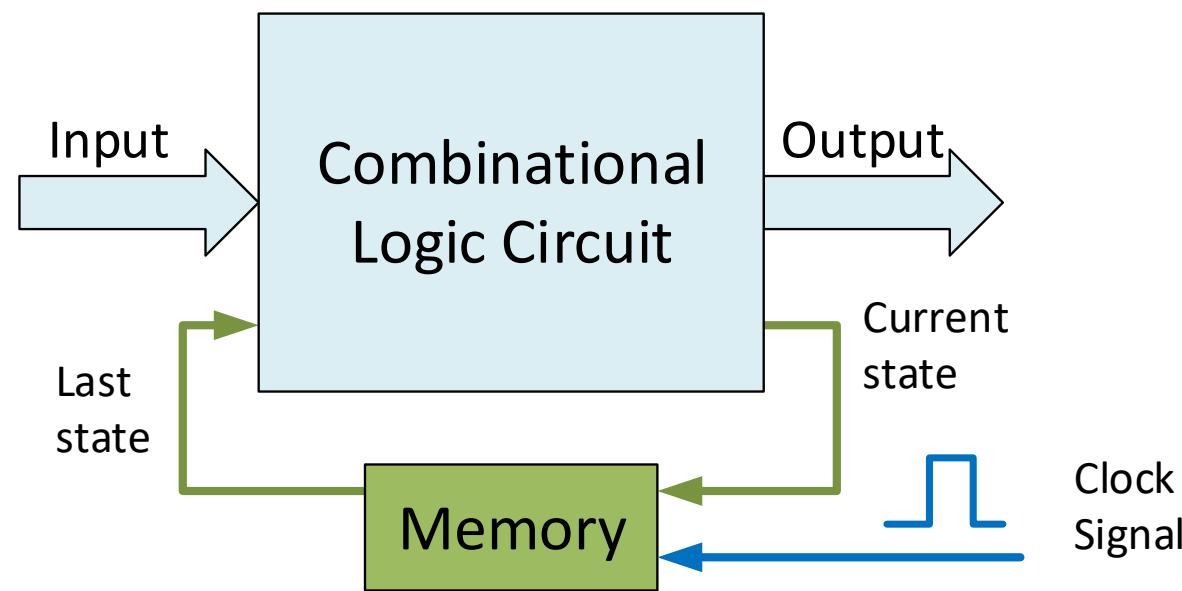
- Resettable flip-flop



Block the effect of input D (reset=0)

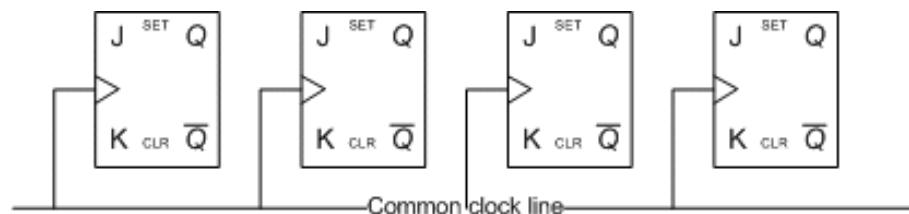
Synchronous and asynchronous circuits

- Synchronous circuit
- Asynchronous circuit or self-timed circuit



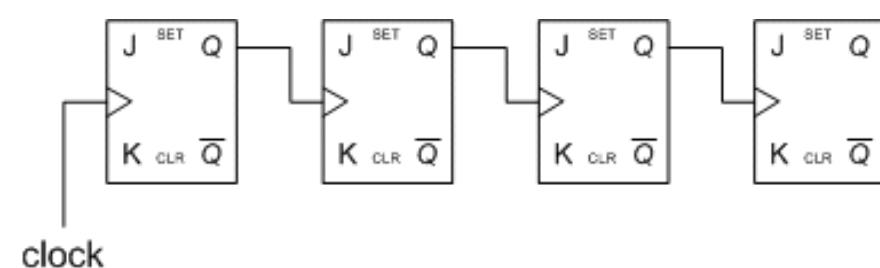
Synchronous and asynchronous circuits

- Synchronous circuit



- Common **clock signal**
- Output only change at the **edge** of clock pulse
- clock signal should be long enough so that the **critical path** can settle before next clock edge
- Easy design

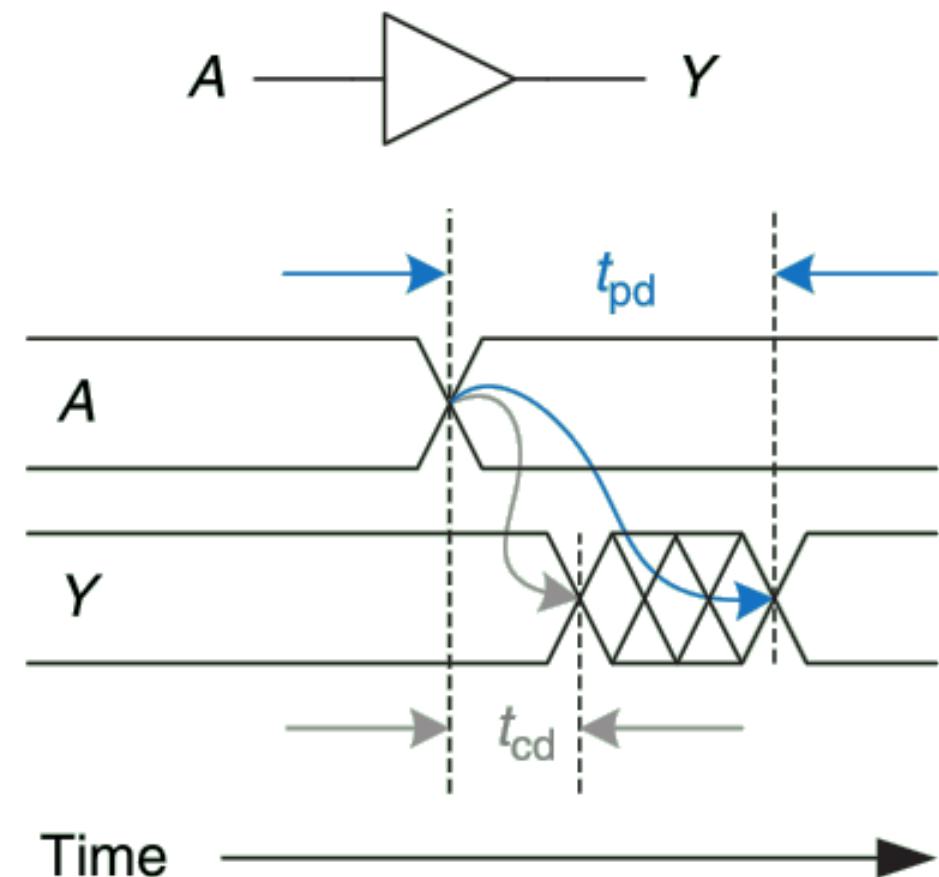
- Asynchronous circuit or self-timed circuit



- Not governed by global clock
- Resulting state can be sensitive to the relative arrival times of inputs at gates, the **race condition**

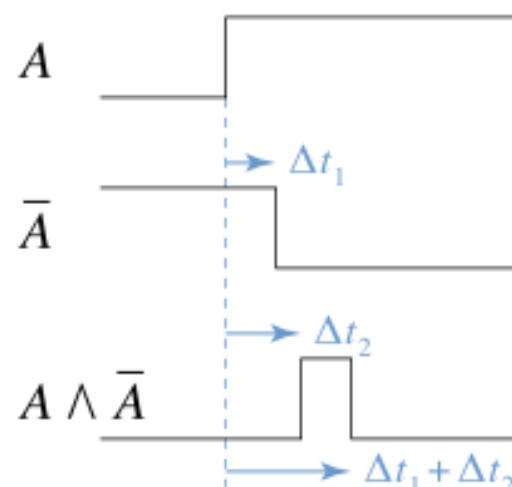
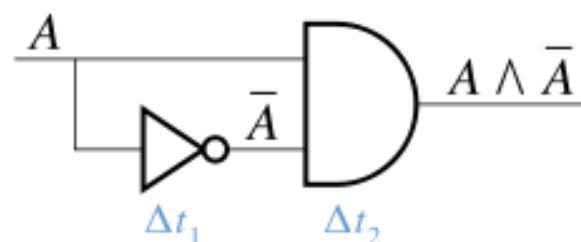
Review: delay in combinational circuit

- contamination delay t_{cd}
 - Y (output) starts to change after the change of A (input)
- propagation delay t_{pd}
 - Y (output) definitely settles in new value



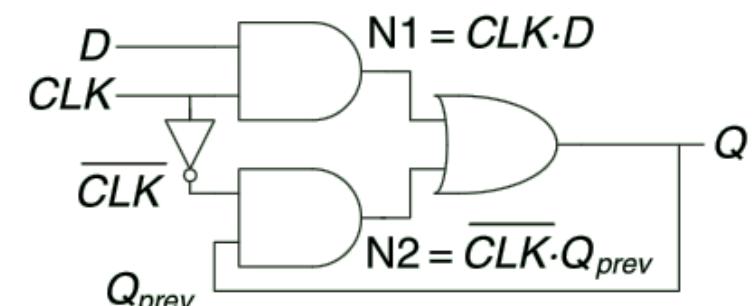
Racing condition 竞争冒险

- In combinational circuit

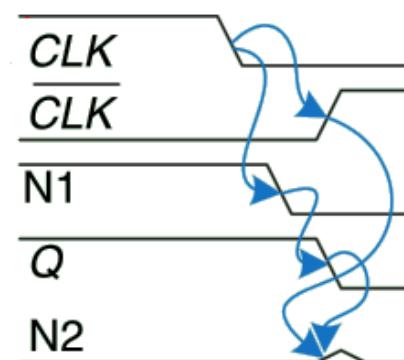


- In asynchronous sequential circuit

$$Q = CLK \cdot D + \overline{CLK} \cdot Q_{prev}$$



- $D = 1$
 $CLK = 1; \overline{CLK} = 0$
 $Q_{prev} = 1 * 1 + 0 * X = 1$
- $CLK = 1 \rightarrow 0; \overline{CLK} = 1$
 $Q = 1 * 0 + 1 * 1 = 1$
- Suppose the delay through the inverter from CLK to \overline{CLK} is rather long compared to the delays of the AND and OR gates
- eventually $Q=0$ because of the race condition**



Rules of synchronous sequential circuit

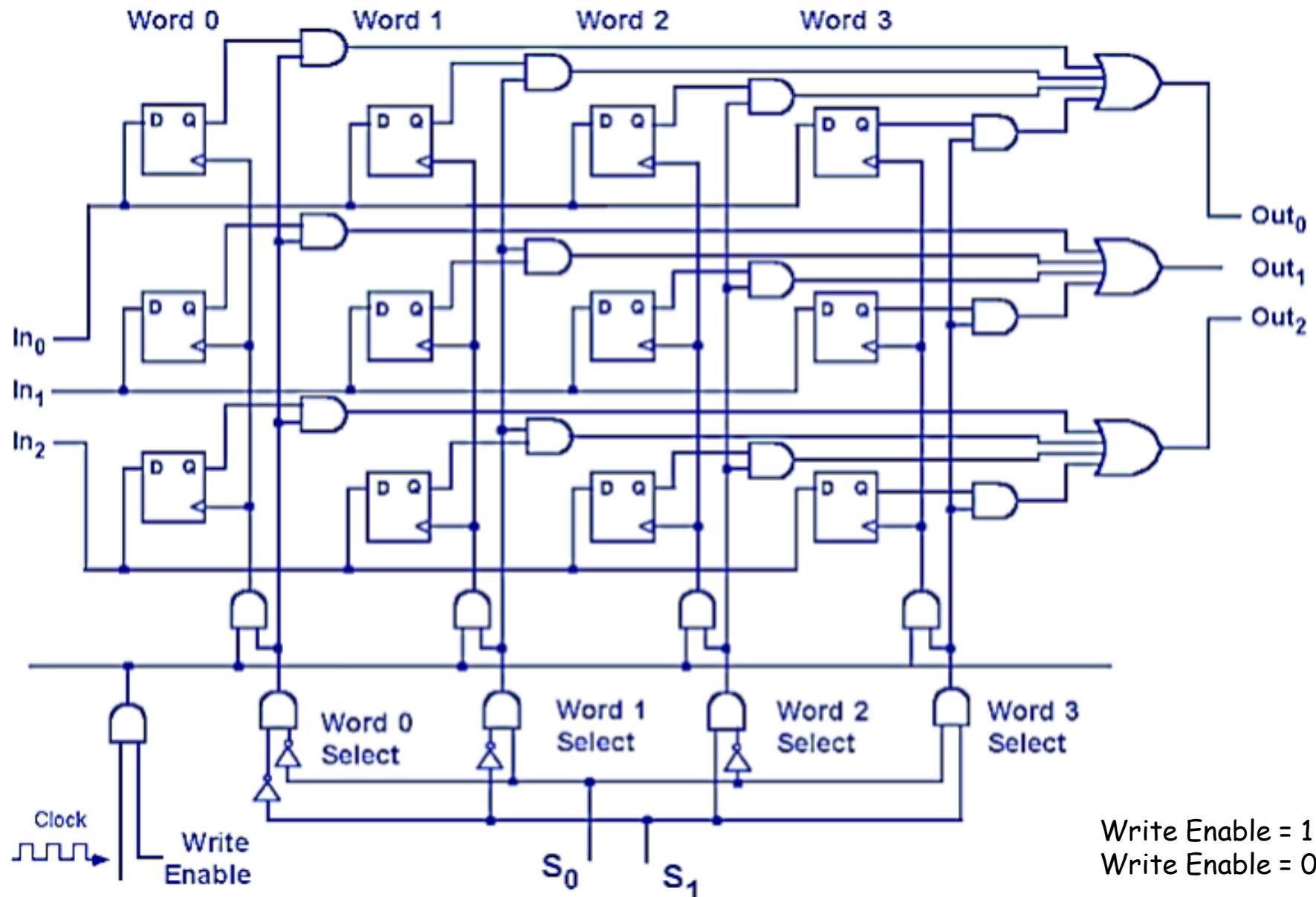
- Every circuit element is either a register or a **combinational circuit**
- At least one circuit element is a **register**
- All registers receive the **same clock signal**
- Every cyclic path contains **at least one register**

Example: 4 × 3 memory

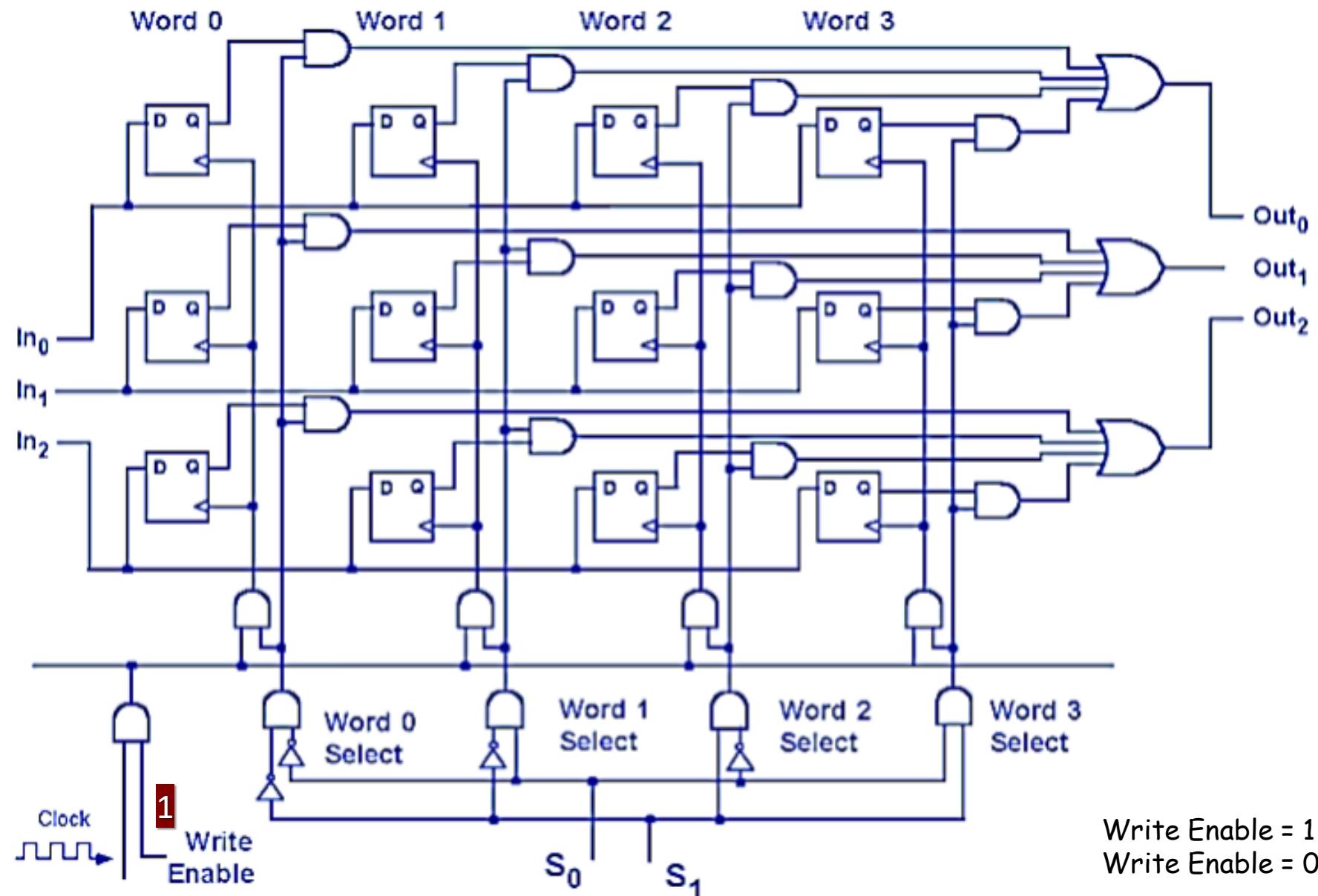
- 4 words and each can store 3 bits of information
- To represent 4 words need 2-bits for address
 - Address decoder performs the address decoding
- To store information we use D Flip-Flop for each bit (total 12, as each location as 3 bits and we have 4 total locations)
- Need select variable to chose if we want to read or write data and that is combined with clock signal (using some combinational logic)
- Also need some other combinational logic...

A simpler example: 2 x 1 memory

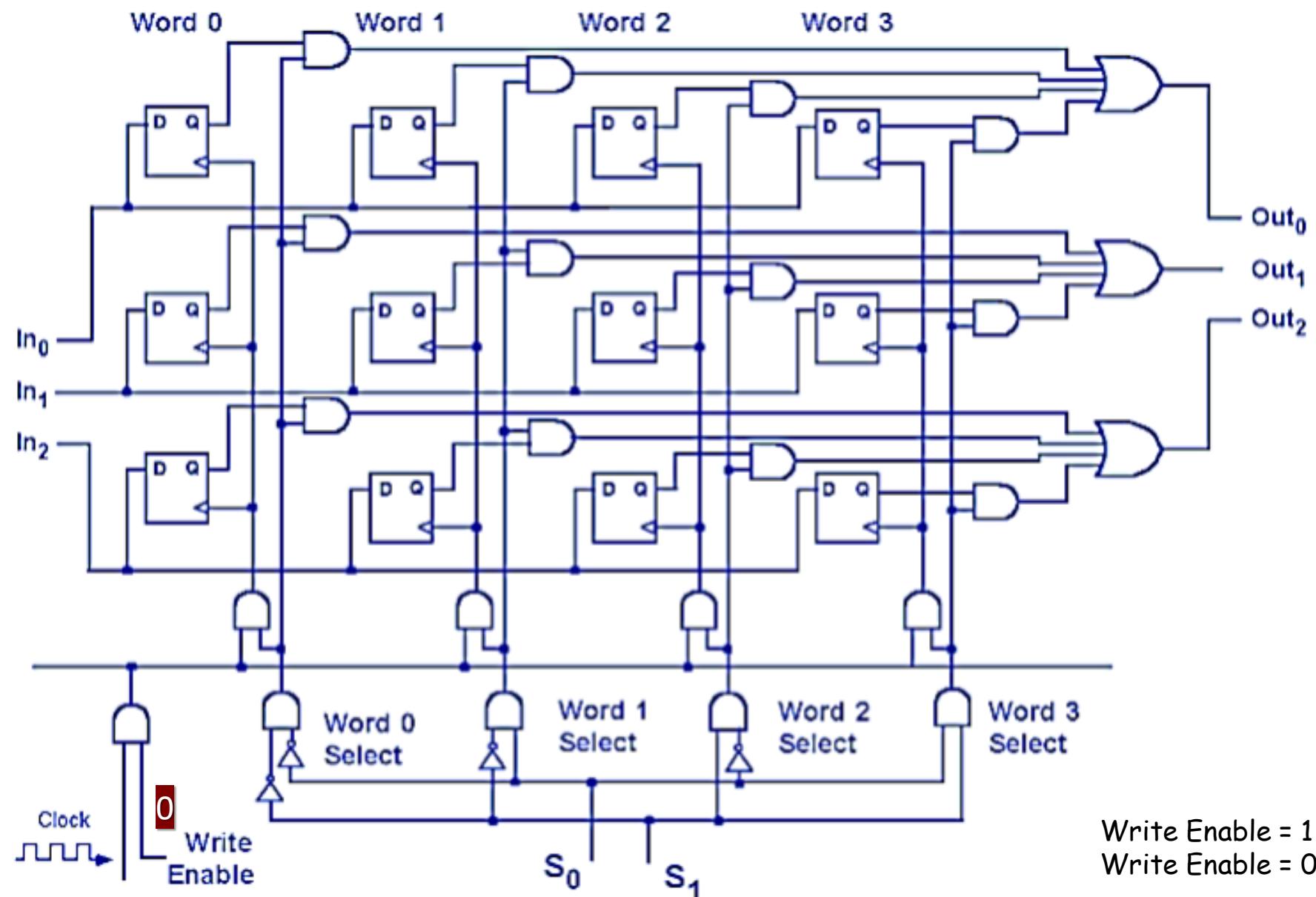
Example: 4 × 3 memory



Write operation to word 1

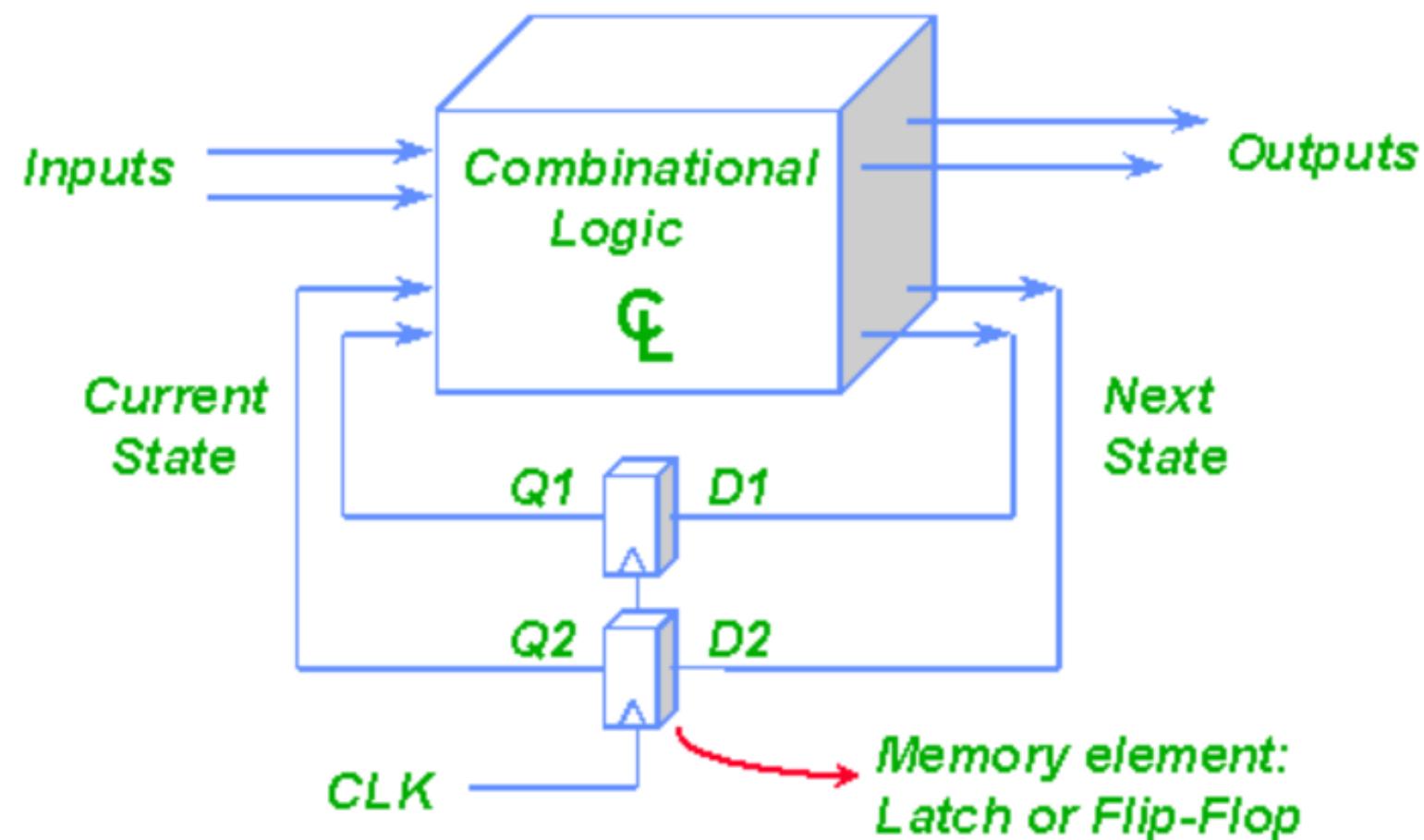


Read operation from word 3

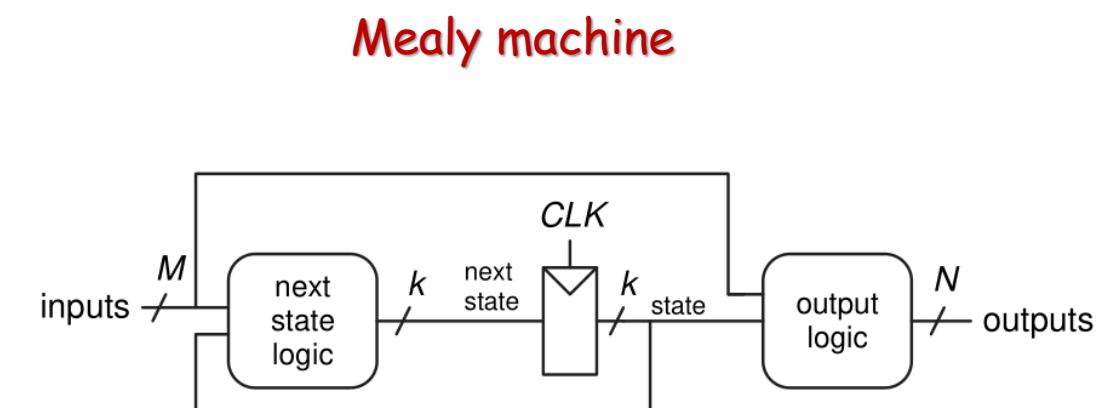
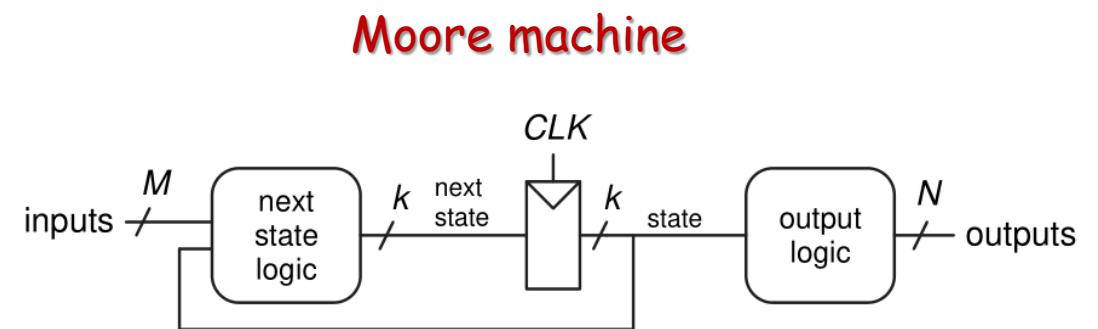
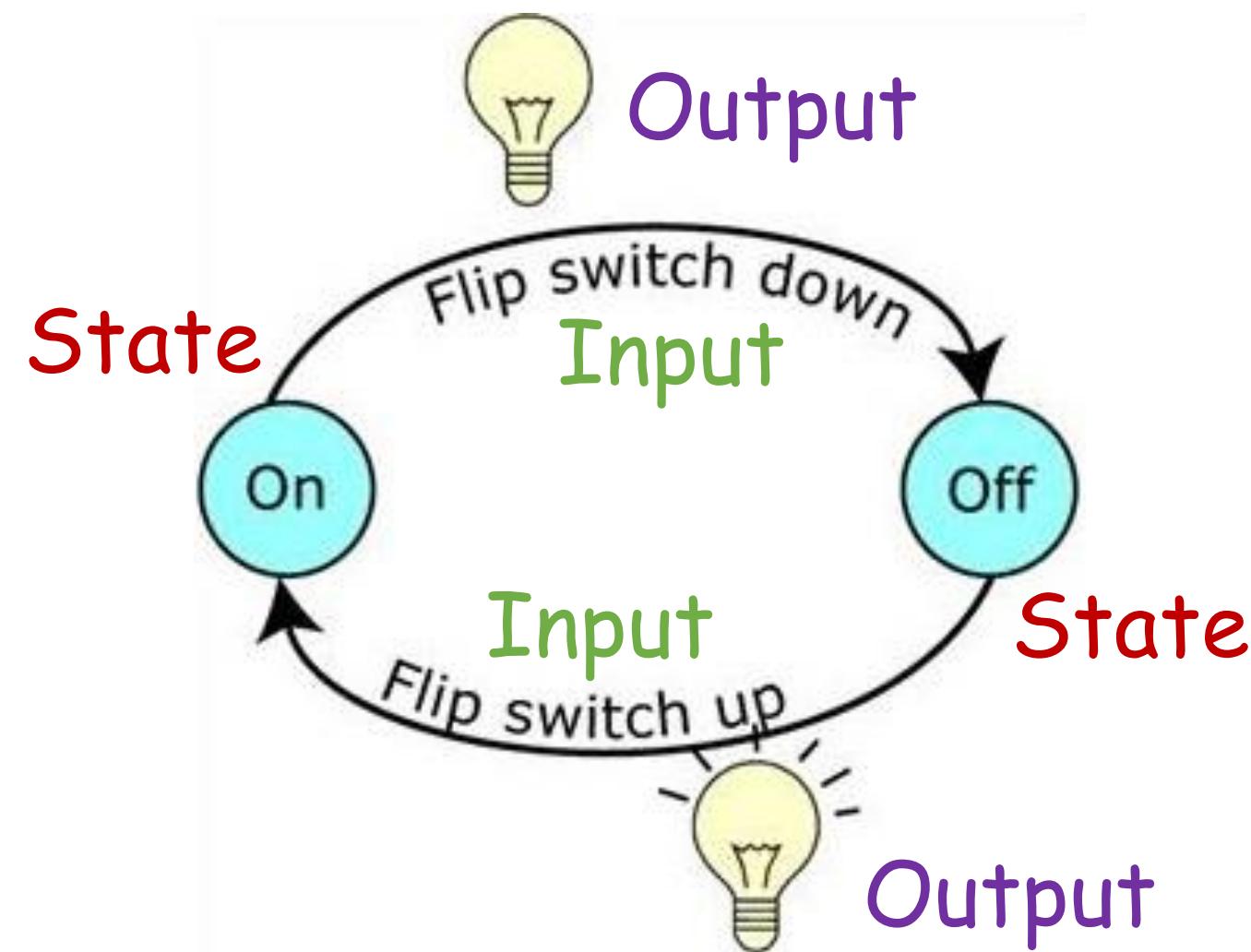


Finite-state machine

- FSM -- a **mathematical model of computation used to design both computer programs and sequential logic circuits**

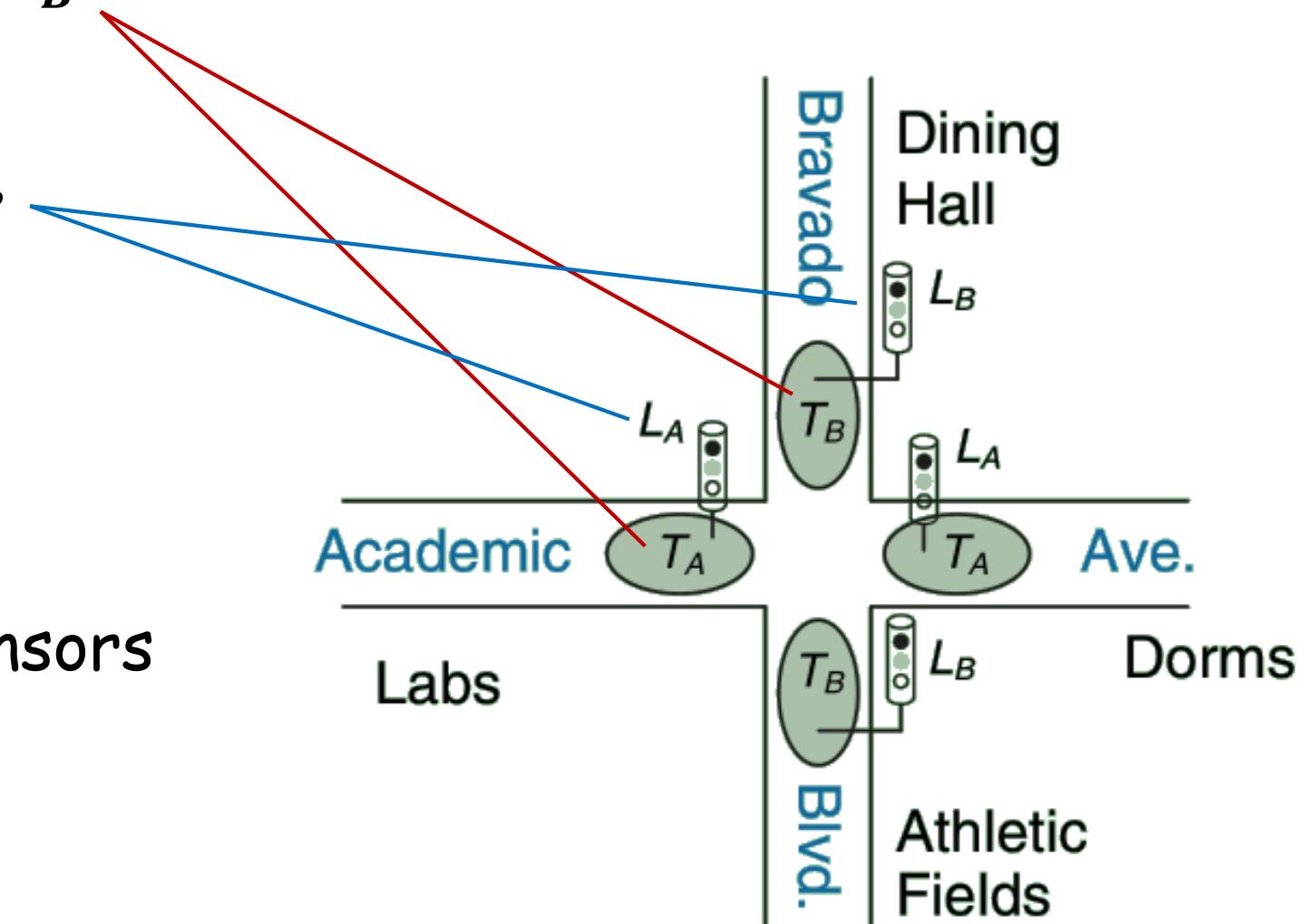


The most seen FSM

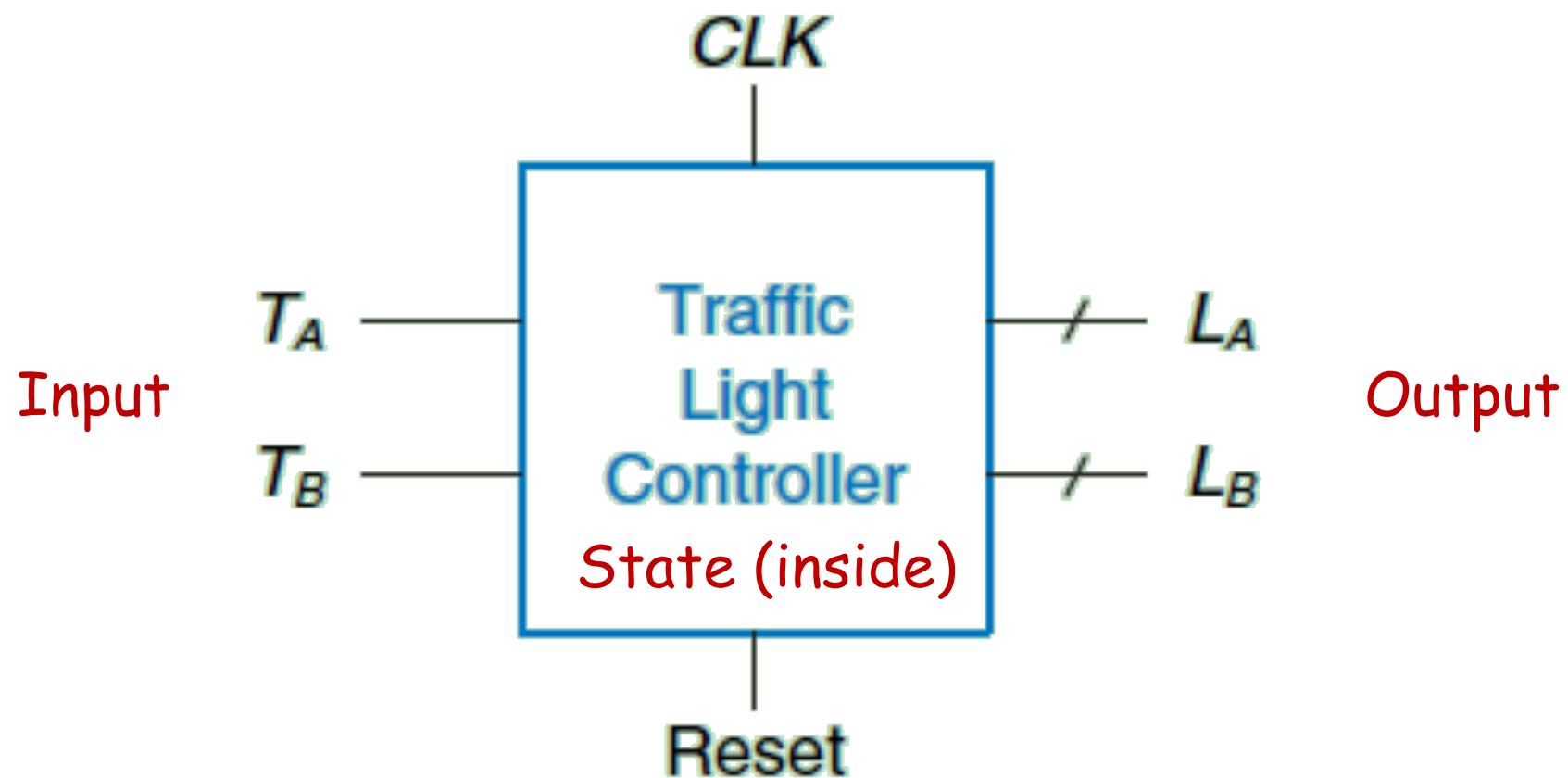


Example: traffic light controller

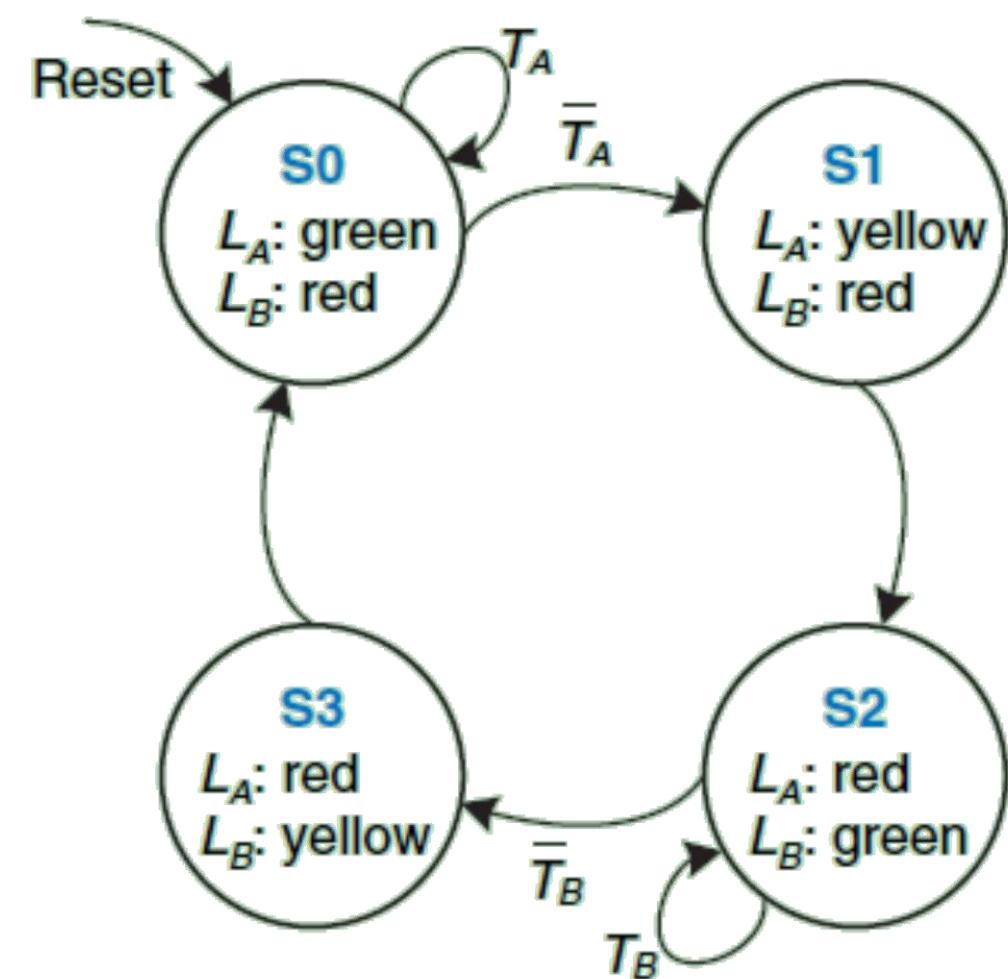
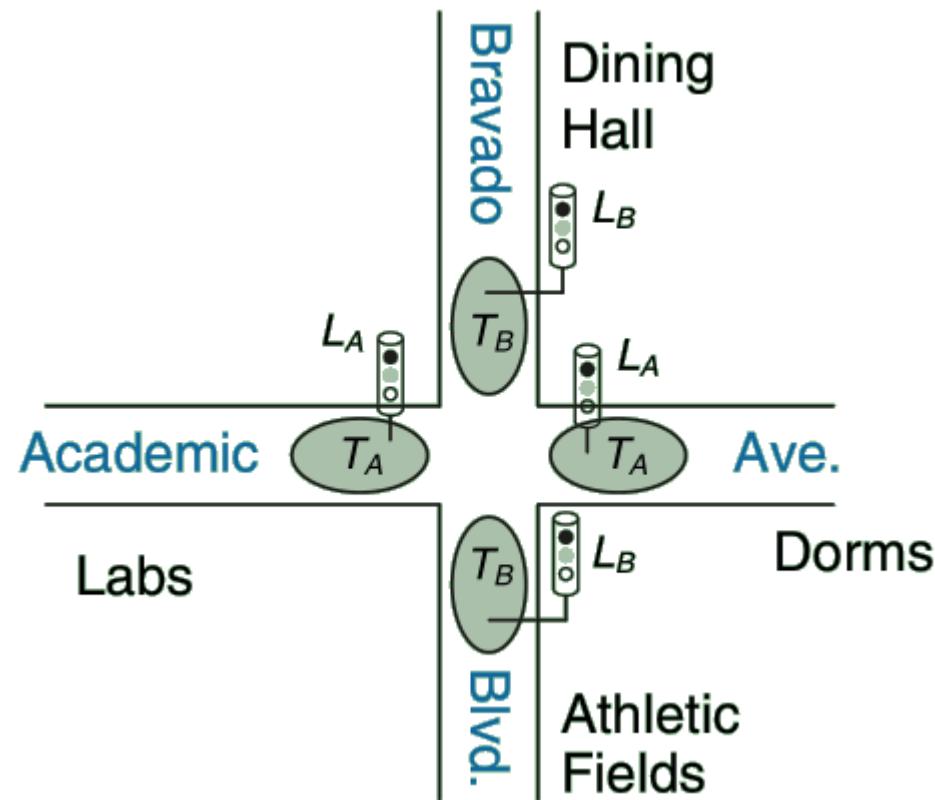
- Traffic sensors T_A and T_B ("1" busy, "0" empty)
- Traffic lights L_A and L_B (each light have red, yellow, and green)
- 5-second clock, at each rising edge, lights may change based on the sensors
- Reset button



Black box view



State transition diagram



State table

State	Encoding $S_{1:0}$
S0	00
S1	01
S2	10
S3	11

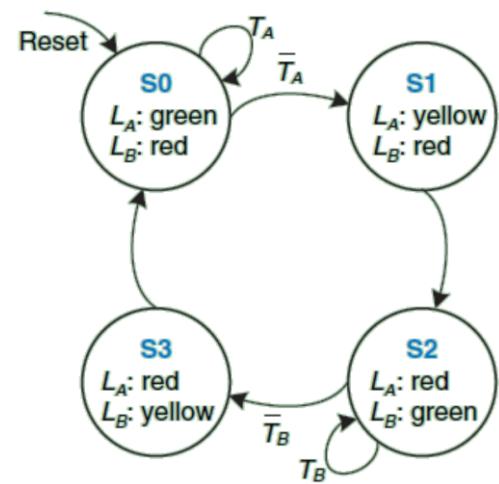
State encoding

State transition table

Current State S	Inputs T_A	Inputs T_B	Next State S'
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0

Binary encoded state transition table

Current State S_1	Current State S_0	Inputs T_A	Inputs T_B	Next State S'_1	Next State S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0



Output table

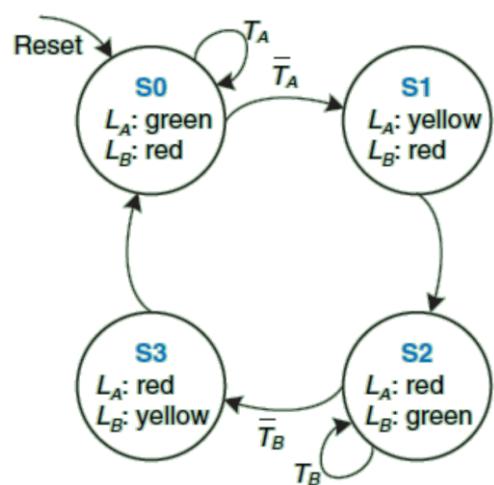
Output encoding

Output	Encoding $L_{1:0}$
green	00
yellow	01
red	10



Output table

Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1



Sum-of-products form

State table

Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

Output table

Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

$$S'_1 = \bar{S}_1 S_0 + S_1 \bar{S}_0 \bar{T}_B + S_1 \bar{S}_0 T_B$$

$$= \bar{S}_1 S_0 + S_1 \bar{S}_0$$

$$= S_1 \oplus S_0$$

$$S'_0 = \bar{S}_1 \bar{S}_0 \bar{T}_A + S_1 \bar{S}_0 \bar{T}_B$$

$$L_{A1} = S_1$$

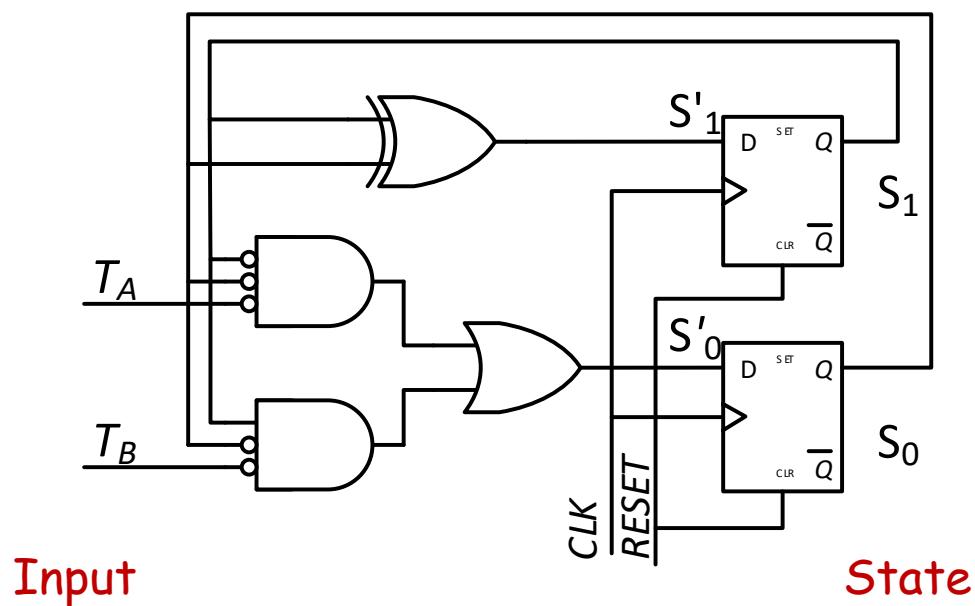
$$L_{A0} = \bar{S}_1 S_0$$

$$L_{B1} = \bar{S}_1$$

$$L_{B0} = S_1 S_0$$

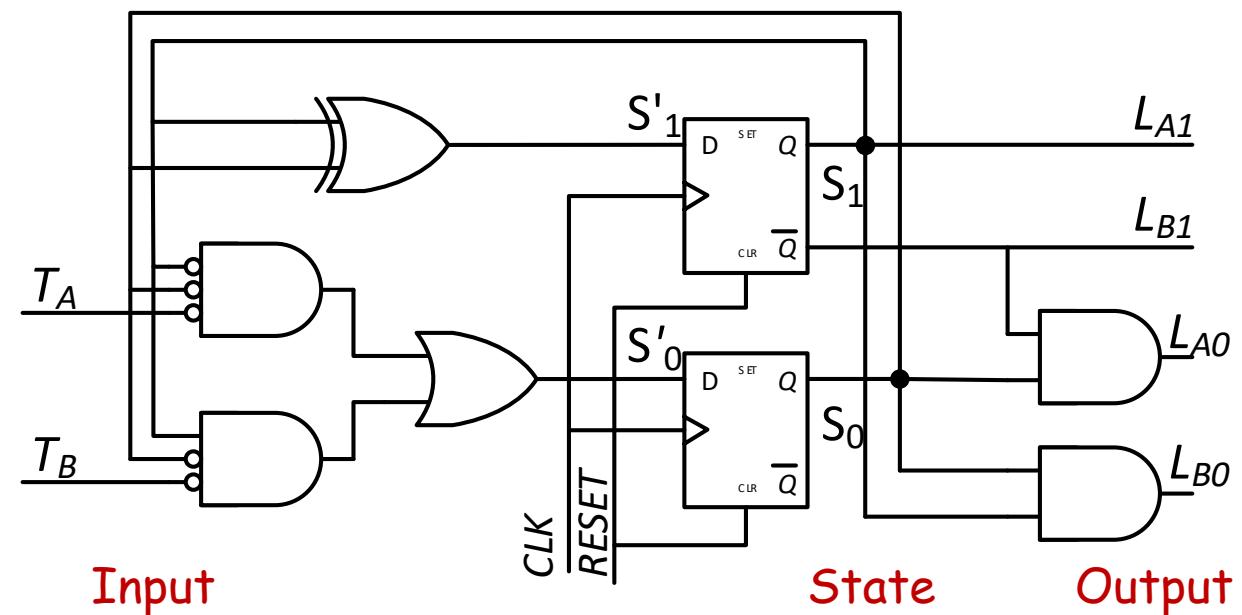
State & output logic

- State logic (sequential)
 - Output Logic (combinational)



$$S'_1 = S_1 \oplus S_0$$

$$S'_0 = \bar{S}_1 \bar{S}_0 \bar{T}_A + S_1 \bar{S}_0 \bar{T}_B$$

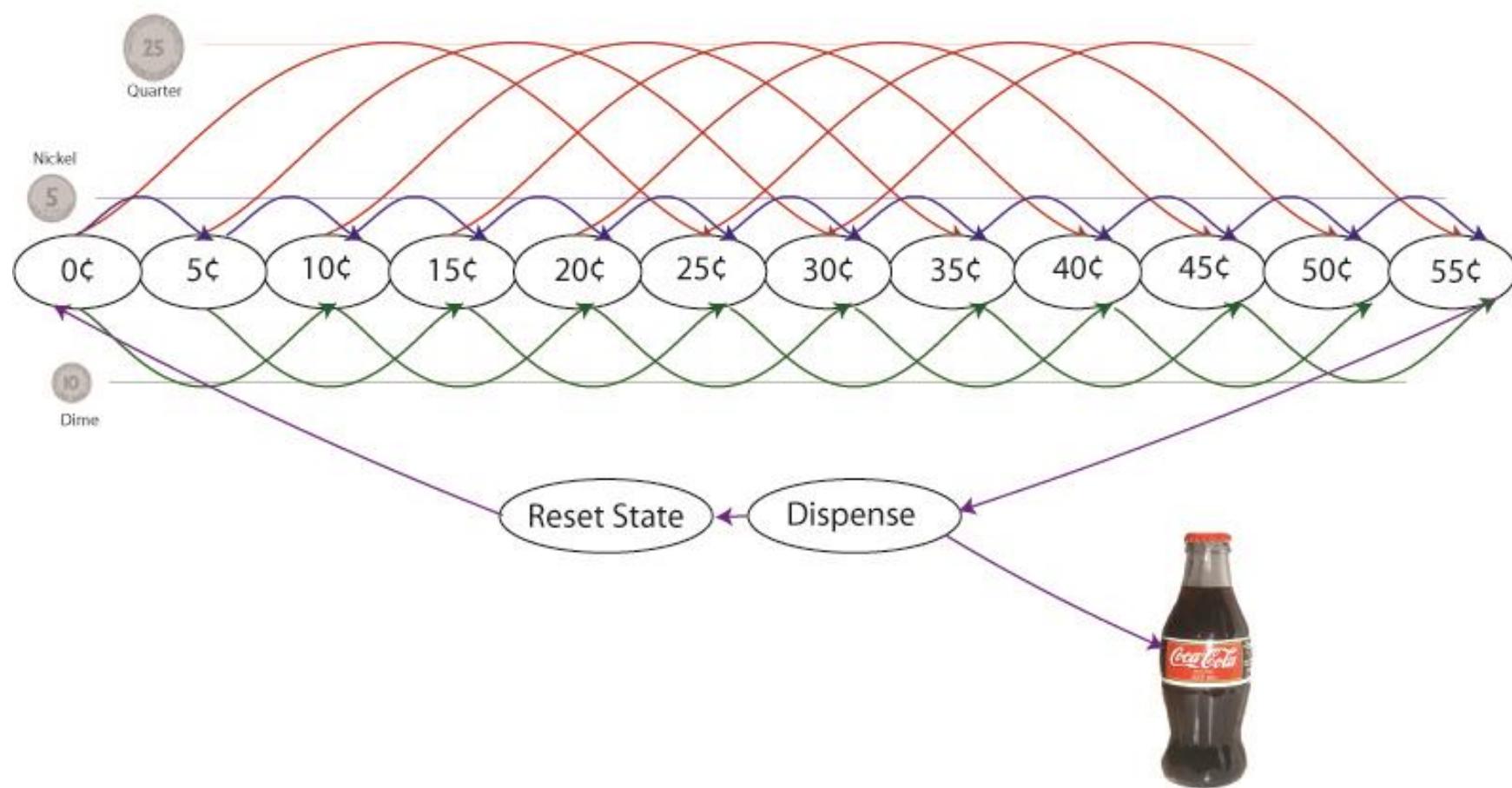


$$L_{A1} = S_1; \quad L_{B1} = \bar{S}_1$$

$$L_{A0} = \bar{S}_1 S_0; \quad L_{B0} = S_1 S_0$$

FSM with more states

Finite State Machine:
Soda Machine State Diagram



FSM with more states

- Digital clock



- Microwave oven

