$$e^{jk\frac{2\pi}{N}n} = e^{j(k+mN)\frac{2\pi}{N}n}$$

$$= e^{jk\frac{2\pi}{N}n} \cdot \boxed{e^{j2\pi mn}}$$

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad = e^{jk\frac{2\pi}{N}n}$$

$$= \cos(2\pi mn) + j\sin(2\pi mn)$$

$$= 1$$

$$x[n] = \sum_{k\in\langle N\rangle} a_k \underline{e^{jk\frac{2\pi}{N}n}}$$

$0 - N-1$ $\qquad$ a_0 $\qquad$ a_{N-1}

$N - 2N-1$ $\qquad$ a_N $\qquad$ a_{2N-1}

$$\frac{1}{N}\sum_{n\in\langle N\rangle} \boxed{a_k e^{jk_1\omega_0 n}} \times e^{-jk_2\omega_0 n}$$

$k_1 = k_2$ $\quad \frac{2\pi}{N}$

$k_1 \neq k_2$

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$k_1 = k_2$ $\quad N$

$$= \sum_{n\in\langle N\rangle} \boxed{e^{j(k_1-k_2)\omega_0 n}}$$

$$= \frac{1-\boxed{e^{j(k_1-k_2)\omega_0 N}}}{1-e^{j(k_1-k_2)\omega_0}} = \frac{\boxed{e^{j(k_1-k_2)\omega_0\frac{N}{2}}}\left(e^{-j(k_1-k_2)\omega_0\frac{N}{2}} - e^{j(k_1-k_2)\omega_0\frac{N}{2}}\right)}{\boxed{e^{j(k_1-k_2)\omega_0/2}}\left(e^{-j(k_1-k_2)\omega_0/2} - e^{j(k_1-k_2)\omega_0/2}\right)}$$

$$= e^{j(k_1-k_2)\omega_0(N-1)/2} \cdot \left(\cancel{\cos(k_1-k_2)\omega_0\tfrac{N}{2}} - j\sin(k_1-k_2)\omega_0\tfrac{N}{2} - \cancel{\left(\cos(k_1-k_2)\omega_0\tfrac{N}{2} + j\sin(k_1-k_2)\omega_0\tfrac{N}{2}\right)} \right)$$

$j\sin(k_1-k_2)\omega_0/2$

$$= e^{j(k_1-k_2)\omega_0(N-1)/2} \cdot \frac{\sin(k_1-k_2)\pi}{\sin(k_1-k_2)\frac{\pi}{N}} = 0 \quad 当 k_1 \neq k_2$$

$\omega_0 = \frac{2\pi}{N}$

$$\text{fft}(\underbrace{[a_1, a_2, a_3, \ldots\ldots]}_{N})$$

$$\text{abs}([b_1, b_2, b_3, \ldots\ldots b_{N-1}, b_N])$$

f_1 f_2 f_3

复数

abs( )

F_s

$$f_1 = \frac{F_s}{N} \times 1$$

$$f_2 = \frac{F_s}{N} \times 2$$