CS 101 Fall 2023 - Quiz 5

11/13/2023 - 20 Minutes **Name:** **ID number:**

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1. (5 points) Notes of discussion

I promise that I will complete this QUIZ independently and will not use any electronic products or paper-based materials during the QUIZ, nor will I communicate with other students during this QUIZ.

True or False: I have read and understood the notes. √ True ◯ False

2. (8 points) True or False

Determine whether the following statements are true or false.

(a)	(b)	(c)	(d)
F	T	T	F

(a) (2') If the pre-order traversal and post-order traversal of two binary trees are equal respectively, then the two binary trees are exactly the same.

(b) (2') The time complexity of running Depth First Traversal which contains n nodes using a stack is $\Theta(n)$.

(c) (2') A tree with n nodes has $n-1$ edges.

(d) (2') Every complete binary tree is also a full binary tree.

3. (8 points) Fill in the blank

(a) (2') There are ____5____ possible values of n, where n is the number of nodes in a full binary tree of height 3.

(b) (2') A full binary tree with n non-leaf nodes contains ____$2n+1$____ total nodes.

(c) (2') Consider a complete binary with height $h > 0$. The range of the possible number of its leaf nodes is ____$2^{h-1}, 2^h$____. (You should not use any Landau symbols here)

(d) (2') Consider a complete binary tree has 1001 nodes. The number of its leaf nodes is ____501____.

4. (8 points) Array Storage

Unlike arbitrary n-ary trees, binary tree can be easily stored within an array.

(a) (6') Firstly, you need to complete the following code:

```
struct BinaryTree {
  int data[SIZE]{};
  size_t head() { return 1; }
  // Return the index of left child
  size_t left_child_idx(size_t idx) { return  2 * idx  ; }
  // Return the index of right child
  size_t right_child_idx(size_t idx) { return  2 * idx + 1 ; }
  // Return the index of parent node
  size_t parent_idx(size_t idx) { return  idx / 2  ; }
};
```

(b) (2') To make sure the code works properly for all tree with n nodes, `SIZE` should be set to **at least** 2^n.

5. (7 points) Perfect Binary Tree

With the following steps, we can calculate the average depth of a node in a perfect binary tree.

(a) (2') At depth k of the tree, there are 2^k nodes.

(b) (2') The total number of nodes in a perfect binary tree with height h is $2^{h+1} - 1$.

(c) (3') Given the height of a perfect binary tree is $h > 0$. Calculate its exact average depth and show the result is $\Theta(h)$.

Solution: Sum the depths up and calculate the average:

$$\frac{\sum_{k=0}^{h} k2^k}{2^{h+1}-1} = \frac{h2^{h+1}-2^{h+1}+2}{2^{h+1}-1} = \frac{(h-1)(2^{h+1}-1)+h+1}{2^{h+1}-1} = h - 1 + \frac{h+1}{2^{h+1}-1}$$

As $h \to \infty$, $\frac{h+1}{2^{h+1}-1} \to 0$.

So the average depth is $\Theta(h)$.