Harmonic and Complex Analysis: modern and classical

dedicated to the memory of Professor Lawrence Zalcman

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Abstracts

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Walter Bergweiler

University of Kiel

Zalcman's lemma and radially distributed values

Zalcman's lemma has proved to be a very powerful tool in the theory of normal families. We show how an extension of this lemma can be used to deal with functions holomorphic in the unit disk for which all zeros lie on one ray while all 1-points lie on a different ray. This family is shown to be normal in the unit disk punctured at the origin. Related problems are also considered, for example meromorphic functions for which zeros, 1-points and poles are on distinct rays.

The results are joint work with Alexandre Eremenko.

Lucian Beznea

Simion Stoilow Institute of Mathematics of the Romanian Academy

Probabilistic characterisation for the differences of superharmonic functions

We identify the linear space spanned by the real-valued superharmonic functions with the set of those functions which are quasimartingales when we compose them with the Brownian motion. In a more general setting, we provide a unifying result which clarifies the relations between harmonic, co-harmonic, invariant, co-invariant, martingale and co-martingale functions, showing that in the conservative case they are all the same. The talk is based on joint works with Iulian Cîmpean (Bucharest) and Michael Röckner (Bielefeld).

Alexander Brudnyi

University of Calgary

Properties of the maximal ideal space of H-infinity

We present some results on the structure of the maximal ideal space of the algebra of bounded holomorphic functions on the open unit disk.

Guy Cohen
Ben-Gurion University of the Negev

Convolution powers of moment-improving measures on the circle

For a probability measure ν on the unit circle $\mathbb{T}$, define the convolution operator $P_\nu f = \nu * f$. It is well-defined for $f \in L^1(\mu)$ (here μ the Lebesgue measure on $\mathbb{T}$), it satisfies $P_\nu \mathbf{1} = \mathbf{1}$, $\|P_\nu f\|_p \leq \|f\|_p$ for every $1 \leq p < \infty$, and it preserves integrals. That is, P_ν is Markov operator with invariant measure μ.

For $f \in L^1(\mu)$, the Fourier coefficients of $P_\nu f$ are $\widehat{P_\nu f}(n) = \hat{\nu}(n)\hat{f}(n)$. That is, $\{\hat{\nu}(n)\}_{n\in\mathbb{Z}}$ is a *multiplier* sequence in any L^p.

It is possible that P_ν maps L^p into L^q, for some $1 \leq p < q$, so $P_\nu f$ has higher moments that $f \in L_p$. In some harmonic analysis works, ν in this case is called *L^p-improving*. Equivalently, in the theory of Markov operators, P_ν is called *hyperbounded*.

We will give sufficient conditions on ν for being L^p-improving and overview some properties of the induced hyperbounded operators.

This is a joint work with Michael Lin.

Arthur Danielyan
University of South Florida

On the boundary behavior of analytic functions

By Fatou's theorem of 1906, a bounded analytic function f in the open unit disc D has radial (even non tangential) limits a.e. on the unit circle T. In the same paper, Fatou proved also the following interpolation theorem: For any closed subset $E \subset T$ of Lebesgue measure zero, there exists a function f in the disc algebra such that f vanishes precisely on E. A new (2022) short proof for this theorem will be described. In 2020 the author has shown that Fatou's interpolation theorem implies the following version of converse to Fatou's first theorem.

Theorem 1. Let E be a subset on T. There exists a bounded analytic function in D which has no radial limits on E but has unrestricted limits at each point of $T \setminus E$ if and only if E is an F_σ set of measure zero.

In 2023 the corresponding problem for Blaschke products was solved by the following theorem which is a joint result with Spyros Pasias.

Theorem 2. Let E be a subset on T. There exists a Blaschke product which has no radial limits on E but has unrestricted limits at each point of $T \setminus E$ if and only if E is a closed set of measure zero.

An extension of Fatou's interpolation theorem which solves a problem of Rubel on bounded analytic functions (with applications) will be presented as well. If time permits, also extensions for univalent functions will be considered.

Serhii Favorov

Jagiellonian University

Crystalline measures and Fourier quasicrystals

A complex measure μ on a d-dimensional Euclidean space is a crystalline measure if it is the temperate distribution, its distributional Fourier transform $\hat{\mu}$ is also a measure, and supports of μ and $\hat{\mu}$ are discrete (locally finite); μ is a Fourier quasicrystal (FQ) if, in addition, $|\mu|$ and $|\hat{\mu}|$ are also temperate distributions. For example, if μ_0 is the sum of the unit masses in all points with integer coordinates, then by Poisson's formula, $\hat{\mu}_0 = \mu_0$. Hence, μ_0 is FQ.

We show that FQ is very often a linear combination of shifts of measures of the μ_0 type. Moreover, support of FQ with integer positive masses (multiset) in many cases (always for $d = 1$) can be described as the zero set of a system of exponential polynomials (zero set of one exponential polynomial for $d = 1$).

FQ of the last type are the particular cases of almost periodic (multi)sets, which were studied earlier in the papers by M.Krein and B.Levin, L.Ronkin, A.Rashkovskii and me. In particular, we showed that any almost periodic set on the real line is the zero set of an almost periodic entire function.

Furthermore, we consider various types of almost periodic measures. Based on their properties, we present the first example of a non-FQ crystalline measure.

Christopher Felder

Indiana University

Interpolating sequences for ℓ^p_A

A well-known theorem of Carleson relates interpolating sequences for H^∞ to certain separation conditions. Analogues of this result are known for various other spaces of functions but have not yet been established in the setting of ℓ^p_A– the space of functions holomorphic on the disk with p-summable Maclaurin coefficients. We will discuss interpolating sequences in this setting and their interaction with various separation conditions. Based on joint work with R. Cheng.

Pavel Gumenyuk

Politecnico di Milano

Commuting univalent self-maps and embeddability

The talk will cover results of a joint work in progress (with Manuel D. Contreras and Santiago Díaz-Madrigal, Universidad de Sevilla, Spain) concerning the structure of the composition semigroup formed by all univalent self-maps of the unit disk $\mathbb{D}$ commuting with a given univalent self-map $\varphi : \mathbb{D} \to \mathbb{D}$. We show that this problem is closely related to the question of embeddability of φ into a continuous one-parameter semigroup in $\mathbb{D}$.

Lior Hadassi
Tel Aviv University

Entire functions of exponential type: Taylor coefficients and zeroes

Let F be an entire function of exponential type represented by the Taylor series

$$F(z) = \sum_{n \geq 0} \omega_n \frac{z^n}{n!}$$

with unimodular coefficients $|\omega_n| = 1$. We show that *either the counting function $n_F(r)$ of zeroes of F grows linearly at infinity, or F is an exponential function.* The same conclusion holds if only a positive asymptotic proportion of ω_n is unimodular.

This improves a classical result of Fritz Carlson (1915) who showed that

$$\int_1^\infty \frac{n_F(r)}{r}\, dr < \infty,$$

provided that F is not an exponential function.

If time permits, we'll discuss what happens if the unimodularity is replaced by a weaker assumption that the absolute values of ω_n are bounded from below and from above by positive constants.

Nathan Hayford
University of South Florida

Painlevé equations and critical phenomena in random matrix theory

The Painlevé equations have been studied for over a century, and have since found applications in a wide range of mathematical disciplines. In particular, these equations arise naturally in the study of critical phenomena (phase transitions) in random matrix theory. The phase transitions appearing in the Hermitian 1-matrix model have been intensively studied for the past few decades, and have been shown to be "universal" in nature. In this talk, we will discuss some of the history of the Painlevé equations in random matrices. We also explain how this story is part of a larger program on critical phenomena in random matrices, and will discuss a new critical phenomena appearing in the Hermitian 2-matrix model, which has not appeared before in the mathematical literature. This new transition is described by a higher-order Painlevé type equation. If time permits, we shall also discuss the physical interpretations of this transition. This is joint work with Seung-Yeop Lee (USF) and Maurice Duits (KTH).

Haakan Hedenmalm
KTH, Stockholm; St. Petersburg State University; University of Reading

Hyperbolic Fourier series and the Klein-Gordon equation

This reports on joint work with A. Bakan, A. Montes-Rodriguez, D. Radchenko, and M. Viazovska. We discuss a discretized Goursat problem for the Klein-Gordon equation. This then connects with hyperbolic Fourier series, a way to represent functions or distributions on the line in terms of a series based on complex exponentials $\exp(i\pi nt)$ and $\exp(-i\pi m/t)$ for integers m, n. This representation is unique in a wide range of ultradistributions dual to Gevrey class. The biorthogonal system solves the discretized Goursat problem.

Some properties of the biorthogonal system are outlined, and a QUE-type conjecture is proposed.

Eran Igra
Technion

Chaotic dynamics in the Rössler system

The Rössler system is a minimal model for chaos, in the sense that it is the least non-linear vector field one can study in $\mathbf{R}^3$. Numerically, despite the apparent simplicity of the vector field, the flow it generates is quite often extremely complex, rich in non-linear phenomena - strange attractors, homoclinic trajectories, period-doubling routes to chaos - to mention a few. And yet, almost everything known about this system was proven or observed numerically.

In this talk we will see how we can analytically prove, for the first time, the existence of chaotic dynamics for the Rössler system. By imposing some mild topological assumptions, I will prove the flow dynamics at one specific Rössler system, the trefoil parameter, can be factored to a distorted Smale Horseshoe map. Using this result we will prove the existence of an attractor for the Rössler system in an open region in the parameter space - and characterize its topology. Time permitting, we will prove this fact allows us to describe the flow dynamics by reducing it to a one-dimensional model: namely, the logistic map.

Oleg Ivrii
Tel Aviv University

Critical values of inner functions

Let $\mathscr{J}$ be the space of inner functions of finite entropy endowed with the topology of stable convergence. We prove that an inner function $F \in \mathscr{J}$ possesses a radial limit (and in fact, a minimal fine limit) in the unit disk at $\sigma(F')$ a.e. point on the unit circle. We use this to show that the singular value measure $\nu(F) = \sum_{c \in \mathrm{crit}\, F}(1 - |c|) \cdot \delta_{F(c)} + F_*(\sigma(F'))$ varies continuously in F. Our analysis involves a surprising connection between Beurling-Carleson sets and angular derivatives. (This is joint work with Uri Kreitner)

Ben Jaye
Georgia Tech

Uncertainty principles associated to sets satisfying the geometric control condition

In this talk we shall introduce some forms of the uncertainty principle suggested by problems in control theory, and describe how they can be used to prove decay rates for the damped wave equation. Joint work with Walton Green and Mishko Mitkovski.

Vladimir Kadets
Holon Institute of Technology

Norm attaining functionals and operators

The classical Bishop-Phelps theorem says that for every bounded closed convex subset C of a real Banach space X the set of all continuous linear functionals that attain their maximum on C is dense in X^*.

We are going to speak about several easy to formulate open questions related to this theorem.

David Kalaj
University of Montenegro

Gaussian curvature conjecture for minimal graphs

In this talk we present a recent solution of the longstanding Gaussian curvature conjecture of a minimal graph S over the unit disk ([1]). This conjecture states the following. For any minimal graph lying above the entire unit disk, the Gaussian curvature at the point above the origin satisfies the sharp inequality $|K| < \pi^2/2$. The conjecture is first reduced to the estimation of the Gaussian curvature of certain Scherk type minimal surfaces over some bicentric quadrilaterals inscribed in the unit disk containing the origin. Then we make a sharp estimate of the Gaussian curvature of those minimal surfaces over those bicentric quadrilaterals at the point above the zero. Our proof uses complex-analytic methods since minimal surfaces that we consider allow conformal harmonic parametrization.

[1] D. Kalaj, P. Melentijević, *Gaussian curvature conjecture for minimal graphs* https://doi.org/10.48550/arXiv.2111.14687

Stanislawa Kanas
University of Rzeszow

Generalized Zalcman conjecture and Chebyshev Polynomials

At the end of 1960's, Lawrence Zalcman posed a conjecture that the coefficients of univalent functions $f(z) = z + a_2 z^2 + \cdots$ on the unit disk satisfy the sharp inequality $|a_n^2 - a_{2n-1}| \leq (n-1)^2 \quad (n \geq 2)$, with equality only for the Koebe function. This remarkable conjecture implied the celebrated Bieberbach conjecture, which was finally proved by Louis de Branges (1985). In 1995, Samuel Krushkal proved the Zalcman conjecture for $n = 3$ and in 2010 for $n = 4, 5, 6$. The Zalcman conjecture for functions in the class $\mathcal{S}$ is still open for $n > 6$. Brown and Tsao in 1986 proved the Zalcman conjecture for starlike functions, and for the subclass $\mathcal{S}$, consisting of functions with real coefficients. They also obtained the truth of the Zalcman conjecture for typically real functions. William Ma in 1988 proved the Zalcman conjecture for close-to-convex functions, and 1999 proposed a generalized conjecture $|a_n a_m - a_{n+m-1}| \leq (n-1)(m-1)$ for $n \geq 2, m \geq 2$. We propose a solution of the generalized Zalcman conjecture for the class of generalized typically-real functions, related to the Chebyshev polynomials.

Oleksiy Karlovych
NOVA University Lisbon

When are the norms of the Riesz projection and the backward shift operator equal to one?

The lower estimate by Gohberg and Krupnik (1968) and the upper estimate by Hollenbeck and Verbitsky (2000) for the norm of the Riesz projection P on the Lebesgue space L^p lead to $\|P\|_{L^p \to L^p} = 1/\sin(\pi/p)$ for every $p \in (1, \infty)$. Hence L^2 is the only space among all Lebesgue spaces L^p for which the norm of the Riesz projection P is equal to one. Banach function spaces X are far-reaching generalisations of Lebesgue spaces L^p. We prove that the norm of P is equal to one on the space X if and only if X coincides with L^2 and there exists a constant $C \in (0, \infty)$ such that $\|f\|_X = C\|f\|_{L^2}$ for all functions $f \in X$. Independently of this, we also show that the norm of P on X is equal to one if and only if the norm of the backward shift operator S on the abstract Hardy space $H[X]$ built upon X is equal to one.

Oleksiy Klurman
University of Bristol

An update on Fekete polynomials

Extremal properties of Littlewood polynomials (with coefficients ± 1) have been extensively studied throughout the past century. Among special classes of Littlewood polynomials, particular attention has been given to so-called "Fekete polynomials" (with coefficients being Legendre symbols). Since their discovery by Dirichlet in the nineteenth century, Fekete polynomials and their extremal properties have attracted considerable attention, particularly due to their intimate connection with the putative Siegel zero and the small class number problem. The goal of this talk is to discuss a general approach to understanding the behavior of such polynomials, resolving several open problems. This is based on joint work with Y. Lamzouri and M. Munsch.

Alexander Koldobsky
University of Missouri-Columbia

Comparison problems for the Radon transform

Given two non-negative functions such that the Radon transform of one of them is pointwise smaller, does it follow that the L^p-norm of this function is smaller for some p¿1? We consider this problem for the classical and spherical Radon transforms. In both cases we point out classes of functions for which the answer is affirmative, and show that in general the answer is negative if the functions do not belong to these classes. The results are in the spirit of the solution of the Busemann-Petty problem from convex geometry, and the classes of functions that we introduce generalize the class of intersection bodies introduced by Lutwak. This is joint work with Michael Roysdon and Artem Zvavitch.

Anna Kononova
Tel Aviv University

Random zero sets for Fock-type spaces

Consider a sequence $\left\{\lambda_n e^{i\theta_n}\right\}$, where $\lambda_n > 0$ are fixed and $\theta_n \in [0, 2\pi]$ are independent random variables. Will this random sequence be a zero set of a given weighted Fock-type space? We will discuss some results in this direction.

Gady Kozma
The Weizmann Institute of Science

Luzin's problem on Fourier convergence and homeomorphisms

The theorem of Pál and Bohr, proved more than a hundred years ago, states that for any continuous function f on $[0,1]$ one can find a homeomorphism φ such that the Fourier expansion of $f \circ \varphi$ converges uniformly. This result can be generalised in many directions, and in particular Luzin asked whether the homeomorphism φ can be taken to be absolutely continuous. We show that this is indeed the case. Joint work with A. Olevskiĭ.

Venky Krishnan
TIFR Centre for Applicable Mathematics, Bangalore

UCP and counterexamples to UCP involving some generalized Radon transforms

We study generalized Radon-type transforms involving functions and symmetric tensor fields. We show in some instances that a unique continuation principle for such transforms holds and we also give explicit counterexamples where such a principle does not hold.

Samuel L. Krushkal
Bar-Ilan University and University of Virginia

The Grunsky norm of univalent functions and abelian holomorphic differentials

We establish that the Grunsky norm of any normalized univalent function on the disk is completely determined by the squares of holomorphic abelian differentials (in contrast to the Teichmüller norm, which relates to all integrable holomorphic quadratic differentials).

This result has important interesting applications. In particular, it provides an explicit representation of Fredholm eigevalues of all quasiconformal Jordan curves.

Aleksei Kulikov
Tel Aviv University

Fourier uniqueness and non-uniqueness pairs

Given discrete sets $\Lambda, M \subset \mathbb{R}$ we call them a Fourier uniqueness pair if there does not exist a non-trivial Schwartz function f such that f is zero on Λ and $\hat{f}$ is zero on M. In their breakthrough paper, Radchenko and Viazovska constructed the first such example with $\Lambda = M = \{\pm\sqrt{n}\}_{n\in\mathbb{N}_0} \cup \{p\}$ for some $p \neq \pm\sqrt{n}$ and moreover provided a way to reconstruct a function from its values of it and the values of its Fourier transform on this set. Later, Ramos and Sousa explored more general Fourier uniqueness sets and they showed, in particular, that if $\gamma < 1 - \frac{\sqrt{2}}{2}$ then $\Lambda = M = \{\pm n^{\gamma}\}_{n\in\mathbb{N}_0}$ is a uniqueness pair.

Motivated by these results, we studied necessary and sufficient conditions for a pair of sets to be a Fourier uniqueness pair. We showed that if $\Lambda = M = \{\pm a\sqrt{n}\}_{n\in\mathbb{N}_0}$ then (Λ, M) is a uniqueness pair if $a < 1$ and a non-uniqueness pair if $a > 1$, and more generally we classified all polynomial uniqueness and non-uniqueness pairs up to the endpoint. Moreover, in the uniqueness case the result can be improved to the frame bound and consequentially the interpolation formula of the form

$$f(x) = \sum_{\lambda\in\Lambda} a_{\lambda}(x) f(\lambda) + \sum_{\mu\in M} b_{\mu}(x) \hat{f}(\mu),$$

thus extending the result of Radchenko and Viazovska. This in turn can be used to construct an abundance of new crystalline measures.

The talk is based on a joint work with Fedor Nazarov and Mikhail Sodin.

Kirill Lazebnik
University of North Texas

A geometric approach to polynomial and rational approximation

We will discuss an alternative approach to the classical theorem of Runge on polynomial and rational approximation in one complex variable, with particular attention towards understanding the behavior of the approximants off of the region where they approximate a given function. Our approach is based on the theory of quasiconformal mappings in the plane. This talk is based on joint work with Christopher Bishop.

Elijah Liflyand
Bar-Ilan University

L^1 convergence of Fourier transforms

The problem of L^1 convergence of trigonometric series is generalized to the non-periodic case in a more or less final form. Roughly, it can now be said that any known condition which guarantees integrability of the Fourier transform of a function of bounded variation f also leads to the L^1 convergence of its partial integrals provided that a universal necessary and sufficient condition $\lim\limits_{|t|\to\infty} |f(t)| \ln |t| = 0$ is added.

Galyna Livshyts
Georgia Institute of Technology

The universal lower bound in the dimensional Brunn-Minkowski conjecture

Many isoperimetric-type inequalities are known to improve when additional symmetry is assumed. For instance, the Poincare inequality on the circle improves by a factor of 4 if the function is assumed to be even, which can be explained simply via a Fourier series. We will discuss this phenomenon in the broader context, and show that log-concavity of measures always improves by a dimensional factor, when symmetry is assumed.

Aleksandr Logunov
Unige/MIT

Sign of Laplace eigenfunctions

The functions $\sin(kx)$, $\cos(kx)$ are positive on half of the circle. This talk will concern a related phenomenon of quasi-symmetry for the sign of Laplace eigenfunctions on Riemannian manifolds. We will talk about the distribution of sign and the question of Nazarov, Polterovich and Sodin at which scale quasi-symmetry holds and at which scale quasi-symmetry breaks down. Based on a joint work in progress with Fedya Nazarov.

Dan Mangoubi
Hebrew University

The inner radius of nodal domains in high dimensions

Let u be an eigenfunction of the Laplace-Beltrami operator on a closed d-dimensional Riemannian manifold with corresponding eigenvalue k^2. It is well known that every ball of radius c_1/k must contain a zero of u, where c_1 is some positive constant depending only on M. Equivalently, the inner radius of every nodal domain of u is at most c_1/k. It is known that in two dimensions the inner radius of every nodal domain is at least c_2/k for some $0 < c_2 < c_1$. We prove an almost optimal lower bound, namely, the inner radius of every nodal domain is at least $ck^{-1}(\log k)^{-(d-2)/2}$.

The talk is based on joint work with Philippe Charron.

Myrto Manolaki
University College Dublin

Holomorphic functions on the unit disc with chaotic radial behaviour

It is known that most holomorphic functions on the unit disc have maximal cluster sets along any radius. This talk is concerned with holomorphic functions on the unit disc that have an even more chaotic radial behaviour, in the sense that they can uniformly approximate any continuous function on any proper compact subset of the unit circle by considering radial limits at a prescribed set of modulii tending to 1. We will discuss a wide range of properties of such functions, focusing on their boundary behaviour and the convergence properties of their Taylor polynomials outside the unit disc. Joint work with Stéphane Charpentier and Konstantinos Maronikolakis.

Alon Nishry
Tel Aviv University

Zeros of the deformed exponential function

The deformed exponential function

$$F_y(z) = \sum_{n=0}^{\infty} y^{n(n-1)/2} \frac{z^n}{n!}, \qquad z \in \mathbb{C}, \quad |y| \leq 1,$$

has found various applications in Statistical Mechanics, Combinatorics and Complex Analysis. We find an asymptotic formula for the zeros of F_y, as $|z| \to \infty$, generalizing results of Eremenko ($|y| < 1$), Nassif and Littlewood (special complex $|y| = 1$) and Zhang ($y \in (0,1)$). Moreover, we address some questions of Benatar, Borichev, and Sodin and Sokal, regarding the distribution of the zeros of F_y.

Joint work in progress with Andrea Sartori.

Shahaf Nitzan
Georgia Tech

The uncertainty principle in finite dimensions

I will give a survey of some results related to the title, and discuss a couple of new observations in the area. The talk is based on joint work with Jan-Fredrik Olsen and Michael Northington.

Jonathan Nurielyan
Ben-Gurion University of the Negev

Projection distance on finite dimensional complete Pick kernels

Recently, Ofek, Pandey, and Shalit have defined a version of Banach-Mazur distances on the space of isomorphism classes of finite-dimensional complete Pick spaces. By the universality theorem of Agler and McCarthy, every finite-dimensional complete Pick space on n points is equivalent to a subspace of the Drury-Arveson space spanned by n kernels at points of the unit ball of some C^d. We propose to study the space of projections on finite-dimensional multiplier coinvariant subspaces of the Drury-Arveson space. The metric on this space is induced by the norm. Firstly, we will focus on the Hardy space, and show that this space is homeomorphic to the symmetrized polydisk. We will later show how it can be generalized to the case of the Drury-Arveson space. When we restrict ourselves to the subspace of projections on spaces spanned by distinct n kernels, then this space can be embedded in the symmetrized polyball. We will show how one can use these methods to construct a hermitian vector bundle on the disk. These tools demonstrate a connection between the interpolation problem and hyperbolic geometry on the unit ball. Joint work with Eli Shamovich.

Valerii Pchelintsev

Tomsk State University

Spectral stability estimates for the Dirichlet-Laplacian

In this talk, we deal with stability estimates for the eigenvalues of the Laplace operator:

$$-\Delta g = -\left(\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2}\right), \quad w = u + iv \in \Omega, \quad g|_{\partial\Omega} = 0,$$

with the Dirichlet boundary condition in bounded doubly connected domains $\Omega \subset \mathbb{C}$.

Our approach to the spectral stability is based on composition operators on Sobolev spaces generated by conformal mappings. In this way we estimate the variation of the eigenvalues of the Laplace operator with the Dirichlet boundary condition upon domain perturbation via energy type integrals for a large class of domains that includes examples with highly non-rectifiable boundaries.

The talk is based on joint works with Vladimir Gol'dshtein and Alexander Ukhlov.

Research was supported by RSF Grant No. 23-21-00080.

Sean Perry

University of South Florida

On the valence of logharmonic polynomials

Here we will discuss recent work concerning the valence of logharmonic polynomials, the product of a complex polynomial and the conjugate of a complex polynomial. Let $P = p\bar{q}$, where p and q are complex polynomials of degree n and m, respectively. Given such a map P and a complex constant w, we are interested in its valence, i.e. the cardinality of the set of preimages of w under P. Under certain nondegeneracy assumptions, this set is finite. Hengartner asked what, then, is its maximum cardinality in terms of the degrees of the constituent polynomials? Here, we present recent progress on two conjectures. Using complex dynamics, we settle the conjecture that the upper bound is $3n - 1$ in the case $m = 1$. We then settle the conjecture that the valence is strictly less than the Bezout bound in the general case, using Sylvester resultants to give a bound of $n^2 + m^2$. We also comment on when this bound improves the Bezout bound in the more general context of polyanalytic polynomials.

Arkady Poliakovsky

Ben-Gurion University of the Negev

Jumps in Besov spaces and fine properties of Besov and fractional Sobolev functions

Jointly with Paz Hashash. In this talk we analyze functions in Besov spaces $B^{1/q}_{q,\infty}(\mathbb{R}^N,\mathbb{R}^d)$, $q \in (1,\infty)$, and functions in fractional Sobolev spaces $W^{r,q}(\mathbb{R}^N,\mathbb{R}^d)$, $r \in (0,1)$, $q \in [1,\infty)$. We prove for Besov functions $u \in B^{1/q}_{q,\infty}(\mathbb{R}^N,\mathbb{R}^d)$ the summability of the difference between one-sided approximate limits in power q, $|u^+ - u^-|^q$, along the jump set $\mathcal{J}_u$ of u with respect to Hausdorff measure $\mathcal{H}^{N-1}$, and establish the best bound from above on the integral $\int_{\mathcal{J}_u} |u^+ - u^-|^q d\mathcal{H}^{N-1}$ in terms of Besov constants. We show for functions $u \in B^{1/q}_{q,\infty}(\mathbb{R}^N,\mathbb{R}^d), q \in (1,\infty)$ that

$$\liminf_{\varepsilon\to 0^+} \frac{1}{\varepsilon^N}\int_{B_\varepsilon(x)} |u(z) - u_{B_\varepsilon(x)}|^q dz = 0$$

for every x outside of a $\mathcal{H}^{N-1}$-sigma finite set. For fractional Sobolev functions $u \in W^{r,q}(\mathbb{R}^N,\mathbb{R}^d)$ we prove that

$$\lim_{\varepsilon\to 0^+} \frac{1}{\varepsilon^N}\int_{B_\varepsilon(x)} \frac{1}{\varepsilon^N}\int_{B_\varepsilon(x)} |u(z) - u(y)|^q dz dy = 0$$

for $\mathcal{H}^{N-rq}$ a.e. x, where $q \in [1,\infty)$, $r \in (0,1)$ and $rq \leq N$. We prove for $u \in W^{1,q}(\mathbb{R}^N), 1 < q \leq N$, that

$$\lim_{\varepsilon\to 0^+} \frac{1}{\varepsilon^N}\int_{B_\varepsilon(x)} |u(z) - u_{B_\varepsilon(x)}|^q dz = 0$$

for $\mathcal{H}^{N-q}$ a.e. $x \in \mathbb{R}^N$.

In addition, we prove Lusin-type approximation for fractional Sobolev functions $u \in W^{r,q}(\mathbb{R}^N,\mathbb{R}^d)$ by Hölder continuous functions in $C^{0,r}(\mathbb{R}^N,\mathbb{R}^d)$.

Marina Prokhorova

Technion

Unbounded Fredholm operators

A linear operator (bounded or unbounded) is called Fredholm if its range is closed and its kernel and cokernel are finite-dimensional. The index theory for norm continuous families of bounded Fredholm operators was developed in the classical work of Atiyah; its analog for self-adjoint operators was developed in the work of Atiyah and Singer. The index theory of elliptic differential operators on closed manifolds is based on these classical results.

However, in some situations (e.g., for elliptic operators on manifolds with boundary) one needs to deal with weaker topologies on the space of unbounded operators. The most important such topology is the graph topology; a family of operators is continuous in the graph topology if the family of their graphs is continuous.

My talk is devoted to an index theory of graph continuous families of unbounded Fredholm operators in a Hilbert space. I will show how this theory is related to the classical index theory of bounded Fredholm operators. The talk is based on my recent preprints arXiv:2110.14359 and arXiv:2202.03337.

Todd Quinto
Tufts University

Microlocal properties of novel ellipsoidal and hyperbolic Radon transforms

Larry Zalcman did beautiful mathematics in a range of areas, and we will describe some of our work in one of those areas, integral geometry.

We present novel microlocal results for generalized ellipsoid and hyperboloid Radon transforms with centers on surfaces in Euclidean Space. We introduce a new Radon transform, R, which integrates compactly supported distributions over ellipsoids and hyperboloids and more general hypersurfaces with centers on a smooth surface, S. Our transform, R, is shown to be a Fourier Integral Operator (FIO). In our main theorem we prove that R satisfies the Bolker condition if and only if the support of the function is in a connected open set that does not meet any plane tangent to S, This implies that standard backprojection type reconstruction operators, such as the normal operator R^*R, do not add artifacts to the reconstruction. We apply our results to a cylindrical geometry that could be used in URT. We describe artifacts that can occur in backprojection reconstruction if our hypotheses are not satisfied, and we characterize object singularities that are visible from the data. Finally, we present reconstructions of image phantoms in two dimensions that illustrate our microlocal theory.

Joint with Sean Holman, University of Manchester, James W. Webber, Brigham and Women's Hospital, Boston.

Antti Rasila
Guangdong Technion - Israel Institute of Technology

Boundary behavior of quasiregular and harmonic mappings

We discuss connections between different conditions involving dilatations and multiplicities of the zeros, and boundary behavior of quasiconformal and related classes of mappings. We compare Caratheódory, Koebe and Lindelöf type results for these classes of mappings to the results from classical function theory as well as those concerning quasiconformal and quasiregular mappings in plane and space.

Sufficient conditions for the existence of angular (non-tangential) limit at a boundary point can be obtained, for example, in the terms of multiplicities of zeroes of the function, which are required grow fast enough on a given sequence of points approaching the boundary. We also discuss sharpness of such conditions. This presentation is based on joint work with Daoud Bshouty, Jiaolong Chen, Stavros Evdoridis, Jie Huang, and Matti Vuorinen.

Oliver Roth
University of Würzburg

Function theory off the complexified unit circle: Möbius–invariant differential operators, strict deformation quantization, and spectral synthesis

We study the Fréchet space structure of $\mathcal{H}(\Omega)$, the space of holomorphic functions on $\Omega = \hat{\mathbb{C}}^2 \setminus \{(z,w) \in \hat{\mathbb{C}}^2 \, : \, zw \neq 1\}$, the complement of the "complexified unit circle" $\{(z,w) \in \hat{\mathbb{C}}^2 \, : \, zw = 1\}$. This offers a unified framework for investigating conformally invariant differential operators on the unit disk $\mathbb{D}$ and the Riemann sphere $\hat{\mathbb{C}}$, which have been studied by Peschl, Aharonov, Minda and many others, within their conjecturally natural habitat. We apply this machinery to a problem in deformation quantization by deriving an explicit formula for the canonical Wick–type star product $\star_\hbar$ for all smooth functions defined on the unit disk $\mathbb{D}$ belonging to the observable algebra $\mathcal{A}(\mathbb{D}) := \{f(z,\overline{z}) \, : \, f \in \mathcal{H}(\Omega)\}$. This formula is given in the form of a factorial series which depends holomorphically on a complex deformation parameter $\hbar$ and easily leads to an asymptotic expansion of the star product $\star_\hbar$ in powers of $\hbar$. As another application we give a function–theoretic Runge–type characterization of the "exceptional" eigenspaces of the invariant Laplacian on the unit disk, which have been introduced by Helgason and Rudin in the 70s and 80s. Finally, we discuss a recent result of A. Moucha, who showed that $\mathcal{H}(\Omega)$ and hence the observable algebra $\mathcal{A}(\mathbb{D})$ admit spectral synthesis by exhibiting a Schauder basis of eigenfunctions of the invariant Laplacian on Ω and the unit disk $\mathbb{D}$, respectively.

This talk is based on collaborations with M. Heins, D. Kraus, A. Moucha, S. Schleißinger, M. Schötz, T. Sugawa and S. Waldmann.

Boris Rubin

Louisiana State University

Shifted Radon transforms in Zalcman's offbeat integral geometry

One of the most attractive problems of L. Zalcman's offbeat integral geometry is whether a function on a constant curvature space X can be reconstructed from its integrals over geodesic spheres (or balls) of fixed radius. The space X can be thought of as the real n-dimensional Euclidean space, the unit sphere, or the hyperbolic space.

The problem acquires a new flavor if the (0-dimensional) centers of the spheres (or of the balls) are replaced by the totally geodesic submanifolds of positive dimension k (k-geodesics). Then the balls become the n-dimensional neighborhoods of the k-geodesics and the spheres become the boundaries of these neighborhoods. The simplest example is the Radon transform over strips of fixed width $d > 0$ in the 2-plane. The case $d = 0$ is the classical problem by J. Radon (1917) for lines in the plane. One can also consider pipes or solid tubes of fixed diameter in the 3-space, slabs of constant thickness, or hoops on the sphere.

In the new setting, the question can be reformulated as the injectivity problem for the shifted totally geodesic Radon transforms with fixed shift. However, the shifted Radon transforms are not new. They naturally arise in the Helgason's double fibration scheme. Their duals, but with variable shift, are well known in integral geometry, since P. Funk (1911), as a tool for explicit inversion of totally geodesic Radon transforms. The term 'shifted Radon transform' is due to F. Rouviere (2001).

I am planning to show some recent injectivity results for the shifted Radon transforms in the cases when X is the Euclidean space and the unit sphere. Open problems will be discussed. If time allows, I will also speak about intriguing connections to the inverse problems for the Euler-Poisson-Darboux Equations with L^p initial data.

References:

1. L. Zalcman, Offbeat integral geometry. Amer. Math. Monthly **87**, 161-175 (1980).

2. B. Rubin, On the injectivity of the shifted Funk-Radon transform and related harmonic analysis, arXiv:2211.10348 [math.FA] (2022).

3. B. Rubin, The inverse problem for the Euler-Poisson-Darboux equation and shifted k-plane transforms, arXiv:2212.11179v2 [math.A] (2022).

Dmitry Ryabogin

Kent State University

On bodies floating in equilibrium in every orientation

We will apply Fedor Nazarov's method to construct a counterexample to Ulam's problem 19 from the Scottish Book.

Andrea Sartori

Tel Aviv University

The growth of harmonic functions and their nodal volume

In this talk, I will discuss the relation between the growth of harmonic functions and their nodal volume, that is volume of their zero set. One way to quantify the growth of a harmonic function u in a ball $B = B(0,1) \subset \mathbb{R}^n$ is via the *doubling index* N, defined by

$$\sup_{B(0,1)} |u| = 2^N \sup_{B(0,\frac{1}{2})} |u|.$$

Since (as we will see) the doubling index gives a notion of "degree" for harmonic functions, it would be reasonable to expect that the nodal volume of u in B is proportional to N. I will present a joint work with A. Logunov and Lakshmi Priya, where we prove the almost sharp lower bound

$$\mathcal{H}^{n-1}(\{u = 0\} \cap B(0,2)) \gtrsim_\varepsilon N^{1-\varepsilon},$$

provided that $u(0) = 0$.

Barry Simon

Caltech

A tale of three coauthors: comparison of Ising models

On Friday, Jan 14, 2022, I had a draft of a single author paper intended for the Lieb Festschrift. Six days later, the paper had three coauthors. This talk will explain the interesting story, expose some underlying machinery and sketch the proof of a lovely inequality on certain finite sums.

Toshiyuki Sugawa

Tohoku University

Uniformly perfect sets and conformal capacities

It is often crucial to compute or estimate the conformal capacities of given condensers in Geometric Function Theory. When the plates of a condenser are continua, it is classically known that its capacity is well controlled in terms of the distance of the plates and diameters of the plates. However, it is not easy to estimate the capacity (especially from below) when the plates are not continua. In this talk, we give lower estimates of the capacity in a manner similar to the case of continua when the plates are uniformly perfect. Furthermore, we show that validity of some of those estimates characterizes uniform perfectness of the plates. Most of the estimates are quantitative with explicit bounds. This talk is based on the joint work with Oona Rainio and Matti Vuorinen.

Xavier Tolsa
ICREA - Univesitat Autonoma de Barcelona - CRM

Carleson's ϵ^2-conjecture in higher dimensions and the Alt-Caffareli-Friedman formula

In this talk I will report on a recent work with Ian Fleschler and Michele Villa where we extend the ϵ^2-conjecture about the characterization of tangent points for Jordan domains in the plane to the higher dimensional setting. The generalization that we obtain does not require any connectivity condition for the domains under consideration, and so it is new even in dimension 2. I will also explain interesting connections with the Alt-Caffarelli-Friedman monotonicity formula and with the so-called Faber-Krahn inequalities that quantify the size of the first eigenvalue of the Laplacian on a given domain.

Alexander Ukhlov
Ben-Gurion University of the Negev

On the non-linear Neumann eigenvalue problem in Hölder domains

We consider the Neumann (p,q)-eigenvalue problems in bounded Hölder γ-singular domains $\Omega_\gamma \subset \mathbb{R}^n$. In the case $1 < p < \gamma$ and $1 < q < p^*_\gamma$ we prove solvability of this eigenvalue problem and existence of the minimizer of the associated variational problem. In addition, we establish some regularity results of the eigenfunctions and some estimates of (p,q)-eigenvalues.

Anna Vishnyakova
Holon Institute of Technology

Entire functions from the Laguerre-Polya class

The famous Laguerre-Polya class consists of entire functions which are uniform on the compacts limits of real polynomials having all real zeros. The Laguerre-Polya class is of interest to many areas of mathematics such as complex analysis, statistical physics, combinatorics, asymptotic analysis, the theory of mock modular forms and others. We present new necessary and new sufficient conditions for an entire function to belong to the Laguerre-Polya class in terms of its Taylor coefficients.

The following statement is an example of our results. Let $f(x) = \sum_{k=0}^{\infty} a_k x^k$, $a_k > 0$, be an entire function. Suppose that there exists $\alpha \in [1+\sqrt{5}, 4)$, such that $\frac{a_k^2}{a_{k-1}a_{k+1}} \in \left[\alpha, \frac{8}{\alpha(4-\alpha)}\right]$ for all $k = 2, 3, \ldots$ Then f belongs to the Laguerre-Polya class.

Matti Vuorinen

University of Turku

Mobile disks, conformal capacity, and metrics

We use numerical simulations to find lower bounds for the conformal capacity of the condenser $\mathrm{cap}(\mathbb{B}^2, E)$ where E is a finite union of disjoint hyperbolic disks. The results are compared to known bounds. Computational results suggest that in the extremal cases a grouping together phenomenon occurs: the disjoint disks of E are tangent to each other. The extremal cases seem to have a connection with packing problems. Two computational methods are used: boundary integral equation method (M.M.S. Nasser) and *hp*-FEM (H. Hakula) arXiv:2303.00145. We also prove with M. Fujimura and R. Kargar a new formula for the visual angle metric and apply it to obtain a sharp distortion result for this metric under quasiregular mappings of $\mathbb{B}^2$ onto itself arXiv:2304.04485. This result is new also for analytic functions.

Oren Yakir

Tel Aviv University

Approximately half of the roots of a random Littlewood polynomial are inside the disk

We will prove that for large n, all but $o(2^n)$ polynomials of the form $\sum_{k=0}^{n} \pm z^k$ have $n/2 + o(n)$ roots inside the unit disk. This problem was suggested in Hayman's book. We will also prove a concentration result for the Mahler measure of random polynomials, which might be of independent interest.