Project 3 Problem2

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(Here are the programming description and test cases.)

The required questions:

1. Explain the stopping criteria you selected for your algorithm (see Resources) and how they may impact your results:

I choose two stopping criteria, the first is when the difference (total distance) between new centroids and last centroids are too small, the algorithm will stop. The second stopping criteria is the iteration time: the algorithm will stop after a given maximum iteration time. When the total difference between new and old centroids is smaller than 0.000000001, or the iteration time is larger than 100 times, the algorithm will stop.

These two stopping criteria will influence the result. The loss function will become minimum when the centroids do not move anymore, but these two stopping criteria will probably stop the algorithm when the centroids are still moving, which will make the result not "exactly" the best, also will make the loss not minimum. However, the result will be very close to the "best" result because we set the difference be very small and the iteration time be very large. These stopping criteria will save lots of time, meanwhile, get a result which is good enough.

One thing that should be noticed is when I was running the program under different K values, the iteration time stopping criteria is rarely be arrived. The algorithm always stops before 100 times iteration because the total difference is small enough. I tried to make the threshold of total difference smaller, but 100 times iteration is still hard to arrive.

2. Explain the centroid initialization method you selected and how that may impact your results.

I choose to initialization the centroids by randomly selecting K points in the input data to become the first group of the centroids.

The randomly selecting will make the result not always the same. Depends on the initial centroids, the final centroids' coordinates are sometimes different. This is because for the same input data and the same K, there will be several "ways" to cluster the data points. In other words, there will be several result. Because we

choose the initial centroids randomly, the centroids will sometimes be changed towards to one result, sometimes towards to another result, depends on the initial location of the centroids.

I found that for 1 specific K value, there will be about 2 to 3 different results. One thing that should be noticed is although the initial centroids are randomly pick, which means they can have lots of (the number of the points in the input file) groups of coordinates, the final centroids (results) always only have 2 to 3 groups of coordinates. This means lots of different groups of initial centroids we pick will finally converged to one single result, and after trying many groups of initial centroids, there will be only 2 to 3 results.

3. Explain why your algorithm may require different amounts of time to run on the same input and parameters.

This is because we choose the initial centroids randomly. Therefore, although the input data and K are the same, different initial centroids will makes iteration time different. For example, if we choose a group of initial centroid: **A**, which is close to one of the final centroid group, the algorithm may only have to loop for 5 times to arrive the stopping criteria. However, if we choose a group of initial centroids **B**: which is far away from the final result, the algorithm then may have to loop 50 times to arrive the stopping criteria. Therefore, although the time of each iteration is similar, sometimes the algorithm loops little times, like **A**, sometimes the algorithm loops many times like **B**, so the run time will be different too.

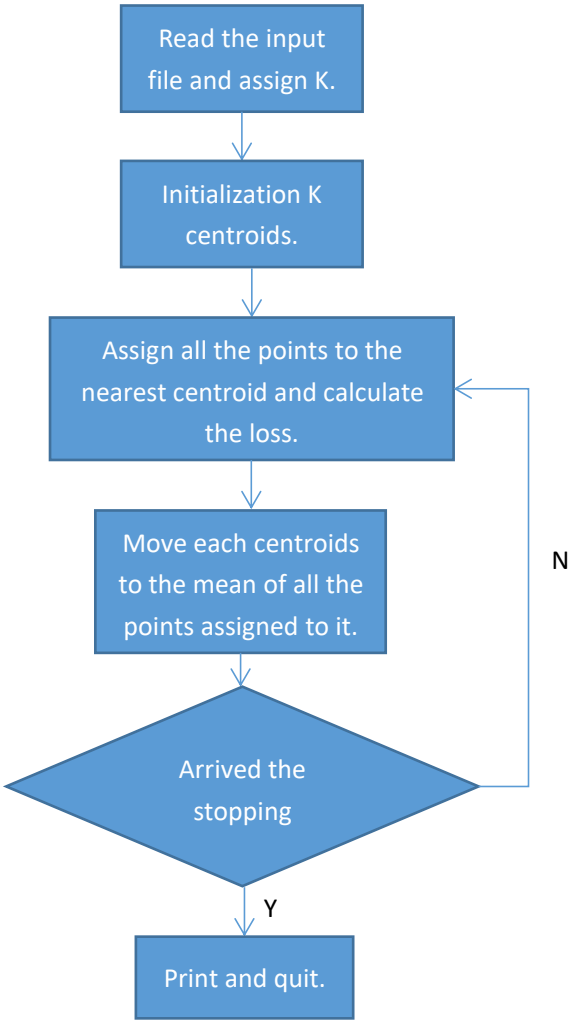
In conclusion, the random initial centroids will make the run time different randomly, depends on whether the initial centroids are close to the result.

4. Explain why your algorithm may produce different results on the same input and parameters.

This is also because we choose the initial centroids randomly, so the initial centroids will randomly be closer to one specific result, even though the input data and K are the same, and the algorithm will change the initial centroids toward to the closer result. Different initial centroids will be closer to different result, and the algorithm will gradually move the centroids to this result, so the final result will be different.

Program Description:

This program implements an algorithm to perform K-Means clustering. The algorithm contains 6 steps as shown in the figure below.



Notice: K, data array and centroids are globally defined as public variables, so every function can directly use them without receiving them as inputs. Therefore, some functions may have no input or outputs, but they can use and change the K, data and the centroid.

1. The first step is read the input file and assign K. The function readData takes care of this step.

public static double[][] readData(String path)
Inputs: the path of the input file.
Outputs: an array called data contains all the points' coordinates in the file.

2. The second step is initialization K centroids. The program will randomly choose K points in the input file as the first group of centroids. The iniCentroid function will do this step. The function pick K random points in the data points by using Math.random() function.

public static void iniCentroid()

Inputs: none.

Outputs: none.

This function has no inputs and outputs because it requires data array and change the initialization the centroid array. The data array and centroid array are globally defined, so the function can directly use and change them.

3. The third step is assigning all the points to the nearest centroid. The program uses distance function to do this step. The function calculate the distance from the point to all the centroids, then pick the nearest centroid, return he index of the centroid and the distance. Therefore, we inputs all the points to this function, then we can get all points' nearest centroids and the distances between them. Then, we assign all the points to the nearest centroid and use the distances to calculate the loss.

public static double[] distance(double[] x)

Inputs: the point needed to be assigned.

Outputs: an array, the first element is the index of the nearest centroid of the point, the second element is the distance between them.

4. The forth step is to moving each centroids to the mean of all the points assigned to it. The program uses moveCentroid function to do this. This function is pretty simple. We store the indexes of the centroids that assigned to each point in an array named c. c[i] means the ith point was assigned to the c[i]th centroid. We input the array c, the moveCentroid function will calculate the mean of each group of points assigned to the centroids, then we will move every centroid to the corresponding mean.

public static void moveCentroid(int[] c)

Inputs: the array stores the index of the centroid assigned to each points.

Outputs: none.

5. The fifth step is judging whether the algorithm can stop. I choose two stopping criteria, the first is when the difference (total distance) between new centroids and last centroids are too small, the algorithm will stop. The second stopping criteria is the iteration time: the algorithm will stop after a given maximum iteration time. The program has a function named stop, to judge whether the distance between new and old centroids are small enough to stop the algorithm. Also, the program uses a

for loop to control the iteration time.

public static boolean stop(double[][] lastCentroid)

Inputs: the array stores the coordinates of the last centroids.

Outputs: whether the algorithm can stop.

6. The sixth, also the last step, is printing the results. When the algorithm arrives the stopping criteria, the program will then print the results of the K-Means clustering.

Test Cases:

The Test Case Dataset.

K=2

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:
The loss after the 1 iteration is 78.49860794746962
The loss after the 2 iteration is 39.333118723243295
The loss after the 3 iteration is 31.891755255606217
The loss after the 4 iteration is 31.189826295566526
The loss after the 5 iteration is 31.11971106581977
Stop at 5 times iteration

The coordinates of the 1 centroid are:
10.168086243285888,10.046715814097372
The coordinates of the 2 centroid are:
0.06026253328707723,-0.08038639294559319

There are 100 points assigned to the 1 centroid.
There are 100 points assigned to the 2 centroid.

Process finished with exit code 0
```

The Electric Power Consumption Dataset.

(The scatter plot and elbow method is at the end)

K=2

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:\IntelliJ IDEA Community Edition 2020.3.2\lib\idea_rt.jar=59323:A:\Intelli
The loss after the 1 iteration is 220.08246889574747
The loss after the 2 iteration is 116.54135583538502
The loss after the 3 iteration is 95.93451281671607
The loss after the 4 iteration is 95.6224299978613
The loss after the 5 iteration is 95.53478673242864
The loss after the 6 iteration is 95.47235009318388
The loss after the 7 iteration is 95.44385263061884
The loss after the 8 iteration is 95.43671840388137
The loss after the 9 iteration is 95.43560890990594
The loss after the 10 iteration is 95.43543769865408
The loss after the 11 iteration is 95.43541621515496
The loss after the 12 iteration is 95.43541442382417
The loss after the 13 iteration is 95.4354142679161
The loss after the 14 iteration is 95.43541426062137

Stop after 14 times iteration

The 1 centroid's coordinates are:
2.042141141688092 0.13821265272952546 239.5334330792764 8.55419039345325 2.1829643537719985 2.1736254343683443 17.759401972872997
The 2 centroid's coordinates are:
0.5836998891883451 0.11596735250075395 241.53794821057284 2.529664570231928 0.5549543276430069 0.830905959868224 0.4197641509433962

There are 713680 points assigned to the 1 centroid.
There are 1335600 points assigned to the 2 centroid.

Process finished with exit code 0
```

K=3

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 23 times iteration
The loss after the last iteration is: 59.924469580809806
```

```
The 1 centroid's coordinates are:
3.948797722095709 0.1956915096293244 237.29577345206144 16.754255539449083 21.732646510664733 17.24602402153655 11.091364671774695
The 2 centroid's coordinates are:
1.8043628362955615 0.13209383363172572 239.81433079249427 7.533830432561683 0.1934380440867955 0.5662516995502729 17.880140947318555
The 3 centroid's coordinates are:
0.5239839634923366 0.11419486107018675 241.61454899245223 2.2780032367840706 0.057141190055031446 0.48156277638561323 0.4136540634827044
```

```
There are 96580 points assigned to the 1 centroid.
There are 650172 points assigned to the 2 centroid.
There are 1302528 points assigned to the 3 centroid.

Process finished with exit code 0
```

K=4

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 28 times iteration
The loss after the last iteration is: 30.338729461996895
```

```
The 1 centroid's coordinates are:
0.5199877041984787 0.11398875117402227 241.61959836182322 2.260452816826988 0.05811405872299805 0.41878088962108734 0.4136945911407412
The 2 centroid's coordinates are:
3.99656026926329 0.1990934682049714 237.17290861966578 16.948098417898287 36.94198959664501 2.449990100613762 11.153926456559693
The 3 centroid's coordinates are:
3.6932114212928417 0.19103110073240077 237.7383874323345 15.69863071860725 0.8422672752361745 35.523935887909985 10.582167498142448
The 4 centroid's coordinates are:
1.7997109765361443 0.1318588528123929 239.81607245430635 7.513770389539041 0.20306314040603055 0.4749102891130203 17.879815204837286
```

```
There are 1298980 points assigned to the 1 centroid.
There are 55559 points assigned to the 2 centroid.
There are 47105 points assigned to the 3 centroid.
There are 647636 points assigned to the 4 centroid.

Process finished with exit code 0
```

K=5

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 45 times iteration
The loss after the last iteration is: 26.858424406524822
```

```
The 1 centroid's coordinates are:
1.7999017000207935 0.1318262719261088 239.81619637135813 7.514409472537053 0.20309263221311666 0.4755485869204211 17.88423483613838
The 2 centroid's coordinates are:
0.4005213554866768 0.10484358168362017 243.4258746172542 1.7383328132401636 0.02502911246927017 0.35523808798942563 0.42613297611889556
The 3 centroid's coordinates are:
3.9977608358854098 0.19905022518465154 237.174001981625 16.952271662763664 36.9578814627995 2.4517744550531435 11.16319582057287
The 4 centroid's coordinates are:
0.7055066083948353 0.12819429620008319 238.83081339208672 3.0713765337428245 0.11108285338675347 0.5221981814198072 0.39968660010016277
The 5 centroid's coordinates are:
3.698297517843814 0.19090064983487942 237.73981271972136 15.719254287844784 0.8441248535208267 35.58359433258762 10.610226909555768
```

```
There are 647345 points assigned to the 1 centroid.
There are 788322 points assigned to the 2 centroid.
There are 55510 points assigned to the 3 centroid.
There are 511168 points assigned to the 4 centroid.
There are 46935 points assigned to the 5 centroid.

Process finished with exit code 0
```

K=6

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 52 times iteration
The loss after the last iteration is: 25.054579698161994
```

```
The 1 centroid's coordinates are:
3.7153031054026626 0.19064918216494764 237.72258043661228 15.788897269572981 0.852811240652544 35.76475658901364 10.70659224619098
The 2 centroid's coordinates are:
1.799647715276202 0.1317707748879111 239.81615432533565 7.513136446609647 0.20300909932593578 0.4769951048810534 17.89038921373601
The 3 centroid's coordinates are:
0.36700645130704507 0.11390653080874528 239.46752390523443 1.6420042449515901 0.026379609639397828 0.39618464815030924 0.42586649550706035
The 4 centroid's coordinates are:
4.001780033963205 0.19889059507894644 237.1699355060158 16.966585973913578 37.00457058207176 2.455179390829931 11.192036709180908
The 5 centroid's coordinates are:
1.8009351831298344 0.15947146028222423 239.40462573331024 7.656740130014362 0.40463770413825906 0.9663627715237039 0.3789281750436023
The 6 centroid's coordinates are:
0.39991799308624104 0.1051664132039131 243.94491399306906 1.7326788030591145 0.022942638611109335 0.3478648425868536 0.419989904088844
```

```
There are 46403 points assigned to the 1 centroid.
There are 646971 points assigned to the 2 centroid.
There are 548416 points assigned to the 3 centroid.
There are 55354 points assigned to the 4 centroid.
There are 126140 points assigned to the 5 centroid.
There are 625996 points assigned to the 6 centroid.

Process finished with exit code 0
```

K=7

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 64 times iteration
The loss after the last iteration is: 24.008851570809096
```

```
The 1 centroid's coordinates are:
4.001799577258671 0.19888757610246963 237.16989648257547 16.96665402055918 37.00487778440193 2.455205679908948 11.19222083717233
The 2 centroid's coordinates are:
1.829891179458122 0.1608526634805901 239.62425832181435 7.77471733302218 0.4170566000033304 0.9903252127287563 0.38118828368274693
The 3 centroid's coordinates are:
0.40529061405122097 0.10953147473722838 236.8681368131103 1.8156993361607219 0.03587728194726166 0.37294278996865204 0.26510349437580677
The 4 centroid's coordinates are:
0.3702196305340695 0.11458146615064911 241.3608830820087 1.6426970003243384 0.024225844876913503 0.3974641077356641 0.4951228724209483
The 5 centroid's coordinates are:
1.799718229652881 0.13176346526022642 239.81631904213899 7.513414558741816 0.20302848557172096 0.47709622768671434 17.89095782503045
The 6 centroid's coordinates are:
0.41539762606235525 0.09963649858357543 245.1018479036785 1.7794118980175133 0.02189801699716714 0.3217053824362606 0.36603682719546743
The 7 centroid's coordinates are:
3.7156854494503078 0.1906438025436531 237.72097327010403 15.790541064884565 0.8529855572321621 35.76908816555292 10.70832075878422
```

```
There are 55353 points assigned to the 1 centroid.
There are 120106 points assigned to the 2 centroid.
There are 173536 points assigned to the 3 centroid.
There are 653971 points assigned to the 4 centroid.
There are 646924 points assigned to the 5 centroid.
There are 353000 points assigned to the 6 centroid.
There are 46390 points assigned to the 7 centroid.

Process finished with exit code 0
```

K=8

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 50 times iteration
The loss after the last iteration is: 23.544097914425496
```

```
The 1 centroid's coordinates are:
1.8426198079803597 0.16092455399700195 239.62097746152628 7.827698147895701 0.4227240228789323 0.9960081029551954 0.3768469971401335
The 2 centroid's coordinates are:
0.4262870786031894 0.0950771247895658 246.23585763154404 1.8090689342605164 0.019969137134717485 0.30592545431000995 0.32198817283204556
The 3 centroid's coordinates are:
0.4154099072080806 0.10538752072822175 235.90279151819064 1.8614119888507992 0.038457467452280986 0.357539780545461 0.18255830363758246
The 4 centroid's coordinates are:
0.38825740809004305 0.10958895279566397 242.90619641616635 1.6990035401109918 0.02425147647497493 0.3655422027549437 0.4609752020777796
The 5 centroid's coordinates are:
3.715836146864148 0.19064773731755313 237.7210387426454 15.791117435267122 0.8530927279391156 35.77159735247828 10.708449216307699
The 6 centroid's coordinates are:
1.7997450318431327 0.13176444382614036 239.81629362211203 7.513532430593412 0.20308848080133557 0.4772166573919496 17.89106535583998
The 7 centroid's coordinates are:
4.001869446602454 0.19887763103217762 237.16968725722234 16.966886483947537 37.006142839075686 2.4553289128981555 11.192325064590147
The 8 centroid's coordinates are:
0.37128081765522103 0.11685154866197846 240.23800938294704 1.6577121930190857 0.026061563502321795 0.4126142946048159 0.49102877112355786
```

```
There are 117488 points assigned to the 1 centroid.
There are 185336 points assigned to the 2 centroid.
There are 113372 points assigned to the 3 centroid.
There are 466652 points assigned to the 4 centroid.
There are 46383 points assigned to the 5 centroid.
There are 646920 points assigned to the 6 centroid.
There are 55349 points assigned to the 7 centroid.
There are 417780 points assigned to the 8 centroid.

Process finished with exit code 0
```

K=9

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 51 times iteration
The loss after the last iteration is: 22.55522057588282
```

```
The 1 centroid's coordinates are:
4.0018407661727355 0.19885442717744845 237.16992518973618 16.966649801228993 37.009342247921936 2.4556559450668596 11.191326346223347
The 2 centroid's coordinates are:
1.839831354148342 0.12353928304497391 239.76540653566505 7.67248216116798 0.21118592453638663 0.4785778652933124 18.250227738463025
The 3 centroid's coordinates are:
3.7162678975863286 0.19062466297103312 237.72030693902457 15.792916459955453 0.8533681326977416 35.7795992321132 10.70783632794806
The 4 centroid's coordinates are:
0.41395501178300187 0.10533986569094733 235.90082880025795 1.8545741269024996 0.0383613586831334 0.3575934227545759 0.16231373035420027
The 5 centroid's coordinates are:
0.38660401927398763 0.10918814025690436 242.90882166893527 1.6906993467708225 0.024236643686674043 0.3657349793955157 0.42924501896818257
The 6 centroid's coordinates are:
1.843806558559769 0.16027740749647634 239.61584953791007 7.830461744597289 0.4231542240698114 1.000146012986567 0.3000824543924142
The 7 centroid's coordinates are:
0.3682004576438545 0.11616307054941745 240.23934378689856 1.6422405279753156 0.025970084543680903 0.4118746537562926 0.43732206084254643
The 8 centroid's coordinates are:
0.42562847985448093 0.09486640390674424 246.23832052017696 1.8058211427896258 0.01994521022598075 0.30588069689127584 0.30679025045206976
The 9 centroid's coordinates are:
1.0795790737079882 0.2619612162289394 240.74380612821375 4.695232043678916 0.071213404876212 0.4602748752706392 11.04937399981173
```

```
There are 55340 points assigned to the 1 centroid.
There are 611447 points assigned to the 2 centroid.
There are 46361 points assigned to the 3 centroid.
There are 112874 points assigned to the 4 centroid.
There are 464462 points assigned to the 5 centroid.
There are 116428 points assigned to the 6 centroid.
There are 415170 points assigned to the 7 centroid.
There are 184706 points assigned to the 8 centroid.
There are 42492 points assigned to the 9 centroid.

Process finished with exit code 0
```

K=10

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 72 times iteration
The loss after the last iteration is: 20.05511182123319
```

```
The 1 centroid's coordinates are:
1.59866749641989 0.11531877729257498 242.26216381733636 6.57666884712652 0.11023106978059863 0.383367218853314 18.48118734862012
The 2 centroid's coordinates are:
1.8421135293716018 0.16078745650542212 239.623155744694 7.825003411339036 0.4236712833458416 1.0079313638534488 0.3498328443747015
The 3 centroid's coordinates are:
2.934579336927564 0.17826551142640798 237.95258732834998 12.400870583739483 0.6509274873524452 0.8873723822261358 18.50436537934357
The 4 centroid's coordinates are:
0.4148118797894783 0.10536398629798324 235.90472004449893 1.8586926581204997 0.03845746371437652 0.35722357594377935 0.17314687290320302
The 5 centroid's coordinates are:
0.3704126230113219 0.11664617796078372 240.239093730694 1.6536106490768232 0.026035500122232394 0.4121508587424923 0.4785685867538431
The 6 centroid's coordinates are:
3.72048722703374 0.1900620868692614 237.7207746678615 15.810087480093248 0.8577630402059382 35.97336329326556 10.645760160561967
The 7 centroid's coordinates are:
1.4772552360176225 0.12871278215073034 237.90759424017293 6.209286859691691 0.10826936695671993 0.4204524179697357 16.86695264995808
The 8 centroid's coordinates are:
4.003010034658571 0.1986126041118512 237.17104012048858 16.968934293854208 37.08602950516249 2.4625015877624343 11.163421582681595
The 9 centroid's coordinates are:
0.4257789902438586 0.09493793014034099 246.23641291312293 1.8068411897539012 0.019960672882656093 0.3057035124301782 0.31481681558391045
The 10 centroid's coordinates are:
0.38831978309335274 0.10960753600823525 242.9066523276686 1.6992798353906762 0.024251971879286693 0.365633573388203 0.4604916838134431
```

```
There are 282815 points assigned to the 1 centroid.
There are 117256 points assigned to the 2 centroid.
There are 120379 points assigned to the 3 centroid.
There are 113268 points assigned to the 4 centroid.
There are 417238 points assigned to the 5 centroid.
There are 45839 points assigned to the 6 centroid.
There are 245702 points assigned to the 7 centroid.
There are 55109 points assigned to the 8 centroid.
There are 185114 points assigned to the 9 centroid.
There are 466560 points assigned to the 10 centroid.

Process finished with exit code 0
```

K=11

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A

Stop after 54 times iteration
The loss after the last iteration is: 19.53417293565287
```

```
The 1 centroid's coordinates are:
4.003152704174214 0.19859749546279504 237.17054791288578 16.969426497277887 37.08885662431942 2.4627949183303084 11.162268602540834
The 2 centroid's coordinates are:
1.6065524742415633 0.11499165049302404 234.97292506284901 6.82392830626153 0.13791666123927004 0.4373396204295893 17.068502429302733
The 3 centroid's coordinates are:
0.38810393915555014 0.10955526203500268 242.90665593124132 1.6982874076104355 0.024253327300914083 0.36555388544264306 0.45777270787723884
The 4 centroid's coordinates are:
0.3703379949468508 0.11662720835351302 240.23882559941936 1.6532545797105107 0.026039993671583973 0.41215007934491305 0.47711222870511977
The 5 centroid's coordinates are:
2.9919251892480054 0.17779123339830274 238.13896885906868 12.636866869135506 0.6917184171161779 0.9135669280630978 18.271655863288156
The 6 centroid's coordinates are:
1.4760112668543026 0.12924008203249943 239.69219400903557 6.1555222363586015 0.10004289045633283 0.4109374267888744 17.14172592637979
The 7 centroid's coordinates are:
1.6564110319128145 0.11628433390384542 243.01287069133483 6.7937885911139775 0.11737114630500249 0.38233637368113177 19.061198734753127
The 8 centroid's coordinates are:
1.8418796656430967 0.16074745820538863 239.62167357557013 7.823971340839046 0.42374616171954965 1.0080433299215286 0.34687819856704194
The 9 centroid's coordinates are:
3.7205336388434183 0.19003059465357497 237.72004386252277 15.81033933442432 0.858025095471904 35.97854882705946 10.643753409710856
The 10 centroid's coordinates are:
0.41462044485253635 0.10535546705989318 235.9049874173324 1.8578926455394889 0.03844557663949987 0.3571359193296306 0.17347308191539149
The 11 centroid's coordinates are:
0.4259014224459229 0.0949624677330459 246.23606159611958 1.8073854860839267 0.019965005886355535 0.30574486698996617 0.3158920798816249
```

```
There are 55100 points assigned to the 1 centroid.
There are 76771 points assigned to the 2 centroid.
There are 466369 points assigned to the 3 centroid.
There are 417166 points assigned to the 4 centroid.
There are 110305 points assigned to the 5 centroid.
There are 277451 points assigned to the 6 centroid.
There are 184628 points assigned to the 7 centroid.
There are 117240 points assigned to the 8 centroid.
There are 45825 points assigned to the 9 centroid.
There are 113251 points assigned to the 10 centroid.
There are 185174 points assigned to the 11 centroid.

Process finished with exit code 0
```

K=12

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A

Stop after 61 times iteration
The loss after the last iteration is: 18.75635550465198
```

```
The 1 centroid's coordinates are:
0.412753548652534 0.0925531783583597 246.96879217239476 1.7456679009804112 0.015317492971267915 0.28852945210176234 0.30609271069053007
The 2 centroid's coordinates are:
2.260425918498308 0.16657436226536076 237.90373415690553 9.637056794481069 0.682737044761752 0.5281164768169421 0.277655222204396
The 3 centroid's coordinates are:
1.5182797739418514 0.11962782167103998 240.91258781597068 6.2842812619615005 0.10415499644215859 0.3895392973857976 17.749091074251826
The 4 centroid's coordinates are:
2.666689604252791 0.19051732624532375 238.7553337271118 11.389230163417958 0.3575999212443394 30.309263634573735 0.5756054341405789
The 5 centroid's coordinates are:
0.3639779837822447 0.11724197942933669 239.36655447410234 1.6334969728713604 0.02699037247282487 0.41677731623036224 0.44534317606883417
The 6 centroid's coordinates are:
0.3690894875999911 0.10332118210114966 243.88418571570648 1.6052331068646415 0.020285150105425215 0.3374175842565012 0.403420462532213 25
The 7 centroid's coordinates are:
0.3370730296309993 0.11045703057125618 241.65517529156233 1.4994802579793374 0.018827233050890766 0.3817369706757157 0.4760775575702444
The 8 centroid's coordinates are:
3.9606227215703504 0.19822098000147595 237.22275791454527 16.78606375913245 37.08729983027083 1.8788096819422921 11.074035126558925
The 9 centroid's coordinates are:
4.363479887230968 0.1914494258406099 237.0896393453905 18.52090352747016 2.2910334869009144 38.33455958192945 16.772467853950353
The 10 centroid's coordinates are:
2.507800744126938 0.16259578615234474 237.0816444559263 10.606637913859743 0.4666892812336785 0.7766273065512247 18.23673682566861
The 11 centroid's coordinates are:
1.3539911261834148 0.15557936978945763 241.3859902030025 5.736279967971374 0.19731524657340682 0.5627714191512411 0.5887240356083087
The 12 centroid's coordinates are:
0.41895712994344864 0.10443511864406901 235.40929118643749 1.8789242937852324 0.03693785310734463 0.35594350282485876 0.15138983050847457
```

```
There are 116664 points assigned to the 1 centroid.
There are 49864 points assigned to the 2 centroid.
There are 462359 points assigned to the 3 centroid.
There are 20316 points assigned to the 4 centroid.
There are 246273 points assigned to the 5 centroid.
There are 298790 points assigned to the 6 centroid.
There are 391348 points assigned to the 7 centroid.
There are 54204 points assigned to the 8 centroid.
There are 29086 points assigned to the 9 centroid.
There are 185721 points assigned to the 10 centroid.
There are 106155 points assigned to the 11 centroid.
There are 88500 points assigned to the 12 centroid.

Process finished with exit code 0
```

K=13

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A

Stop after 70 times iteration
The loss after the last iteration is: 17.35701307723265
```

```
The 1 centroid's coordinates are:
0.4184094652651516 0.09196748119342893 246.9584918236029 1.7682266637231057 0.016216950534055596 0.28687021345242747 0.2972117033741446
The 2 centroid's coordinates are:
4.363504417477393 0.19140534222558284 237.09047750008688 18.521035442950843 2.2923441850871464 38.33438756918423 16.767609749389802
The 3 centroid's coordinates are:
0.35941946419026544 0.11171448825309463 241.66939003929943 1.5916474114619483 0.02261076034752577 0.38923965247423237 0.4557915237856957
The 4 centroid's coordinates are:
1.845234541935799 0.15834047065684947 239.63153719037305 7.830218891667512 0.42739989925857846 0.5260557966083721 0.29512818020342696
The 5 centroid's coordinates are:
3.959290641302131 0.19823425316595694 237.2222053862745 16.781315784623036 37.073494442293864 1.877767332116721 11.06224999539162
The 6 centroid's coordinates are:
0.41365491874673727 0.10594026206982286 243.88626704310806 1.7888353208205674 0.025692166722249384 0.3502423514403479 0.38676181564453826
The 7 centroid's coordinates are:
0.41814224640491915 0.10432861617247097 235.3576293408903 1.8764362954664093 0.03862508858965273 0.3544500582977707 0.12991243913033537
The 8 centroid's coordinates are:
1.059074781358562 0.26177855516952336 240.79388834311908 4.607269677728634 0.06224991014735833 0.4467712950760752 11.022666826404697
The 9 centroid's coordinates are:
1.5643961979459233 0.10709270919889335 240.97566652465537 6.460348427461376 0.11270811608599346 0.3917419725391516 18.173121511782554
The 10 centroid's coordinates are:
2.4824838286177693 0.15119353728199164 236.822880215654 10.505281153906573 0.47058343009516695 0.7794063588570629 17.409264629514013
The 11 centroid's coordinates are:
0.3831066416334501 0.11843783149495977 239.38598994758897 1.7123038410385851 0.03006336540718141 0.4259015880466244 0.40783071266525855
The 12 centroid's coordinates are:
2.344463337250301 0.268519271445358 238.99807226792043 9.862972972972933 0.25863689776733256 0.51786133960047 28.335135135135136
The 13 centroid's coordinates are:
2.663769748146678 0.19052943197996947 238.76088860523367 11.377171191516481 0.35490205704747413 30.268005302174874 0.5738622416417104
```

```
There are 459764 points assigned to the 1 centroid.
There are 20317 points assigned to the 2 centroid.
There are 296271 points assigned to the 3 centroid.
There are 392346 points assigned to the 4 centroid.
There are 20987 points assigned to the 5 centroid.
There are 30315 points assigned to the 6 centroid.
There are 248021 points assigned to the 7 centroid.
There are 88656 points assigned to the 8 centroid.
There are 115955 points assigned to the 9 centroid.
There are 186071 points assigned to the 10 centroid.
There are 49959 points assigned to the 11 centroid.
There are 106013 points assigned to the 12 centroid.
There are 34605 points assigned to the 13 centroid.

Process finished with exit code 0
```

K=14

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:

Stop after 92 times iteration
The loss after the last iteration is: 16.903904199315438
```

```
The 1 centroid's coordinates are:
0.41263688778191693 0.09257807692638859 246.9769568140123 1.745169046736528 0.015323227013004922 0.2888230072479381 0.30673170563546576
The 2 centroid's coordinates are:
0.36934582496490553 0.1032166813324189 243.8936952637347 1.606130057163113 0.02031933258549233 0.3369986498559175 0.4037837619984819
The 3 centroid's coordinates are:
1.653723284086569 0.11588740118846011 243.03388905849528 6.781849206782852 0.11705827836231805 0.3808973450362536 19.03260099220411
The 4 centroid's coordinates are:
3.9394146277785893 0.1977309413695361 237.24347419066115 16.69445770550528 37.10909294345815 1.5527189318215724 11.070043264210055
The 5 centroid's coordinates are:
0.36429712285751825 0.11723781387285052 239.37507082725637 1.6346364470379724 0.027090161781169044 0.4171149361645375 0.43947654516576207
The 6 centroid's coordinates are:
1.6032962129341126 0.11500264335627669 234.96851985134307 6.8101782302599405 0.13749378418697167 0.43744929205161087 17.052579235258708
The 7 centroid's coordinates are:
2.2603316952820025 0.16654007158890874 237.87853566300475 9.637303079941042 0.684058322716334 0.5245202127444438 0.2671034803534955
The 8 centroid's coordinates are:
1.3650485888466766 0.15455393390691777 241.39726593229634 5.778533625549213 0.20041344064925495 0.557063422944042 0.5037850874254707
The 9 centroid's coordinates are:
0.4177993522394595 0.10436549941882237 235.41693950097303 1.8737318452145066 0.03696974484556442 0.3553880356155417 0.14088226332479434
The 10 centroid's coordinates are:
2.9843794826974226 0.17714465718005054 238.13103187392056 12.603676289842301 0.6907109496626881 0.6270723513506131 18.34003404983658
The 11 centroid's coordinates are:
1.4759381818574757 0.1291796481642295 239.6918789442798 6.154854591128538 0.10003640177467663 0.41354938927913676 17.151809816945928
The 12 centroid's coordinates are:
4.744449681763665 0.18830956553823422 236.58146065861175 20.104621344894255 3.9111705562217507 42.80712111428835 16.197214279125543
The 13 centroid's coordinates are:
0.3382317836781595 0.11069622699644135 241.66330647728887 1.504589379975803 0.018950322247148668 0.38251538663291834 0.4819676556042484
The 14 centroid's coordinates are:
2.882270013836755 0.19461823812347426 238.66148876589602 12.304796731896927 0.2891546418923371 28.325492521578706 6.277492258022007
```

```
There are 116033 points assigned to the 1 centroid.
There are 297746 points assigned to the 2 centroid.
There are 183430 points assigned to the 3 centroid.
There are 53624 points assigned to the 4 centroid.
There are 247433 points assigned to the 5 centroid.
There are 76418 points assigned to the 6 centroid.
There are 49449 points assigned to the 7 centroid.
There are 104489 points assigned to the 8 centroid.
There are 88613 points assigned to the 9 centroid.
There are 109839 points assigned to the 10 centroid.
There are 277459 points assigned to the 11 centroid.
There are 21682 points assigned to the 12 centroid.
There are 392711 points assigned to the 13 centroid.
There are 30354 points assigned to the 14 centroid.

Process finished with exit code 0
```

K=15

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A

Stop after 94 times iteration
The loss after the last iteration is: 16.678635040565393
```

```
The 1 centroid's coordinates are:
1.4781665356513474 0.12786804412044092 237.85875230655455 6.213106221711046 0.10922540980215306 0.42333948415817824 16.89755980504919
The 2 centroid's coordinates are:
0.3780043561819143 0.0996964583142691 244.5071903083982 1.6320023794277478 0.020074128306030933 0.32073304658186147 0.36528781916353986
The 3 centroid's coordinates are:
3.9947538856982123 0.19408618820032947 237.20004919876214 16.879734125868794 38.09616852082413 1.6596248845335153 10.95184545564079
The 4 centroid's coordinates are:
0.3983985087378719 0.10271885048543708 234.81913972815394 1.7970330097086857 0.035697087378640774 0.3456776699029126 0.11684660194174758
The 5 centroid's coordinates are:
1.3573028225885881 0.155158206779727 241.56471971084454 5.7461383915381035 0.20565699541058519 0.5557426776182316 0.5483267348184916
The 6 centroid's coordinates are:
0.32991328637283907 0.11160217540789498 240.5882893174155 1.4775599372247763 0.01928980104822286 0.3911555785130765 0.47456266471015945
The 7 centroid's coordinates are:
0.4271214031202563 0.11923881719971827 238.34439525574237 1.9052027880699676 0.0388683048706046 0.42695512684014025 0.3660051389066337
The 8 centroid's coordinates are:
2.8818603823335813 0.19452030323005726 238.6620744891235 12.302386288727734 0.2733355306526038 28.321588661832564 6.275313117996045
The 9 centroid's coordinates are:
0.3478757829763533 0.11013927695296029 242.46502483608165 1.5380893247378806 0.019272326619064143 0.37510120102022804 0.472588980806808
The 10 centroid's coordinates are:
0.413316233818454 0.091794851767192 247.40546813081173 1.7444307883157035 0.013599972432488313 0.284542666467568 0.30349533075270796
The 11 centroid's coordinates are:
3.002084068627457 0.2472710784313718 238.19106249999996 13.434558823529391 19.912867647058825 0.7131127450980392 12.912867647058823
The 12 centroid's coordinates are:
2.2355369807897127 0.16404524476351953 237.89644649126126 9.503333201846118 0.26225789909668257 0.5261330913967891 0.2644866080233521
The 13 centroid's coordinates are:
2.9081133590372077 0.17658977811603 08 237.9974396781674 12.266573552902795 0.26804218555286996 0.6199703931686836 18.70334624101968
The 14 centroid's coordinates are:
4.7410932371839225 0.18816752184163052 236.58773771552848 20.08998289650058 3.810520963343041 42.820228354828274 16.23025008089493
The 15 centroid's coordinates are:
1.5875577149775104 0.11498039174038652 242.27561375374623 6.53050838457568 0.1084631641550111 0.3823380522013714 18.36748760102378
```

```
There are 241702 points assigned to the 1 centroid.
There are 218540 points assigned to the 2 centroid.
There are 49798 points assigned to the 3 centroid.
There are 64375 points assigned to the 4 centroid.
There are 101538 points assigned to the 5 centroid.
There are 303943 points assigned to the 6 centroid.
There are 140886 points assigned to the 7 centroid.
There are 30340 points assigned to the 8 centroid.
There are 328554 points assigned to the 9 centroid.
There are 87059 points assigned to the 10 centroid.
There are 8160 points assigned to the 11 centroid.
There are 50702 points assigned to the 12 centroid.
There are 119567 points assigned to the 13 centroid.
There are 21633 points assigned to the 14 centroid.
There are 282483 points assigned to the 15 centroid.

Process finished with exit code 0
```

K=16

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A

Stop after 77 times iteration
The loss after the last iteration is: 16.470816333809505
```

```
The 1 centroid's coordinates are:
0.3458308847645073 0.11481617653242461 239.37833979464412 1.5531161437350909 0.02492578607889802 0.40665444484623214 0.3917780225879424
The 2 centroid's coordinates are:
1.3879650459261923 0.15303470328223417 238.37588686165765 5.925000783723779 0.16607103670961473 0.5946424652810433 0.33988526286090476
The 3 centroid's coordinates are:
0.3945414339918285 0.09169144284821681 247.0834226742643 1.670045773630069 0.01306086680899065 0.2844211173986408 0.29973803686129
The 4 centroid's coordinates are:
0.3490263211090376 0.12058819273775563 241.7123651229311 1.560374708755167 0.02171438298596704 0.4209302820422797 1.0778728203164065
The 5 centroid's coordinates are:
4.33508313172749 0.1909368535465089 237.12402720802294 18.39761579187258 1.7879457242032186 38.462115634094175 16.783422741138107
The 6 centroid's coordinates are:
1.5594391779283434 0.11202374913952143 237.54699988224422 6.5424386797575504 0.11865059236984167 0.4161669867033803 17.665460490561937
The 7 centroid's coordinates are:
0.35471080890273443 0.10114569167317018 235.31558837735972 1.6074140130543222 0.030914427971612233 0.33202046099159016 0.12020213279827592
The 8 centroid's coordinates are:
2.6713816692167067 0.19067617590444683 238.7540767977888 11.409545432110905 0.3734761364197226 30.356102857706926 0.553329055821529
The 9 centroid's coordinates are:
1.3816829963210027 0.15006337307404974 242.61869298221316 5.815042460789528 0.1971867998119001 0.5301374789079141 0.3471688196730381
The 10 centroid's coordinates are:
0.3589829778922072 0.10083767145090598 244.01600490478458 1.5588894033224516 0.018743850917298455 0.32803909369755196 0.37066308814167487
The 11 centroid's coordinates are:
1.0703194981399007 0.2639561358718133 240.78752498358952 4.6500255306733775 0.06433729666642352 0.4487076616334768 11.110705862328883
The 12 centroid's coordinates are:
1.6178044292751887 0.11259958495296438 242.1585001773282 6.653842345331877 0.11176071941486093 0.3815026669054416 18.728098543823982
The 13 centroid's coordinates are:
2.5644450457820533 0.17197627513294067 238.2646225732341 10.93213555268853 1.0033667914791857 0.5131053144960825 0.22145306944400742
The 14 centroid's coordinates are:
0.3424008946859071 0.10563476420228023 241.7125714505994 1.5163837208303432 0.019042069089631415 0.36652126592909356 0.0
The 15 centroid's coordinates are:
3.9812469042945353 0.1982618187154886 237.19857698446245 16.873475078424757 37.11839812148007 2.1395131257911246 11.110252976463466
```

```
The 16 centroid's coordinates are:
3.019094296535595 0.1779006906538909 237.9525378745642 12.761681705098049 0.707985221491961 1.0343656009802829 18.14401618952137
```

```
There are 237826 points assigned to the 1 centroid.
There are 63798 points assigned to the 2 centroid.
There are 107267 points assigned to the 3 centroid.
There are 187986 points assigned to the 4 centroid.
There are 28521 points assigned to the 5 centroid.
There are 220808 points assigned to the 6 centroid.
There are 80739 points assigned to the 7 centroid.
There are 20261 points assigned to the 8 centroid.
There are 72302 points assigned to the 9 centroid.
There are 276464 points assigned to the 10 centroid.
There are 41127 points assigned to the 11 centroid.
There are 284787 points assigned to the 12 centroid.
There are 31781 points assigned to the 13 centroid.
There are 233378 points assigned to the 14 centroid.
There are 54511 points assigned to the 15 centroid.
There are 107724 points assigned to the 16 centroid.

Process finished with exit code 0
```

K=17

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A
Stop after 100 times iteration
The loss after the last iteration is: 15.758909434945528
```

```
The 1 centroid's coordinates are:
1.4876496610339307 0.1286107910163203 239.6342681414959 6.203854753407354 0.101567986699345 0.41730088432156626 17.226494532112195
The 2 centroid's coordinates are:
1.3800566853385132 0.15401147710667296 238.50850280077913 5.893378592304029 0.1649263273258646 0.582059790550414 0.4212433024841695
The 3 centroid's coordinates are:
4.744419926199253 0.18829206642066393 236.5813390221412 20.10448339483382 3.9107472324723247 42.80839483394834 16.19732472324723
The 4 centroid's coordinates are:
1.5916097953615127 0.11462769440654785 234.9056278308317 6.76230559345166 0.13598908594815826 0.4357980900409277 16.959836289222373
The 5 centroid's coordinates are:
3.0082462004001984 0.1693106935428745 238.14549016545095 12.703388159500259 0.7103323734686905 0.6227634340377763 17.6122309531944
The 6 centroid's coordinates are:
0.3489806436175767 0.11626327396428648 240.71943415079866 1.5618996805190952 0.021956204001914295 0.41552843635611086 0.49924333553218736
The 7 centroid's coordinates are:
0.3983719228532977 0.09127645861601064 247.47439123376938 1.6831096142660766 0.011642275634812948 0.28308538476448925 0.3028687730180267
The 8 centroid's coordinates are:
0.3443172728896867 0.10826241354781338 242.55131291383972 1.5202386660271434 0.018464026173039137 0.36402836666071253 0.46013092787058885
The 9 centroid's coordinates are:
0.3426775640219706 0.11076379283847877 238.54326096371483 1.5424046830688842 0.02743199631166436 0.3830140967070428 0.3546460455174148
The 10 centroid's coordinates are:
0.36905490013934955 0.1022585849202669 234.86367611085177 1.6732899829694974 0.03314754605976157 0.336940702895185 0.11435206688341848
The 11 centroid's coordinates are:
0.3673024871929233 0.0984083603610554 244.58115983864369 1.5863018847923038 0.018963581249427718 0.31636169115627244 0.3579514513043185
The 12 centroid's coordinates are:
3.9398855074356565 0.19768059261470722 237.2431770940238 16.69599761162866 37.11857891888866 1.5532065754856044 11.072024331535834
The 13 centroid's coordinates are:
1.6173774110342793 0.10915195507772207 243.2169656349148 6.624523499738753 0.11639731562007806 0.3757561231710379 18.434596772352712
The 14 centroid's coordinates are:
2.88280343121085 0.19454147146156178 238.6622873639065 12.307093368525225 0.2898713295941933 28.34084460574068 6.268261299901023
The 15 centroid's coordinates are:
1.369904599941942 0.15212769781685873 242.67427295477532 5.774391306151155 0.19480968379719885 0.5291661481880902 0.5362865873048792
```

```
The 16 centroid's coordinates are:
2.339871142202109 0.26871271530186314 238.9941655399451 9.843924519428562 0.25471753571218625 0.5462347892540121 28.339486214802186
The 17 centroid's coordinates are:
2.553398668639035 0.17182451923076897 238.24412999260193 10.88459072978289 0.9825875246548323 0.5105091222879684 0.24503821499013806
```

```
There are 279084 points assigned to the 1 centroid.
There are 65696 points assigned to the 2 centroid.
There are 21680 points assigned to the 3 centroid.
There are 73300 points assigned to the 4 centroid.
There are 102445 points assigned to the 5 centroid.
There are 309252 points assigned to the 6 centroid.
There are 82544 points assigned to the 7 centroid.
There are 319107 points assigned to the 8 centroid.
There are 143154 points assigned to the 9 centroid.
There are 64590 points assigned to the 10 centroid.
There are 207503 points assigned to the 11 centroid.
There are 53593 points assigned to the 12 centroid.
There are 175236 points assigned to the 13 centroid.
There are 30310 points assigned to the 14 centroid.
There are 72327 points assigned to the 15 centroid.
There are 17011 points assigned to the 16 centroid.
There are 32448 points assigned to the 17 centroid.

Process finished with exit code 0
```

K=18

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:\
Stop after 100 times iteration
The loss after the last iteration is: 15.212004501768995
```

```
The 1 centroid's coordinates are:
3.5579629042732748 0.18320893622548234 236.44744901935366 15.131170281854832 1.0207559423301729 0.8553058838810235 17.26842447071048
The 2 centroid's coordinates are:
1.563533519001749 0.11321924957786003 237.28395638254017 6.5685072695775135 0.1259182912655424 0.43030854586522227 17.55110633538738
The 3 centroid's coordinates are:
1.491869923480615 0.10747351867596464 239.7590761421965 6.190082528399767 0.09625330017707488 0.3927865960071853 18.00810581022943
The 4 centroid's coordinates are:
0.4255564673044191 0.094863813988776 246.2400023847581 1.8055293894479825 0.01987480014091759 0.30547681634644047 0.3065770575323162
The 5 centroid's coordinates are:
4.415339031173957 0.19993730443852162 236.64351315331936 18.729887588365095 36.96856530304786 2.6866091088190984 17.6257967319504
The 6 centroid's coordinates are:
1.5419123160202413 0.10376034536751001 242.10537765251283 6.334708241996644 0.09924842303046572 0.35522450379516546 18.35774467260174
The 7 centroid's coordinates are:
3.7176740873833136 0.18967662912485392 237.72390989964734 15.797559051667347 0.8078355756168105 36.020750251982996 10.621784477847408
The 8 centroid's coordinates are:
3.247603513174401 0.31660163111668704 237.43201380175665 13.790276035131708 0.8723337515683814 1.1010037641154329 28.10194479297365
The 9 centroid's coordinates are:
3.301416325753958 0.1968504423936831 238.06421653505785 13.992265245932819 36.97359908667111 2.173247074493388 0.3974407763295595
The 10 centroid's coordinates are:
1.056335753703159 0.26234035240584935 240.83224731339163 4.595827282408853 0.06249394907541873 0.45043082583018684 11.001839481072707
The 11 centroid's coordinates are:
0.4137670302020367 0.1053641349126894 235.90130420915827 1.8537967450142436 0.038600905442399865 0.3573928220212097 0.16209368050818176
The 12 centroid's coordinates are:
0.3678771368035809 0.11611723407329334 240.24080437715236 1.6408542398986603 0.026022054220638514 0.41155438917513365 0.4370587952039293
The 13 centroid's coordinates are:
0.3864435730092359 0.10918292743362559 242.91060151298805 1.6899574026150415 0.02428072473729458 0.36562431993932515 0.4284215266540692
The 14 centroid's coordinates are:
1.5965826347486591 0.11106612595708447 233.07369215870378 6.8345307629454535 0.14111914777713888 0.460944829464637 16.721120963592895
The 15 centroid's coordinates are:
2.1385848012811204 0.2578092881059839 239.36695443296026 8.9667054884262 0.1056922405008007 0.42313291599941766 28.42859222594264
```

```
The 16 centroid's coordinates are:
2.5710992393736096 0.15828519015659956 239.5316328859074 10.7809105145412 0.43860178970917224 0.8900559284116331 17.89244966442953
The 17 centroid's coordinates are:
1.840087450380603 0.1601315965837795 239.62221658962255 7.811568401697577 0.3671146014125410 1.011487635969962 0.29679686560239205
The 18 centroid's coordinates are:
1.636592705167211 6 0.10338957669211883 245.17035603084133 6.643843131440228 0.11746608347542442 0.36431536807769294 18.766161316628363
```

```
There are 38495 points assigned to the 1 centroid.
There are 91202 points assigned to the 2 centroid.
There are 156431 points assigned to the 3 centroid.
There are 184505 points assigned to the 4 centroid.
There are 34516 points assigned to the 5 centroid.
There are 134118 points assigned to the 6 centroid.
There are 45638 points assigned to the 7 centroid.
There are 3188 points assigned to the 8 centroid.
There are 21022 points assigned to the 9 centroid.
There are 41316 points assigned to the 10 centroid.
There are 112873 points assigned to the 11 centroid.
There are 415340 points assigned to the 12 centroid.
There are 464113 points assigned to the 13 centroid.
There are 33043 points assigned to the 14 centroid.
There are 13738 points assigned to the 15 centroid.
There are 89400 points assigned to the 16 centroid.
There are 116386 points assigned to the 17 centroid.
There are 53956 points assigned to the 18 centroid.

Process finished with exit code 0
```

K=19

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A:\I

Stop after 96 times iteration
The loss after the last iteration is: 14.672464509027618
```

```
The 1 centroid's coordinates are:
4.741020274929881 0.18834885752379174 236.58603374557583 20.09027630913508 3.8869017516436024 42.766677394142796 16.21084088087904
The 2 centroid's coordinates are:
0.3425234070358177 0.11085723921336896 238.53629878661107 1.5413373240874464 0.027520744191275145 0.3839063015970041 0.34051387981367115
The 3 centroid's coordinates are:
1.0545050863094456 0.26262754618951845 240.84269610818623 4.587486950399489 0.060841486804729415 0.4438563694190196 11.017213333657045
The 4 centroid's coordinates are:
2.8794751545530772 0.19442780284043276 238.66411395154597 12.294121971595663 0.28945697577276525 28.452932330827068 6.11953216374269
The 5 centroid's coordinates are:
1.5847920747996265 0.11113291777975581 234.7353014247547 6.732528940338456 0.13018699910952805 0.43171564262392403 17.037102997922233
The 6 centroid's coordinates are:
0.34668189097895497 0.1156177193357941 240.7184476777822 1.5499782787705452 0.02193519941385101 0.4149208634026467 0.4549235866612201
The 7 centroid's coordinates are:
3.9394001979051794 0.1975640484681018 237.2446104441667 16.693553145012494 37.131252217098265 1.5565990179421594 11.067567819868934
The 8 centroid's coordinates are:
0.39787870116645974 0.0911978805441712 247.47437859204413 1.6807890971697663 0.011627906976744186 0.2829740281785775 0.29701481679074615
The 9 centroid's coordinates are:
2.3359723166875543 0.2688798997673174 239.00574607720344 9.825058170753504 0.24228864626215618 0.5425093968140325 28.4067179762544
The 10 centroid's coordinates are:
0.36715854764336087 0.09834969035170671 244.58109852795542 1.5853809349258998 0.019012984006328015 0.31636698628286997 0.35128682499565916
The 11 centroid's coordinates are:
0.34439194537279644 0.10840173254266162 242.55210458701924 1.520048621755152 0.01872000402040355 0.36602319270296757 0.44633075357439
```

```
The 12 centroid's coordinates are:
1.3822438265423838 0.15331704536701787 238.50032041900965 5.899089578679905 0.16634059924516675 0.5835477162443194 0.3459446969113456
The 13 centroid's coordinates are:
0.36871254758760913 0.1022586900784717 234.8585090513547 1.6716587677725225 0.03301996736850284 0.33694351643228965 0.11026338279853935
The 14 centroid's coordinates are:
1.5758333696693523 0.10289107785815392 243.40952703729397 6.439686849669195 0.10447593565223136 0.3557031017738579 18.5335183166518
The 15 centroid's coordinates are:
2.5334700856533354 0.15605186368362967 239.50045227962354 10.621182852180747 0.3995261730433198 0.6483174855011096 17.894107179228797
The 16 centroid's coordinates are:
1.385117521155526 0.14978341294702854 242.67533969298046 5.8286580957916065 0.19733802038299622 0.5239904180073424 0.35742533558236117
The 17 centroid's coordinates are:
2.556547327672311 0.17171259990009952 238.21286650849 10.899044705294573 0.9869193306693307 0.510989010989011 0.21544080919080918
The 18 centroid's coordinates are:
3.543708283708786 0.18344515111542447 236.4702088031832 15.072614854420593 1.1195803664191775 0.6675366799280339 17.308719560094264
The 19 centroid's coordinates are:
1.4985563732125446 0.10820763292395474 239.56559064097877 6.224233549095501 0.10064564823246618 0.4014668602790972 17.971872872109145
```

```
There are 21751 points assigned to the 1 centroid.
There are 142329 points assigned to the 2 centroid.
There are 41189 points assigned to the 3 centroid.
There are 29925 points assigned to the 4 centroid.
There are 67380 points assigned to the 5 centroid.
There are 308454 points assigned to the 6 centroid.
There are 53561 points assigned to the 7 centroid.
There are 82474 points assigned to the 8 centroid.
There are 16761 points assigned to the 9 centroid.
There are 207332 points assigned to the 10 centroid.
There are 318376 points assigned to the 11 centroid.
There are 64915 points assigned to the 12 centroid.
There are 64355 points assigned to the 13 centroid.
There are 151365 points assigned to the 14 centroid.
There are 93283 points assigned to the 15 centroid.
There are 70549 points assigned to the 16 centroid.
There are 32032 points assigned to the 17 centroid.
There are 39463 points assigned to the 18 centroid.
There are 243786 points assigned to the 19 centroid.

Process finished with exit code 0
```

K=20

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:A
Stop after 100 times iteration
The loss after the last iteration is: 13.985296210489484
```

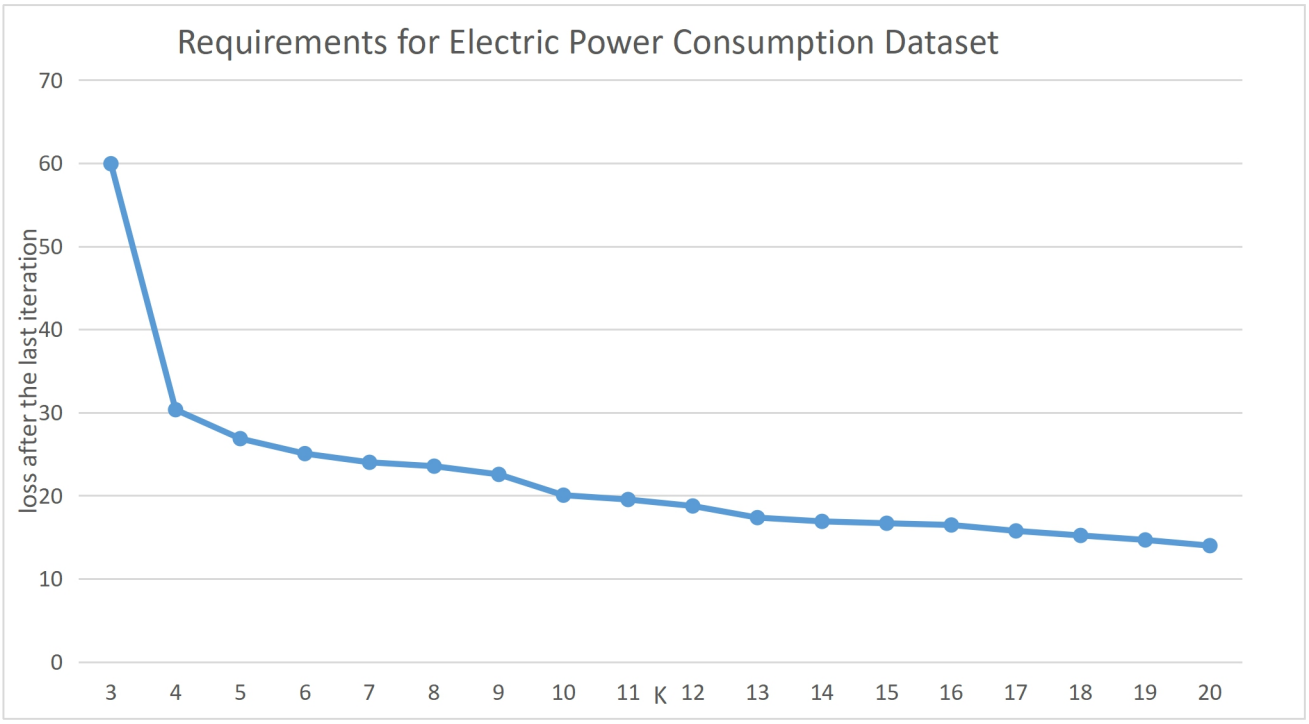
```
The 1 centroid's coordinates are:
0.3418720098803119 0.11166199843048442 241.70070494397746 1.5196006741189068 0.019708996410698433 0.3896746471806615 0.4562504020275051
The 2 centroid's coordinates are:
2.54376099238245 0.15571172705060318 239.46336609581164 10.65908965273243 0.28537628940065674 0.6505212882397815 17.886477462437394
The 3 centroid's coordinates are:
1.3668526771050167 0.15050785612613476 242.62760196540074 5.754825906054881 0.19196868498850322 0.5317256104237381 0.360889083543195
The 4 centroid's coordinates are:
2.87746840360339 0.19426764006563577 238.6658631660028 12.285127758614916 0.27494055791835503 28.463983121797664 6.0962124510230735
The 5 centroid's coordinates are:
1.3811336177367992 0.15352360341284146 238.4267434763657 5.9001401258019746 0.2128978015818646 0.585616864918727 0.34298125428162174
The 6 centroid's coordinates are:
1.5078603111293207 0.11033580333808991 238.36038001576892 6.299406286455671 0.11331391218774643 0.42444327135511367 17.758871488063814
The 7 centroid's coordinates are:
3.560883391281073 0.1787006686279746 236.4441021663555 15.108280288847288 0.2865472051350629 0.6623696175447981 17.291120620486762
The 8 centroid's coordinates are:
0.3463881091532048 0.11520026794918982 239.45082771429347 1.5561782744551182 0.024725131467583362 0.4077039402168463 0.40855030665741415
The 9 centroid's coordinates are:
1.5976271456379387 0.112213988674571 233.78651367014714 6.816036984604618 0.12957883560431782 0.4364935409662007 16.850844983188818
The 10 centroid's coordinates are:
1.6169945768604788 0.102788228237049 244.5091752665645 6.5810055455578595 0.10813226094727435 0.3570457967608065 18.675529766058247
The 11 centroid's coordinates are:
2.57559957667884 0.16785427490218663 238.22990154576243 10.927336283753224 0.2567827592842024 0.5069270733115259 0.20354691809377204
The 12 centroid's coordinates are:
1.5152751795144688 0.10512728022708116 241.19333693358075 6.247938908096446 0.09590929567374232 0.3715012998854798 18.232481397254773
The 13 centroid's coordinates are:
0.3580388475167408 0.10164637893805387 243.9239017080865 1.5573992303862403 0.018745501356514035 0.33111054205193513 0.38666878910359337
The 14 centroid's coordinates are:
0.3551165741089232 0.1012376323048709 235.35230754273988 1.6089001227350934 0.03101189680523994 0.3328310507831962 0.12158073179327022
The 15 centroid's coordinates are:
2.994773155027249 0.24617558197127215 238.1938026250621 13.40668647845467 20.1180039623576 0.7226349678058445 12.691431401684001
```

```
The 16 centroid's coordinates are:
0.3935399217116295 0.09167588968130279 247.0181860510875 1.6663735320927124 0.013360859384774876 0.28493038053872416 0.29857722487354105
The 17 centroid's coordinates are:
3.9959716588611185 0.1940462704267254 237.19888926855654 16.8842314727345 38.1053848163856 1.662137444473478 10.95706618962433
The 18 centroid's coordinates are:
4.737714680419958 0.188274544114938 236.59200681525235 20.07588874562521 3.7889114017314425 42.77035365629029 16.245256953398417
The 19 centroid's coordinates are:
1.0531658486806885 0.26251508819670694 240.83910394091254 4.581311045240405 0.04786432771271685 0.4440206035278682 11.012221196365227
The 20 centroid's coordinates are:
2.3321451062556173 0.2684093385214013 239.01617599521143 9.80681233163721 0.1886261598323855 0.5429512122119126 28.406046093983836
```

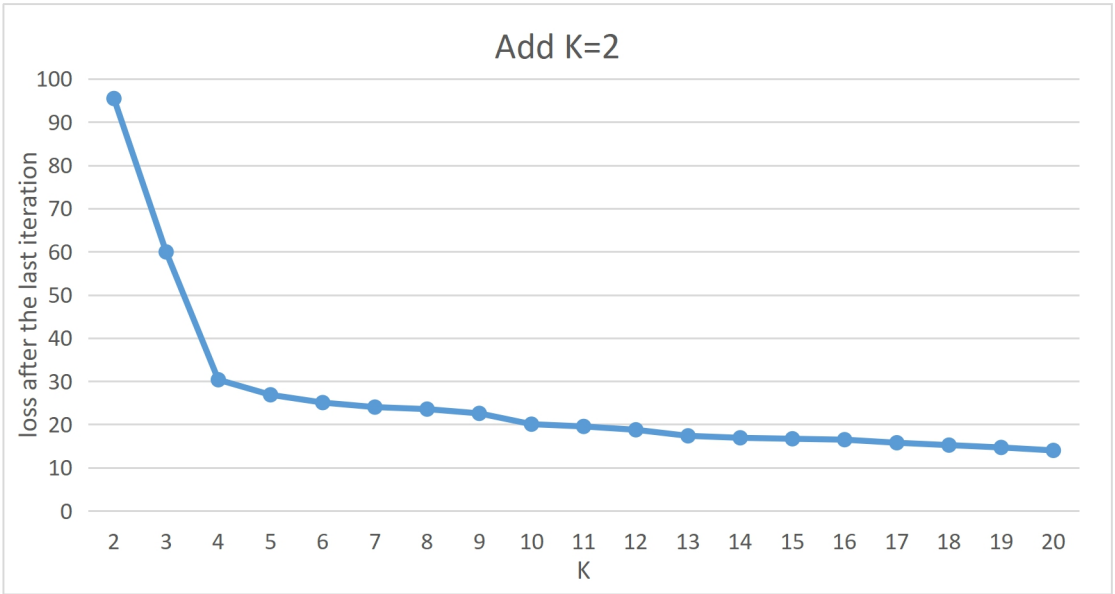
```
There are 388655 points assigned to the 1 centroid.
There are 90449 points assigned to the 2 centroid.
There are 73064 points assigned to the 3 centroid.
There are 29861 points assigned to the 4 centroid.
There are 64228 points assigned to the 5 centroid.
There are 153441 points assigned to the 6 centroid.
There are 37390 points assigned to the 7 centroid.
There are 249301 points assigned to the 8 centroid.
There are 45208 points assigned to the 9 centroid.
There are 81687 points assigned to the 10 centroid.
There are 31182 points assigned to the 11 centroid.
There are 184247 points assigned to the 12 centroid.
There are 288976 points assigned to the 13 centroid.
There are 82291 points assigned to the 14 centroid.
There are 8076 points assigned to the 15 centroid.
There are 111894 points assigned to the 16 centroid.
There are 49751 points assigned to the 17 centroid.
There are 21716 points assigned to the 18 centroid.
There are 41158 points assigned to the 19 centroid.
There are 16705 points assigned to the 20 centroid.

Process finished with exit code 0
```

Scatter plot containing the loss of the last iteration of training for each value of K:



As we can see in the scatter plot, there is an obvious “elbow” in the plot: K=4. The line obviously “turns” when K=4. This trend is more specific when we add K=2 to the plot just like the plot below.



Therefore, if we want to use the elbow method to choose a proper K, K=4 is the best.