Problem 1

$X_i \in \{0, 1\}$

The generation model is

$$t \sim \text{Categorical}(\pi_1, \pi_2, \dots \pi_k)$$

$$X_i | t = k \sim \text{Bernoulli}(P_{k,i})$$

Parameter of naïve Bayes decompose:

$$\pi, P_{1,i}, P_{2,i}, P_{3,i}, \dots P_{k,i}$$

Problem 2.

MLE for naïve Bayes:

$$\arg\max_\theta L(\theta) = \arg\max_\theta \log L(\theta)$$

$$= \arg\max_\theta \sum_{m=1}^{m} \log P(x^{(m)}, t^{(m)})$$

$$= \arg\max_\theta \sum_{m=1}^{m} \log \left[P(t^{(m)}) \cdot P(x^{(m)} | t^{(m)}) \right]$$

$$= \arg\max_\theta \sum_{m=1}^{m} \left[\log P(t^{(m)}) + \log P(x^{(m)} | t^{(m)}) \right]$$

$$= \arg\max_\theta \left[\sum_{m=1}^{m} \log P(t^{(m)}) + \sum_{m=1}^{m} \log P(x^{(m)} | t^{(m)}) \right]$$

$$= \underset{\theta}{\operatorname{argmax}} \left[\sum_{m=1}^{m} \log P(t^{(m)}; \pi) + \sum_{k=1}^{k} \sum_{m=1}^{m} \log P(x^{(m)} \mid t^{(m)}; P_{k,i}^{(m)}) \right]$$

$$\hat{\pi}_k = \frac{\sum_{m=1}^{m} \mathbb{1}\{t^{(m)} = k\}}{m} \quad \text{for } k = 1, \cdots k$$

$$\hat{P}_{k,i} = \frac{\sum_{m=1}^{m} \mathbb{1}\{x_i^{(m)} = x \mid t^{(m)} = k\}}{\sum_{m=1}^{m} \mathbb{1}\{t^{(m)} = k\}}$$