Problem 1

		Actual class	
		Positive $(y=1)$	Negative $(y=0)$
Predicted class	Positive $(\hat{y}=1)$	TP	FP
	Negative $(\hat{y}=0)$	FN	TN

From the question, we know that

$$P(t=1) = 90\%$$

$$P(t=0) = 10\%$$

Because we alway predict $t=1$

$P(TP)+P(FN)=0.9$ $\quad$ $P(FN)=P(TN)=0$

$P(FP)+P(TN)=0.1$ $\quad$ $P(TP)+P(FP)=1$

$$\begin{cases} P(TP)=0.9 \\ P(FN)=0 \\ P(FP)=0.1 \\ P(TN)=0 \end{cases}$$

$$\text{Precision (P)} = \frac{|\{\hat{t}=1\} \cap \{t=1\}|}{|\{\hat{t}=1\}|}$$

$$= \frac{P(TP)}{P(TP)+P(FP)}$$

$$= \frac{0.9}{0.9+0.1}$$

$$= 0.9 = 90\%$$

$$\text{Recall (R)} = \frac{|\{\hat{t}=1\} \cap \{t=1\}|}{|\{y=1\}|}$$

$$= \frac{P(TP)}{P(TP)+P(FN)}$$

$$= \frac{0.9}{0.9+0} = 1 = 100\%$$

$$F_\beta\text{-score} = \frac{(1+\beta^2) \cdot P \cdot R}{\beta^2 \cdot P + R}$$

$$= \frac{(1+1) \cdot 0.9 \cdot 1}{1^2 \cdot 0.9 + 1}$$

$$= \frac{2 \cdot 0.9 \cdot 1}{1 \cdot 0.9 + 1} = \frac{1.8}{1.9} \approx 0.9474$$

$$= 94.74\%$$

Problem 2.

$$\sigma(z) = \frac{1}{1+\exp\{-z\}} \qquad \sigma(-z) = \frac{1}{1+\exp\{z\}}$$

$$1-\sigma(z) = 1 - \frac{1}{1+\exp\{-z\}}$$

$$= \frac{\exp\{-z\}}{1+\exp\{-z\}}$$

$$= \frac{\exp\{-z\} \cdot \exp\{z\}}{(1+\exp\{-z\}) \cdot \exp\{z\}}$$

$$= \frac{1}{\exp\{z\}+1} = \sigma(-z)$$

Problem 3.

$$J = -t\log y - (1-t)\log(1-y)$$
$$= -t_1\log y_1 - t_2\log y_2$$
$$= t_1\log\frac{1}{y_1} + t_2\log\frac{1}{y_2}$$
$$= \sum_{k\in\{1,2\}} t_k\log\frac{1}{y_k}$$

$$KL(P\|Q) = \sum_{k=1}^{K} p_k\log\frac{p_k}{q_k}, \quad p_k \in \{0,1\},$$

binary classification.

Thus, minimizing the loss $J = -t\log y - (1-t)\log(1-y)$ is equivalent to minimi e the kullback-Leibler divergence between t and y.