Problem 1.

$$\frac{1}{m}\sum_{i=1}^{m}(x_i^2)$$

$$= \frac{1}{m}(x_1, x_2, x_3, \cdots x_m) \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{pmatrix}$$

$$\boxed{= \frac{1}{m} \cdot X^T \cdot X}$$

$$\frac{1}{m}\sum_{i=1}^{m}[(x_i-\mu)^2] = E[(X-\mu)^2]$$

and $$\frac{1}{m}\sum_{i=1}^{m}[(x_i-\mu)^2] = \frac{1}{m}\cdot\|X-\mu\|_2^2$$

$$= \frac{1}{m}\sum_{i=1}^{m}[x_i^2 - 2\mu x_i + \mu^2]$$

$$= \frac{1}{m}\left[\sum_{i=1}^{m}x_i^2 - \sum_{i=1}^{m}2\mu x_i + \sum_{i=1}^{m}\mu^2\right]$$

$$= \frac{1}{m}\sum_{i=1}^{m}x_i^2 - \frac{1}{m}\sum_{i=1}^{m}2\mu x_i + \frac{1}{m}\sum_{i=1}^{m}\mu^2$$

$$= \frac{1}{m}\cdot X^T\cdot X - \frac{1}{m}\cdot 2\mu\cdot\sum_{i=1}^{m}x_i + \frac{1}{m}\cdot m\cdot\mu^2$$

$$\boxed{= \frac{1}{m}\cdot X^T\cdot X - \frac{2\mu}{m}\cdot\underbrace{(1,1,1,\cdots 1)}_{1\times m \text{ matrix}}\cdot X + \mu^2}$$