Linear Mixed Effects Models

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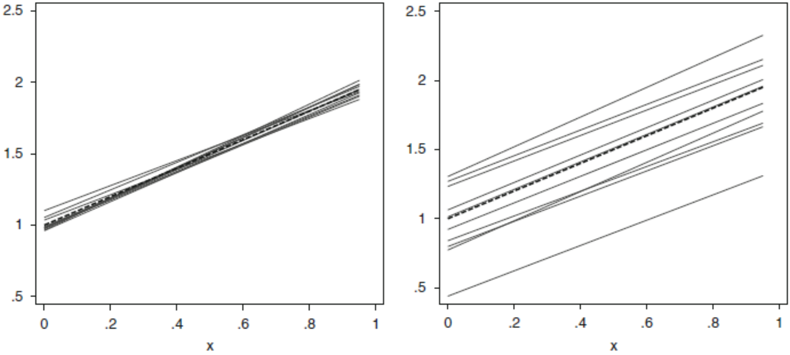
Overview

Random Effects

- Standard assumption for OLS: i.i.d errors
- Problematic with certain data structures
- Take longitudinal data: repeated measurements of same individual
  $\rightarrow$ Errors of an individual probably correlate
- Why "Random Effect"?
  $\rightarrow$ often belong to individuals who have been selected randomly from the population
- RE useful in many cases of grouped data

Let's have a look



Random Intercept Model

To model this type of individual-specific heterogeneity we introduce individual-specific parameters γ_{0i}:

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + \gamma_{0i} + \epsilon_{ij} \tag{1}$$

- $i = 1, ..., m$ number of individuals and $j = 1, ..., n_i$ number of repeated measurements
- $\epsilon_{ij} \sim \mathcal{N}(0, \epsilon^2)$ are i.i.d.
- β_0 is the "fixed" population intercept.
- γ_{0i} is the individual- or cluster-specific (random) deviation from the population intercept β_0
- $\beta_0 + \gamma_{0i}$ is the (random) intercept for individual i
- β_1 is a "fixed" population slope parameter that is common across individuals

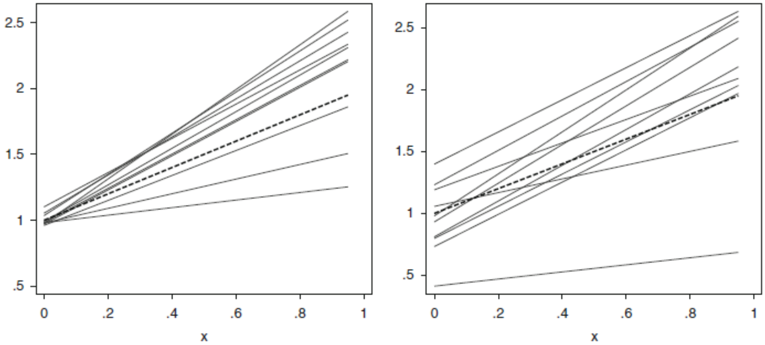
Random Intercept Model

Individuals or clusters are a random sample from a larger population, γ_{0i} are assumed to be random with

$$\gamma_{0i} \sim_{i.i.d.} \mathcal{N}(0, \tau^2) \tag{2}$$

- Mean is zero because the populationmean is already represented by the fixed effect β_0
- We assume mutual independence between the γ_{0i} and the ϵ_{ij}

Expanding the Model



Random Slope Model

In case of individual-specific slope parameters (random slopes) we can model this by

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + \gamma_{1i} * x_{ij} + \epsilon_{ij} \tag{3}$$

But in most cases

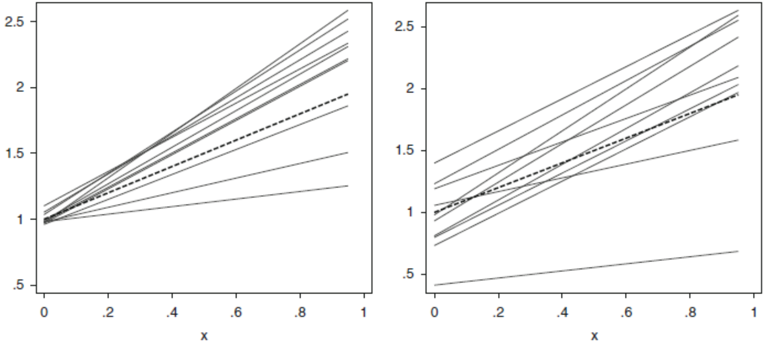
$$y_{ij} = \beta_0 + (\beta_1 + \gamma_{1i}) * x_{ij} + \gamma_{0i} + \epsilon_{ij} \tag{4}$$

Where

- β_0 is the "fixed" population intercept.
- γ_{1i} is the individual- or cluster-specific (random) deviation from the population slope β_1
- β_1 is a "fixed" population slope parameter that is common across individuals
- $\beta_1 + \gamma_{1i}$ is the (random) slope for individual i

Random Slope Model

Let's have another look



$$y_{ij} = \beta_0 + \beta_1 x_{ij} + \gamma_{1i} * x_{ij} + \epsilon_{ij} \tag{5}$$

$$y_{ij} = \beta_0 + (\beta_1 + \gamma_{1i}) * x_{ij} + \gamma_{0i} + \epsilon_{ij} \tag{6}$$

Blocks of Highlighted Text

Block 1

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Block 2

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End