GOV 2000/ E-2000 Section 5[1]

Sampling, Interval Estimation, and Hypothesis Testing

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October 3, 2013

[1]These notes and accompanying code draw on the notes from TF's from previous years.

Logistics - This Week

Problem Set 4- Due by 1pm on Tuesday to course dropbox.

Problem Set 3 Corrections- Due by 1pm on Tuesday to course dropbox.

No Reading Quiz

Practice Midterms- Not due, but recommended as midterm preparation.

Logistics - Next Week

Midterm- Posted October 8, due by 11:59pm on Sunday, October 13 with 5 hours to complete.

Office Hours- Cancelled Friday October 6. Rescheduled for Tuesday October 15 from 9:30am to 12pm.

Problem Set 5- Due by 1pm on Tuesday, October 15 to course dropbox.

Problem Set 4 Corrections- Due by 1pm on Tuesday, October 15 to course dropbox.

Reading Quiz (ALZ ch. 11 & 12)- Due by 9pm on Sunday, October 13 on course website.

Mid-semester Evaluation- Due by 9pm on Monday, October 14 on course website.

Problem Set Expectations

- Must be typeset using LaTeX or Word and submitted as one document containing graphics and explanation electronically in **pdf** form
- Must be accompanied by source-able, commented code
- Corrections do not need to include any of your original work, just 1-2 pages explaining where you made a mistake and how to correct it

MIDTERM

- Window of exam: Tuesday, October 8th (after class) - Sunday, October 13th at 11:59pm
- Needs to be completed in 5 hours
- Open note / book, but **no collaboration** allowed

ESTIMATION

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  ex. μ or σ^2

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  ex. $\bar{X}_n$ or $\hat{\mu}$ or S^2

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Estimation

Which are random variables?

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ESTIMATION

Which are random variables?

Parameter?No

ESTIMATION

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Estimand?

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Estimand?No

Statistic?Yes

Estimator?Yes

Estimate?No

ESTIMATION

Which are random variables?

Parameter?No

Estimand?No

Statistic?Yes

Estimator?Yes

Estimate?No

Point Estimation

Point Estimation

Estimand μ

Point Estimation

Estimand μ

Estimator $\bar{X}_n$

Point Estimation

Estimand μ

Estimator $\bar{X}_n$

Estimate $\bar{x}_n$

Point Estimation

Estimand μ

Estimator $\bar{X}_n$

Estimate $\bar{x}_n$

Sampling Distribution of $\bar{X}_n$ with **Known** Population

1. Start with the population.

Sampling Distribution of $\bar{X}_n$ with **Known** Population

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2. Define the quantity of interest (the estimand). For us, it's μ.

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2. Define the quantity of interest (the estimand). For us, it's μ.
3. Choose a plausible estimator. For us, it's $\hat{\mu} = \bar{X}_n$.

Sampling Distribution of $\bar{X}_n$ with **Known** Population

1. Start with the population.
2. Define the quantity of interest (the estimand). For us, it's μ.
3. Choose a plausible estimator. For us, it's $\hat{\mu} = \bar{X}_n$.
4. Draw a sample of size n from the population and calculate the estimate, $\bar{x}_n$, using the estimator.

SAMPLING DISTRIBUTION OF $\bar{X}_n$ WITH **KNOWN** POPULATION

1. Start with the population.
2. Define the quantity of interest (the estimand). For us, it's μ.
3. Choose a plausible estimator. For us, it's $\hat{\mu} = \bar{X}_n$.
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5. Repeat step 4 many times (we will have $s = 1,000$).

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6. Create a density plot of your 1,000 estimates ($\bar{x}_n$) of $\bar{X}_n$.

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Sampling Distribution of $\bar{X}_n$

The sampling distribution of $\bar{X}_n$ is the distribution of the column vector:

		Observations				$\hat{\mu} =$
		X_1	X_2	$\ldots$	X_n	$\bar{X}_n$
Samples (s)	1	$x_{1,1}$	$x_{2,1}$	$\ldots$	$x_{n,1}$	$\bar{x}_{n,1}$
	2	$x_{1,2}$	$x_{2,2}$	$\ldots$	$x_{n,2}$	$\bar{x}_{n,2}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	10,000	$x_{1,10000}$	$x_{2,10000}$	$\ldots$	$x_{n,10000}$	$\bar{x}_{n,10000}$

SAMPLING DISTRIBUTION OF $\bar{X}_n$ WITH **UNKOWN** POPULATION

In reality, we only get to observe one sample (not the entire population), so we cannot directly observe the sampling distribution of $\bar{X}_n$:

		Observations				$\hat{\mu} =$
		X_1	X_2	$\dots$	X_n	$\bar{X}_n$
Samples (s)	1	$x_{1,1}$	$x_{2,1}$	$\dots$	$x_{n,1}$	$\bar{x}_{n,1}$
	2	$x_{1,2}$	$x_{2,2}$	$\dots$	$x_{n,2}$	$\bar{x}_{n,2}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	10,000	$x_{1,10000}$	$x_{2,10000}$	$\dots$	$x_{n,10000}$	$\bar{x}_{n,10000}$

Central Limit Theorem

Remember that if our samples are sufficiently large ($n \geq 30$), the Central Limit Theorem states that the sample mean will be distributed as:

$$\bar{X}_n \sim N(\mu, \frac{\sigma^2}{n})$$

Estimated Sampling Distribution of $\bar{X}_n$ with **Unkown** Population

We could apply the CLT to get the sampling distribution of our estimator, except σ^2 is **unkown**.

ESTIMATED SAMPLING DISTRIBUTION OF $\bar{X}_n$ WITH **UNKOWN** POPULATION

We could apply the CLT to get the sampling distribution of our estimator, except σ^2 is **unkown**.

We can estimate σ^2 using the sample variance!

		Observations				$\hat{\mu} =$	$\hat{\sigma^2} =$	
		X_1	X_2	$\dots$	X_n	$\bar{X}_n$	S_n^2	$\hat{SE}[\bar{X}_n]$
	1	$x_{1,1}$	$x_{2,1}$	$\dots$	$x_{n,1}$	$\bar{x}_{n,1}$	$s_{n,1}^2$	$\frac{s_{n,1}}{\sqrt{n}}$
s	2	$x_{1,2}$	$x_{2,2}$	$\dots$	$x_{n,2}$	$\bar{x}_{n,2}$	$s_{n,2}^2$	$\frac{s_{n,2}}{\sqrt{n}}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	1000	$x_{1,1000}$	$x_{2,1000}$	$\dots$	$x_{n,1000}$	$\bar{x}_{n,1000}$	$s_{n,1000}^2$	$\frac{s_{n,1000}}{\sqrt{n}}$
	E[·]	μ	μ	$\dots$	μ	μ	σ^2	
	V[·]	σ^2	σ^2	$\dots$	σ^2	$\frac{\sigma^2}{n}$		

Estimated Sampling Distribution of $\bar{X}_n$ with **Unkown** Population

We could apply the CLT to get the sampling distribution of our estimator, except σ^2 is **unkown**.

We can estimate σ^2 using the sample variance!

		Observations				$\hat{\mu} =$	$\hat{\sigma}^2 =$	
		X_1	X_2	$\ldots$	X_n	$\bar{X}_n$	S_n^2	$\hat{SE}[\bar{X}_n]$
s	1	$x_{1,1}$	$x_{2,1}$	$\ldots$	$x_{n,1}$	$\bar{x}_{n,1}$	$s_{n,1}^2$	$\frac{s_{n,1}}{\sqrt{n}}$
	2	$x_{1,2}$	$x_{2,2}$	$\ldots$	$x_{n,2}$	$\bar{x}_{n,2}$	$s_{n,2}^2$	$\frac{s_{n,2}}{\sqrt{n}}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	1000	$x_{1,1000}$	$x_{2,1000}$	$\ldots$	$x_{n,1000}$	$\bar{x}_{n,1000}$	$s_{n,1000}^2$	$\frac{s_{n,1000}}{\sqrt{n}}$
	E[$\cdot$]	μ	μ	$\ldots$	μ	μ	σ^2	
	V[$\cdot$]	σ^2	σ^2	$\ldots$	σ^2	$\frac{\sigma^2}{n}$		

Central Limit Theorem

So, by the CLT again with n large we can estimate the sampling distribution of our estimator as :

$$\bar{X}_n \sim_{approx} N(\bar{x}_n, \frac{s_n^2}{n})$$

Generally, we report our point estimate $\bar{x}_n$ and the estimated standard error of our estimate $\frac{s_n}{\sqrt{n}}$.

BOOTSTRAPPED SAMPLING DISTRIBUTION OF $\bar{X}_n$ WITH **UNKNOWN** POPULATION

Or we can estimate the sampling distribution using boostrapping.

Bootstrapped Sampling Distribution of $\bar{X}_n$ with **Unknown** Population

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Or we can estimate the sampling distribution using boostrapping.

1. Start with observed sample.
2. Define a quantity of interest (the estimand). For us, it's μ.

Bootstrapped Sampling Distribution of $\bar{X}_n$ with **Unknown** Population

Or we can estimate the sampling distribution using boostrapping.

1. Start with observed sample.
2. Define a quantity of interest (the estimand). For us, it's μ.
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Interval Estimation for **Large** Samples and σ **Known**

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **KNOWN**

Estimand $[LB, UB]$ s.t. $P(LB \leq \mu \leq UB) = 100(1-\alpha)\%$

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **KNOWN**

Estimand $[LB, UB]$ s.t. $P(LB \leq \mu \leq UB) = 100(1-\alpha)\%$

Estimator $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ KNOWN

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Estimator $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Estimate $\bar{x}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ KNOWN

		Observations				
		X_1	X_2	$\dots$	X_n	$\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
	1	$x_{1,1}$	$x_{2,1}$	$\dots$	$x_{n,1}$	$\bar{x}_{n,1} \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
s	2	$x_{1,2}$	$x_{2,2}$	$\dots$	$x_{n,2}$	$\bar{x}_{n,2} \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	1000	$x_{1,1000}$	$x_{2,1000}$	$\dots$	$x_{n,1000}$	$\bar{x}_{n,1000} \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Interval Estimation for **Large** Samples and σ **Known**

How did we choose our estimator?

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By the CLT, if n is large, then $\bar{X}_n \sim N(\mu, \frac{\sigma^2}{n})$.

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If we standardize, we get that $Z = \frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$

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Which implies that:

$$P(-z_{1-\alpha/2} \leq \frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{1-\alpha/2}) = 1 - \alpha$$

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Which implies that:

$$
\begin{aligned}
& P(-z_{1-\alpha/2} \leq \frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{1-\alpha/2}) = 1 - \alpha \\
\Rightarrow \quad & P(\bar{X}_n - z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X}_n + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}) = 1 - \alpha
\end{aligned}
$$

Interval Estimation for **Large** Samples and σ Known

How did we choose our estimator?

By the CLT, if n is large, then $\bar{X}_n \sim N(\mu, \frac{\sigma^2}{n})$.

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& P(-z_{1-\alpha/2} \leq \frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{1-\alpha/2}) = 1 - \alpha \\
\Rightarrow \quad & P(\bar{X}_n - z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X}_n + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}) = 1 - \alpha
\end{aligned}
$$

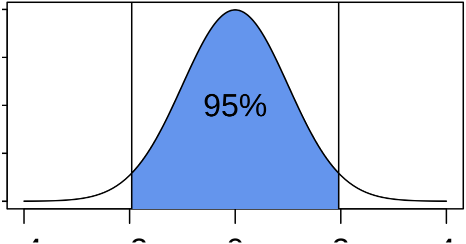
Interval Estimation for **Large** Samples and σ **Known**

How do we get $z_{1-\alpha/2}$?

Interval Estimation for **Large** Samples and σ **Known**

How do we get $z_{1-\alpha/2}$?

```
## alpha = .05
qnorm(1-.025, mean = 0, sd=1)
```



INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **KNOWN**

How do we interpret our estimated confidence interval?

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **KNOWN**

How do we interpret our estimated confidence interval?
The true population mean μ will lie within the estimated intervals in 100(1-α)% of many repeated samples.

Quiz Break!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$?

Quiz Break!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$? $\bar{X}_n$

Quiz Break!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$? $\bar{X}_n$

What is random in our estimate: $\bar{x}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$?

QUIZ BREAK!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$? $\bar{X}_n$

What is random in our estimate: $\bar{x}_n \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$? Nothing!

Interval Estimation for **Large** Samples and σ **Unknown**

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **UNKNOWN**

Estimand $[LB, UB]$ s.t. $P(LB \leq \mu \leq UB) = 100(1-\alpha)\%$

Interval Estimation for **Large** Samples and σ **Unknown**

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Estimator $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **UNKNOWN**

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Estimator $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$

Estimate $\bar{x}_n \pm z_{1-\alpha/2} \cdot \frac{s_n}{\sqrt{n}}$

Interval Estimation for **Large** Samples and σ **Unknown**

		Observations				
		X_1	X_2	$\dots$	X_n	$\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$
	1	$x_{1,1}$	$x_{2,1}$	$\dots$	$x_{n,1}$	$\bar{x}_{n,1} \pm z_{1-\alpha/2} \cdot \frac{s_{n,1}}{\sqrt{n}}$
s	2	$x_{1,2}$	$x_{2,2}$	$\dots$	$x_{n,2}$	$\bar{x}_{n,2} \pm z_{1-\alpha/2} \cdot \frac{s_{n,2}}{\sqrt{n}}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	1000	$x_{1,1000}$	$x_{2,1000}$	$\dots$	$x_{n,1000}$	$\bar{x}_{n,1000} \pm z_{1-\alpha/2} \cdot \frac{s_{n,1000}}{\sqrt{n}}$

Interval Estimation for **Large** Samples and σ **Unknown**

How did we choose our estimator?

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **UNKNOWN**

How did we choose our estimator?

Same as before except we *estimated* $\sigma/\sqrt{n}$ with $\hat{SE}[\bar{X}_n] = \frac{S_n}{\sqrt{n}}$.

INTERVAL ESTIMATION FOR **LARGE** SAMPLES AND σ **UNKNOWN**

How did we choose our estimator?

Same as before except we *estimated* $\sigma/\sqrt{n}$ with $\hat{SE}[\bar{X}_n] = \frac{S_n}{\sqrt{n}}$.

Are we still getting 95% coverage over repeated samples?

Interval Estimation for **Large** Samples and σ **Unknown**

How did we choose our estimator?

Same as before except we *estimated* $\sigma/\sqrt{n}$ with $\hat{SE}[\bar{X}_n] = \frac{S_n}{\sqrt{n}}$.

Are we still getting 95% coverage over repeated samples?
No, but as n increases, this matters less.

Quiz Break!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$?

Quiz Break!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$? $\bar{X}_n$ **and** S_n

What is random in our estimate: $\bar{x}_n \pm z_{1-\alpha/2} \cdot \frac{s_n}{\sqrt{n}}$?

Quiz Break!

What is random in our estimator: $\bar{X}_n \pm z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$? $\bar{X}_n$ **and** S_n

What is random in our estimate: $\bar{x}_n \pm z_{1-\alpha/2} \cdot \frac{s_n}{\sqrt{n}}$? Nothing!

Interval Estimation for **Small** Samples and σ **Unknown**

INTERVAL ESTIMATION FOR **SMALL** SAMPLES AND σ **UNKNOWN**

Assuming $X_i \sim_{i.i.d} N(\mu, \sigma^2)$,

Estimand $[LB, UB]$ s.t. $P(LB \leq \mu \leq UB) = 100(1-\alpha)\%$

INTERVAL ESTIMATION FOR **SMALL** SAMPLES AND σ **UNKNOWN**

Assuming $X_i \sim_{i.i.d} N(\mu, \sigma^2)$,

Estimand $[LB, UB]$ s.t. $P(LB \leq \mu \leq UB) = 100(1-\alpha)\%$

Estimator $\bar{X}_n \pm t_{n-1,1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$

Interval Estimation for **Small** Samples and σ **Unknown**

Assuming $X_i \sim_{i.i.d} N(\mu, \sigma^2)$,

Estimand $[LB, UB]$ s.t. $P(LB \leq \mu \leq UB) = 100(1-\alpha)\%$

Estimator $\bar{X}_n \pm t_{n-1,1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$

Estimate $\bar{x}_n \pm t_{n-1,1-\alpha/2} \cdot \frac{s_n}{\sqrt{n}}$

INTERVAL ESTIMATION FOR **SMALL** SAMPLES AND σ **UNKNOWN**

		Observations				
		X_1	X_2	$\dots$	X_n	$\bar{X}_n \pm t_{n-1,1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}$
	1	$x_{1,1}$	$x_{2,1}$	$\dots$	$x_{n,1}$	$\bar{x}_{n,1} \pm t_{n-1,1-\alpha/2} \cdot \frac{s_{n,1}}{\sqrt{n}}$
s	2	$x_{1,2}$	$x_{2,2}$	$\dots$	$x_{n,2}$	$\bar{x}_{n,2} \pm t_{n-1,1-\alpha/2} \cdot \frac{s_{n,2}}{\sqrt{n}}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	1000	$x_{1,1000}$	$x_{2,1000}$	$\dots$	$x_{n,1000}$	$\bar{x}_{n,1000} \pm t_{n-1,1-\alpha/2} \cdot \frac{s_{n,1000}}{\sqrt{n}}$

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When we standardize $\bar{X}_n$ and estimate σ with S_n,

$$\frac{\bar{X}_n - \mu}{\frac{S_n}{\sqrt{n}}} \sim t_{n-1}$$

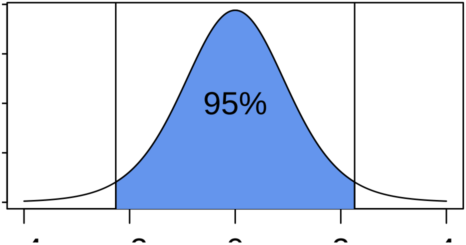
Interval Estimation for **Small** Samples and σ **Unknown**

How do we get $t_{n-1,1-\alpha/2}$?

Interval Estimation for **Small** Samples and σ **Unknown**

How do we get $t_{n-1,1-\alpha/2}$?

```
## alpha = .05
## 9 degrees of freedom
qt(1-.025, df=9)
```



Quiz Break

You are told that your estimand is $[LB, UB]$ s.t. $P(LB \leq \mu_1 - \mu_2 \leq UB) = 100(1-\alpha)\%$. Assuming n_1 and n_2 are both large and σ_1 and σ_2 are both unkown, what is your estimator and estimate?

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   ex.

$$\begin{array}{lll} \text{Reject null} & \text{if} & \frac{\bar{x}_n - \mu_0}{\frac{s_n}{\sqrt{n}}} \in [-\infty, -1.96] \cup [1.96, \infty] \\ \text{Fail to reject null} & \text{if} & \frac{\bar{x}_n - \mu_0}{\frac{s_n}{\sqrt{n}}} \notin [-\infty, -1.96] \cup [1.96, \infty] \end{array}$$

P-Values

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How do we find p-values?

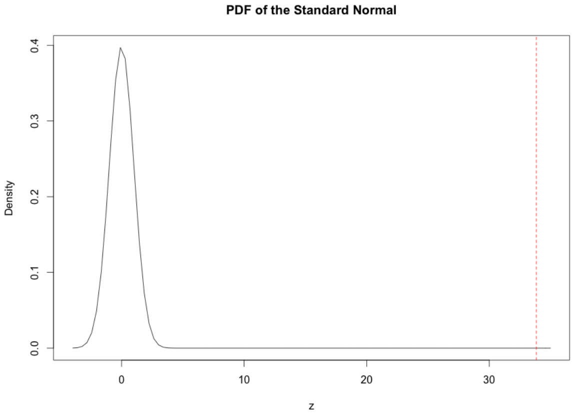
```
# create a population with mean 12, sd 4
pop <- rnorm(n=1e6, mean=12, sd=4)

# draw a single sample of size 100 from population
my.sample <- sample(pop, size=100, replace=T)

# calculate our test statistic
test.statistic <- mean(my.sample) /(sd(my.sample)/10)

# find the p-value
p.value <- 2*(pnorm(test.statistic))
```

P-Values



Questions

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