

## Section 5: Interval Estimation and Hypothesis Testing

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### This Week

**Problem Set 4-** Due by 1pm on Tuesday to course dropbox.

**Problem Set 3 Corrections-** Due by 1pm on Tuesday to course dropbox.

**No Reading Quiz**

**Practice Midterms-** Not due, but recommended as midterm preparation.

### Next Week

**Midterm-** Posted October 8, due by 11:59pm on Sunday, October 13 with 5 hours to complete.

**Office Hours-** Cancelled Friday October 6. Rescheduled for Tuesday October 15 from 9:30am to 12pm.

**Problem Set 5-** Due by 1pm on Tuesday, October 15 to course dropbox.

**Problem Set 4 Corrections-** Due by 1pm on Tuesday, October 15 to course dropbox.

**Reading Quiz (ALZ ch. 11 & 12)-** Due by 9pm on Sunday, October 13 on course website.

**Mid-semester Evaluation-** Due by 9pm on Monday, October 14 on course website.

### Sampling

**Parameter-** characteristic of the **population** distribution (the distribution of  $X_i$ )

ex.  $\mu$  or  $\sigma^2$

**Estimand-** parameter being estimated

ex.  $\mu$

**Statistic-** function of the sample used to estimate a parameter

ex.  $\bar{X}_n$  or  $\hat{\mu}$  or  $S^2$

**Estimator-** a statistic (random variable) that describes the estimation procedure

ex.  $\bar{X}_n$

**Estimate-** realized values of an estimator

ex.  $\tilde{x}_n$

Notation	Description	Formula <sup>1</sup> /Distribution	Random Variable	Observed?	Estimand/te/tor
n	Sample size		No	Yes	
N	Population size		No	?	
s	Number of samples		No	Yes	
$Y_i$	Random variable	$\sim?(\mu, \sigma^2)$	Yes	No	
$y_i$	Realization of $Y_i$		No	Yes	
$\mu, E[Y_i], \bar{y}_N$	Population mean	$E[Y_i] = \frac{1}{N} \sum_{i=1}^N y_i$	No	No	Estimand
$\hat{\mu}$	Estimator for $\mu$		Yes	No	Estimator
	Estimate of $\hat{\mu}$ (realization)		No	Yes	Estimate
$\bar{Y}_n$	Sample mean	$\sim N(\mu, \frac{\sigma^2}{n})$ for $n$ large	Yes	No	Estimator
$\bar{y}_n$	Realization of $\bar{Y}_n$	$\frac{1}{n} \sum_{i=1}^n y_i$	No	Yes	Estimate
$\sigma^2$	Population variance	$V[Y_i] = \frac{1}{N} \sum_{i=1}^N (y_i - \mu)^2$	No	No	Estimand
$\sigma$	Population standard deviation	$SD[Y_i] = \sqrt{\sigma^2}$	No	No	Estimand
$S_n^2$	Sample variance	$\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$	Yes	No	Estimator
$S_n$	Sample standard deviation	$\sqrt{S_n^2}$	Yes	No	Estimator
$s_n^2$	Realization of $S_n^2$	$\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x}_n)^2$	No	Yes	Estimate
$s_n$	Realization of $S_n$	$\sqrt{s_n^2}$	No	Yes	Estimate
$SE[Y_n]; SE[\hat{\mu}]$	Standard deviation of $\bar{Y}_n$	$\frac{\sigma}{\sqrt{n}}$	No	No	Estimand
$\hat{SE}[\bar{Y}_n]; \hat{SE}[\hat{\mu}]$	Estimator of standard error	$\frac{S_n}{\sqrt{n}}$	Yes	No	Estimator
	Estimated standard error	$\frac{s_n}{\sqrt{n}}$	No	Yes	Estimate

<sup>1</sup>The formulas in this table assume that  $Y_i$  is a discrete random variable and each outcome has equal probability. These can be adapted to more general formulas found in Lecture 3.

## Practice Conceptual Questions

1. What is the difference between an estimand, estimator, and estimate? How do these relate to parameters and statistics?
2. What is the difference between a statistic and a test statistic?
3. Given that you have only one sample from the population, what are some point estimates you might calculate and how would you calculate them?
4. What do we do when we don't know  $\sigma$ ? What are the implications of this?
5. What is bootstrapping and why do we do it?

6. Why can't we use the t-distribution with small samples unless the underlying population distribution is normal?
7. Why do we get our critical values (eg. 1.96) from the standard normal distribution?
8. Can we interpret our confidence intervals the same when we don't know  $\sigma$  as when we do? Do these confidence intervals truly cover the population mean in 95% of samples? Why is this not a problem in most instances?
9. Why and how is hypothesis testing related to interval estimation?
10. How are p-values related to hypothesis testing?