GOV 2000 / E-2000 Section 3[1]

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Harvard University

September 19, 2013

[1] These notes and accompanying code draw on the notes from TF's from previous years.

LOGISTICS

Problem Set 2- Due by 1pm on Tuesday to course dropbox.

Problem Set 1 Corrections- Due by 1pm on Tuesday to course dropbox.

Reading Quiz- Due by 9pm on Sunday on course website.

Logistics

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LaTeX Tutorial- Next Friday, September 27, at 4pm. Location TBD.

Outline

Quick R Code

Contents of your lm object:

```
## load some data
load(file="Leinhardt.RData")

## run a model
my.lm <- lm(linfant ~ lincome, data=Leinhardt)

## our lm object has many components we may want to extract
names(my.lm)

## Two ways to get the coefficients:
my.coefs <- my.lm$coefficients
my.coefs <- coefficients(my.lm)

## Two ways to get the residuals:
my.resids <- my.lm$residuals
my.resids <- residuals(my.lm)

## One way to get the fitted values:
my.fvs <- my.lm$fitted.values
```

Outline

Why Add More Variables?

1. Improve fit of model.

2. Theory calls for it.

Review: Additive Models

$$\boxed{\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z}$$

[2] Note that this is not a causal effect, just an expected relationship. We will discuss this more later in the semester. See the interpretation slide for a more precise definition.

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What is the intercept when plotting Y on X? $\beta_0 + \beta_2 Z$

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Additive Multiple Regression with Two Continuous Variables

We estimate that $\hat{Y} = 2 + \frac{1}{3}X + 8Z$.

What is the relationship between X and Y?
(Suppose Z can be any positive integer)

Z	Intercept $\beta_0 + \beta_2 Z$	Slope β_1
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$\vdots$	$\vdots$	$\vdots$
1253		

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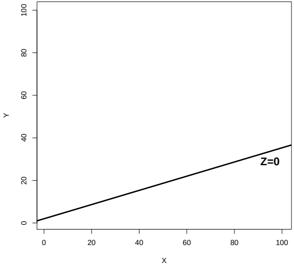

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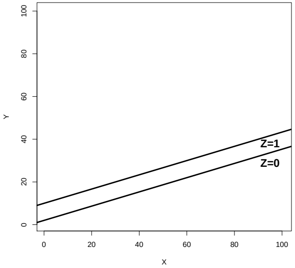

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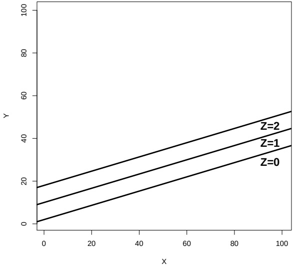

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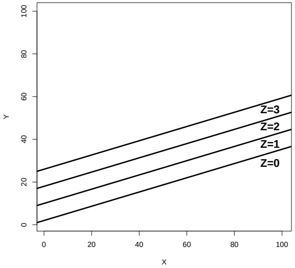

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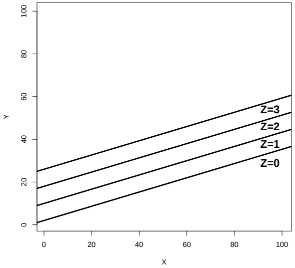
1253	$2 + 8 \cdot 1253$	$\frac{1}{3}$



Interpretation

How do we interpret β_1 in the additive multiple regression model?

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The expected change in Y associated with a one-unit increase in X, holding constant the value of Z.

Review: Interaction Models

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Y &= \beta_0 + \beta_1 X + \beta_3 X \cdot Z + \beta_2 Z \\
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\end{aligned}$$

or...

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

Quiz Break!

What is the conditional intercept when plotting Y on X?

What is the conditional effect of Z on Y?

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$\beta_0 + \beta_2 Z$

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What is the conditional intercept when plotting Y on X?

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What is the conditional effect of Z on Y? $\beta_2 + \beta_3 X$

Interactions with Two Continuous Variables

We estimate that $\hat{Y} = 2 + \frac{1}{3}X + 8Z + \frac{1}{2}X \cdot Z$.

What is the relationship between X and Y conditional on Z?
(Suppose Z can be any positive integer)

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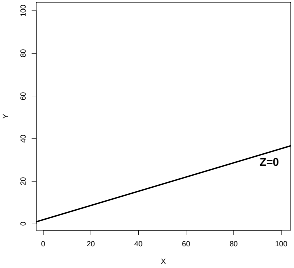

INTERACTIONS WITH TWO CONTINUOUS VARIABLES

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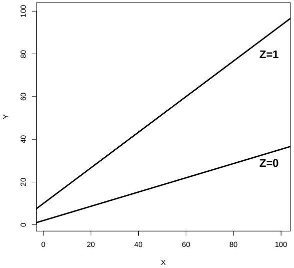

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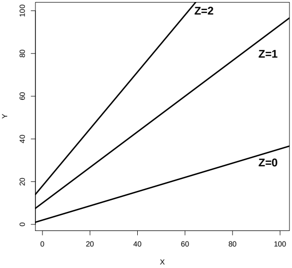

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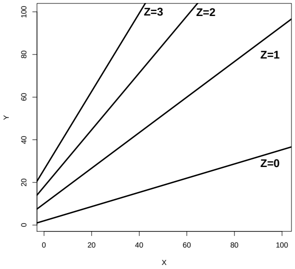

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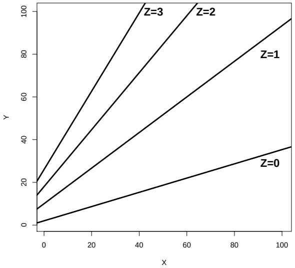
1253	$2 + \frac{4}{3} \cdot 1253$	$\frac{1}{2} + 8 \cdot 1253$



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The expected change in Y associated with a one-unit increase in X, conditional on Z being z.

When Interaction?

When the hypotheses we are testing are of the form:[3]

> *1. The effect of X on Y is increasing(decreasing) in Z. An increase in X is associated with a larger(smaller) increase in Y when condition Z is met, than when condition Z is absent.*

Example?

[3] See Brambor (2006) for more.

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More education is associated with higher wages when "ability" is low. More education is associated with even higher wages when "ability" is high.

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$$Wages = \beta_0 + \beta_1 education + \beta_2 ability + \beta_3 education \cdot ability$$

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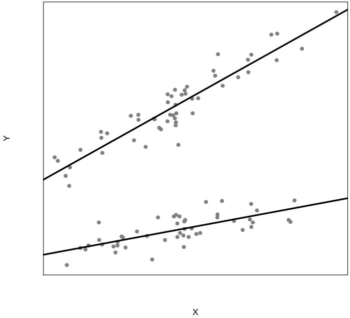

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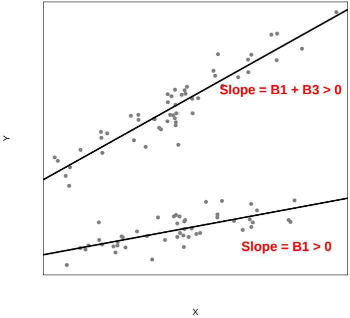

When Interaction?

When the hypotheses we are testing are of the form:

> *2. An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.*

Example?

[4] See Boix (1999).

When Interaction?

When the hypotheses we are testing are of the form:

> *2. An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.*

Example?

High ethnic or religious fragmentation encourages the adoption of PR systems in **small** countries, but not in large countries.[4]

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When the hypotheses we are testing are of the form:

> *2. An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.*

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High ethnic or religious fragmentation encourages the adoption of PR systems in **small** countries, but not in large countries.[4]

$$PR = \beta_0 + \beta_1 fragmentation + \beta_2 countrysize + \beta_3 fragmentation \cdot countrysize$$

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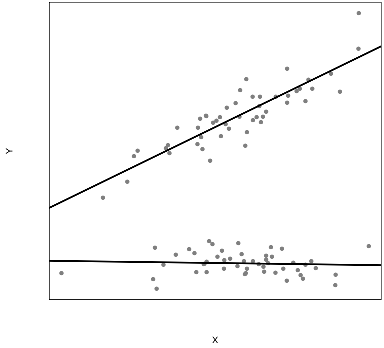
> *2. An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.*



When Interaction?

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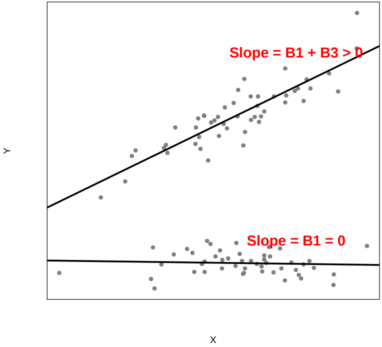

WHEN INTERACTION?

When the hypotheses we are testing are of the form:

> *3. An increase in X is associated with an increase in Y when condition Z is present, but a decrease in Y when condition Z is absent.*

Example?

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Example?

Aspirin is expected to make you feel better when you have not been drinking and is expected to make you feel worse when you have been drinking.

When Interaction?

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Aspirin is expected to make you feel better when you have not been drinking and is expected to make you feel worse when you have been drinking.

$$feel = \beta_0 + \beta_1 Aspirin + \beta_2 drinking + \beta_3 Aspirin \cdot drinking$$

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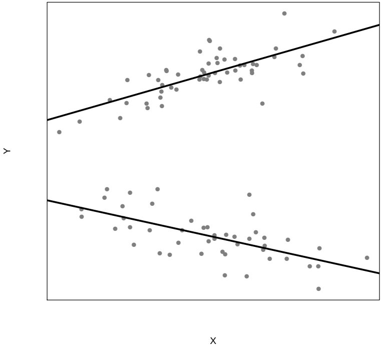

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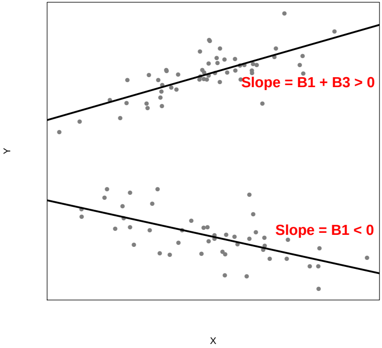

Questions?

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PROBABILITY

Probability is a formal model of uncertainty.

Suppose I roll a die.

What is the probability we roll a 2?

Probability

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What is the probability we roll a 2?
$P(X = 2) = \frac{1}{6}$

PROBABILITY

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What is the probability we roll something less than 4?

Probability

Probability is a formal model of uncertainty.

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What is the probability we roll something less than 4?
$P(X < 4) = P(X = 1) + P(X = 2) + P(X + 3)$

Probability

Probability is a formal model of uncertainty.

Suppose I roll a die.

What is the probability we roll something less than 4?
$P(X < 4) = P(X = 1) + P(X = 2) + P(X + 3) = \frac{3}{6}$

Notation and Definitions

Sample Space (S): The set of all possible outcomes from some process.

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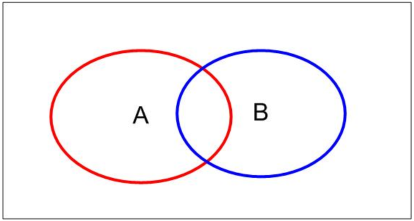

Axioms of Probability

1. For any event A, $P(A) \geq 0$
2. $P(S) = 1$ where S is the sample space
3. For any sequence of disjoint events (A and B), $P(A \cup B) = P(A) + P(B)$

Probability Example

	Speeding Violation	No Speeding Violation	Total
Car Phone	25	280	305
No Car Phone	45	405	450
Total	70	685	755

Source: `http://cnx.org/content/m16835/1.12/`

Let's define C to be the event of having a car phone and S to be the event of receiving a speeding violation.

Marginal Probability

	Speeding Violation	No Speeding Violation	Total
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What is the probability of having a car phone?

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What is the probability of having a car phone? $P(C) = \frac{305}{755}$

Joint Probability

	Speeding Violation	No Speeding Violation	Total
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Joint Probability: The probability of some event A and another event B both occurring (P(A∩B) or P(A,B)).

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Joint Probability: The probability of some event A and another event B both occurring ($P(A \cap B)$ or $P(A,B)$). Look inside the table.

What is the probability of getting a speeding violation and having a car phone?

Joint Probability

	Speeding Violation	No Speeding Violation	Total
Car Phone	25	280	305
No Car Phone	45	405	450
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Conditional Probability

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Conditional Probability

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What is the probability of getting a speeding violation given that you have a car phone?

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What if we don't have such a nice table?

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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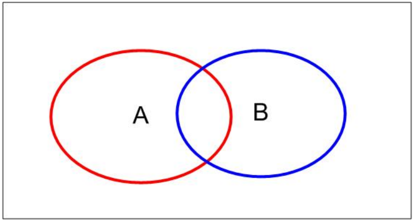


CONDITIONAL PROBABILITY

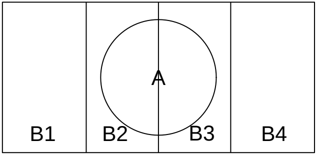
Note that if:

$$\begin{aligned} P(B|A) &= \frac{P(A \cap B)}{P(A)} \\ P(B|A)P(A) &= P(A \cap B) \end{aligned}$$

Law of Total Probability

Law of Total Probability: Let S be the sample space and let the disjoint k events $B_1, ..., B_k$ partition S such that $P(B_1 \cup ... \cup B_k) = P(S) = 1$. If A is some other event in S, then

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(B_i)P(A|B_i)$$



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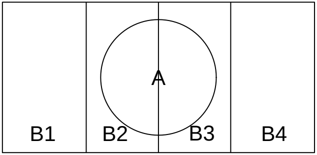

What is the P(A) in this figure?

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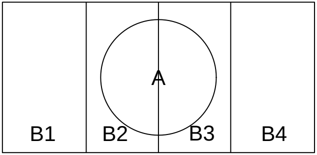

What is the P(A) in this figure?

$P(A) =$
$P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3) + P(B_4)P(A|B_4)$

Bayes' Rule

$$
\begin{aligned}
P(B|A) &= \frac{P(A \cap B)}{P(A)} && \text{Definition of Conditional Prob.} \\
&= \frac{P(B)P(A|B)}{P(A)} && \text{Rearranged Conditional Prob.} \\
&= \frac{P(B)P(A|B)}{P(B)P(A|B) + P(B^C)P(A|B^C)} && \text{Law of Total Prob.}
\end{aligned}
$$

Quiz Break!

Write a function in R that takes in $P(A)$, $P(B)$, and $P(A|B)$ and outputs $P(B|A)$.

OUTLINE

Discrete Random Variables

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What is the distribution of X?

Discrete Distributions

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What is the random variable? (Call it X) The outcome of a roll of a die.

What is the distribution of X? $P(X) = \begin{cases} \frac{1}{6} & \text{if } x = 1 \\ \frac{1}{6} & \text{if } x = 2 \\ \frac{1}{6} & \text{if } x = 3 \\ \frac{1}{6} & \text{if } x = 4 \\ \frac{1}{6} & \text{if } x = 5 \\ \frac{1}{6} & \text{if } x = 6 \\ 0 & \text{otherwise} \end{cases}$

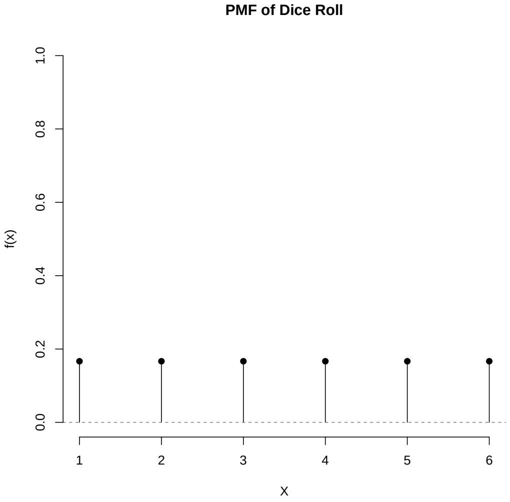
Discrete Distributions in R

How would we draw this PMF?

```
## Create an empty plot
plot(0,0,type="n",xlab="X",ylab="f(x)",
     xlim=c(1,6), ylim=c(0,1),
     main="PMF of Dice Roll")

## Add each vertical line and each point
## for each line we specify the starting point (x0,y0)
##  and end point(x1,y1)
segments(x0=1,x1=1,y0=0,y1=1/6)
points(x=1,y=1/6,pch=19)
segments(x0=2,x1=2,y0=0,y1=1/6)
points(x=2,y=1/6,pch=19)
segments(x0=3,x1=3,y0=0,y1=1/6)
points(x=3,y=1/6,pch=19)
segments(x0=4,x1=4,y0=0,y1=1/6)
points(x=4,y=1/6,pch=19)
segments(x0=5,x1=5,y0=0,y1=1/6)
points(x=5,y=1/6,pch=19)
segments(x0=6,x1=6,y0=0,y1=1/6)
points(x=6,y=1/6,pch=19)
```

Discrete Distributions in R

How would we draw the PMF?



Discrete Distributions in R

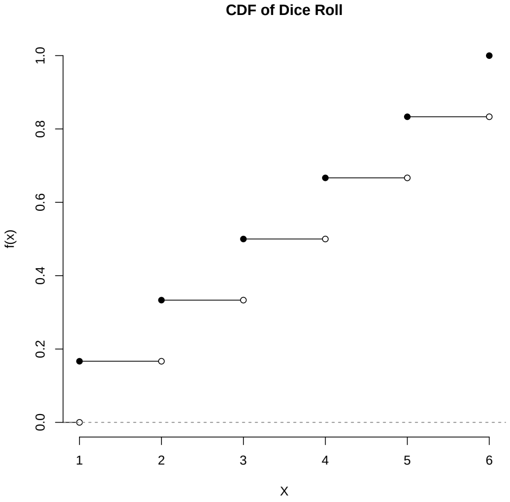
How would we draw the CDF?

```
## Create an empty plot
plot(0,0,type="n",xlab="X",ylab="f(x)",
     xlim=c(1,6), ylim=c(0,1),
     main="PMF of Dice Roll")

## First, add the horizontal line for each value of x
segments(x0=0,x1=1,y0=0,y1=0)

## Then add the open point
points(x=1,y=0,pch=21,bg="white")

## Then add the closed point for the cumulative probability
points(x=1,y=1/6,pch=19)

## Repeat for all values of x
```

See if you can finish this CDF on your own!

Discrete Distributions in R

How would we draw the CDF?



Discrete Distributions in R

What if we aren't certain about the distribution of X?

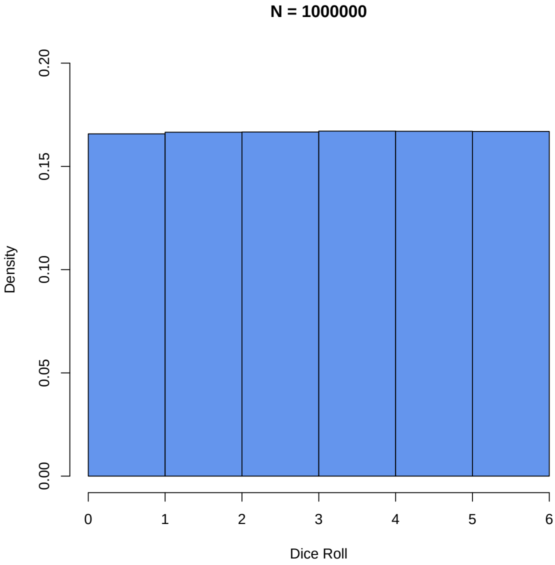
Discrete Distributions in R

What if we aren't certain about the distribution of X?

We can sample from our population or sample space and then look at the distribution of our sample.

```
## Define the sample space
S <- 1:6

## Set the sample size
n <- 1000000

## Sample from the population
## i.e. Roll the "die" repeatedly
dice.rolls <- sample(S, size=n, replace=T)

## Plot the distribution
hist(dice.rolls, freq=F)
```

Discrete Distributions in R



Continuous Random Variables

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CONTINUOUS RANDOM VARIABLES

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Continuous Distributions in R

Suppose our random variable of interest is people's salary.

CONTINUOUS DISTRIBUTIONS IN R

Suppose our random variable of interest is people's salary.

Let's assume that this random variable is normally distributed with mean salary $40,000$ and standard deviation $10,000$.

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Continuous Distributions in R

Suppose our random variable of interest is people's salary.

Let's assume that this random variable is normally distributed with mean salary $40,000$ and standard deviation $10,000$.

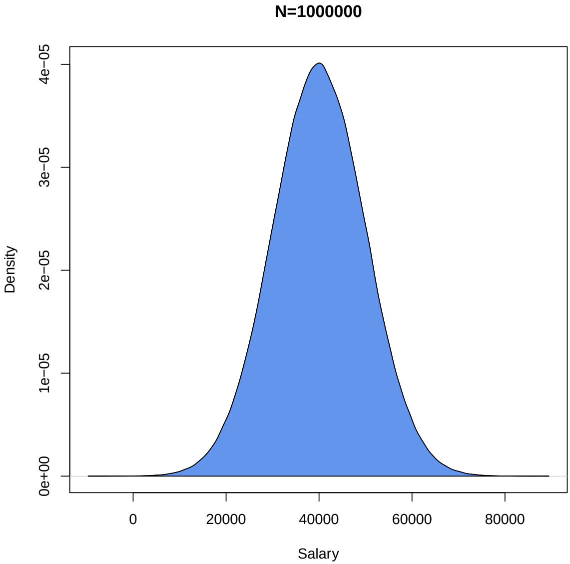
What if we want to look at the distribution of X?

Again, we can sample from our population or sample space and then look at the distribution of our sample.

```
## Sample from the population
salaries <- rnorm(n=1000000 , mean=40000 , sd=10000)

## Plot the distribution
plot(density(salaries))
```

Continuous Distributions in R



Other Cool Things in R

What is f(20000)?

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## dnorm provides the PDF of the normal distribution
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What is the probability that a salary is less that $20,000?

```
## pnorm provides the CDF of the normal distribution
pnorm(20000, mean=40000, sd=10000)
```

What value of p satisfies $P(X \leq p) = .95$?

```
## qnorm provides the inverse CDF of the normal distribution
qnorm(0.95, mean=40000, sd=10000)
```

Other Cool Things in R

For a list of the distributions that have similar commands in R, go to `http://www.stat.umn.edu/geyer/old/5101/rlook.html`.

Outline

Expectation

Expectation of a Discrete Random Variable:
$E[X] = \sum_x x \cdot P(X = x)$

Conditional Expectation of a Discrete Random Variable:
$E[X|Y = y] = \sum_x x \cdot P(X = x|Y = y)$

Expectation of a Continuous Random Variable:
$E[X] = \int_x x \cdot f(x) dx$

Conditional Expectation of a Discrete Random Variable:
$E[X|Y = y] = \int_x x \cdot f(x|y) dx$

Variance

Variance of a Discrete Random Variable:
$V[X] = \sum_x (x - E[X])^2 \cdot P(X = x)$

Variance of a Continuous Random Variable:
$V[X] = \int_x (x - E[X])^2 \cdot f(x) dx$

Outline

Questions?

REFERENCES

Boix, Carles. 1999. "Setting the Rules of the Games: The Choice of Electoral Systems in Advanced Democracies." *American Political Science Review* 93:609-624.

Brambor, Thomas, William Roberts Clark, and Matt Golder. 2006. "Understanding Interaction Models: Improving Empirical Analyses." *Political Analysis* 14:63-82.