Vectors $v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ $w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$

1. Transpose: $v^T = (v_1 \; \cdots \; v_n)$
2. Length: $\|v\| = \sqrt{\sum v_i^2}$
3. Dot product (inner product): $v \cdot w = \sum v_i w_i = v^T w$
4. Outer product: $v w^T$

Matrices

1. Identity Matrix $\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$
2. Diagonal Matrix $\begin{pmatrix} * & & 0 \\ & \ddots & \\ 0 & & * \end{pmatrix}$
3. Multiplication
4. Q: Suppose A is a $m \times n$ matrix and B is a $p \times q$ matrix, when can we compute AB and/or BA?
   $n = p$ $m = q$
5. Q: Is $AB = BA$? No
6. Matrix Inverse $A^{n \times n}$, A^{-1} is a matrix s.t $A^{-1}A = I$ and $AA^{-1} = I$
7. Does $(AB)^{-1}$ equal to $A^{-1}B^{-1}$ or $B^{-1}A^{-1}$?
   $(AB)^{-1} = B^{-1}A^{-1}$
8. Q: Check that the inverse of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
9. Matrix Transpose $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$ $A^T = \begin{pmatrix} a_{11} & \cdots & a_{m1} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{mn} \end{pmatrix}$
10. Q: Given a random matrix A, is $A^TA = AA^T$ ture?
    No
11. How does $(AB)^T$ relate to A^T and B^T?
    $(AB)^T = B^T A^T$
12. Orthogonal Matrices
    $Q = (q_1 \; \cdots \; q_n)$ $\|q_i\| = 1$ $q_i \cdot q_j = 0$.