Volumes, Matrix calculus

1. Geometric meaning of the determinate of a linear transformation A.

2. Matrix calculation: Given a function $f(x, y, z)$, write df in a matrix multiplication form.

3. Given a $n \times n$ matrix A, trace $tr(A) =$

Eigenvalues

1. Suppose A is a $n \times n$ matrix, λ is an **eigenvalue** of A if

   ____,

   ________________________ is an **eigenvector** of A.

2. If A is diagonalizable, then

$$P^{-1}AP = \Sigma,$$

   where $\Sigma = diag(\lambda_1, \cdots, \lambda_n)$, $P = (u_1, \cdots, u_n)$, and u_i is the eigenvector for the eigenvalue λ_i.

Problems

1. What is area of the triangle whose vertices are $(-1, 1), (1, 2), (0, 3)$?

2. Suppose A is a 2×2 matrix with eigenvalues being $2, 1$ and the corresponding eigenvectors being u and v.

   (a) What is $(2A + I)u$?

   (b) Is $2A + I$ diagonalizable? If yes, what are the eigenvalues of $2A + I$?

3. Suppose A, B are $n \times n$ matrices.

   (a) Is $tr(AB) = tr(A)tr(B)$? Is $tr(A^T) = tr(A)$?

   (b) If A is diagonalizable, is A^2 also diagonalizable?

   (c) If the eigenvector matrix of A is the identity matrix, what can you say about A?