Recitation 2/25

Sungwoo Jeong Tuesday 10AM, 11AM

March 12, 2020

Intuition for some matrix multiplications

-Blackboard

Matrix Decompositions

LU decomposition : A square matrix $A = LU$ where L is a ________________ matrix and U is a ________________ matrix.

QR decomposition : A m-by-n matrix $A = QR$ where Q is a ________________ matrix and R is a ________________ matrix.

Singular Value Decomposition : A m-by-n matrix $A = U\Sigma V^T$, where U and V are ________________ matrices and Σ is non-zero only on diagonals.(there can be zeros on the diagonal of Σ)

Dimensions of SVD - Blackboard
1. Rank r
2. Partial
3. Full

Problems

1. Find a LU decomposition of $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$

2. We have learned four fundamental vector spaces, col(A), null(A), row(A), null(A^T). Let A be a n-by-n matrix. If $\text{null}(A) = \mathbb{R}^n$, what can we say about A?

3. (a) Find a QR decomposition of $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$

(b) Describe $\text{col}(A)$. Is $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in it?

(c) Find a vector x that minimizes $||Ax - b||$

4. Full SVD of 4-by-3 matrix A is given as $A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(a) Write Rank-r and partial SVD of A.

(b) What is the rank of A?

(c) Compute the rank-1 approximation of A.

(d) Describe $\text{col}(A)$ and $\text{row}(A)$.

Answers

1. $L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$

2. A is a zero matrix since any linear combination of columns of A becomes zero vector.

3(a) $Q = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix}, R = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 1 \end{pmatrix}$

3(b) Column space is consisting of vectors of form $\begin{pmatrix} x \\ y \\ x \end{pmatrix}$. b is not in it.

3(c) From lectures, we know that $x = R^{-1}Q^T b$. First multiplying Q^T to b we have a vector $c = \begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$. Then solving $Rx = c$ we obtain $x = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

4(a) Rank-r version $A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

partial version $A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

4(b) Rank is number of nonzero singular values, 2

4(c) $\sqrt{5} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

4(d) Column space of A is column space of U_1, which is a U of reduced(rank-r) SVD(see lecture slide 10 page 8). So column space of A is vectors of form $\begin{pmatrix} x \\ 0 \\ y \\ 0 \end{pmatrix}$. Similarly rowspace of A is columnspace of $V1$, which has the form $\begin{pmatrix} x \\ 0 \\ y \end{pmatrix}$ (Look at 4(a), there are U_1 and V_1.