— *course overview* —

18.06: Linear Algebra

Prof. Steven Johnson,

MIT Applied Math, Physics

Fall 2018

http://web.mit.edu/18.06

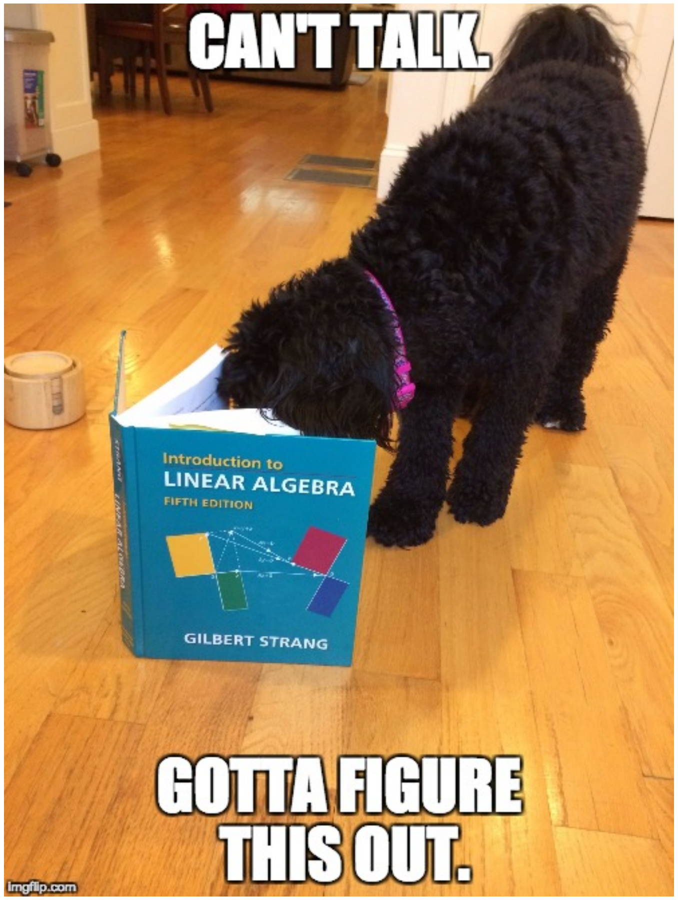
https://github.com/stevengj/1806

Textbook: Strang, *Introduction to Linear Algebra*, 5th edition
+ supplementary notes

Help wanted:

arrive 10 minutes early
and get paid $10 to erase the boards

Apply at MIT math office (2-110).



The textbook

Gil Strang, *Introduction to Linear Algebra*, 5^{th} edition

For a discounted, autographed copy of the textbook, see Professor Strang in 2-245. (Cash or check only.)

Administrative Details

Lectures MWF11, 54-100
Tuesday recitations — use Stellar to switch sections

Weekly psets, due Wednesday 10:55am. Electronic submission through Stellar
— no extensions or makeup, but lowest pset score will be dropped
— pset 1 to be posted shortly

Grading: homework 15%, 3 exams 45% (9/28, 10/26, & 11/30), & final exam 40%

Collaboration policy: talk to anyone you want, read anything you want, but:

- Make an effort on a problem before collaborating.
- Write up your solutions independently (from "blank sheet of paper").
- List your collaborators and external sources (not course materials).

18.06 piazza.com forum: bit.ly/2Neh71U

Syllabus and Calendar

- Significant overlap with Strang's OCW video lectures: these are a useful supplement but not a replacement for attending lecture. Likely topics:
- Exam 1: Friday 9/28. Elimination, LU factorization, nullspaces and other subspaces, bases and dimensions, vector spaces, complexity. (Book: 1–3.5, 11.1)
- Exam 2: Friday 10/26. Orthogonality, projections, least-squares, QR, Gram–Schmidt, orthogonal functions, determinants, infinite dimensional vector spaces (Book: 1–5, 10.5). Concentrating on material since exam 1, but linear algebra is cumulative!
- Exam 3: Friday 11/30. Eigenvectors, determinants, similar matrices, Markov matrices, ODEs, symmetric matrices, definite matrices, matrices from graphs and engineering. (Book: 1–7, 10.1–3.)
- Other topics: defective matrices, SVD and principal-components analysis, sparse matrices and iterative methods, complex matrices, symmetric linear operators on functions.
- Final exam: all of the above.

Before each exam, a definitive list of topics will be announced, along with a review session.

What is 18.06 about?

High school:

3 “linear” equations
(only ± and × constants)
in 3 unknowns

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 &= 2 \\ \boxed{4x_1 + 9x_2 - 3x_3 = 8} \\ -2x_1 - 3x_2 + 7x_3 &= 10 \end{aligned}$$

Method: eliminate unknowns one at a time.

Equivalent matrix problem

Ax = b

Ax is a “linear operation:”

A(x+y) = Ax + Ay

A(3x) = 3Ax

take “dot products” of rows × columns

$$\underbrace{\begin{pmatrix} 2 & 4 & -2 \\ \boxed{4 \quad 9 \quad -3} \\ -2 & -3 & 7 \end{pmatrix}}_{\text{A}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\text{x}} = \underbrace{\begin{pmatrix} 2 \\ \boxed{8} \\ 10 \end{pmatrix}}_{\text{b}}$$

A: matrix of coefficients

x: vector of unknowns

b: vector of right-hand sides

What is 18.06 about?

Linear system of equations,
in matrix form

$$
\underbrace{\begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{pmatrix}}_{A}
\underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x}
=
\underbrace{\begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix}}_{b}
$$

Ax = b

Will we learn faster methods to solve this? No. (Except if A is special.) The standard "Gaussian elimination" (and "LU factorization") matrix methods are just a slightly more organized version of the high-school algebra elimination technique.

Will we get better at doing these calculations by hand? Maybe, but who cares? Nowadays, all important matrix calculations are done by computers.

Will we learn more about the computer algorithms? A little. But mostly the techniques for "serious" numerical linear-algebra are topics for another course (e.g. 18.335).

How do we *think* about linear systems?
(imagine someone gives you a $10^6 \times 10^6$ matrix)

- All the formulas for 2×2 and 3×3 matrices would fit on one piece of paper. They aren't the reason why linear algebra is important (as a class or a field of study).
- Large problems are solved by computers, but must be understood by human beings. (And we need to give computers the right tasks!)
- Understand non-square problems: #equations > #unknowns or vice versa
- Break up matrices into simpler pieces
  - Factorize matrices into products of simpler matrices: $A=LU$ (triangular: Gauss), $A=QR$ (orthogonal/triangular), $A=X\Lambda X^{-1}$ (diagonal: eigenvecs/vals), $A=U\Sigma V^*$ (orthogonal/diagonal: SVD)
  - Submatrices (matrices of matrices).
- Break up vectors into simpler pieces: subspaces and basis choices.
- Algebraic manipulations to turn harder/unfamiliar problems (e.g. minimization or differential equations) into simpler/familiar ones: algebra on whole matrices at once

Don’t expect a lot of “turn the crank” problems
on psets or exams of the form
“solve this system of equations.”

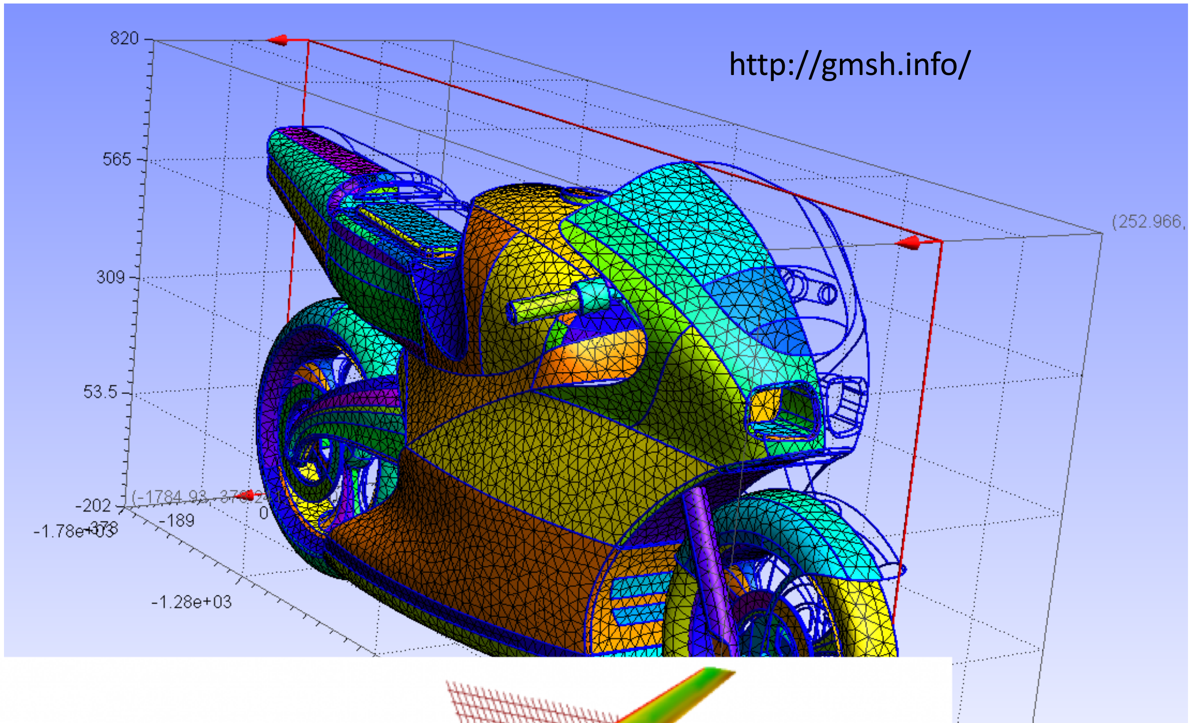
… we will turn it upside-down, give you the answer and ask the question, ask about properties of the solution from partial information, … the general goal is to require you to understand the crank rather than just turn it.

Where do big matrices come from?

Lots of examples in many fields, but here are a couple that are relatively easy to understand…

Engineering & Scientific Modeling

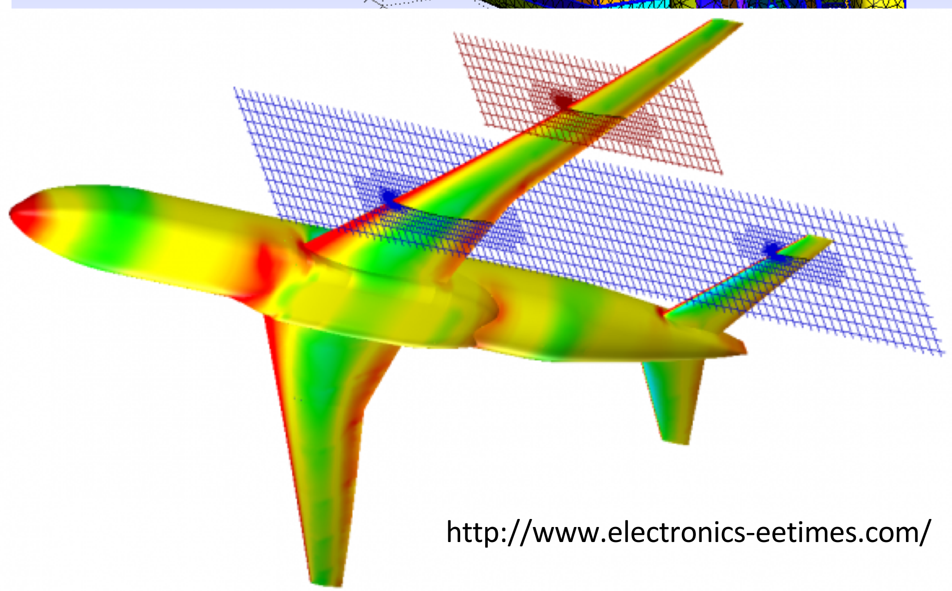
[18.303, 18.330, 6.336, 6.339, …]



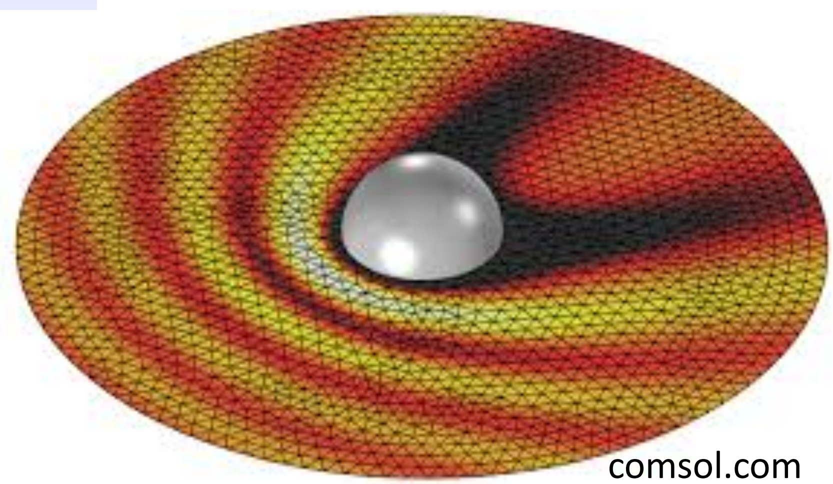
http://gmsh.info/

Unknown functions
(fluid flow, mechanical stress, electromagnetic fields, …)
approximated by values on a discrete mesh/grid

e.g. 100x100x100 grid
= 10^6 unknowns!

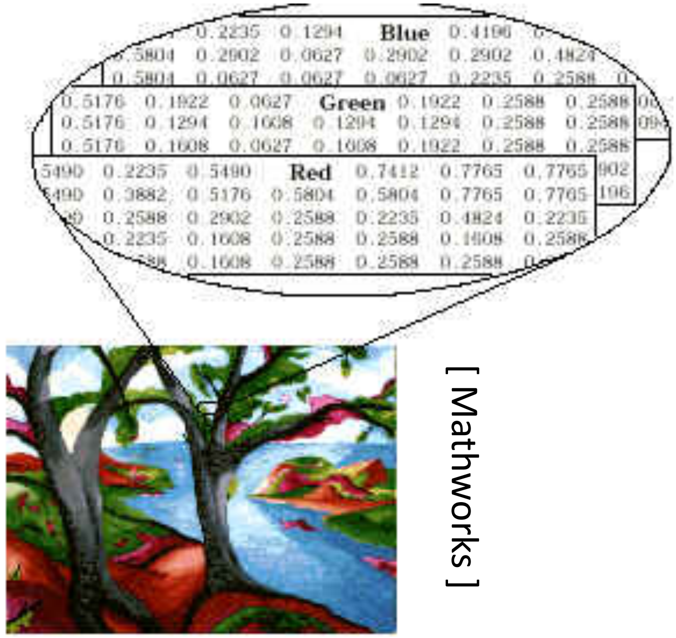


http://www.electronics-eetimes.com/



comsol.com

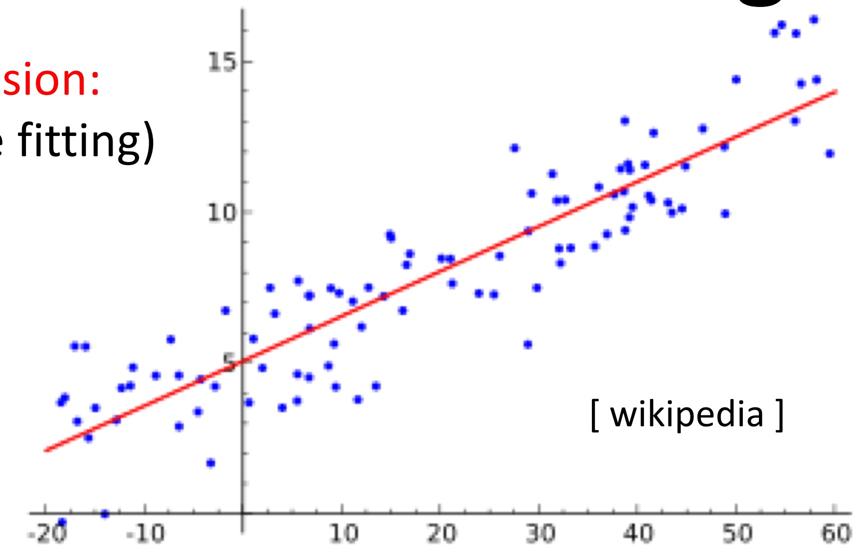
Data analysis
and Machine Learning



[Mathworks]

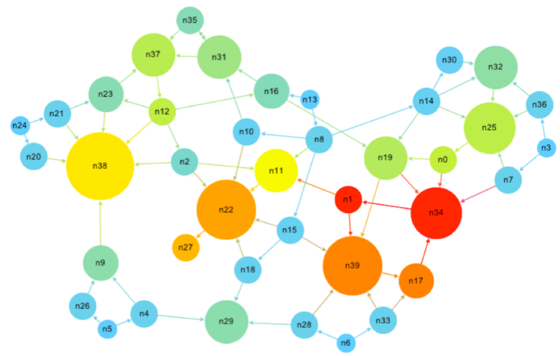
Image processing:
images are just matrices of numbers
(red/green/blue intensity)

Regression:
(curve fitting)



[wikipedia]

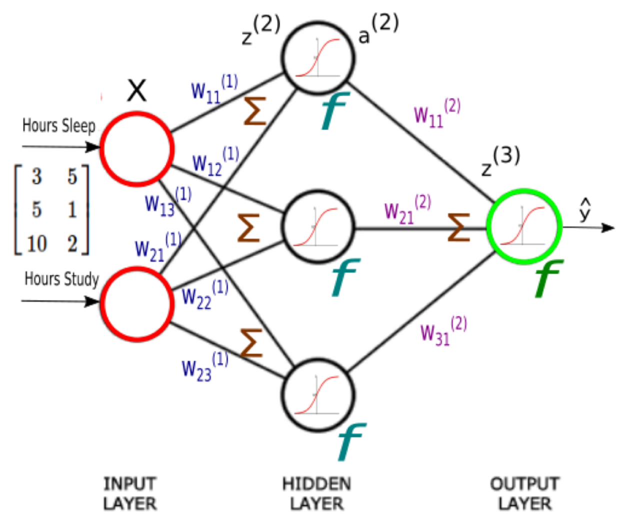
Google "page rank" problem
(also for gene networks etc.)



[computationalculture.net]

Determine the "most important" web pages just from how they link.

matrix = (# web pages) × (# web pages)
(*entry = 1 if they link, 0 otherwise*)



Machine Learning

Not just matrices of numbers

- There are lots of surprising and important generalizations of the ideas in linear algebra.
- Instead of vectors with a finite number of unknowns, similar ideas apply to functions with an infinite number of unknowns.
- Instead of matrices multiplying vectors, we can think about linear operators on functions

Poisson's equation
(e.g. 18.303)

$$\nabla^2 u = f$$

"A"
linear
operator
∇^2

"x"
unknown
function
$u(x,y,z)$

"b"
right-hand side
$f(x,y,z)$

18.06 vs. 18.700

"applied" vs. "pure" math

few proofs vs. formal proofs expected

(deduce patterns from examples, informal arguments)

(definitions to lemmas to theorems … training in proof writing)

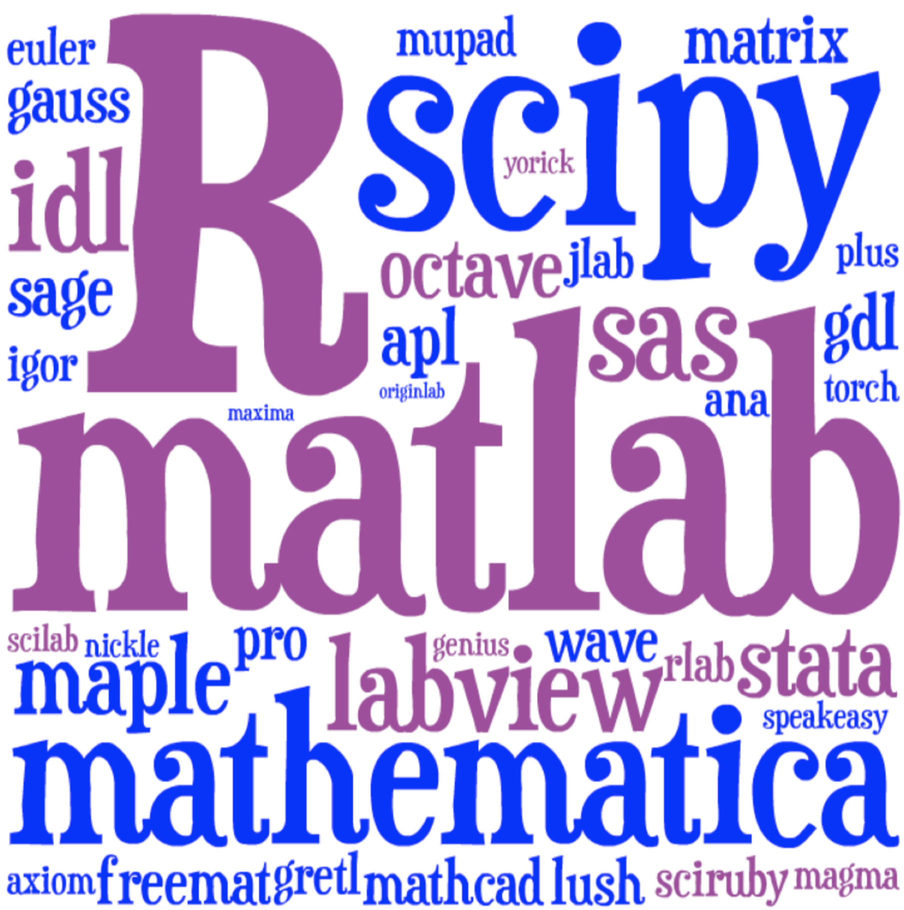
more applications vs. more theorems

more concrete vs. more abstract

some computers vs. only pencil-and-paper

Computer software

Lots of choices:



[image: Viral Shah]

This semester: a relatively new language that scales better to real problems.

julia

julialang.org

No programming required for 18.06, just a "glorified calculator" to turn the crank.

Use it online: log in at juliabox.com

see "Julia" link on Stellar

Optional tutorial: Friday 5pm 32-141

18.06 vs. 18.700

puppies vs. no puppies

“Cookie”
2-year old mini labradoodle