Problem 1. *Suppose that a symmetric* 3×3 *matrix* P *satisfies* $P^2 = P$. *Let*

$$P = Q\Lambda Q^T,$$

where Q *is an orthogonal matrix, and* $\Lambda = diag[1, 0, 1]$. *Show that* P *is the projection to span*$\{q_1, q_3\}$, *where* $Q = [q_1, q_2, q_3]$.

Problem 2. *Suppose that a non-zero symmetric square matrix* P *satisfies* $P^2 = P$. *Then,* P *is*

(1) *i) not ii) possible iii) always a projection matrix.*
(2) *i) not ii) possible iii) always positive definite.*
(3) *i) not ii) possible iii) always positive semi-definite.*
(4) *i) not ii) possible iii) always a Markov matrix.*

Problem 3. *Given a unit vector* x, *the matrix* $M = xx^T$ *is*

(1) *i) not ii) possible iii) always symmetric.*
(2) *i) not ii) possible iii) always a projection matrix.*
(3) *i) not ii) possible iii) always positive definite.*
(4) *i) not ii) possible iii) always positive semi-definite.*
(5) *i) not ii) possible iii) always has rank* 1.

In addition, if M *is a Markov matrix, what is* x*?*

Problem 4. *Suppose that a symmetric* 4 *matrix* A *has eigenvalues* $-3, 0, 1, 3$. *Find the dimension of the null space of* $A^4 - 10A^2 + 9I = 0$.

Problem 5. *Suppose that a positive* 3×3 *Markov matrix* M *has a steady-state eigenvector* $v = (1\ 2\ 3)^T$*. Find the limit*

$$\lim_{n\to\infty} M^n \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$$

Problem 6. *Find the eigenvalues of the matrix* M*.*

(1) *Given a vector* v*,* $M = -3I + 2vv^T$*.*
(2) *Suppose that a symmetric Markov matrix* S *has eigenvalues* $\lambda_1, \cdots, \lambda_n$*.* $M = S - 5I + uu^T$*, where* $u = (1\ \cdots\ 1)^T$*.*