

Projection

Consider the projection of b onto the column space of A .

1. If A is a matrix with linearly independent columns, the projection is _____.
2. If $A = QR$, then the projection is _____.
3. If $A = U_1 \Sigma_r V_1^T$, then the projection is _____.

Determinant

1. Formula of the determinate of a 2×2 matrix.

Geometric meaning.

2. Formula of the determinate of a 3×3 matrix.

Geometric meaning:

3. Axiomatic Approach:

4. Properties:

- $|AB| =$
- $|A^T| =$
- $|A^{-1}| =$

Problems

1. What is the projection of $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ on

(a) the column space of $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$?

(b) the column space of $B = \begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{pmatrix}$?

2. Suppose the determinate of $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is a .

(a) What is the determinate of $2A$?

(b) What is the determinate of $B = \begin{pmatrix} a_{21} & a_{22} & a_{23} + a_{21} \\ a_{11} & a_{12} & a_{13} + a_{11} \\ a_{31} & a_{32} & a_{33} + a_{31} \end{pmatrix}$?

(c) Given a random 3×3 matrix D , is the $|A + D| = |A| + |D|$?

(d) Suppose further that $a_{11} = a_{12} = a_{13}$. What is the determinate of

$$C = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + 1 & a_{22} + 1 & a_{23} + 1 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}?$$