Problem 1. *Let P denote the set of square matrices M such that $y^T My > 0$ for $y \neq 0$.*

1. *Show that if $M \in P$ then $\frac{1}{2}(M + M^T) \in P$.*
2. *Provide an example $M \in P$ such that the eigenvalues of M are different from those of $\frac{1}{2}(M + M^T)$.*
3. *Find a matrix $M \in P$ having complex eigenvalues.*
4. *Find a matrix $M \in P$ such that $M^2 \notin P$.*

Problem 2. *A Markov matrix is a square matrix such that all entries are non-negative and the sum of each column vector entries is equal to 1. In particular, if all entries are positive, we call it a positive Markov matrix.*

1. *A Markov matrix has an eigenvalue 1.*
2. *Show that the product of two Markov matrices is a Markov matrix.*
3. *Is a positive Markov matrix invertible?*
4. *Is a positive Markov matrix is positive definite?*
5. *Show that a 2×2 Markov matrix is diagonalizable.*
6. *Is a positive Markov matrix diagonalizable?*

Problem 3. *Suppose that $f : \mathbb{R}^n \to \mathbb{R}$ has the positive definite Hessian $D^2 f(x)$ for all $x \in \mathbb{R}^n$.*

(1) *Show that given $x, v \in \mathbb{R}$ with $v \neq 0$, the function $g(t) = f(x + tv)$ satisfies $g'' > 0$.*
(2) *Show that f has at most one critical point.*