1. Linearly independent and span.

   - The vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_n}$ are **linearly independent** if ____

   - The vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_n}$ **span** the vector space S if ____.

   - The vectors $\vec{v_1}, \vec{v_2}, ..., \vec{v_n}$ are a **basis for** S if ____.

   - The **dimension of a space** S is ____.

2. Solve $Ax = b$. Let A be a rank r $m \times n$ matrix

   - $Ax = b$ is solvable if

   - Suppose $Ax = b$ is solvable. Then
     - if $r < n$, $Ax = b$ has ______________ solutions.
     - if $r = n$, $Ax = b$ has ______________ solutions.

Problems

1. (a) Find a basis for symmetric 2×2 matrices. What is the dimension of symmetric 2×2 matrices? Also, do this for skew-symmetric 2×2 matrices.

   (b) Let $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}$ be a basis for symmetric 2×2 matrices. Let $\vec{w_1}, \vec{w_2}, ..., \vec{w_n}$ be a basis for skew-symmetric 2×2 matrices. Are $\vec{v_1}, \vec{v_2}, ..., \vec{v_m}, \vec{w_1}, \vec{w_2}, ..., \vec{w_n}$ linearly independent? What subspace of 2×2 matrices do they span?

2. A set F consists of functions $f(x) = ae^x + be^{-x}$, where $a, b \in \mathbb{R}$.

   (a) Is F a vector space? Provide a basis of F.

   (b) Define a map $V : F \to \mathbb{R}^2$ that sends $f(x) = ae^x + be^{-x}$ to $V(f) = \begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2$. What is $V(f')$ for $f \in F$? Find a matrix A satisfying $AV(f) = V(f')$ for all $f \in F$.

   (c) Show that $A^2 = I_2$ and thus $\frac{d^2}{dx^2} f(x) = f(x)$.

3. (a) Find a basis for the space of all the degree ≤ 3 polynomials. $V = \{f(x) = a + bx + cx^2 + dx^3 | a, b, c, d \in \mathbb{R}\}$.

   (b) Find a basis for the subspace W of V consisting all $f \in V$ such that $f(1) = 0$.

   (c) Find a basis for the subspace U of V consisting all $f \in V$ such that $f(-1) = f(1)$.