**Problem 1.** *Let* $A$ *be a* $m \times n$ *matrix. For* $\lambda > 0$, *what are the ranks of* (a) $A^T A + \lambda I$ and (b) $AA^T + \lambda I$. In addition, (c) express trace($A^T A$) in terms of the singular values of $A$.

**Problem 2.** *Suppose that* $A$ *has the inverse matrix* $A^{-1}$. *Then, the first column of* $A^{-1}$ *is orthogonal to the space spanned by which rows of* $A$*?*

**Problem 3.** *Suppose that* $A$ *and* $B$ *are* $m \times n$ *and* $m \times l$ *matrices, respectively. If* $A^T B = 0$, *show that rank*$(A)$ + *rank*$(B) \leq m$.

**Problem 4.** (A) We define a function $f : \mathbb{R}^n \to \mathbb{R}^m$ by $f(x) = Ax$ where $A$ is a $m \times n$ matrix. Show that $df = Adx$.
(B) We consider a differentiable function $g : \mathbb{R}^m \to \mathbb{R}^l$, and define a differentiable function $h : \mathbb{R}^n \to \mathbb{R}^l$ by $h(x) = g(f(x))$. Show that $dh = JAdx$ where $J_{ij} = \partial_j g_i(Ax)$.

**Problem 5.** *Show that the matrix of any connected graph with $n$-nodes and $m$-edges (in pset 6) has rank $n-1$. Also, show that if the graph is planar then we can find $m-n+1$ independent loop vectors.*