Rec 7 solu

Tuesday, April 7, 2020 9:45 AM

Projection

Consider the projection of b onto the column space of A.

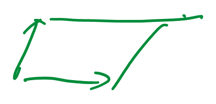
1. If A is a matrix with linearly independent columns, the projection is $A(A^TA)^{-1}A^Tb$.
2. If $A = QR$, then the projection is QQ^Tb.
3. If $A = U_1\Sigma_r V_1^T$, then the projection is $U_1U_1^Tb$.

Determinant

1. Formula of the determinate of a 2×2 matrix.

$$\det\begin{pmatrix} a & b \\ c & d\end{pmatrix} = ad - bc$$

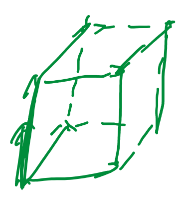
Geometric meaning.

Area. 

2. Formula of the determinate of a 3×3 matrix.

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{pmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33}\end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33}\end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32}\end{vmatrix}$$

Geometric meaning:

Volumn 

3. Axiomatic Approach:

- $\det(I_n) = 1$
- $\det\left(\begin{smallmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1\end{smallmatrix}\right) A = -\det A$
- $\det\left(\begin{smallmatrix} 1 & & \\ & 1 & \\ & k & 1 & \\ & & & 1\end{smallmatrix}\right) A = \det A$

4. Properties:

- $|AB| = |A|\,|B|$
- $|A^T| = |A|$
- $|A^{-1}| = |A|^{-1}$

Problems

1. What is the projection of $b = \begin{pmatrix}1\\2\\3\end{pmatrix}$ on

(a) the column space of $A = \begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}$?

$$A(A^TA)^{-1}A^Tb$$

$$= \begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}\left(\begin{pmatrix}1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}\right)^{-1}\begin{pmatrix}1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}\begin{pmatrix}2&\\&1\end{pmatrix}^{-1}\begin{pmatrix}4\\2\end{pmatrix}$$

$$= \begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}\begin{pmatrix}\frac12&\\&1\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}2\\2\\2\end{pmatrix}$$

(b) the column space of $B = \begin{pmatrix}1&2\\1&2\\1&2\end{pmatrix}$?

$$\frac{\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\3\end{pmatrix}}{\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{6}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\2\\2\end{pmatrix}$$

2. Suppose the determinate of $A = \begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$ is a.

(a) What is the determinate of $2A$?

$$\det(2A) = 2^3\cdot a = 8a$$

(b) What is the determinate of $B = \begin{pmatrix}a_{21}&a_{22}&a_{23}+a_{21}\\a_{11}&a_{12}&a_{13}+a_{11}\\a_{31}&a_{32}&a_{33}+a_{31}\end{pmatrix}$?

$$\det B = \det\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}A\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}$$

$$= \det\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}\det A\,\det\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}$$

$$= -\det A$$

(c) Given a random 3×3 matrix D, ~~how is the $|A+D|$ relate to $|D|$~~?

is $|A+D| = |A| + |D|$?

No

$$D = \begin{pmatrix}-a_{11}&-a_{12}&-a_{13}\\-a_{11}&-a_{12}&-a_{13}\\-a_{11}&-a_{12}&-a_{13}\end{pmatrix} \quad \det D = 0$$

$$\det(A+D) = 0 \neq \det A + \det D$$

if $a \neq 0$.

(d) Suppose further that $a_{11} = a_{12} = a_{13}$. What is the determinate of

$$C = \begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}+1&a_{22}+1&a_{23}+1\\a_{31}&a_{32}&a_{33}\end{pmatrix}?$$

$\det C = \det A + \det D$, $\quad D = \begin{pmatrix}a_{11}&a_{12}&a_{13}\\1&1&1\\a_{31}&a_{32}&a_{33}\end{pmatrix}$

Since $a_{11} = a_{12} = a_{13}$ $\quad \det D = 0$

$\det C = \det A = a$.