Recitation 2. February 26

Focus: QR decomposition, SVD, least squares.

1. Say that a square matrix A is factored as a product $A = BC^{-1}$. Perform the same column operation on both B and C, for example add the first column to the second: $B \mapsto B'$, $C \mapsto C'$. Show that this operation does not change the result, that is $A = BC^{-1} = B'(C')^{-1}$.

Solution:

2. *Finding a QR decomposition.* Write the following matrix A as a product $A = QR$ for some orthogonal Q and upper-triangular R:

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}.$$

Solution:

3. *Finding an SVD.* Consider matrix A:

$$A = \begin{pmatrix} 1 & 3 & -1 \\ -2 & -6 & 2 \\ 3 & 9 & -3 \end{pmatrix}.$$

   a) Describe its column space.
   b) Express A as an outer product of two vectors. Is this decomposition unique?
   c) Find a compact and a full form SVD for this matrix.

Solution:

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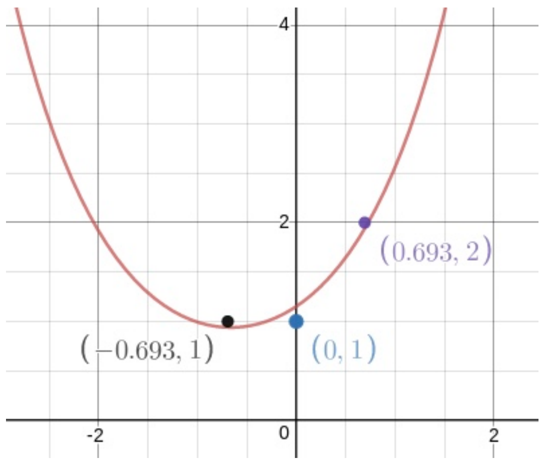
4. *Least squares approximation.* Consider the set of functions of the form $f(x) = ae^x + be^{-x}$, where a and b vary over real numbers. In this space of functions, use the least squares algorithm to approximate the unknown function U that takes the following values:

$$U\left(\ln \frac{1}{2}\right) = 1,\ U(0) = 1,\ U(\ln 2) = 2.$$

You will get the following picture:



a) Write the sum $S(a, b)$ of squared errors.

b) Write the condition of finding local minimum using partial derivatives with respect to a and b.

c) Write the condition above as a matrix equation and solve this equation.

Solution: