**Problem 1.** *Find the eigenvalues and singular values of*

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}.$$

*Moreover, describe the null space and the eigenvectors whose the eigenvalue is* $0$*. In addition, express* $A$ *by* $X\Lambda X^{-1}$ *in several ways, where* $\Lambda$ *is diagonal.*

**Problem 2.** *Provide examples satisfying*

(1) *diagonlizable and invertible.*
(2) *diagonlizable and non-invertible.*
(3) *non-diagonlizable and invertible.*
(4) *non-diagonlizable and non-invertible.*

**Problem 3.** *Let $\lambda_1, \cdots, lambda_k$ be eigenvalues of diagonalizable matrix $A$. Is there a relation between the ranks of $A - \lambda_i I$?*

**Problem 4.** *A sequence $a_n$ with $a_0 = -1$, $a_1 = 4$ has the recurrence relation $a_{n+2} = a_{n+1} + 2a_n$. What is $a_n$?*