18.06 Recitation Feb 25

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LU factorization

- A square matrix $A = LU$ where L is a _____ matrix and U is a _____ matrix.
- example: Given $A_{2\times 2} = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, and $B_{2\times 2}$, then AB is to add λ times the first row of B to the second row.

  1. Find a LU factorization of $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$.

QR factorization

- A $m \times n$ matrix $A = QR$ where Q is a _____ matrix and R is a _____ matrix.

  2. (a) Write down a QR decomposition for

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(b) Let $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Can you solve $Ax = b$?

(c) Find a vector $\hat{x}$ that minimizes $||Ax - b||$.

SVD

- **SVD:** A $m \times n$ matrix A has the SVD as $A = U\Sigma V^T$. What are U, V, Σ and what are their matrix size?

3. Suppose that a 3×3 matrix A has the SVD as

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \Sigma = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}, V = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

(a) What is the rank of A?

(b) Write the column space of A as all the linear combinations of two column vectors of U, Σ or V.

(c) Let $B = U\Sigma$. How are the columns of B related to columns of A?

(d) What is the norm of the third column of A?