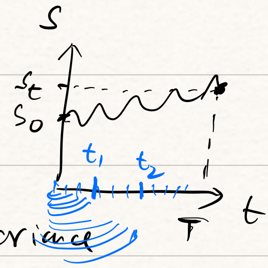
$$dS = \sigma dW, \quad S(0) = S_0$$



$$\underline{A(T)} = \frac{1}{T}\int_0^T S(t)\,dt, \quad S(T)$$

larger variance

$$S(T) = \frac{1}{T}\int_0^T S(T)\,dt$$

normal $\mathbb{E}S = 0$.

$$\mathbb{E}A = \mathbb{E}\frac{1}{T}\int_0^T S(t)\,dt$$

$$Var\,S(t_1) < Var\,S(t_2)$$

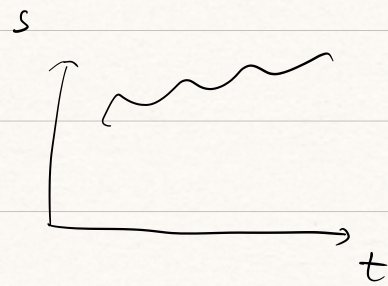
$$= \frac{1}{T}\int_0^T \underline{\mathbb{E}S(t)}\,dt = 0$$

$$dS = \sigma dW$$

$$S = \int dS \qquad dS = \sigma dW \qquad Var(dW) = dt$$

$$Var(dS) = \sigma^2 dt \qquad Var(\sigma dW) = \sigma^2 dt$$

$$Var(S) = \int_0^T \sigma^2 dt = \sigma^2 T$$



$$D(T) = \int_0^T q\,S(t)\,dt \qquad q = 2\%$$

1 day

$$S(t_1),\ S(t_2),\ S(t_3) \text{ --- } \qquad qS\cdot\frac{T}{365}$$

dt

$Y(t) = tS(t)$ $\qquad\qquad A = \frac{1}{T}\int_0^T S(t)\,dt$

$dY = Y_t\,dt + Y_x\,dS + \frac{1}{2}Y_{xx}\,dS\,dS$ $\qquad Y(t,x) = tx$

$= S\,dt + t\,dS$ $\qquad Y_t = x$

$\qquad Y_x = t$

$Y(T) - Y(0) = \int_0^T t\,dS + \int_0^T S\,dt$ $\qquad Y_{xx} = 0.$

$= \int_0^T t\cdot\sigma\,dW + \int_0^T S\,dt$ $\qquad dY(t,S) = tS.$

$$\underbrace{TS(T) - 0}_{N\sim(0,\sigma^2T^3)} = \underbrace{\sigma\int_0^T t\,dW}_{N\sim(0,\frac{1}{3}\sigma^2T^3)} + \underbrace{\int_0^T S(t)\,dt}_{N\sim(0,\ )}$$

$ES = 0$

$\mathrm{Var}\,S(T) = \sigma^2 T$

$\mathrm{Var}\int_0^T t\,dW$

$\mathrm{Var}(TS(T)) = \sigma^2T^3$ $\qquad \mathrm{Var}\,dW = dt$

$= \int_0^T \mathrm{Var}(t\,dW)$

$= \int_0^T t^2\,dt$

$= \frac{1}{3}T^3$

$\sigma\int_0^T \cancel{t}\,dW$

$dS = \sigma S\,dW.$ S log-normal distributed

$A = e^{N_1},\ B = e^{N_2}$ $\qquad N_1 \sim N(0,\sigma_1^2)$ e

$\qquad N_2 \sim N(0,\sigma_2^2)$

$A+B \quad A-B$ $\qquad$ B-S-M; $\quad dS = \alpha S\,dt + \sigma S\,dW.$

$AB = e^{N_1+N_2}$

$$\left\{\begin{aligned} \mathbb{E}(A+B) &= \mathbb{E}(A) + \mathbb{E}(B) \\ Var(A+B) &= Var(A) + Var(B) + 2\,cov(A,B) \end{aligned}\right.$$

$$cov(A,B) = \mathbb{E}\Big((X-\mathbb{E}X)(Y-\mathbb{E}Y)\Big)$$