Mathematical Blossoms

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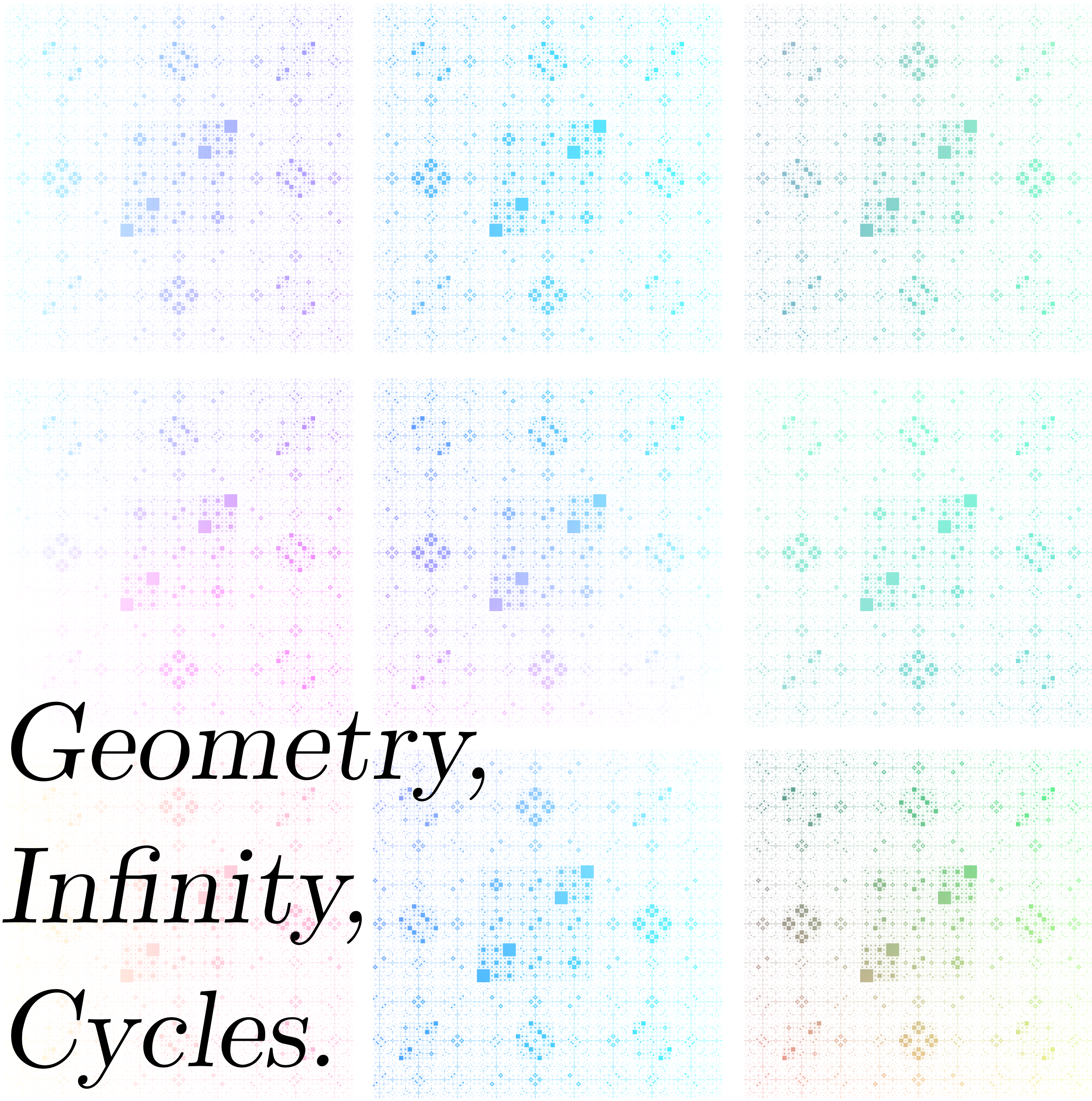


*Geometry,
Infinity,
Cycles.*

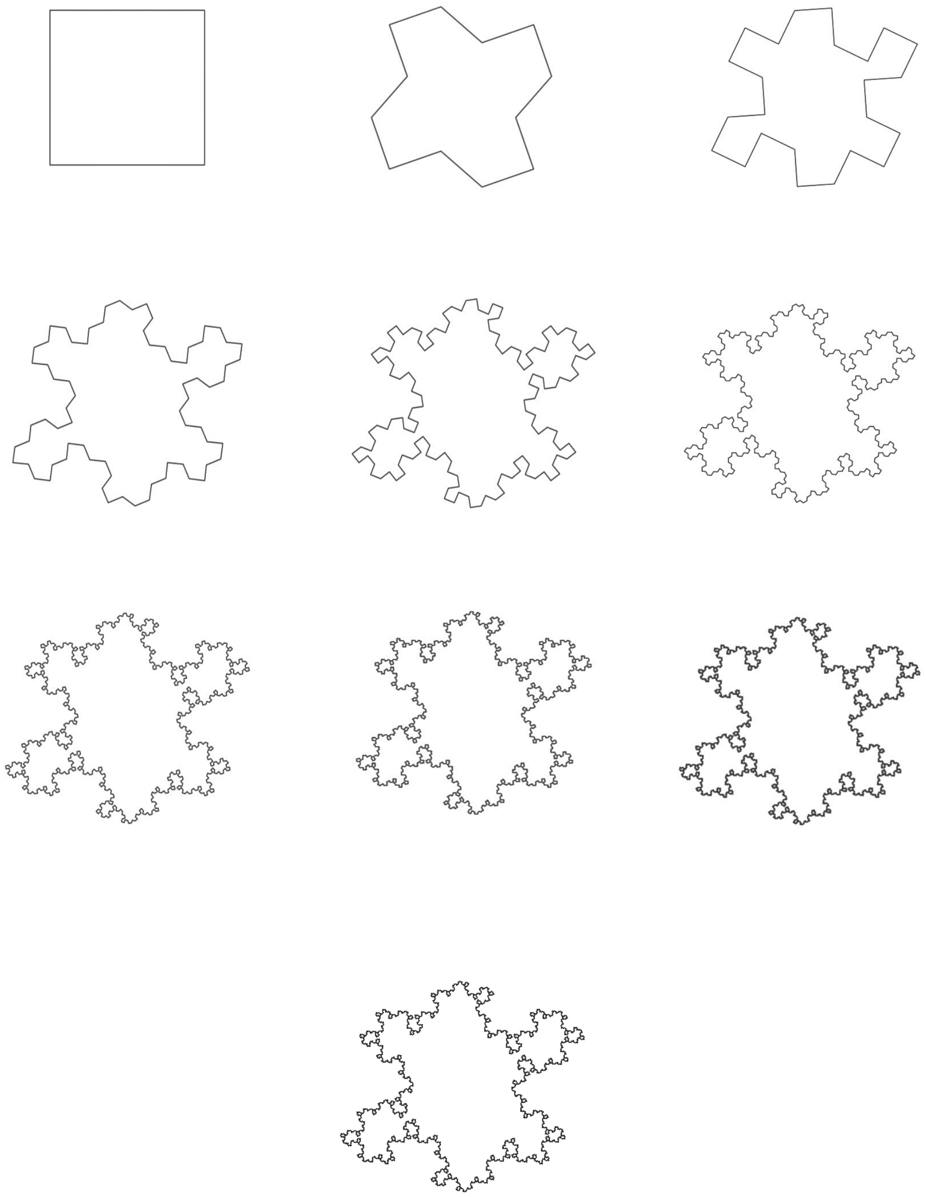
Fractal Butterfly

$$f1 : x' = \frac{y}{2.05}, \quad y' = \left| \frac{\text{width}}{1.5} - x + \frac{y}{1.45 \cdot j \cdot \left(0.5 - \frac{y}{\text{width}}\right) \cdot 800} \right|$$

$$f2 : x' = \frac{\left|0.5 - \frac{j}{\text{ifs}}\right| \cdot 2.0 \cdot x}{1 + j \cdot \left(1 + \frac{y}{10}\right)}, \quad y' = \frac{\frac{\text{height}}{2} - \frac{y \cdot 2}{j} + x}{1.25} + j \cdot \left(20 - \frac{x}{10}\right)$$

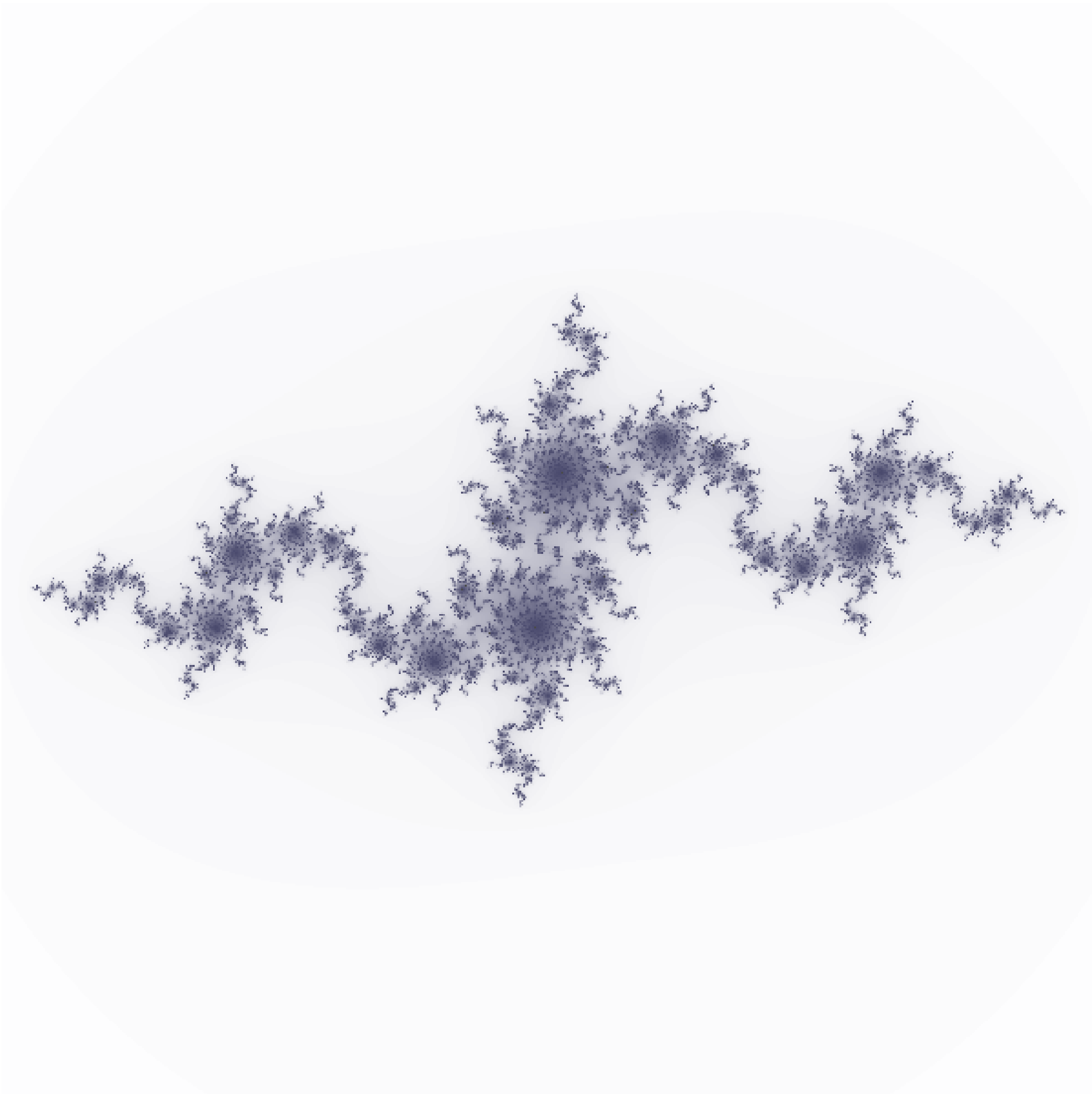
$$f3 : x' = \frac{|\text{width} \cdot 0.75 - x \cdot j|}{8 \cdot \frac{j}{4} + \frac{x}{\text{width}/100}}, \quad y' = \frac{y}{2.25}$$

i is the index in the inner loop, j is the index in the outer loop



Dragon Curve

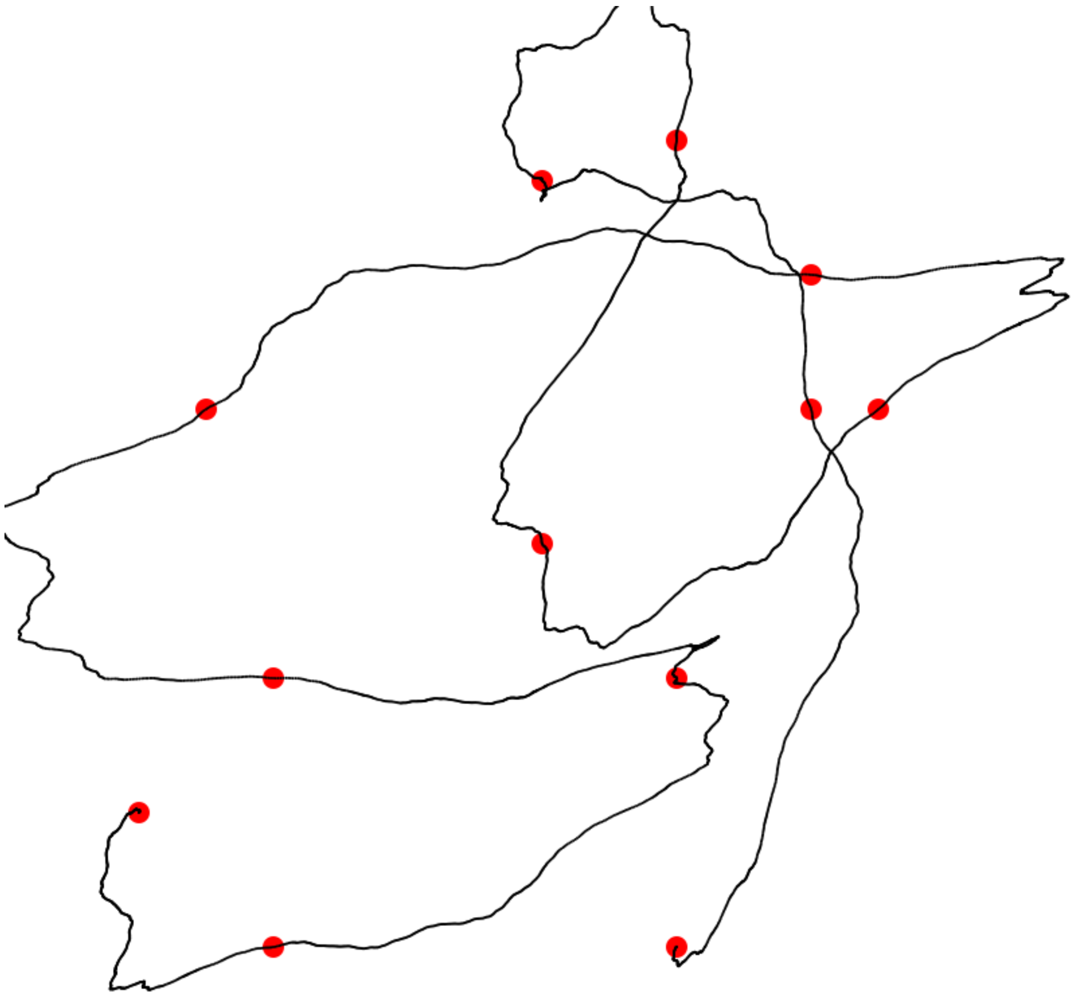
$$f_1(x,y) = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix},\ f_2(x,y) = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}$$



Mandelbrot & Julia Set

$$M = \left\{ c \in \mathbb{C} : \lim_{n\to\infty} |z_n| \leq 2, \text{ with } z_0 = 0 \text{ and } z_{n+1} = z_n^2 + c \right\}$$
$$J_c = \left\{ z_0 \in \mathbb{C} : \lim_{n\to\infty} |z_n| \leq 2, \text{ with } z_{n+1} = z_n^2 + c \right\}$$

*The Mandelbrot Set and Julia Set are fractals generated by iteratively applying a function to points in the complex plane and examining whether these points escape to infinity.



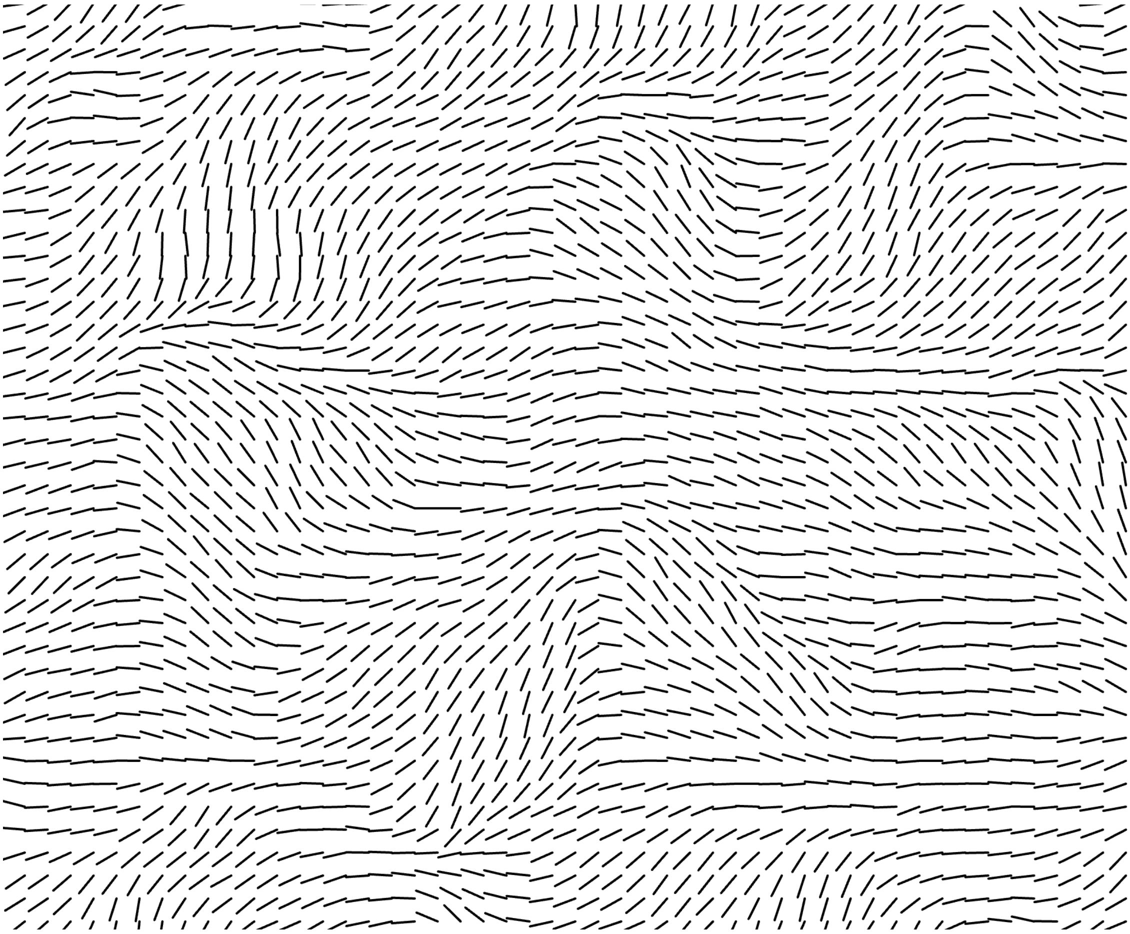
Fractal Interpolation

Given a set of initial points (x_i, y_i) for $i = 0, 1, \ldots, n$, fractal interpolation over N iterations is defined by

$$x_{\text{new}}^{(N)} = \sum_{i=1}^{n} w_i \cdot x_i^{(N-1)}$$

$$y_{\text{new}}^{(N)} = \sum_{i=1}^{n} w_i \cdot y_i^{(N-1)}.$$

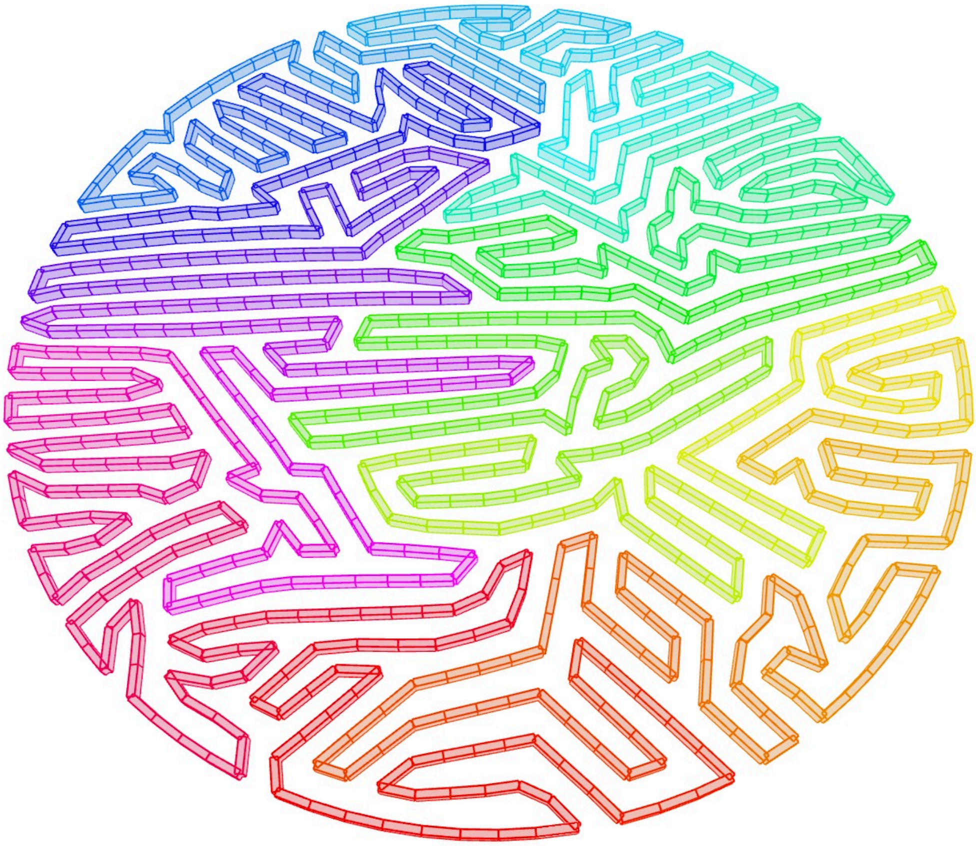
Fractal interpolation creates complex, self-similar curves or surfaces by recursively applying transformations and generating structures with infinite detail and patterns at every scale.



Perlin Noise Vector Field

$$
\begin{aligned}
\theta(x, y, z) &= \mathrm{PerlinNoise}(x, y, z) \times 2\pi \\
\vec{F}(x, y) &= \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \\
\nabla \cdot \vec{F} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \\
\nabla \times \vec{F} &= \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}
\end{aligned}
$$

Perlin noise is a gradient-based, smooth, pseudo-random noise function that generates natural-looking patterns and textures.

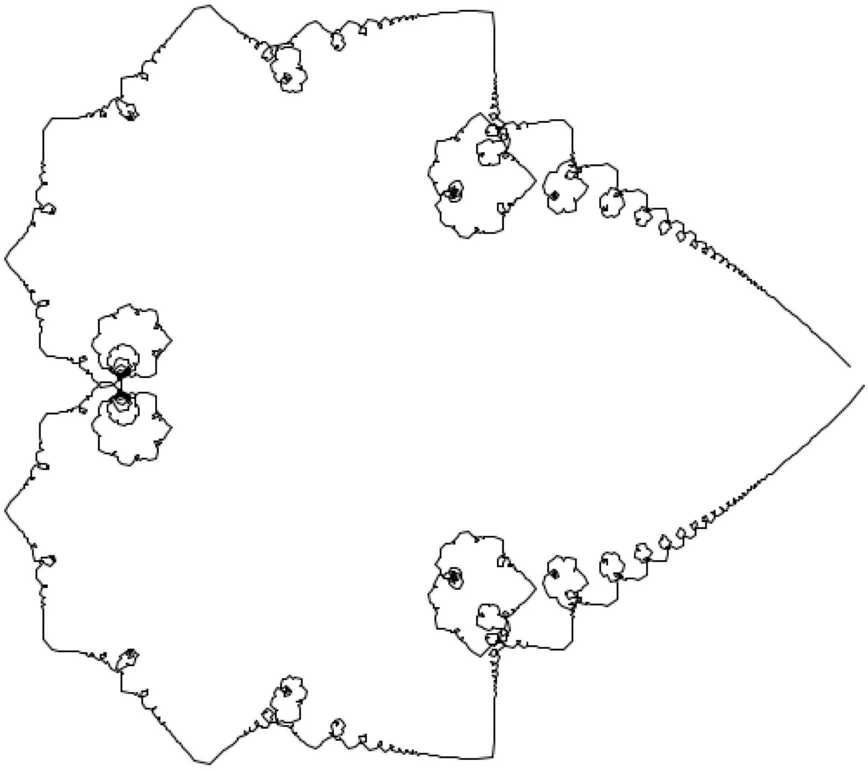


Differential Growth

The Lennard-Jones potential $V(r)$ describes the interaction between two particles based on their separation distance r.

$$V(r) = 4\epsilon \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} \right]$$
$$F(r) = -\frac{dV(r)}{dr} = 24\epsilon \left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7} \right] \approx k\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

where ϵ is the depth of the potential well, and σ is the distance at which $V(r) = 0$.



Power Series Fractal

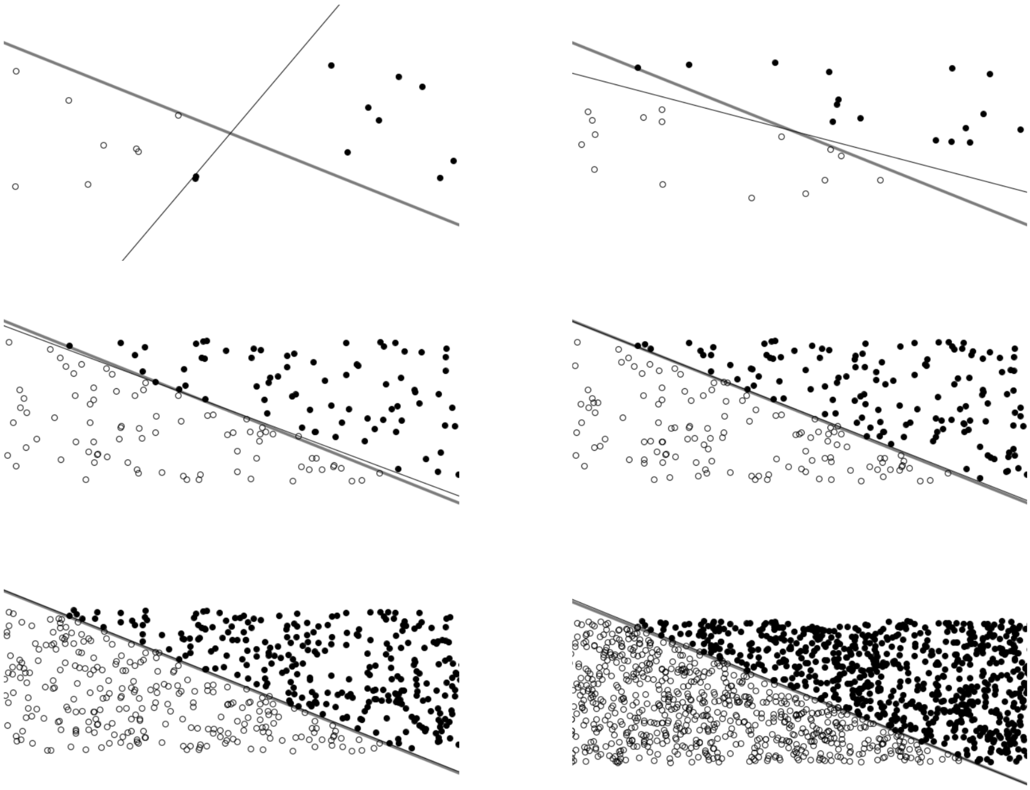
$$f(z) = \sum_{n=0}^{\infty} c_n z^n$$

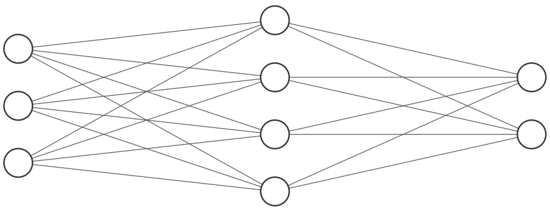
$$z^n = r^n \left(\cos(n \cdot \theta) + i \sin(n \cdot \theta)\right)$$

$$r = |z| = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$a_{\text{accum}} = \sum_{k=1}^{N} \frac{\mathrm{Re}(z^k)}{k^2}, \quad b_{\text{accum}} = \sum_{k=1}^{N} \frac{\mathrm{Im}(z^k)}{k^2}$$

$$|z| \le R \Rightarrow \text{convergent}; \quad |z| > R \Rightarrow \text{divergent}$$



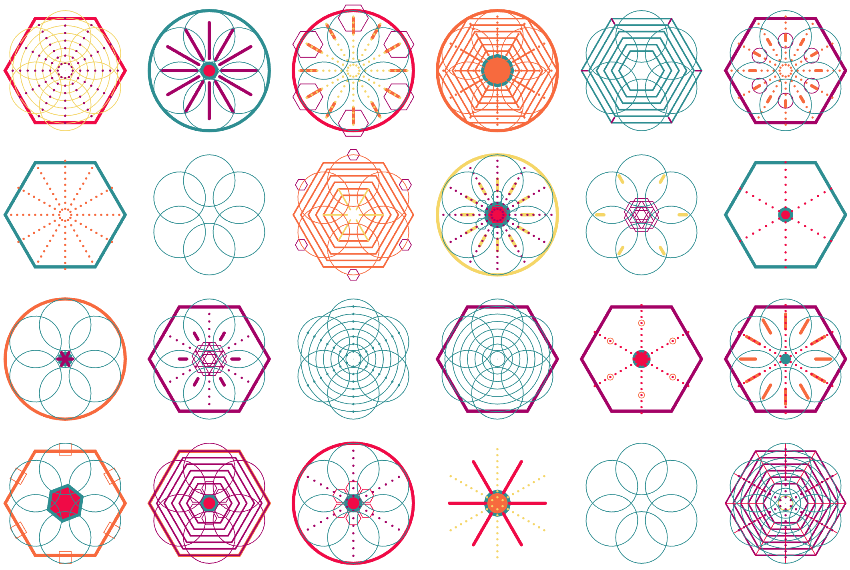


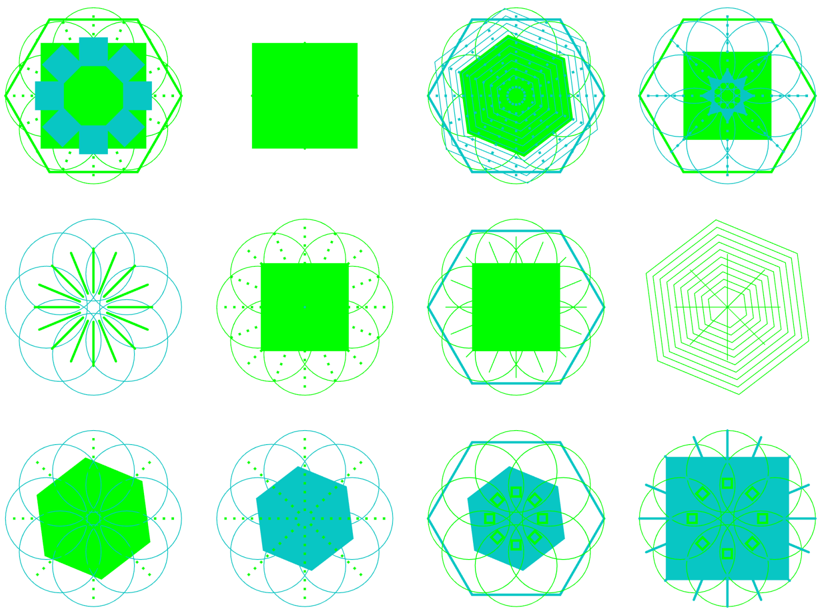
Two-dimensional Neural Network

$$z = \sum_{i=1}^{n} w_i x_i + b$$

$$f(z) = \frac{1}{1 + e^{-z}}$$

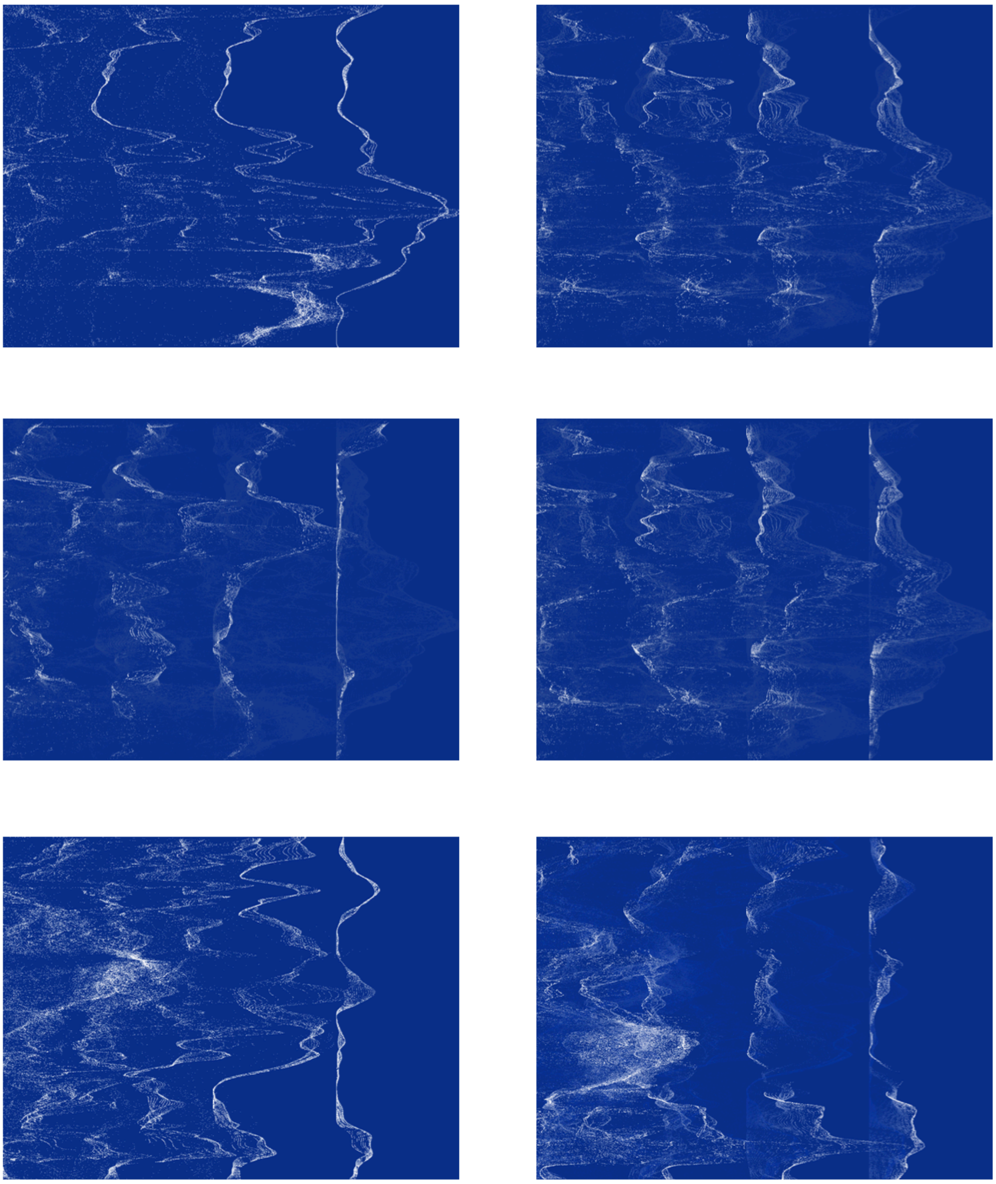
$$L = \frac{1}{m} \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$$





Crystals

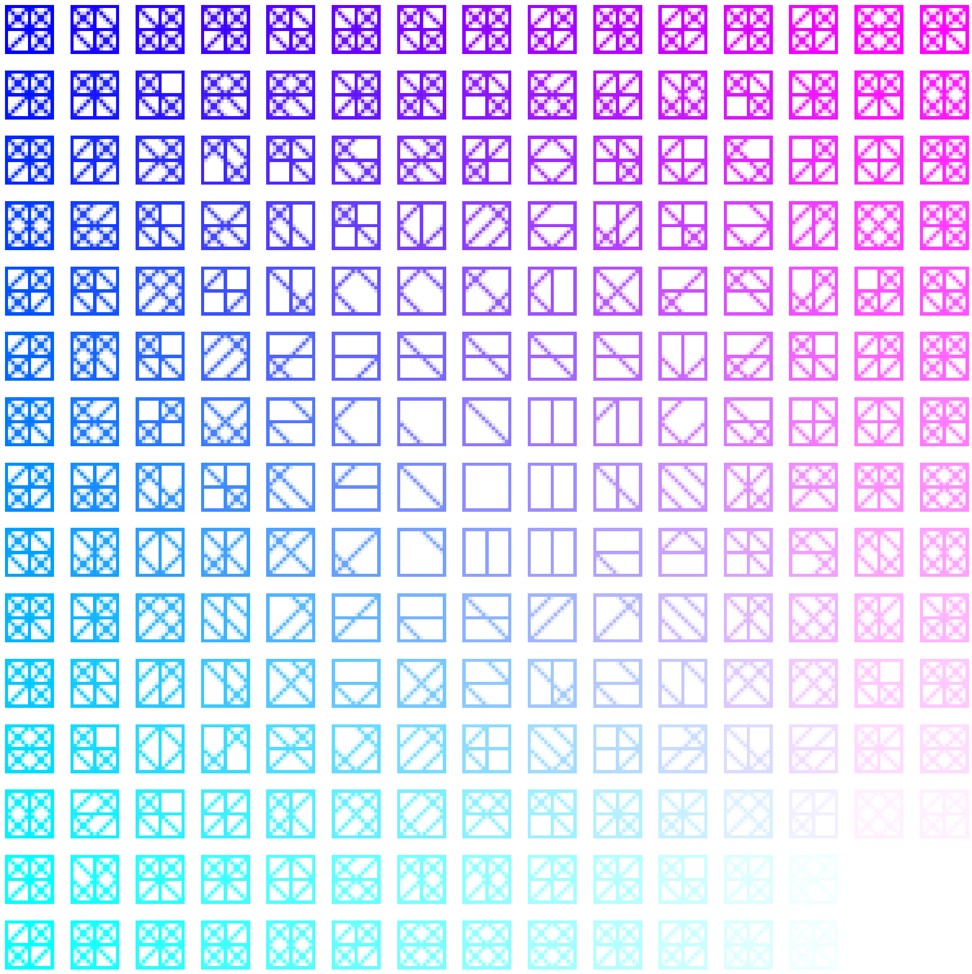
$$s_x = x + r\cos\left(\frac{2\pi k}{n}\right)$$
$$s_y = y + r\sin\left(\frac{2\pi k}{n}\right)$$



Frothy Waves

$$\theta = \text{noise}(x \cdot k_x, y \cdot k_y, t) \cdot 2\pi$$
$$F(x, y) = (\cos(\theta(x, y)), \sin(\theta(x, y)))$$

This particle system simulates natural fluid motion by combining a smooth noise-driven flow field, trigonometric vector updates, and spatial wrapping.



Tiling and Tessellation

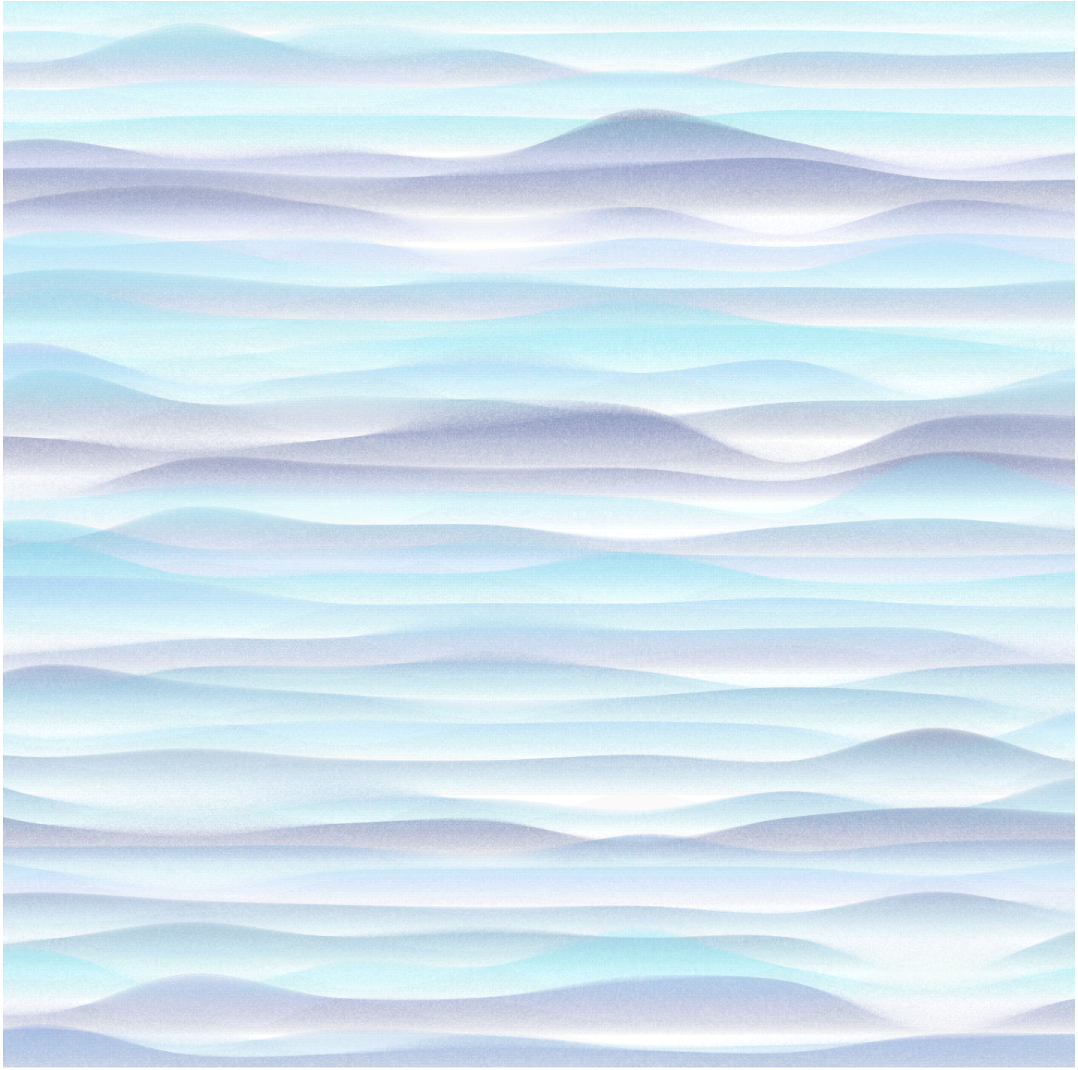
Tessellations are patterns created by repeating shapes to cover a plane without gaps or overlaps by translations, rotations, and glide reflections.

$$
\begin{aligned}
T(x, y) &= (x + a, y + b) \\
R(x, y) &= (x \cos\theta - y \sin\theta, x \sin\theta + y \cos\theta) \\
M(x, y) &= (-x, y) \\
G(x, y) &= T(M(x, y))
\end{aligned}
$$

Dihedral Group

The dihedral group D_n represents the symmetries of a regular n-sided polygon, including n rotations and n reflections.

$$D_4 = \{e, R_{90^\circ}, R_{180^\circ}, R_{270^\circ}, M_x, M_y, M_{\text{diag1}}, M_{\text{diag2}}\}$$
$$(A \circ B) \circ C = A \circ (B \circ C)$$
$$e \circ A = A \circ e = A$$
$$A \circ A^{-1} = e$$



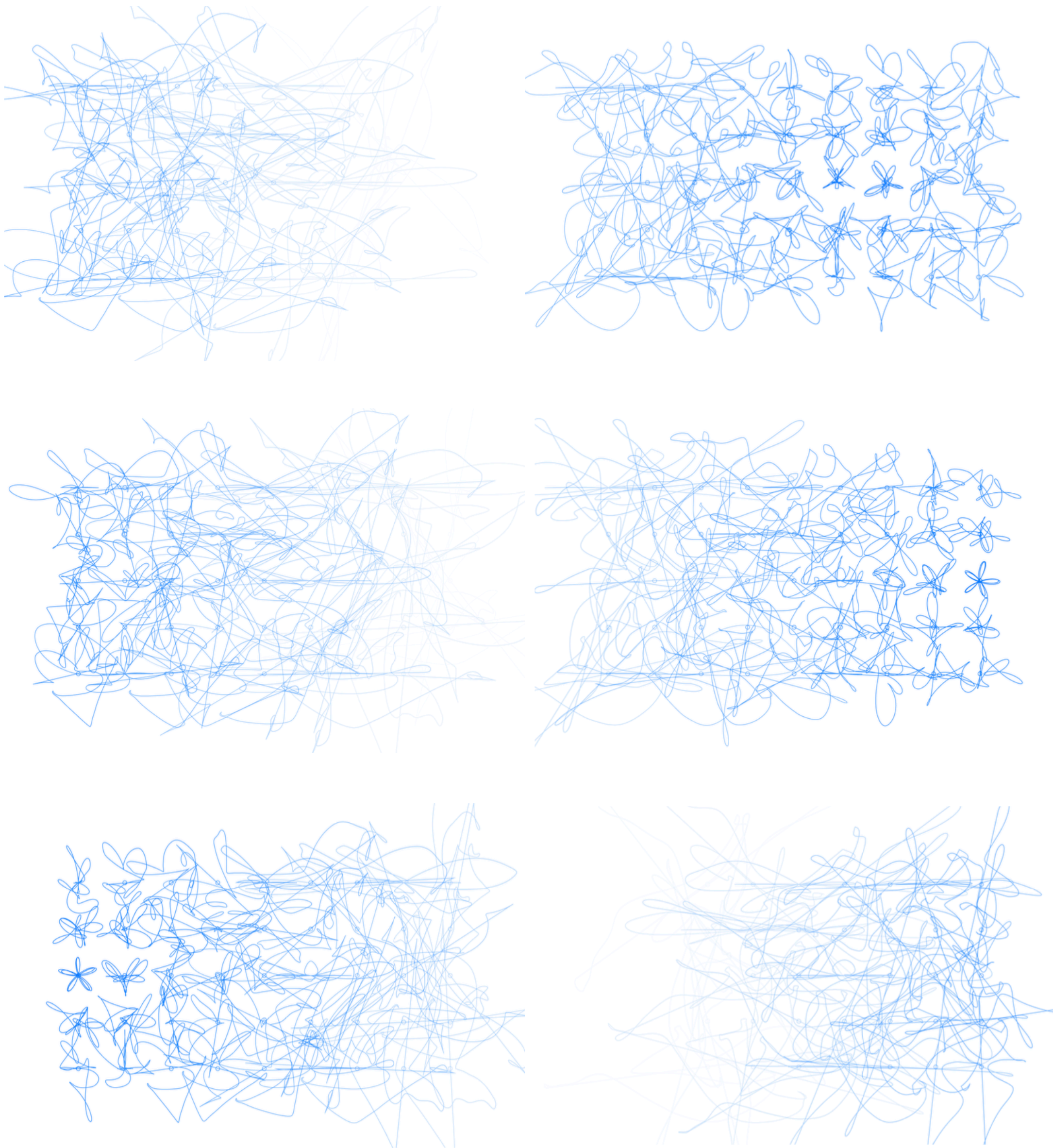
Ocean Waves

This dynamic, layered ocean wave effect uses Perlin noise, cosine modulation, and gradient color blending to simulate the natural flow and texture of waves. The displacement is defined by

$$y = A\cos(kx + \omega t)$$

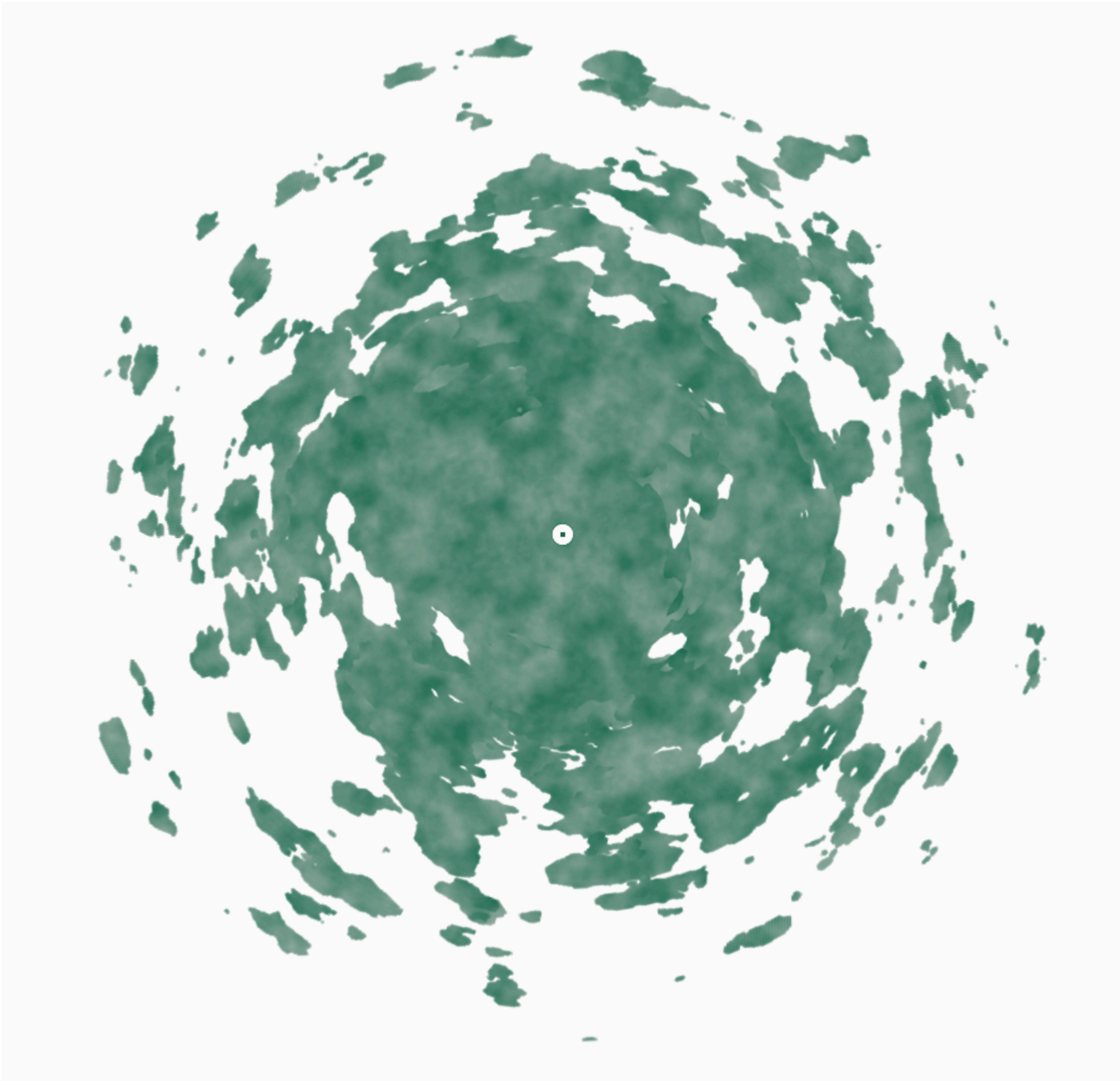
Linear gradients are created between color stops, which fade over the vertical distance of the wave

$$C = C_1 \cdot (1 - \alpha) + C_2 \cdot \alpha$$



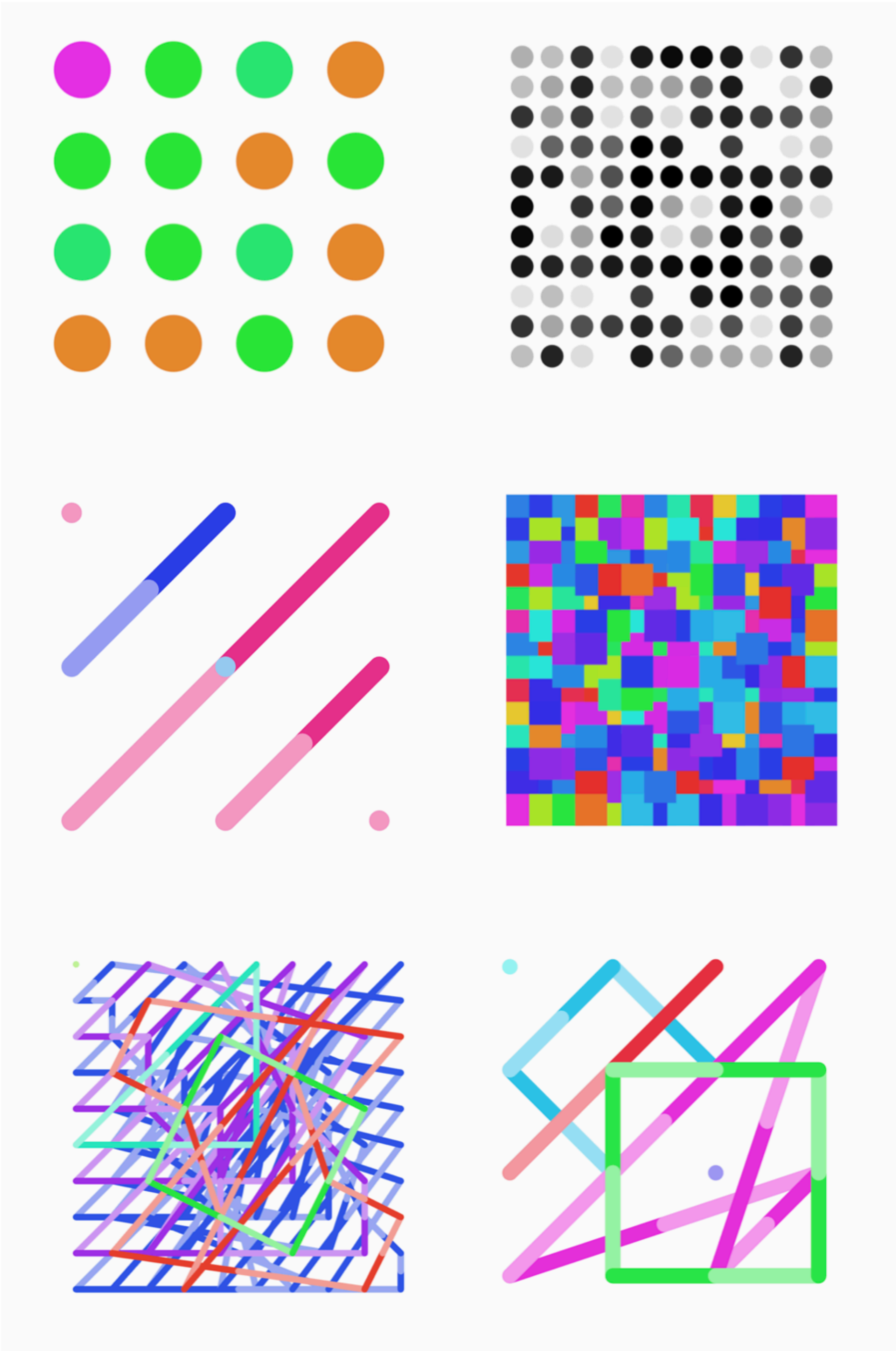
Polar Grid

$$\text{offsetX} = 4\left(\sin(5\theta) + \text{offsetX} \cdot \cos\left(N(\cos(\theta)) \cdot \theta\right)\right)$$
$$\text{offsetY} = 4\left(\sin(5\theta) + \text{offsetY} \cdot \sin\left(N(\sin(\theta)) \cdot \theta\right)\right)$$
$$r(\theta) = e^{\sin(\theta)} - 2 \cdot \cos(4 \cdot \theta) + \sin^5\left(\frac{\theta}{12}\right)$$



Polar Oasis

A similar adaptation of ***Polar Grid***, using perlin noise, perlin coordinates, and spiral growth.



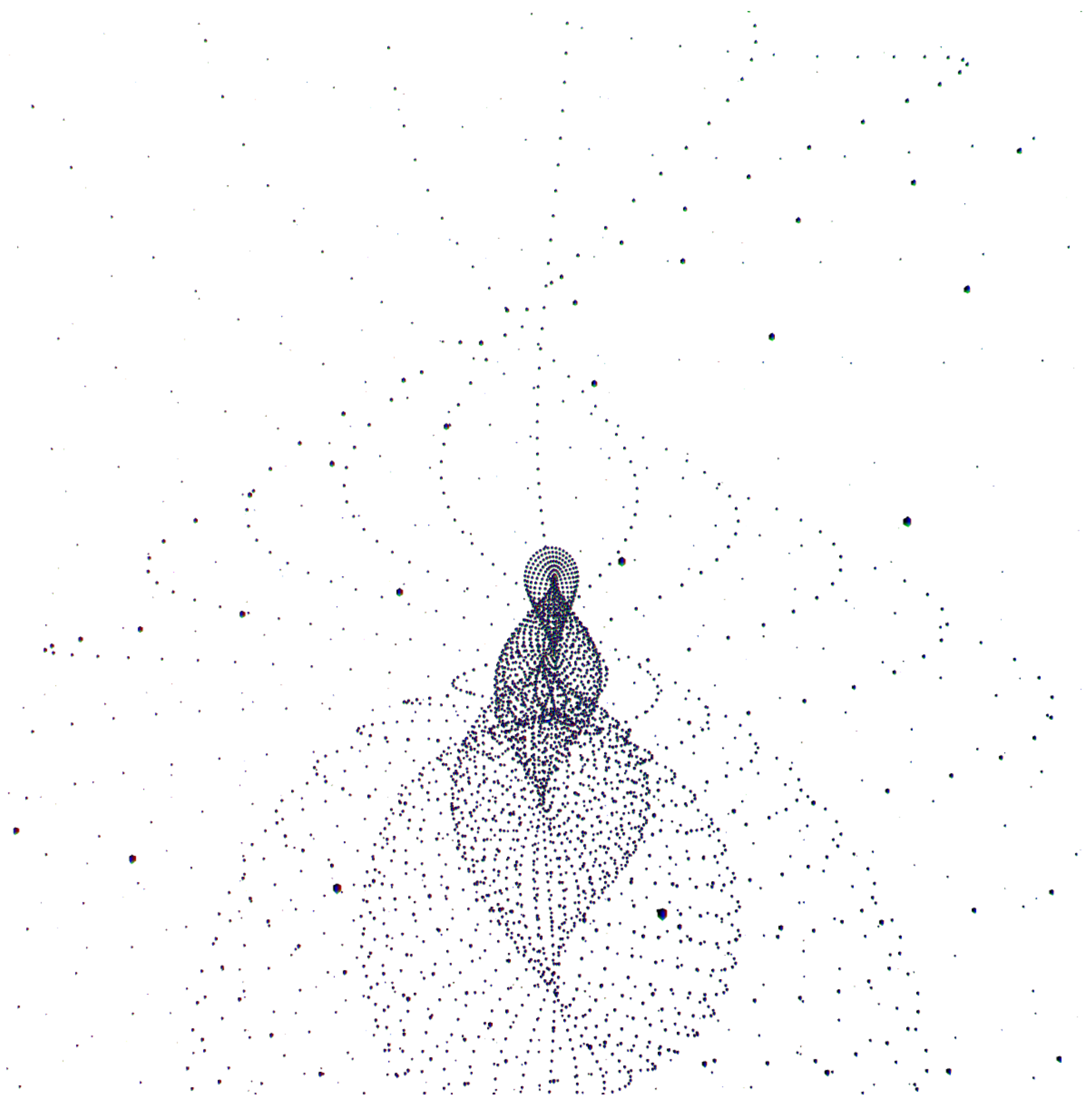
Fibonacci Sequence Modulo

$$F(n) \equiv (F(n-1) + F(n-2)) \pmod{N}$$

Tree Fractal

Tree fractals are recursive structures where each branch splits into smaller branches at specific angles and scales, creating a self-similar, tree-like pattern.

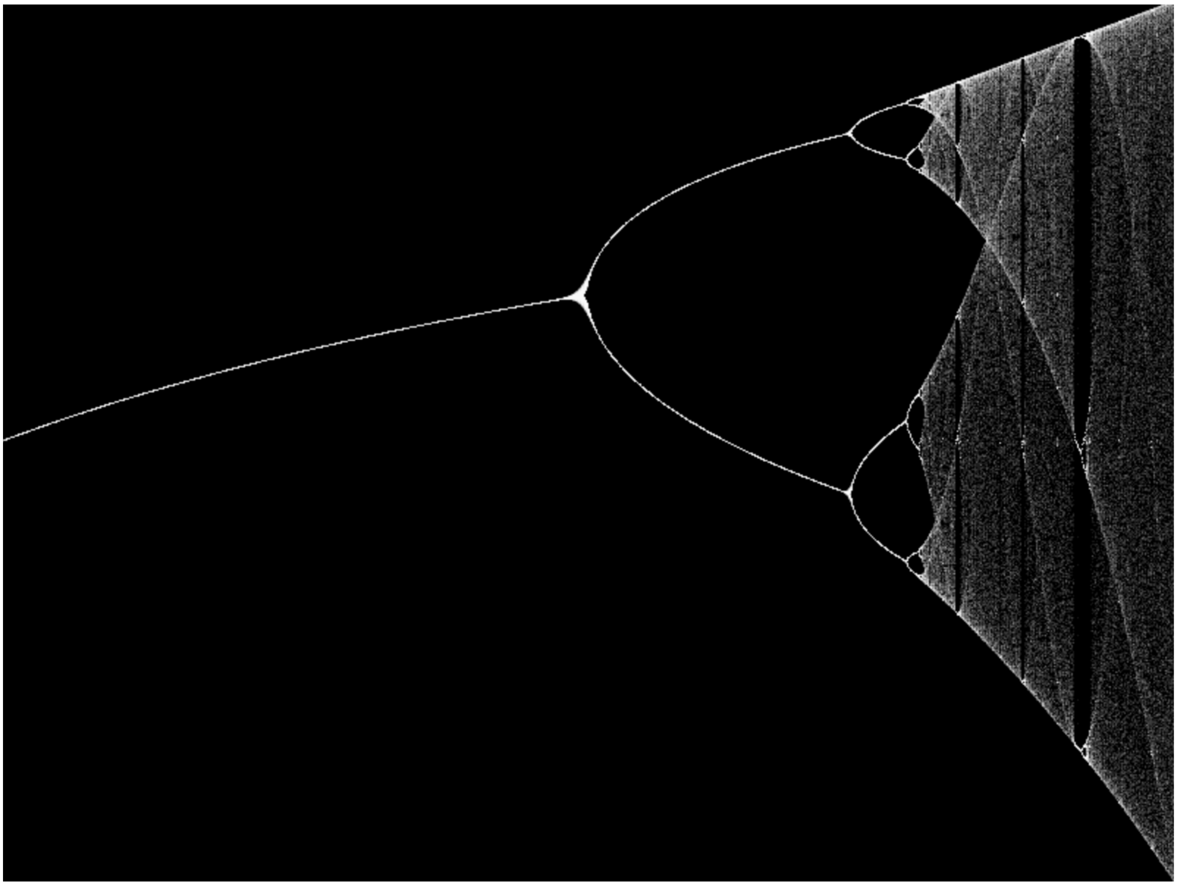
$$L_n = L_0 \cdot s^n$$
$$\theta_{\text{left}} = \theta + \alpha, \theta_{\text{right}} = \theta - \alpha$$
$$x_{\text{new}} = x + L \cdot \cos(\theta), y_{\text{new}} = y + L \cdot \sin(\theta)$$
$$D = \frac{\log(N)}{\log(s^{-1})}$$



Shell Structure

The shell structure is a spiraling form generated by mathematical transformations.

$$
\begin{aligned}
b &= a \cdot \frac{|\exp(2m\pi) - 1|}{\exp(2m\pi) + 1} \cdot \sqrt{1 + k^2} \\
x &= (a + b\cos(v + t)\exp(mu)\cos(u + t)) + 3\sin(t + u) \\
y &= \left(a + b\cos(v + t)\exp(mu)\sin(u + \frac{t}{2})\right) + 3\sin(t + u) \\
z &= \frac{kau}{1.5} + b\sin(v + t)\exp(mu) + 30\sin(t)
\end{aligned}
$$



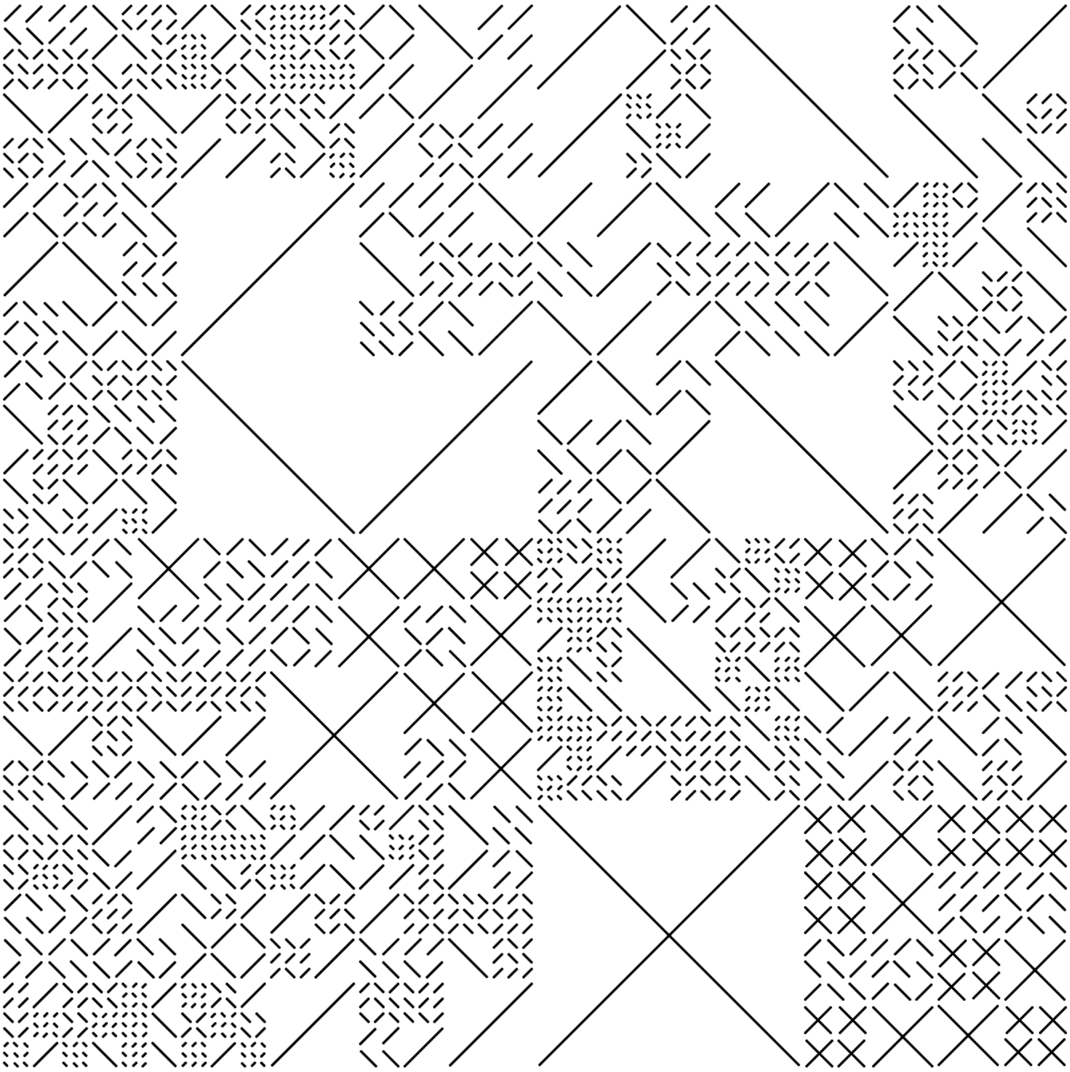
Bifurcation

The bifurcation diagram for the logistic map plots the value of r on the horizontal axis and the long-term values of x (after a sufficient number of iterations) on the vertical axis. The logistic map is defined as:

$$x_{n+1} = r \cdot x_n \cdot (1 - x_n)$$

The bifurcation diagram shows points where

- points where bifurcations occur as branching points,
- regions of stable periodic behavior interspersed with chaotic regimes.

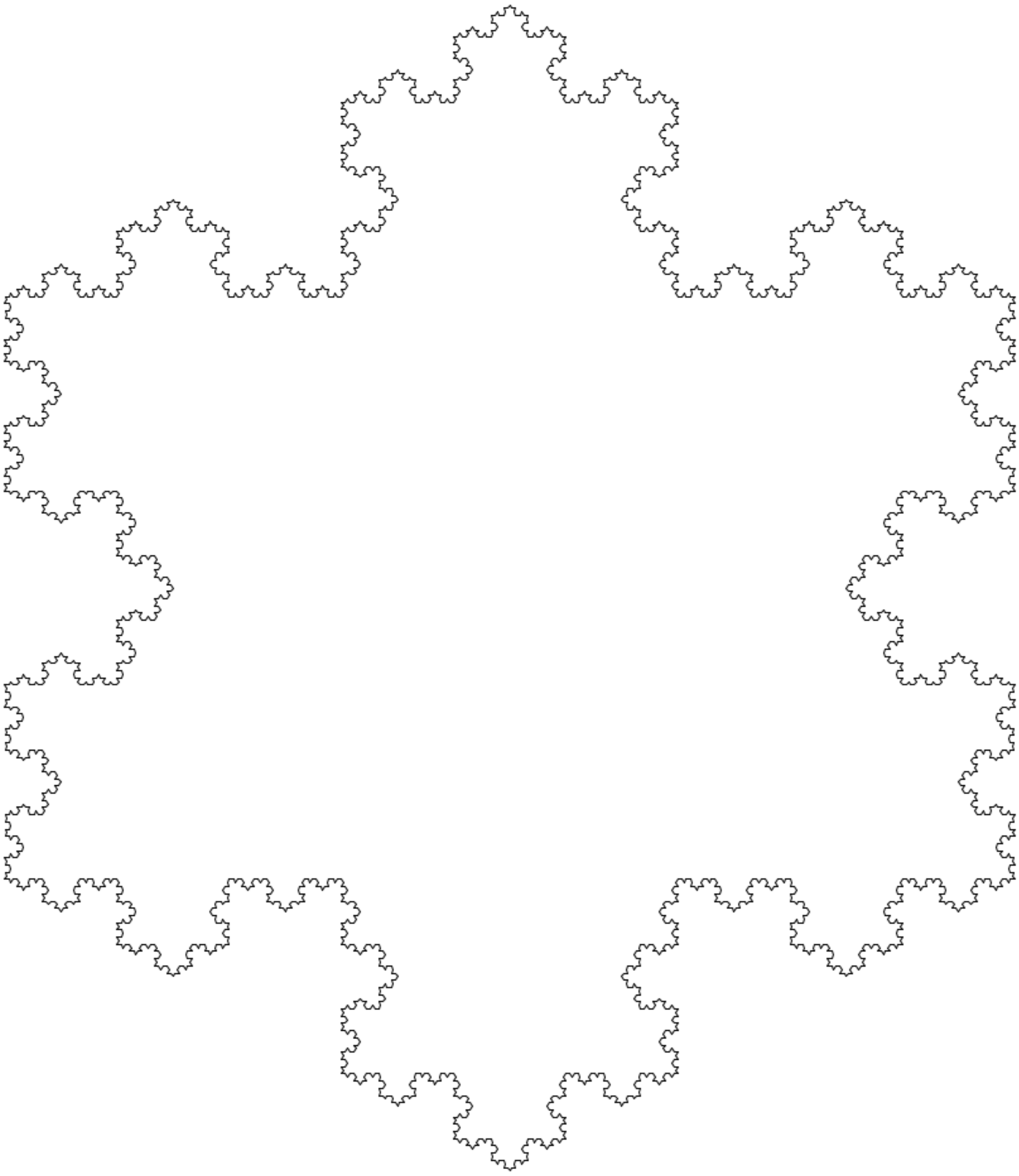


Recursive Grid Subdivision

$$w = \frac{d}{s}$$

$$\theta = k \times \frac{\pi}{2}$$

$$f(d,n) = \begin{cases} \text{draw pattern} & \text{if } d \leq d_{\min} \text{ or } p < P \\ f\left(\frac{d}{n}, n\right) & \text{otherwise} \end{cases}$$



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